Nonlinear surface magnetostatic waves

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A theory is derived for the propagation of intense surface magnetostatic waves in thin ferromagnetic films. At certain angles between the wave propagation direction and the external magnetic field, these waves become unstable with respect to the formation of longitudinal solitons (envelope solitons). The regions in which they exist are found. These nonlinear waves exhibit bistable properties during propagation in ferromagnetic films with periodic structures. The threshold power level for the occurrence of this instability is determined. The properties of nonlinear magnetostatic Love waves in a structure consisting of a ferromagnetic film and a nonmagnetic half-space are analyzed.

1. INTRODUCTION

Nonlinear wave phenomena in thin ferromagnetic films have become the subject of intense research interest, primarily because ferromagnetic films are an excellent arena for studying the physics of spin magnetostatic waves and are also of applied importance. Numerous experiments carried out on the physics of magnetostatic waves, however, have shown that these waves become very nonlinear even at low power levels of the excited microwave signal. This nonlinearity leads in turn to several new effects: an instability of the waves with respect to decay into new waves, self-modulation, self-focusing, and self-channeling of magnetostatic waves. The mechanism for the occurrence of the instability of magnetostatic waves involves primarily a magnetic dipole interaction, so there is a quadratic nonlinearity. This situation is responsible for second-harmonic generation and for three-magnon decay processes. Furthermore, since the magnitude of the magnetic moment is conserved during its precession, its average projection onto the magnetizing field depends on the oscillation amplitude. This circumstance is responsible for the cubic nonlinearity. The cubic nonlinearity is responsible in turn for phenomena such as self-modulation and four-magnon scattering of waves.

It has been established[1] that the range in which magnetostatic waves exist in ferromagnetic films is limited by the frequencies

$$\omega_H<\omega<[\omega_H^2+\omega_H\omega_M]^{1/2}$$

for bulk magnetostatic waves and

$$[\omega_H^2+\omega_H\omega_M]^{1/2}<\omega<\omega_H+\omega_M/2$$

for surface magnetostatic waves ($\omega_H = gH_0$, $\omega_M = 4\pi M_0 g$, g is the gyromagnetic ratio, H_0 is the internal magnetic field, and $4\pi M_0$ is the saturation magnetization). As a result, certain nonlinear processes involving the propagation of magnetostatic waves in thin magnetic films cannot occur, because conservation laws do not hold. For example, second-harmonic generation and three-magnon decays are forbidden for bulk magnetostatic waves by energy and momentum conservation. In this connection the most interesting subject for a study of nonlinear properties is a surface magnetostatic wave. In the first place, the frequencies of these waves are higher than those of bulk magnetostatic waves, and in the case of an instability the surface waves may decay into bulk waves. Second, for surface waves there may also be an entire frequency range in which three-magnon decay processes are forbidden by the conservation laws. In this case these waves may be unstable with respect to four-magnon scattering or self-modulation.

Self-modulation of surface magnetostatic waves was discussed theoretically in Refs. 2 and 3. Zvezdin and Popkov[3] found that these waves are stable with respect to longitudinal perturbations and unstable with respect to transverse perturbations (self-focusing). Kalinikos and Slavin[4] suggest that solitons of dipole-exchange surface magnetostatic waves might exist in thin ferromagnetic films. Envelope solitons of surface magnetostatic waves were seen experimentally in Refs. 5 and 6 in a regime of pulsed excitation of dipole-exchange waves. Three-magnon decays of surface magnetostatic waves were studied experimentally in Refs. 7 and 8. Temiryazev[8] proposed a mechanism of decay into bulk magnetostatic waves followed by coalescence of these waves and the formation of new surface magnetostatic waves. A theory has recently been developed[9] for instability of surface magnetostatic waves which are propagating in thin ferromagnetic films with respect to three- and four-magnon decay processes.

We thus have a fair body of experimental evidence now, which requires theoretical confirmation. Our purpose in the present study is thus to derive a theory for the nonlinear self-effects of surface magnetostatic waves propagating in thin ferromagnetic films.

2. NONLINEAR DISPERSION RELATION

Surface magnetostatic waves propagate anisotropically. It has been established that they can propagate near an axis running perpendicular to the external magnetic field, within an angle (Fig. 1)

$$0<\theta<|\varphi|,\quad \cos^2\varphi=\omega_M/(\omega_H+\omega_M). \tag{1}$$

The equations which serve as our starting point for describing the properties of these nonlinear surface waves are Maxwell's equations in the magnetostatic approximation and the Landau-Lifshitz equation:

$$\begin{aligned}&\mathrm{rot}\,\mathbf{H}=0,\quad \mathrm{div}\,\mathbf{B}=0\\ &\partial\mathbf{M}/\partial t=-g[\mathbf{M}\mathbf{H}_{\mathrm{eff}}],\end{aligned} \tag{2}$$

where **H** is the total magnetic field, **B** is the magnetic induc-

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tion, $\mathbf{M}=\mathbf{M}_0=\delta\mathbf{M}$ is the total magnetization in the film ($\mathbf{M}_0$ and $\delta\mathbf{M}$ are its static and oscillatory parts), $\mathbf{H}_{\text{eff}}$ is the effective magnetic field, given by

$$\mathbf{H}_{\text{eff}}=-\delta\mathscr{H}/\delta\mathbf{M}\ , \tag{3}$$

and $\mathscr{H}$ is the Hamiltonian of the system, which we write in the form

$$\mathscr{H}=-\int\left[\mathbf{MH}_e+\frac{1}{2}\mathbf{MH}^{(m)}\right]dV. \tag{4}$$

