Motion of Abrikosov vortices in anisotropic superconductors

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An analysis is made of the motion of the Abrikosov vortices in anisotropic superconductors in the presence of a transport current. An expression is obtained for the viscosity tensor. The anisotropy affects the relaxation and ohmic losses differently. It is shown that a vortex moves along a direction which is not generally perpendicular to the transport current. The resistance in the case of viscous flow of the flux depends strongly on the mutual orientation of the magnetic field and the anisotropy axis.

A new class of type II superconductors in the form of high-temperature Y–Ba–Cu–O materials is now attracting much attention. One of the characteristic properties of these superconductors is a strong anisotropy of the normal and superconducting parameters. This can be allowed for using the Ginzburg–Landau theory if we assume that the effective mass is a tensor. The free energy density is then

$$F=\frac{1}{2}\left(-i\hbar\nabla+\frac{2e}{c}\mathbf{A}\right)\psi^{*}\hat{m}^{-1}\left(i\hbar\nabla+\frac{2e}{c}\mathbf{A}\right)\psi$$
$$+a|\psi|^{2}+\frac{1}{2}b|\psi|^{4}, \qquad (1)$$

where the mass tensor $\hat{m}$ is of the form $m_{ij}=m_0(\delta_{ij}+\mu\nu_i\nu_j)$ in the case of uniaxial anisotropy case ($\boldsymbol{\nu}$ is a unit vector along the anisotropy axis) and ψ is the order parameter.

Magnetic properties of such a superconductor, particularly the structure of an Abrikosov vortex and the features of a vortex lattice, were considered in Refs. 1–3. The Ginzburg–Landau phenomenological approach can be used also to describe the transport properties of anisotropic superconductors. The present paper deals with this subject.

In the presence of a transport current the vortex lines are subject to the Lorentz force. Then, in the absence of pinning, the vortices move and give rise to a resistance. The energy dissipation is then associated with two different processes (see, for example, Ref. 4): 1) the losses due to relaxation of the order parameter; 2) the ohmic losses associated with the flow of the normal currents and the appearance of an electric field. The anisotropy should affect these two energy dissipation processes and, as shown below, the influence of the anisotropy is different for these two processes. In the isotropic case the equation of motion of a vortex line is

$$\eta\mathbf{V}_L=\frac{\phi_0}{c}[\mathbf{j}_{tr},\mathbf{n}], \qquad (2)$$

where η is the viscosity; $\mathbf{V}_L$ is the vortex velocity; $\mathbf{j}_{tr}$ is the transport current; $\mathbf{n}$ is the vector directed along the vortex axis; ϕ_0 is a quantum of the flux. If we allow for the anisotropy, we find that the viscosity should naturally depend on the direction of motion and, consequently, it should be a tensor. The equations of motion of a vortex will be obtained for the anisotropic case using an approach developed in Ref. 4 and based on the time-dependent Ginzburg–Landau equation. Strictly speaking, this equation applies only to zero-gap superconductors with a high concentration of paramagnetic impurities. A detailed study of the validity of this method can be found in Ref. 4. We shall consider only the case of sufficiently weak magnetic fields $B\ll H_{c2}$ when the distance between the Abrikosov vortices in a lattice is large, and we shall discuss the motion of a single vortex with a transport current $\mathbf{j}_{tr}$ flowing around it. We shall also assume that the Ginzburg–Landau parameter satisfies $\varkappa\gg 1$ and, consequently, the potential $\mathbf{A}$ can be ignored in the region where the distance to the center of a vortex obeys $r\ll\delta$ (δ is the depth of penetration of the magnetic field).

