Retardation effects in the relaxation of a two-dimensional electron plasma

A. O. Govorov and A. V. Chaplik

Institute of the Physics of Semiconductors, USSR Academy of Sciences, Siberian Branch
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The characteristic velocity with which charge spreads in structures with a highly mobile two-dimensional electron gas that are now available may be comparable to the velocity of light. One must take into account retardation effects in this case which qualitatively change the spreading kinetics. Retardation effects are most important in the quantum Hall effect regime which results in slow spreading even for $\sigma_{xx} = 0$. In these conditions a linear current introduced into the system induces electric polarization of the system that is proportional to σ_{xy}.

Relaxation of an initial density perturbation in a two-dimensional plasma, usually called charge spreading, differs qualitatively from the analogous process in the three-dimensional case.[1,2] The main difference is due to the unique character of two-dimensional Maxwellian relaxation. In the quasistatic approximation used in Refs. 1 and 2, the speed of spot spreading of the nonequilibrium charge is $v = 2\pi\sigma/\bar{\varepsilon}$ where $\bar{\varepsilon}$ is the average dielectric permittivity of the surrounding media and σ is the two-dimensional conductivity of the electron plasma. Of course, the speed of light does not enter into the quasistatic results so that the characteristic times t and distances x related to the relaxation process should also satisfy the inequality $x \ll ct$. However, this is not the only condition for the validity of the calculations.[1,2]

Let $\bar{c}$ be the velocity of light in the materials of structures to be considered. In the currently widely studied heterogeneous structures GaAs–GaAlAs for typical two-dimensional electron gas (TEG) densities of 3–4×10^{11} cm^{-2} the dimensionless parameter $\beta = v/\bar{c}$ is comparable to unity even for a mobility of 2×10^5 cm^2 V^{-1} s^{-1}, which is an order of magnitude lower than the highest mobilities that have been reached at the present time. Thus the situation $\beta > 1$ and even $\beta \gg 1$, for which the results of Refs. 1 and 2 are inapplicable, are completely realizable. Because v is the velocity of the physical signal (change of the local density in the monopolar case, or local conductivity in the bipolar case, see Ref. 1), the question that arises is what is the nature of the spreading kinetics for $\beta \gtrsim 1$ (including distances $x \ll \bar{c}t$). This is the subject of the first part of this paper. The second part of the paper considers spreading in the quantum Hall effect (QHE) regime. It is clear that in the quasistatic approximation for $\sigma_{xx} \to 0$ spreading due to electric fields stops. It turns out that due to retardation effects slow spreading still occurs; the charge fraction withdrawn from the injection region is $\sim (\sigma_{xy}/c)^2$, i.e., it is quantized according to a j^2 law, where j is the number of filled Landau levels. The first-order effect in σ_{xy}/c arises due to the reaction of the Hall conductor on the linear current carried by the TEG: in the direction transverse to the excited current an inhomogeneous flow of electrons occurs and, accordingly, a charge density; i.e., local neutrality is destroyed.

1. We begin with the simplest case of a monopolar injection where an excess charge is introduced into the system. Although this situation is difficult to realize experimentally, it is the clearest case of how the quasistatic result generalizes taking into account retardation effects. We will subsequently consider the more realistic case: separation of electrons from donors in some part of the structure containing the TEG.

We write the expressions for the potentials $\mathbf{A}$ and φ following from the wave equations and the continuity equation in a Fourier representation of the coordinates x, y and time (for time the transformation is one-sided):

$$\varphi_{\mathbf{k}\omega} = \frac{2\pi}{\varepsilon R}\rho(\mathbf{k},\omega)e^{-R|z|}, \qquad \mathbf{A}_{\mathbf{k}\omega} = \frac{2\pi}{cR}\mathbf{j}(\mathbf{k},\omega)e^{-R|z|} \tag{1}$$

$$\mathbf{k}\mathbf{j}(\mathbf{k},\ \omega) = \omega\rho(\mathbf{k},\ \omega) - i\rho_0(\mathbf{k}).$$