Here $\mathbf{H}_e$ is the external magnetic field, and $\mathbf{H}^{(m)}$ is the field produced by the magnetization. The integration is over the volume of the film. The Hamiltonian (4) has been written neglecting the exchange energy. This simplification is justified for dipole magnetostatic waves near the beginning of the spectrum.[10] Furthermore, we are ignoring the anisotropy energy, as we are completely justified in doing for the real yttrium iron garnet (YIG) films used experimentally. We assume that the nonlinearity is slight, so that magnetization component δM_z can be written in the form

$$\delta M_z=[M_0^2-(\delta M_x^2+\delta M_y^2)]^{1/2}\approx M_0\left[1-\frac{\delta M_x^2+\delta M_y^2}{2M_0^2}\right], \tag{5}$$

where $\delta M_x, \delta M_y \ll M_0$. Expressing the magnetostatic potential in terms of the magnetic field $\delta\mathbf{H}=-\nabla\Psi$, and working from (2), we thus find an equation for the nonlinear potential of a surface magnetostatic wave in a film in which the wave is propagating in a direction perpendicular to the external field:

$$\frac{\partial^2\Psi}{\partial x^2}-q^2\Psi=4q^4A|A|^2(\chi_1^2-\chi_2^2)\times[(1+4\pi\chi_1)M_0^2]^{-1}\{(\chi_1+\chi_2)e^{3qx}+\alpha^2\alpha_1(\chi_1-\chi_2)e^{-3qx}+2\alpha(\chi_2-\chi_1)e^{qx}-2\alpha\alpha_1(\chi_1+\chi_2)e^{-qx}\}, \tag{6}$$

where $|A|$ is the potential amplitude of the surface wave,

$$\alpha=\frac{\mu_1+\mu_2-1}{\mu_1-\mu_2+1},\quad \alpha_1=\frac{\mu_1-\mu_2-1}{\mu_1+\mu_2+1},\quad \mu_1=1+4\pi\chi_1,\quad \mu_2=4\pi\chi_2,$$
$$\chi_1=\frac{\omega_H\omega_M}{\omega_H^2-\omega^2},\quad \chi_2=\frac{\omega_M\omega}{\omega_H^2-\omega^2},$$

ω is the frequency of the wave, and q is the wave number. A general solution of (6) can be written in the form

$$\Psi=[A(1+L_1qx)e^{qx}+A\alpha(1+L_2qx)e^{-qx}+AL_3e^{-3qx}+AL_4e^{3qx}]e^{iqy}+\text{c.c.} \tag{7}$$

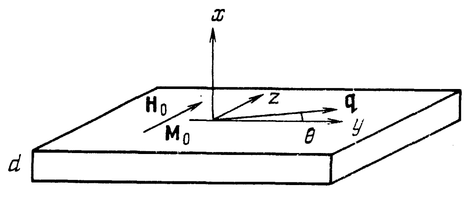

FIG. 1. Geometry of the problem. The ferromagnetic film.

where

$$L_1=-\frac{4(\chi_2-\chi_1)^2(\chi_1+\chi_2)\alpha}{\mu_1}\frac{q^2|A|^2}{M_0^2},$$
$$L_2=\frac{4(\chi_1+\chi_2)^2(\chi_1-\chi_2)\alpha_1}{\mu_1}\frac{q^2|A|^2}{M_0^2},$$
$$L_3=\frac{(\chi_1-\chi_2)^2(\chi_1+\chi_2)\alpha_1\alpha^2}{2\mu_1}\frac{q^2|A|^2}{M_0^2},$$
$$L_4=\frac{(\chi_1-\chi_2)(\chi_1+\chi_2)^2}{2\mu_1}\frac{q^2|A|^2}{M_0^2}. \tag{8}$$