We assume these approximations in writing down the following form of the Ginzburg–Landau equations:

$$\hbar\gamma\frac{\partial\rho}{\partial t}=\frac{\hbar^2}{2}\nabla(\hat{m}^{-1}\nabla\rho)-\frac{\hbar^2}{2}\rho\nabla\theta\hat{m}^{-1}\nabla\theta-a\rho-b\rho^3, \qquad (3)$$

$$\gamma\rho^2\left(\hbar\frac{\partial\theta}{\partial t}+2e\Phi\right)=\frac{\hbar^2}{2}[\nabla(\rho^2\hat{m}^{-1}\nabla\theta)], \qquad (4)$$

where γ is the relaxation parameter; ρ and θ are the amplitude and phase of the order parameter $\psi=\rho e^{i\theta}$; Φ is the scalar potential. The expression for the total current is

$$\mathbf{j}=\mathbf{j}_s-\hat{\sigma}_n\nabla\Phi=2e\hbar\hat{m}^{-1}\rho^2\nabla\theta-\hat{\sigma}_n\nabla\Phi, \qquad (5)$$

where $\hat{\sigma}_n$ is the tensor of the normal conductivity such that the anisotropy axis of $\hat{\sigma}_n$ clearly coincides with $\boldsymbol{\nu}$ [$\sigma_{nij}=\sigma_{n0}(\delta_{ij}-\beta\nu_i\nu_j)$]. In the case of high-temperature superconductors we have $\beta>0$. Using the condition $\operatorname{div}\mathbf{j}=0$, we readily find from Eqs. (4) and (5) that

$$\frac{\hbar}{4e}\nabla(\hat{\sigma}_n\nabla\Phi)=\gamma\rho^2\left(\hbar\frac{\partial\theta}{\partial t}+2e\Phi\right). \qquad (6)$$

We use a system of coordinates x, y, and z such that the z axis is directed along the vector $\boldsymbol{\nu}$ and the vector along the vortex axis $\mathbf{n}$ lies in the xz plane. We substitute the variables in accordance with $\tilde{x}=x$, $\tilde{y}=y$, $\tilde{z}=z(1+\mu)^{1/2}$. Then, Eq. (3) expressed in terms of the new variables is exactly the same as in the isotropic case.

However, this is not true of the expression for the current (5) or the equation for the scalar potential Φ given by Eq. (6). These equations transform as follows:

$$\mathbf{j}=2e\hbar\rho^2\left(\frac{\partial\theta}{\partial\tilde{x}},\frac{\partial\theta}{\partial\tilde{y}},\frac{1}{(1+\mu)^{1/2}}\frac{\partial\theta}{\partial\tilde{z}}\right)$$
$$-\sigma_{n0}\left(\frac{\partial\Phi}{\partial\tilde{x}},\frac{\partial\Phi}{\partial\tilde{y}},(1-\beta)(1+\mu)^{1/2}\frac{\partial\Phi}{\partial\tilde{z}}\right), \qquad (7)$$

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$$\frac{\hbar\sigma_{n0}}{4e}\left[\frac{\partial^2}{\partial\tilde{x}^2}+\frac{\partial^2}{\partial\tilde{y}^2}+(1+\mu)(1-\beta)\frac{\partial^2}{\partial\tilde{z}^2}\right]\Phi=\gamma\rho^2\left(\hbar\frac{\partial\theta}{\partial t}+2e\Phi\right). \quad (8)$$

Still ignoring the ohmic losses, i.e., assuming that $\sigma_{n0}=0$, we can reduce the solution to the familiar isotropic case. It is convenient to introduce a current $\tilde{\mathbf{j}}$, which is expressed in terms of $\mathbf{j}$ as follows:

$$\tilde{\mathbf{j}}=(j_x,\ j_y,\ (1+\mu)^{1/2}j_z).$$

If $\sigma_{n0}=0$, we find that $\tilde{\mathbf{j}}=2e\hbar\rho^2\tilde{\nabla}\theta$. Having found the value of $\tilde{\mathbf{j}}$ obtained in this way and allowing for the fact that at large distances from a vortex the value of the current is $\tilde{\mathbf{j}}_\infty=(j_{tr,x},j_{tr,y},(1+\mu)^{1/2}j_{tr,z})$, we can see that the system of equations describing the motion of a vortex in a current $\tilde{\mathbf{j}}$ in no way differs from the equations for the isotropic case. Consequently, the equation of motion of a vortex is similar to Eq. (2):