Here, $R = (k_2 - \omega_2/\bar{c}^2)^{1/2}$, $\bar{c} = c/\varepsilon^{1/2}$, ρ and $\mathbf{j}$ are the surface charge density and current assuming that the conducting plane $z = 0$ is immersed in a medium with dielectric constant ε and ρ_0 is the injected charge density for $t = 0$. Using the equation for the current $\mathbf{j} = \sigma[(i\omega/c)\mathbf{A} - i\mathbf{k}\varphi]$ and solving Eq. (1) for the density ρ we obtain, after transforming to an x-, t-representation for the initial density $\rho_0(x,y) = N_0\delta(x)$

$$\rho(x,t) = \frac{N_0}{\pi}\frac{vt}{x^2+v^2(t^2-x^2/\bar{c}^2)}\frac{\theta((\bar{c}t)^2-x^2)}{[1-x^2/(\bar{c}t)^2]^{1/2}}, \tag{2}$$

where $\theta(x) = 0$ for $x < 0$ and $\theta(x) = 1$ for $x > 0$. For $\bar{c} \to \infty$ one obtains the quasistatic result [Eq. (4) of Ref. 1]. The density distribution in space is critically dependent on the parameter $\beta = v/\bar{c}$. For $\beta > (2/3)^{1/2}$ the curvature of the $\rho(x)$ function changes sign at $x = 0$. Considering the dependence of the density on t for fixed x in the case $\beta \ll 1$ together with the singularity due to the perturbation front at $t = x/\bar{c}$, a maximum occurs in the region $t \sim x/v$; for $\beta \gg 1$ the $\rho(t)$ function monotonically increases in the region $t > x/\bar{c}$. We neglected diffusion in Eq. (2) which is allowable for $x \sim vt$ if $x \gg D/\sigma \sim a_0$ where a_0 is the effective Bohr radius.

Now let electrons be injected into the two-dimensional system separated from donors situated at a distance Δ from the plane $z = 0$ (the GaAs–GaAlAs lattice). Thus we are considering the kinetics of shielding of a linearly charged inhomogeneity in the TEG. One can again suppose that the diffusion contribution can be neglected if the inequality $x \gg \Delta \gg a_0$ is satisfied. Because of the complexity of the general equation, we present only the results for $\beta \gg 1$ valid inside a small vicinity of the perturbation front $\bar{c}t - |x| \gg |x|/\beta$:

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$$\rho(x,t)\approx\frac{N_0}{\pi}\frac{\Delta(\bar{c}t)}{x^2}\frac{(\bar{c}t)^2+x^2}{[(\bar{c}t)^2-x^2]^{1/2}}\theta((\bar{c}t)^2-x^2). \qquad (3)$$

For $t\to\infty$ we obtain the static shielding of a linear charge in a two-dimensional system: $\rho\sim\Delta/x^2$. We recall that for small β the function $\rho(t)$ has a maximum for $t\sim x/v$ (see Ref. 1) whereas Eq. (3) describes a monotonic decrease of $\rho(t)$.

2. We consider the relaxation of an initial perturbation in the idealized QHE regime: $\sigma_{xx}=\sigma_{yy}=0$, $\sigma_{xy}=-\sigma_{yx}=\sigma_1$. The potential part of the electric field does not contribute to the spreading, due to the antisymmetry of the $\sigma_{\alpha\beta}$ tensor. The role of the rotational part of the field can be understood from the following qualitative arguments. The linear charge $\rho(x,t)$ creates the x-component of the electric field which generates the y-component of the current $j_y(x,t)$ proportional to σ_{yx}. This current creates the vector potential $A_y(x,t)$ and the electric field $E_y(x,t)$ appears which produces the current $j_x(x,t)$ driving the spreading. One can see that to lowest order the effect is proportional to $(\sigma_1/c)^2$.

Solving the system of equations (1) taking into account the tensorial relation between the current and the field we obtain for monopolar injection with the same initial conditions.

$$\rho(x,t)=\frac{N_0}{1+\alpha^2}\left[\delta(x)+\frac{\alpha^2}{2}\delta(x-\bar{c}t)+\frac{\alpha^2}{2}\delta(x+\bar{c}t)\right],$$

$$\alpha=2\pi\sigma_1/(c\varepsilon^{1/2}), \qquad (4a)$$

and for the case of donor ionization (we suppose $x\gg\Delta$)

$$\rho(x,t)=\frac{N_0}{\pi}\frac{\alpha^2}{1+\alpha^2}\frac{\Delta}{x^2[1-x^2/(\bar{c}t)^2]^{1/2}}. \qquad (4b)$$

One can show from the general formula valid for all x in the case (4b) that the fraction of charge withdrawn from the injection region for $t\to\infty$ is equal to $\alpha^2/(1+\alpha^2)$ as in case (4a).