The solution of Maxwell's equations in vacuum and in the substrate is written in the standard form. The boundary conditions are the continuity of the normal components of the induction and of the magnetostatic potential at the surfaces of the film. As a result, after the appropriate substitutions, we find a dispersion relation for nonlinear surface magnetostatic waves which are propagating in the direction perpendicular to the magnetic field:

$$\omega^2-[\omega_H^2+\omega_H\omega_M+{}^1/_4\omega_M^2(1-e^{-2qd})]=G, \tag{9}$$

where

$$G={}^1/_4(\omega_H^2-\omega^2)e^{qd}[F_1(\mu_1-\mu_2-1)e^{-qd}+F_2(\mu_1+\mu_2+1)e^{qd}+F_3(\mu_1-\mu_2-1)+F_4(\mu_1+\mu_2-1)],$$
$$F_1=-\mu_1(3L_4-3L_3+L_1)-(\mu_2+1)(L_3+L_4)-L_5,\quad F_2=\mu_1L_2, \tag{10}$$

$$F_3=\mu_1L_1(1+qd)e^{qd}+\mu_1(3L_4e^{3qd}-3L_3e^{-3qd})+\mu_2(L_1qde^{qd}+L_3e^{-3qd}+L_4e^{3qd})+L_5e^{-qd}+L_3e^{-3qd}+L_4e^{3qd}+L_1qde^{qd},$$
$$F_4=\mu_1L_2(1-qd)e^{-qd}+\mu_2L_2qde^{-qd}+L_2e^{-qd},$$
$$L_5=q^2|A|^2M_0^{-2}(\chi_1^2-\chi_2^2)[-(\chi_1+\chi_2)e^{3qd}+(\chi_1-\chi_2)\alpha_1\alpha^2e^{-3qd}+2(\chi_1-\chi_2)\alpha e^{qd}-2(\chi_1+\chi_2)\alpha\alpha_1e^{-qd}]. \tag{11}$$

For the case in which a wave is propagating at an angle from the magnetic field, on the other hand, we will write only the dispersion relation; all the explanatory expressions are in the Appendix. The dispersion relation is

$$(\mu_1+\mu_2s\cos\theta-1)(\mu_1-\mu_2s\cos\theta-1)e^{-pd}-(\mu_1-\mu_2s\cos\theta+1)(\mu_1+\mu_2s\cos\theta+1)e^{pd}=F. \tag{12}$$

If there is no nonlinearity, and we have $|A|^2=0$, then we have $G=F\equiv0$. In this case Eqs. (9) and (12) describe linear surface magnetostatic (Damon–Eschbach) waves.[1] From Eq. (12) we can find the nonlinear Damon–Eschbach shift of the wave number, defined as $\Delta q=q-q_{\text{DE}}$:

$$2d\Delta q=s\ln[1-F/(\mu_1+\mu_2\cos\theta-1)(\mu_1-\mu_2s\cos\theta-1)]. \tag{13}$$

Here $s=\cos^2\theta+(\sin^2\theta)/(1+4\pi\chi_1)$.

We know that solitons can arise from a longitudinal or transverse modulational instability. The former is manifested as a self-modulation, and the latter as a self-focusing. The self-focusing of surface magnetostatic waves was analyzed in Refs. 2 and 3 in the limit $q\to0$ with $\theta=0$. It is interesting to examine the modulational instability in the case $\theta\neq0$ for arbitary q. The condition for the modulational instability is the so-called Lighthill criterion[11]

$$\frac{\partial\omega}{\partial|A|^2}\bigg/\left(\frac{\partial^2\omega}{\partial q^2}\right)_l<0, \tag{14}$$

where

$$\partial\omega/\partial|A|^2=(\partial q/\partial|A|^2)\,v_{gr}$$

is the nonlinear frequency shift, and $(\partial^2\omega/\partial q^2)_l$ is the dispersion of the linear group velocity v_{gr}. Figure 2 shows $(\partial^2\omega/\partial q^2)_l$ as a function of the angle θ. We see that we have $(\partial^2\omega/\partial q^2)_l<0$ for arbitrary θ. Figure 3 shows the nonlinear shift of the wave number as a function of the angle θ. We see that at a certain value of θ (smaller than the cutoff angle for surface magnetostatic waves, φ) the nonlinear shift changes sign, becoming positive. Consequently, at certain angles (greater than θ_0) the Lighthill condition begins to hold. This circumstance means that the original wave becomes unstable against amplitude modulation. There is accordingly the possibility that envelope solitons of surface magnetostatic waves will exist. Let us estimate the distance over which a pulse of surface magnetostatic waves evolves during the formation of a soliton. This evolution distance is[12]

$$L_{\rm DE}=-\left(\frac{t_1}{\tau}\right)^2\left(\frac{\partial^2 q}{\partial\omega^2}\right)_l^{-1}, \tag{15}$$

where t_1 is the half-width of the pulse. For the ordinary normalization[12] and for an initial sechτ_1 pulse, the value of τ_1 is the dimensionless half-width of the pulse, ≈ 2.64. We assume an initial pulse of surface magnetostatic waves of width 10 ns for a film with $d=10$ μm, $M_0=140$ G, $H_0=570$ Oe, $\omega_0=3.78$ GHz, and $(\partial^2 q/\partial\omega^2)_l\approx 0.4\times10^{-18}$ s^2/cm. We then have $L_{ev}\approx 1$ cm. The threshold power for the formation of a soliton of surface magnetostatic waves is given by[12]

$$|A_{\rm peak}|^2t_1^2=\frac{1}{\omega_2 v_{gr}^{\ 2}}\left(\frac{\partial^2\omega}{\partial q^2}\right)_l, \tag{16}$$

where $\omega_2=v_{gr}\,\partial q/\partial|A|^2$. Using the expression for the power of surface magnetostatic waves,[9]