$$\eta_{p0}\mathbf{V}_L=\frac{\phi_0}{c}[\tilde{\mathbf{j}}_\infty\mathbf{n}].$$

Here, $\eta_{p0}=2\pi\hbar\gamma\alpha_1|a|/b$ is the viscosity associated with relaxation of the order parameter ($\alpha_1=0.279$),

$$\tilde{\mathbf{n}}=(1+\mu\nu_z^2)^{-1/2}(-\nu_x,\ 0,\ \nu_z(1+\mu)^{1/2}).$$

is the vector directed along a vortex in the new coordinate system, $\nu_z=\mathbf{vn}$ and $\nu_x^2=1-\nu_z^2$.

We can now return to the old variables and find the equation for $\mathbf{V}_L$. We do this after calculating the total viscosity allowing for the fact that $\sigma_{n0}\neq 0$. We rotate the axes in the coordinate system $\tilde{x},\tilde{y},\tilde{z}$ in such a way that the z' axis is directed along $\tilde{\mathbf{n}}$. Following Ref. 4, we seek the equation for the order parameter in the form

$$\psi=\psi_0(x'-\tilde{V}_{Lx'}t,\ y'-\tilde{V}_{Ly'}t)+\psi_1.$$

Here, $\psi_0=\rho_0e^{i\theta_0}$ is the solution of the stationary Ginzburg–Landau equation describing a single vortex. We assume that $\partial\theta/\partial t=-\tilde{\mathbf{V}}_L\tilde{\nabla}\theta$ and rewrite Eq. (8) as follows:

$$\frac{\hbar}{4em_0}\tilde{\nabla}(\hat{\sigma}_n\hat{m}\tilde{\nabla}\Phi)=\gamma\rho_0^2(2e\Phi-\hbar\tilde{\mathbf{V}}_L\tilde{\nabla}\theta_0). \quad (8a)$$

Repeating then the procedure described in Ref. 4, which can easily be generalized to the anisotropic case, we obtain an equation analogous to Eq. (2), except that now $\hat{\eta}$ is a tensor. The total current can then be described by

$$\tilde{\mathbf{j}}=2e\hbar\rho^2\tilde{\nabla}\theta-\frac{1}{m_0}\hat{\sigma}_n\hat{m}\tilde{\nabla}\Phi,$$

because it is in this case that the condition $\tilde{\nabla}\mathbf{j}=0$ is satisfied. The vector $\hat{\eta}_{0h}\tilde{\mathbf{V}}_L$ of interest to us ($\hat{\eta}_{0h}$ is the ohmic viscosity tensor), projected along an arbitrary direction $\mathbf{d}$ in the $x'y'$ plane can be written in the form

$$(\mathbf{d},\hat{\eta}_{0h}\tilde{\mathbf{V}}_L)=\frac{\hbar}{2e}\int\left(\frac{1}{m_0}\hat{\sigma}_n\hat{m}\tilde{\nabla}\Phi\right)\tilde{\nabla}\theta_d\,dx'\,dy', \quad (9)$$

where $\theta_d=\mathbf{d}\tilde{\nabla}\theta_0$.

The integral in Eq. (9) can be calculated if we solve Eq. (8a). In general, this cannot be done analytically. However, the solution can be obtained in some limiting cases. Equation (8a) contains two characteristic scales: ξ is the coherence length and

$$l_E=\left[\hbar\sigma_{n0}\Big/\left(8e^2\gamma\frac{|a|}{b}\right)\right]^{1/2}$$

is the length representing the penetration of an electric field. The parameter $u=(\xi/l_E)^2$ represents the ratio for the two different dissipation mechanisms mentioned above. If $u\ll 1$, which is true of semiconductors with a finite gap, the main contribution to the dissipation comes from ohmic losses. However, if $u\gg 1$, then the relaxation of the order parameter is the more important contribution. Zero-gap superconductors with a high concentration of paramagnetic impurities are close to the latter case because they are characterized by $u=12$. We shall first consider the situation when $u\ll 1$. Then, in a region where the distance to the center of a vortex r satisfies the condition $\xi\ll r\ll l_E$ we can find the potential from the approximate equation

$$\frac{1}{4em_0}\tilde{\nabla}(\hat{\sigma}_n\hat{m}\tilde{\nabla}\Phi)=-\gamma\frac{|a|}{b}\tilde{\mathbf{V}}_L\tilde{\nabla}\theta_0. \quad (10)$$

