Gravitational radiation in the field of a cosmic string

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We calculate the gravitational radiation emitted by a freely moving object in the field of a rectilinear, infinitely thin cosmic string. Expressions are derived for the total energy radiated as a point mass passes near a string, and for the radiation due to a cosmic test string moving parallel to the string responsible for shaping the spacetime background.

1. INTRODUCTION

The theory of cosmic strings (which may be produced in cosmological phase transitions[1]) has of late been the center of a great deal of attention. The cosmic string hypothesis holds out the intriguing possibility of deducing for the origin of structure in the universe a stochastic theory that lacks the usual difficulties associated with the theory of condensations at inhomogeneities.[2] String theory provides a good description of the observed relationship between the scale and distribution of structures, and the structure correlation function that it yields contains absolutely no free parameters. Furthermore, using a single parameter—the phase-transition energy—it is possible to describe a number of the finer details of the structure correlation function. This energy is of order 10^{15}–10^{16} GeV, which is quite consistent with our present understanding of the cosmological phase transition associated with the breaking of grand unified symmetry.[3]

Cosmic strings are described by topologically nontrivial solutions of the equations of gauge theories with spontaneous symmetry breaking, and such solutions exist if the manifold of spatial coordinates in which the Higgs potential has a true minimum is multiply connected.[4] Far from a string, the vacuum expectation value $\langle\Phi\rangle$ of the Higgs field is nonzero—i.e., $\langle\Phi\rangle\sim\eta$; on the other hand, at the very core of the string $\langle\Phi\rangle=0$—the interior of the string "remembers" the state of unbroken symmetry.

String solutions show up even in the simplest model with the gauge group $U(1)$ (scalar electrodynamics).[5] One realistic grand unified theory permitting of string solutions is the model based on $SO(10)$, which is spontaneously broken to $SU(5)\times Z_2$, with the subsequent phase transition $SU(5)\rightarrow SU(3)\times SU(2)\times U(1)$, preserving Z_2 (Ref. 6).

Topologically stable strings have no ends, so they are either closed or infinite. The linear mass density of a string turns out to be of the order of the square of the vacuum expectation value of the Higgs field far from the string, i.e., $\mu\sim\eta^2$; under these circumstances, the tension in the string is negative, and is equal in absolute magnitude to the linear mass density μ (taking $c=1$, which we shall do for the remainder of the paper). The characteristic radius of the string is of order $\delta\sim\lambda^{-1/2}\eta^{-1}\approx 10^{-28}$ cm, where λ is the self-action constant of the Higgs field.

A cosmological scenario taking advantage of strings can be divided into several stages. First, a system of strings is formed that consists of rectilinear string segments whose length is of the order of the radius of the horizon, and which move at relativistic speeds. When these segments collide, they give rise to closed oscillating loops, which slowly relax by radiating gravitational waves. As the universe expands, the repetition of this stochastic process leads to a hierarchy of closed loops over a range of scale sizes. These loops then serve as seeds for the gravitational condensation of matter, which finally results in a structural hierarchy of galaxies, clusters of galaxies, and superclusters.[7]

It is just possible that some primordial cosmic strings are still in existence, making the study of physical effects due to strings an intriguing topic. One effect by which a string would divulge its presence is the focusing of light passing nearby.[7] A string could give rise to double images separated by a large angular distance (of the order of arc minutes). Moreover, a cosmic string located within a system of gravitating bodies should induce periastron advance, precession of the plane of motion, and other kinematic anomalies, if it is encompassed by the corresponding orbits.[8]