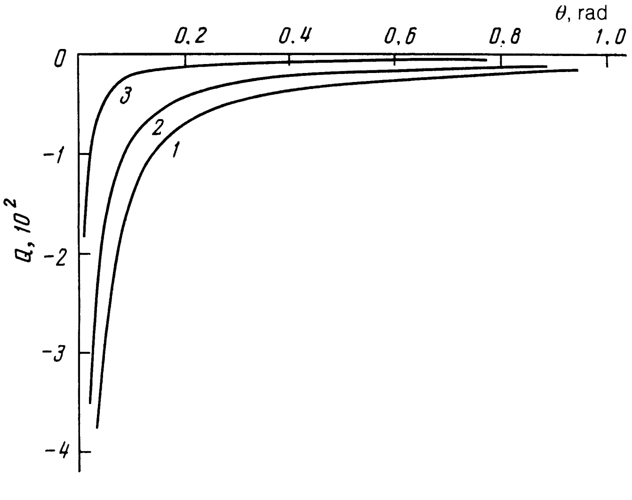

FIG. 2. The dimensionless dispersion of the group velocity, $Q=(\partial^2\omega/\partial q^2)_d/\omega_H d^2$, versus the angle θ for surface magnetostatic waves with the following frequencies: a—3.78 GHz; 2—3.8 GHz; 3—3.85 GHz.

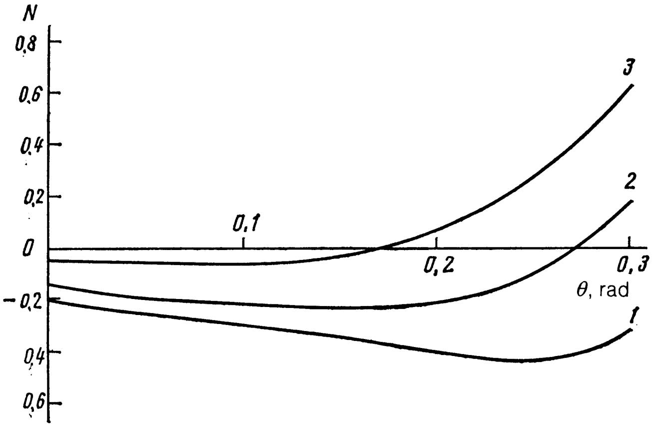

FIG. 3. Dimensionless nonlinear shift of the wave number, $N=2\Delta gd$, versus the angle θ for the frequencies in Fig. 2.

$$P=q_{\rm DE}^2 d^2\omega^2|A|^2/\pi\omega_H, \tag{17}$$

we find that for the parameter values of the field and film which we are using the threshold power for the formation of a surface-magnetostatic-wave soliton is $P\approx 2$ mW/mm. This value could be lowered by using a broader initial phase.

3. THEORY OF THE MULTISTABILITY OF SURFACE MAGNETOSTATIC WAVES

Let us use the results derived in the preceding section to study the properties of nonlinear surface magnetostatic waves which are propagating through a ferromagnetic film with a periodic structure. A theory has been derived previously[13] for linear surface magnetostatic waves propagating in ferromagnetic films with periodic structures consisting of a system of etched grooves. Let us assume that the upper surface of a ferromagnetic film has a sinusoidally varying region, the equation of whose surface is

$$\xi(x)=d(1+\varepsilon e^{iQy}+\varepsilon^* e^{-iQy}), \tag{18}$$

where $|\varepsilon|d$ is the amplitude of the variation, $Q=2\pi/\Lambda$, and Λ is the period of the variation. We assume $|\varepsilon|\ll1$ and $|\varepsilon|Qd\ll1$, and we assume that the period of the structure, Λ, is approximately $\lambda/2$, where λ is the wavelength of the surface magnetostatic wave (this is the case of Bragg reflection).

The problem of the propagation of nonlinear surface magnetostatic waves in ferromagnetic films with a periodically varying surface region is solved by the method of two coupled modes.[14] According to this method, the solutions in the region of a periodic variation of the surface of a film can be written in the form of a Bloch function; in the Fourier expansion of the periodic coefficient we retain only the two terms corresponding to waves whose phase velocities are directed along and opposite the y axis. Accordingly, the solution for the magnetostatic potential (7) can be written in the form

$$\Psi=A_+[(e^{qx}+\alpha e^{-qx})+L_1qxe^{qx}+\alpha L_2qxe^{-qx}$$
$$+L_3e^{-3qx}+L_4e^{3qx}]e^{iqy}+\text{c.c.} \tag{19}$$