Using Eq. (10) we can readily see that the integral of Eq. (9) diverges logarithmically at the upper and lower limits of integration. Consequently, the main contribution to this integral comes specifically from the region $\xi\ll r\ll l_E$ and in the ohmic viscosity case we have

$$\eta_{0h}=2\pi\hbar\gamma\frac{|a|}{b}\ln\frac{l_E}{\xi}. \quad (11)$$

We can see that in this limiting case the ohmic viscosity, like the relaxation contribution η_{p0}, is independent of the direction of motion of a vortex expressed in terms of the new variables. This is not true if $u\gg 1$. We can then find the solution using a model function $\rho_0(\mathbf{r}')$ described by

$$\rho_0(\mathbf{r}')=\begin{cases}0; & |\mathbf{r}'|<\xi,\\ (|a|/b)^{1/2}; & |\mathbf{r}'|>\xi.\end{cases} \quad (12)$$

Obviously, the assumptions about the nature of $\rho_0(\mathbf{r}')$ agree with those described by the Bardeen–Stephen model. In we ignore Eq. (8a) the left-hand side of the equation for $|\mathbf{r}|>\xi$ and obtain $\Phi=(\hbar/2e)\tilde{\mathbf{V}}_L\tilde{\nabla}\theta_0$. We use the condition of continuity of the function Φ for $|\mathbf{r}'|=\xi$, which leads to the following expression in the case when $|\mathbf{r}'|<\xi:\Phi=(\hbar/2e\xi^2)[\mathbf{r}'\tilde{\mathbf{V}}_L]z_0'$. We now calculate the integral in Eq. (9):

$$(\mathbf{d},\hat{\eta}_{0h}\tilde{\mathbf{V}}_L)=\frac{\pi\hbar^2\sigma_{n0}}{2e\xi^2(1+\mu\nu_z^2)}\left\{\tilde{V}_{Lx'}d_{x'}\left[1+\frac{\mu}{4}(1+3\nu_z^2-\beta\nu_x^2)-\frac{\beta\nu_x^2}{4}\right]\right.$$
$$\left.+\tilde{V}_{Ly'}d_{y'}\left[1+\frac{\mu}{4}(3+\nu_z^2-3\beta\nu_x^2)-\frac{3}{4}\beta\nu_x^2\right]\right\}. \quad (13)$$

Having obtained the expression for the viscosity tensor, associated with the ohmic losses, we can now write down the complete equation of motion for a vortex line:

$$(\eta_{p0}\hat{I}+\hat{\eta}_{0h})\tilde{\mathbf{V}}_L=\frac{\phi_0}{c}[\tilde{\mathbf{j}}_\infty,\tilde{\mathbf{n}}], \quad (14)$$

where $\hat{I}$ is a unit tensor.

In Eq. (14) we return from the coordinates $\tilde{x},\tilde{y},\tilde{z}$ to the coordinates x, y, and z. We then carry out rotation in the xz plane, so that the new axis z is directed along the vortex axis. The equation for $\mathbf{V}_L$ then becomes

$$\hat{\eta} V_L = \frac{\phi_0}{c}[\mathbf{j}_{tr}\mathbf{n}], \quad \hat{\eta} = \begin{pmatrix} \eta_x & 0 \\ 0 & \eta_y \end{pmatrix}.$$