The qualitative arguments given above are also correct in the case of perturbations of a linear current system. Consider a thin wire with current $I(t)$ situated parallel to the y-axis at a distance Δ from the plane of the TEG. A calculation shows that the surface current excited in the x-direction differs only by the factor $(1+\alpha^2)^{-1}$ from the value $(-\sigma_1/c)\partial A_y/\partial t$ where A_y is the vector potential component created by the given current without taking into effect the screening action of the TEG. If a current is turned on for $t>0$ according to $I(t)=I_0(1-e^{-\gamma t})$, at the perturbation front $j_x(x,t)$ and $\rho(x,t)$ are proportional to $\gamma(\bar{c}t-|x|)^{1/2}$. In the interior part of the region subjected to perturbations we have for $|x|\ll\bar{c}t$ and $x^2/\bar{c}^2t^2\ll\gamma t$

$$j_x=\frac{2I_0\sigma_1}{(1+\alpha^2)c(\bar{c}^2t^2-x^2-\Delta^2)^{1/2}},$$

$$\rho=\frac{-2\sigma_1 I_0}{(1+\alpha^2)c^2}\frac{x}{x^2+\Delta^2}\frac{\bar{c}t}{(\bar{c}^2t^2-x^2-\Delta^2)^{1/2}}. \qquad (5)$$

It is easy to see that all the square-root singularities in the above obtained equations are smoothed out due to the finite sizes of the regions of initial perturbations.

We consider the magnitude of the correction due to the finite value of σ_{xx} considering, of course, that $\sigma_{xx}\ll\sigma_{xy}$. Without writing the complicated general formula, we consider again the chain leading to spreading due to the rotational fields: $E_x\to j_y\to A_y\to E_y\to j_x$, and compare the result with the current of direct spreading $j_x=\sigma_{xx}E_x$. In the indicated chain the factor σ_{xy} occurs twice, A_y and j_y are related by Eq. (1) for $z=0$ through the coefficient $2\pi/cR$, and the factor $i\omega/c$ arises from the transition $A_y\to E_y$. It follows from this that in the region of growth $|x|\sim\bar{c}t$ together with the terms $\alpha^2\delta(x\pm\bar{c}t)$ in Eq. (4a) a smearing out of the density occurs proportional to $(\bar{c}^2t^2-x^2)^{-1/2}$. Its integrated contribution to the total charge is $\sim\alpha^2(\sigma_{xx}c/\sigma_{xy}^2)$. Thus, for $\sigma_{xx}/\sigma_{xy}\ll\sigma_{xy}/c\sim 10^{-2}$ the influence of direct spreading in the region of the perturbation front is negligibly small.

As one might expect at small distances, $|x|\sim\sigma_{xx}t\ll\bar{c}t$, the finiteness of σ_{xx} leads to a slow spreading according to a quasistatic law. Together with the term $\delta(x)$ in Eq. (4a) one obtains

$$\rho=\frac{N_0}{\pi(1+\alpha^2)}\frac{v^*t}{(v^*t)^2+x^2}. \qquad (6)$$

Hence, the difference between the present work and Ref. 1 is a weak renormalization of the velocity of spreading $v^*=2\pi\sigma_{xx}/\varepsilon(1+\alpha^2)$. In the best presently available samples where $\sigma_{xx}/\sigma_{xy}\sim 10^{-8}$, the velocity v^* all told may be several cm s^{-1}.

A calculation of the effect of finite σ_{xx} in the case of a linear current in a TEG leads to analogous changes. At distances $x\ll\bar{c}t$ for $\gamma t\gg 1$ the current is described by the equation [see Eq. (5)]

$$j_x=\frac{2I_0\sigma_1}{(1+\alpha^2)c}\left(\frac{v^*}{c}\right)\frac{v^*t}{(v^*t)^2+x^2}. \qquad (7)$$

It was assumed for simplicity in the last equation that the exciting linear current was placed directly in the plane of the TEG; this is valid for $v^*t\gg\Delta$.

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[1]A. O. Govorov and A. V. Chaplik, Surfaces 12, 5 (1987).
[2]M. I. Dyakonov and A. S. Furman, Zh. Eksp. Teor. Fiz. 92, 1012 (1987) [Sov. Phys. JETP 65, 574 (1987)].

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