The complex-conjugate quantity here stands for the reflected wave propagating in the opposite direction ($-y$), with an amplitude A_-. The boundary conditions can be "carried" away from the varying surface to the $x=d$ surface through a series expansion in the small parameter of the variation, $|\varepsilon|Qd$:

$$\Psi^{(1)}+\frac{\partial\Psi^{(1)}}{\partial x}\xi=\Psi^{(2)}+\frac{\partial\Psi^{(2)}}{\partial x}\xi|_{x=d},$$
$$B_x^{(1)}+\frac{\partial B_x^{(1)}}{\partial x}\xi-B_y^{(1)}\xi_y'=B^{(2)}+\frac{\partial B^{(2)}}{\partial x}\xi-B_y^{(2)}\xi_y'|_{x=d}. \tag{20}$$

The superscript (1) here corresponds to the vacuum region ($x>d$), (2) corresponds to the region of the ferromagnetic film ($0<x<d$), and ξ_y' is the derivative of the function ξ with respect to y. In writing (20) we need to allow for the circumstance that the nonlinearity must be incorporated in the boundary conditions. From (19) and (20) we find equations for the coupled modes:

$$A_+(D_++\eta_1)=\varepsilon Qd\omega_M(\omega_H+\omega_M/2+\omega)A_-,$$
$$A_-(D_-+\eta_2)=\varepsilon^*Qd\omega_M(\omega_H+\omega_M/2-\omega)A_+, \tag{21}$$

where $D_\pm$ are Damon–Eschbach determinants incorporating the nonlinearity of the medium and the variation of the film surface, and

$$\eta_1=\frac{\omega_H^2-\omega^2}{4}(\mu_1-\mu_2+1)e^{-Qd/2}\Big[\mu_1\frac{\partial\eta_+}{\partial x}+(\mu_2+1)\eta_+-(\mu_1+1)\eta_+\times(e^{Qd/2}-\alpha e^{-Qd/2})-\mu_2(e^{Qd/2}+\alpha e^{-Qd/2})\eta_+\Big],$$
$$\eta_2=\frac{\omega_H^2-\omega^2}{4}(\mu_1+\mu_2+1)e^{-Qd/2}\times\Big[\mu_1\frac{\partial\eta_-}{\partial x}-(\mu_2-1)\eta_--(\mu_1-1)\eta_-\times(e^{Qd/2}-\alpha_1e^{-Qd/2})+\mu_2\eta_-(e^{Qd/2}+\alpha_1e^{-Qd/2})\Big]. \tag{22}$$

For thin films, with $qd\ll1$, we have

$$\eta_\pm\approx-2QdP_\pm,\quad P_\pm=|A_\pm|^2\omega_H/2d^2M_0^2, \tag{23}$$

where $P_\pm$ is the dimensionless power of the waves. From the determinant of system (21) we find a nonlinear dispersion relation. We denote by Ω the resonant frequency at which the wavelength of the linear surface wave in the ferromagnetic film is equal to twice the period of the variation, so that $D(Q/2,\Omega)=0$ is the Damon–Eschbach equation. Linearizing the determinant of system (21) for $\Delta q\ll Q$, $\Delta\omega\ll\Omega$,

$$D_\pm=\pm\frac{\partial D}{\partial q}\Delta q+\frac{\partial D}{\partial\omega}\Delta\omega=\pm D_q'\delta+D_\omega'\Delta\omega,$$

and solving Eqs. (2), we find (24):

$$\delta_{1,2}=\frac{\eta_2-\eta_1}{2D_q'}\pm\Big[\frac{(\eta_1+\eta_2)^2}{4D_q'^2}+\frac{D_\omega'}{D_q'^2}\Delta\omega(\eta_1+\eta_2)+\Big(\frac{D_\omega'}{D_q'}\Delta\omega\Big)^2-\Big(\frac{|\varepsilon|Q}{2}\Big)^2e^{-Qd}\Big]^{1/2}. \tag{24}$$

With $P=0$, Eq. (24) corresponds to the linear case, described in Ref. 13. We now consider the reflection and transmission of surface magnetostatic waves in a ferromagnetic film with a periodically varying surface region of finite length L. According to Ref. 13, the transmission coefficient of a wave through such a structure is

$$T=\frac{E}{E_0}=\frac{(1-R_\infty^{\rightarrow}R_\infty^{\leftarrow})\exp(i\delta_1L)}{1-R_\infty^{\rightarrow}R_\infty^{\leftarrow}\exp[i(\delta_1-\delta_2)L]}, \tag{25}$$

where $E_0=|A_+|^2|_{y=0}=A_0^2$ is the input power of the wave, $E=|A_+|^2|_{y=L}$ is the output power, and

$$R_\infty^{\rightarrow}=\frac{\Psi^{\leftarrow}|_{x=0,y=0}}{\Psi^{\rightarrow}|_{x=d,y=0}}=\varepsilon^*Qd\omega_M\frac{\omega_H+\omega_M/2+\omega}{D_-+\eta_2}, \tag{26}$$

$$R_\infty^{\leftarrow}=\frac{\Psi^{\rightarrow}|_{x=d,y=0}}{\Psi^{\leftarrow}|_{x=0,y=0}}=\varepsilon Qd\omega_M\frac{\omega_H+\omega_M/2-\omega}{D_++\eta_1}. \tag{27}$$

Figure 4 shows a parametric plot of $E(E_0)$. For example, curve 1 in Fig. 4a is a plot of $T(|A_+|^2)$ versus E, while lines 2–4 are plots of T versus E/E_0 for various values of E_0. It can be seen from Fig. 4a that for certain power levels $E_0\geqslant E_1$ lines 2 and 3 cross curve 1 twice. This result means that $E(E_0)$ is a double-valued function. Figure 4b shows the functional dependence $E(E_0)$. We see that it is unstable between points B and A. In this region, a filter operating on the basis of surface magnetostatic waves will exhibit hysteretic properties. The physical reason is that the nonlinearity gives rise to a frequency shift, which causes a deviation from the Bragg condition, which in turn changes the wave reflection and transmission coefficients of the finite periodic structure.