The expressions for η_x and η_y are different for different values of u. If $u \ll 1$, then

$$\eta_x = 2\pi\hbar\gamma \frac{|a|}{b}\left(\alpha_1 + \ln\frac{l_E}{\xi}\right)\left(\frac{1+\mu}{1+\mu\nu_z^2}\right)^{1/2},$$
$$\eta_y = 2\pi\hbar\gamma \frac{|a|}{b}\left(\alpha_1 + \ln\frac{l_E}{\xi}\right)\left(\frac{1+\mu\nu_z^2}{1+\mu}\right)^{1/2}. \qquad (15)$$

However, if $u \gg 1$, we find that

$$\eta_x = 2\pi\hbar\gamma \frac{|a|}{b}\left(\frac{1+\mu}{1+\mu\nu_z^2}\right)^{1/2} \times \left\{ \alpha_1 + 2\left(\frac{l_E}{\xi}\right)^2 \left[1 + \frac{\mu}{4}(1+3\nu_z^2-\beta\nu_x^2) - \frac{\beta\nu_x^2}{4}\right] \Big/ (1+\mu\nu_z^2) \right\},$$
$$\eta_y = 2\pi\hbar\gamma \frac{|a|}{b}\left(\frac{1+\mu\nu_z^2}{1+\mu}\right)^{1/2} \times \left\{ \alpha_1 + 2\left(\frac{l_E}{\xi}\right)^2 \left[1 + \frac{\mu}{4}(3+\nu_z^2-3\beta\nu_x^2) - \frac{3}{4}\beta\nu_x^2\right] \Big/ (1+\mu\nu_z^2) \right\}. \qquad (16)$$

Figure 1 shows the dependences of η_x and η_y on the angle between the direction of the vortex axis n and the anisotropy axis $\boldsymbol{\nu}$ ($\cos\alpha = \boldsymbol{\nu}\mathbf{n}$) for $u = 12$, $\mu = 9$, and $\beta = 0.9$. It follows from our results that, in contrast to the isotropic case, the direction of motion of vortices is no longer perpendicular to the current. Since the average electric field is given by the expression

$$\mathbf{E} = \frac{1}{c}[\mathbf{B}\mathbf{V}_L],$$

this field is also not parallel to the current. Consequently, there is a Hall field which does not vanish even for $\mathbf{B}\perp\mathbf{j}$, as found also in the isotropic case. We can readily obtain an expression for the energy dissipated per unit time per unit volume:

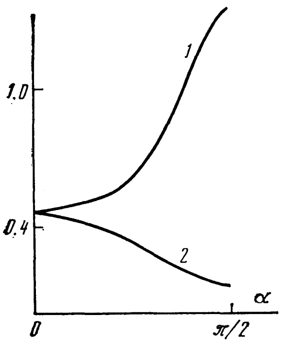

FIG. 1. Components of the viscosity tensor versus the angle between the direction of the vortex axis and the anisotropy axis: 1) $\eta_x/(2\pi\hbar\gamma|a|/b)$; 2) $\eta_y/(2\pi\hbar\gamma|a|/b)$.

$$W = \mathbf{E}\mathbf{j} = \frac{B\phi_0}{c^2}\left\{\frac{(\mathbf{j}[\boldsymbol{\nu}\mathbf{n}])^2}{(1-(\boldsymbol{\nu}\mathbf{n})^2)\eta_x} + \frac{(\mathbf{j}(\boldsymbol{\nu}-\mathbf{n}(\boldsymbol{\nu}\mathbf{n})))^2}{(1-(\boldsymbol{\nu}\mathbf{n})^2)\eta_y}\right\}. \qquad (17)$$

Obviously, the resistance in the case of viscous flow of the flux depends strongly on the orientation of the Abrikosov vortices relative to the anisotropy axis $\boldsymbol{\nu}$. The above expression for the viscosity tensor makes it possible to find the resistance anisotropy and its dependence on the direction of the applied magnetic field.

Unfortunately, there are as yet no experimental data on the flow of the flux in high-temperature superconductors. One would hope that later investigations would make it possible to check experimentally our results.

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