Multistable properties may also be manifested in a more complex structure, nonlinear Fabry–Perot resonator. For surface magnetostatic waves, such a resonator would be made up of two periodic gratings separated by a resonator cavity of length D. The transmission coefficient through the resonator is[13]

$$|T_{\rm res}|=|T|^2/|1-R^{\rightarrow}R^{\leftarrow}e^{2iqD}|, \tag{28}$$

where $R^{\rightleftarrows}$ are the reflection coefficients at a grating of finite length. Figure 5 shows the transmission coefficient $|T_{\rm res}|$ versus the input power near the resonant frequency, i.e., for $qD\approx\pi n$, where $n=0, 1, 2, \ldots$ (in our case, we have $n=1$

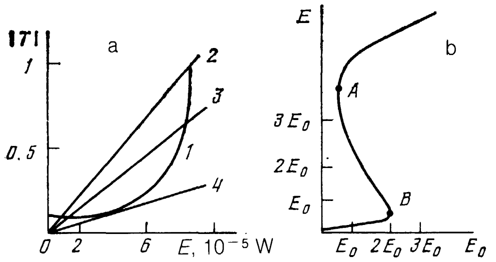

FIG. 4. a—Transmission coefficient of the periodic structure for surface magnetostatic waves versus the output power E (curve 1) and E/E_0 for various values of E_0 (curves 2–4; the points at which these curves cross curve 1 give the values of the output power E). b—Output power of the surface magnetostatic waves as a function of the input power for a grating with $\Lambda=150\,\mu$m, $N=20\,\Lambda$, $H_0=380$ Oe, $M_0=140$ G, $|\varepsilon|=0.01$, and $d=10\,\mu$m.

and $qD = 0.9\pi$). We are also assuming $|R^{=}| \approx 0.9$. It can be seen from Fig. 5 that between points E_a and E_b the transmission coefficient is a multistable quantity; again in this case, a hysteresis may appear.

From Figs. 4 and 5 we can easily estimate the threshold power required for observation of multistable states. For a reflecting grating with $|\varepsilon| = 0.01$, $\Lambda = 150\ \mu$m, $L = \Lambda N$, $N = 20$ periods, $d = 10\ \mu$m, $H_0 = 380$ Oe, and $M_0 = 140$ G, this power is $P_{\rm thr} = 10^{-5}$ W. These values are lower than the threshold power levels for the onset of three- and four-magnon decays of surface magnetostatic waves.[9] Consequently, such effects are easily achieved experimentally. In addition, the threshold power levels for multistable phenomena depend on the value of the small paramter ε. If ε is sufficiently small, i.e., if the coupling between the surface magnetostatic wave modes is weak, multistability cannot set in regardless of the wave power levels.

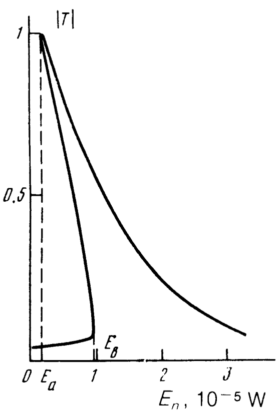

FIG. 5. Transmission coefficient of a nonlinear Fabry–Perot resonator for surface magnetostatic waves versus the input power.

4. NONLINEAR MAGNETOELASTIC LOVE WAVES

The nonlinearity of magnetostatic waves has a substantial effect on the interactin with other possible oscillations in a ferromagnetic film. In this section of the paper we consider the interaction of nonlinear surface magnetostatic waves with Love acoustic waves, which can propagate in a structure consisting of a ferromagnetic film and a nonmagnetic substrate under the inequalities $v_{t_2} > v_{t_1}$, where v_{t_2} is the velocity of a shear bulk acoustic wave in the substrate, and v_{t_1} is that in the film. The propagation of linear magnetoelastic waves in layered structures was studied in Refs. 15 and 16. It was found that when the frequencies of the magnetostatic and acoustic waves are equal these waves begin to interact effectively by virtue of magnetostriction. As a result, there is a restructuring of the spectra, "gaps" appear in the spectra, there is a change in attenuation, and so forth. If one of the interacting waves (e.g., the magnetostatic wave) is nonlinear; the dispersion characteristics may become dependent on the power of this wave, with the further consequence that the interaction will be restructured.

The equations which serve as the starting point for a description of the magnetoelastic interaction are Eqs. (2) and the elasticitiy equation

$$\rho_1 \frac{\partial^2 u_z}{\partial t^2} = \sum_i \frac{\partial}{\partial x_i}\left(\frac{\partial \mathscr{H}_e}{\partial e_{zi}}\right), \tag{29}$$

where ρ_1 is the density of the ferromagnet, e_{zi} are elements of the stress tensor, $\mathscr{H}_e$ is the Hamiltonian of the elastic subsystem, and the summation is over $i = x, y$. In addition, the magnetoelastic energy must be added to expression (4) for the Hamiltonian.

We write a solution of Eqs. (2) and (29) for the potential of the surface magnetostatic wave, taking the magnetoelasticity into account:

$$\Psi = \Psi_{\rm nl} + \frac{\omega_M \omega_H}{v_{t_1}^2} \frac{b}{M_0} u_z, \tag{30}$$

where $\Psi_{\rm nl}$ is solution (7), b is the magnetostrictive constant, and the elastic displacement is given by

$$u_z = B(\cos \varkappa x + \eta \sin \varkappa x)\, e^{iqy}, \tag{31}$$

where B is the amplitude of the elastic displacement,

$$\eta = \frac{c_{44}^{(2)}}{c_{44}^{(1)}} \frac{r}{\varkappa}, \quad r^2 = q^2 - \frac{\omega^2 \rho_2}{c_{44}^{(2)}}, \quad \text{and} \quad \varkappa^2 = \frac{\omega^2 \rho_1}{c_{44}^{(1)}} - q^2.$$

Here $c_{44}^{(1)}$ and $c_{44}^{(2)}$ are the elastic moduli of the ferromagnet and the substrate, and ρ_2 is the density of the substrate.

Solutions in the substrate and in vacuum are written in standard form. The boundary conditions are the continuity of the magnetostatic potential and of the normal component of the magnetic induction, the absence of elastic stresses at the ferromagnet–vacuum interface, and the continuity of these stresses at the ferromagnet-substrate interface. Calculations lead to the following dispersion relation for nonlinear magnetoelastic waves:

$$\Delta_{\rm DE} \Delta_l = b^2 \Phi / M_0^2 + \Delta_l N_1 + N_2, \tag{32}$$

where

$$\Phi = -\frac{\omega_H \omega_M}{\omega^2} \left\{ (\mu_1 - \mu_2 + 1) \left[\cos(\varkappa d) \left(\frac{\varkappa}{q} + \frac{\varkappa}{r} \frac{c_{44}^{(1)}}{c_{44}^{(2)}} \right) + \sin(\varkappa d) \left(\frac{\varkappa}{q} - \frac{c_{44}^{(1)}}{c_{44}^{(2)}} \frac{\varkappa}{r} \right) \right] + (\mu_1 - \mu_2 - 1) \times e^{-qd} \left(\frac{\varkappa}{r} \frac{c_{44}^{(1)}}{c_{44}^{(2)}} - \frac{\varkappa}{q} \right) \right\}$$
$$\times \left\{ \frac{c_{44}^{(1)}}{(c_{44}^{(2)})^2} \frac{\varkappa q}{r^2} \sin(\varkappa d) \left[(\chi_1 + \chi_2) - \alpha (\chi_1 - \chi_2) \right] - \frac{q^2}{2\varkappa c_{44}^{(2)}} \left[(\chi_1 + \chi_2) e^{qd} + (\chi_1 - \chi_2) \alpha e^{-qd} \right] \right\}, \tag{33}$$

$$N_1 = (\mu_1 - \mu_2 - 1) \left[L_1 (1 + 2qd) e^{qd} + L_2 e^{-qd} + 4 L_3 e^{-3qd} - 2 L_4 e^{3qd} + L_5 \right], \tag{34}$$

$$N_2 = \left[2(L_3 - 2L_4) + L_1 + L_2 + L_5 \right] (\mu_1 - \mu_2 - 1) e^{-qd}, \tag{35}$$

$\Delta_{\rm de}$ is the Damon–Eschbach determinant, and Δ_L is the Love-wave determinant, given by

$$\Delta_l = \cos(\varkappa d) - \frac{c_{44}^{(1)}}{c_{44}^{(2)}} \frac{\varkappa}{r} \sin(\varkappa d).$$

For thin ferromagnetic films we have

$$L_1 - L_5 \approx 4qdP, \tag{36}$$

where P is the dimensionless power of the surface magnetostatic wave.

Expanding the determinants Δ_{DE} and Δ_L in small deviations of the wave numbers and frequencies near a crossing point of the dispersion curves for the waves, we find a linearized dispersion relation. We are interested in studying this relation in two cases.

1) $\Delta q = 0$:

$$\Delta\omega \approx N_1/2 \frac{\partial\Delta_l}{\partial\omega} \pm \left[N_2 \Big/ \frac{\partial\Delta_l}{\partial\omega}\frac{\partial\Delta_{DE}}{\partial\omega} + b^2\Phi/M_0^2 \frac{\partial\Delta_l}{\partial\omega}\frac{\partial\Delta_{DE}}{\partial\omega} \right]^{1/2}. \tag{37}$$

We see that even with $\Delta q = 0$ the quantity $\Delta\omega$ has two values. In the absence of a nonlinearity ($N_1 = N_2 \equiv 0$) the expression in the radical is greater than zero. This result means that with $\Delta q = 0$ the dispersion branches for the acoustic and magnetostatic waves repel each other. Under the conditions $N_1, N_2 \neq 0$, and at high power levels of the magnetostatic wave, the expression in the radical in (37) may go negative; i.e., the expression for $\Delta\omega$ may become complex.

2) $\Delta\omega = 0$:

$$\Delta q \approx N_1 \Big/ \frac{\partial\Delta_l}{\partial q} \pm \left[\left(N_2 + \frac{b^2\Phi}{M_0^2} \right) \Big/ \frac{\partial\Delta_l}{\partial q}\frac{\partial\Delta_{DE}}{\partial q} \right]^{1/2}. \tag{38}$$

Here again, for a single frequency ($\Delta\omega = 0$) we can find two distinct values of Δq. Figure 6 shows a plot of $\Delta qd \equiv \delta$ versus P (for $\Delta\omega = 0$) and of the dimensionless frequency Ω versus P for $\Delta q = 0$. We see from these results that the wave power restructures the wave repulsion regions and changes the group velocities of the waves. In principle, the waves may be bistable again in this case, and we may observe effects similar to those described in Sec. 3.

5. CONCLUSION

A theory has been derived here for the propagation of nonlinear surface magnetostatic waves in thin ferromagnetic films. The dispersion relation of these waves has been derived. It has been found that under certain conditions a change in the angle between the wave propagation direction and the external magnetic field may be accompanied by a change in the nonlinear frequency shift, which may change sign at certain angles. This effect leads in turn to conditions corresponding to the existence of longitudinal solitons of surface magnetostatic waves (envelope solitons).

A nonlinear surface magnetostatic wave propagating in a ferromagnetic film with a periodic structure has bistable properties. The threshold power levels for the onset of this instability depend on the small parameter of the periodic structure and may be lower than the threshold power levels for the three-magnon decay of the wave. The propagation of nonlinear Love magnetoelastic waves has also been analyzed.

In summary, this paper predicts the existence of some new effects, which have not been recognized previously, for nonlinear surface magnetostatic waves. Future experiments should resolve the validity of this theory. This effort may in turn involve further research in such an interesting field as the physics of nonlinear magnetostatic waves.

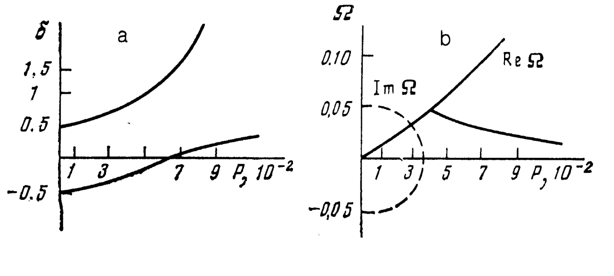

FIG. 6. a—The dimensionless quantity $\delta = \Delta gd$ versus the dimensionless power P ($\Delta\omega = 0$); b—the dimensionless frequency Ω versus the power p ($\Delta q = 0$).

APPENDIX

$$F = F_1'(\mu_1 - \mu_2 s\cos\theta - 1)e^{-pd} + F_2'(\mu_1 + \mu_2 s\cos\theta + 1)e^{pd} + F_3'(\mu_1 - \mu_2 s\cos\theta + 1) + F_4'(\mu_1 + \mu_2 s\cos\theta - 1), \tag{A1}$$

$$F_1' = -\mu_1(3L_4' - 3L_3' + L_1') - \mu_2 s\cos\theta(L_3' + L_4') - (L_3' + L_4' + L_5'), \tag{A2}$$

$$F_2' = \mu_1 L_2', \tag{A3}$$

$$F_3' = \mu_1 L_1'(1 + pd)e^{pd} + \mu_1(3L_4'e^{3pd} - 3L_3'e^{-3pd}) + \mu_2 s\cos\theta(L_1'pde^{pd} + L_3'e^{-3pd} + L_4'e^{3pd}) + L_1'pde^{pd} + (L_3'e^{-3pd} + L_4'e^{3pd} + L_5'e^{-pd}), \tag{A4}$$

$$F_4' = \mu_1 L_2'(1 - pd)e^{-pd} + \mu_2 s\cos\theta L_2'pde^{-pd} - L_2'pde^{-pd}. \tag{A5}$$

Here $L_1 - L_5$ are complex functions which depend on $|A|^2$, k_y, k_z, θ, and other parameters and which were found in Ref. 17;

$$p^2 = k^2\cos^2\theta + k^2\sin^2\theta/(1 + 4\pi\chi_1), \quad k_y = k\cos\theta, \quad k_z = k\sin\theta.$$

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