Our objective in the present paper is to demonstrate that the interaction of a rectilinear cosmic string with matter, as well as the interaction among a number of strings, should result in the generation of gravitational radiation. What makes this effect nontrivial is that a rectilinear cosmic string does not create a Newtonian gravitational field. However, although the spacetime of the cosmic string is locally pseudoeuclidean, the global conical aspect of space changes the behavior of the retarded potentials induced by the moving bodies in such a way that a portion of the field is "detached" from the source in the form of radiation. Gravitational radiation associated with string oscillations was heretofore an active topic of discussion in cosmic string theory.[9] We must emphasize, however, that there is a significant difference between these two types of radiation. While the latter conforms to the usual dynamics and is due to the accelerated motion of different parts of the string relative to each other, the former arises as a result of distortion of the field of a uniformly moving source by virtue of the global conical structure of three-dimensional space around a string. As an indication of the magnitude of this effect, we point out that when a rectilinear cosmic string passes with relativistic velocity ($\sim 0.5c$) near a black hole at the center of a galaxy whose mass is $10^9 M_\odot$, an energy of order $10^{-2} M_\odot$ is liberated in the form of gravitational waves over a period of 10^4–10^5 sec.

2. THE SPACETIME OF A RECTILINEAR COSMIC STRING

In gauge models that permit the existence of string solutions, the energy-momentum tensor of a system of matter fields comprising a rectilinear string has a particular feature that is reflected in the equality of the mixed components,

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$$T_t{}^t = T_z{}^z \tag{2.1}$$

(the z-axis is directed along the string). What this means physically is that the tension in the string is negative, and is equal to the linear energy density. The energy density falls off exponentially with distance ρ in any plane orthogonal to the z-axis. Furthermore, the nonzero tension terms T^ρ_ρ and T^φ_φ also rapidly tend to zero with increasing ρ. This behavior is typical of a so-called local string, which occurs in models with spontaneous breaking of local (gauge) symmetry. In principle, string solutions exist in models like the Goldstone model with spontaneously broken global symmetry. For these solutions, the string energy density in the spacetime plane falls off as ρ^{-2}, so that the total energy per unit length turns out to be infinite.

Negative string tension is a source of "antigravity," which completely nullifies the contribution of the positive energy density in the Newtonian approximation. Indeed, the Einstein equations imply that the static Newtonian potential Ψ satisfies the equation

$$\Delta\Psi = 8\pi G(T_t{}^t - {}^1/_2 T), \quad T = T_\mu{}^\mu, \tag{2.2}$$

whereupon we obtain the above result (G is the gravitational constant).

Moreover, the Newtonian potential vanishes even when other components of the energy-momentum tensor are taken into consideration, given their rapid falloff with distance.[10] It can be shown rigorously in the model based on the $U(1)$ gauge group that at large distances, the gravitational field contains no long-range component, and is local.[11]

However, even in the simplest $U(1)$ model, it has not been possible to obtain an exact solution for a self-gravitating string. Straight-string models have therefore been proposed in which the energy-momentum tensor is chosen phenomenologically so as to satisfy Eq. (2.1). Thus, one can obtain an exact solution of the Einstein equations by choosing

$$T_t{}^t = T_z{}^z = \frac{\mu}{\pi\rho_0{}^2}\theta(\rho_0 - \rho), \tag{2.3}$$

and setting all other components of the energy-momentum tensor to zero; $\theta = (\rho_0 - \rho)$ is the Heaviside step function.

The corresponding solution of the Einstein equations[12] is the spacetime $R^2 \times M$, where R^2 is the pseudoeuclidean plane defined by the variables (t,z), and M is the manifold defined by the variables (ρ,ϕ) and is the union of two parts:

$$M = M_{in} \cup M_{out}, \tag{2.4}$$

where $M_{\rm in}$ takes care of radial values $\rho \leqslant \rho_0$, and $M_{\rm out}$ is responsible for $\rho > \rho_0$.

The manifold $M_{\rm in}$ can be represented as part of a spherical surface of radius

$$R = \rho_0/(8G\mu)^{1/2} \tag{2.5}$$

in three-dimensional Euclidean space, and the manifold $M_{\rm out}$ as part of a conical surface with vertex angle q in this same space. The two surfaces join smoothly at $\rho = \rho_0$. We may write the resulting metric in four-space in the form

$$ds^2 = dt^2 - dz^2 - \begin{cases} d\chi^2 + R^2 \sin^2(\chi/R)\, d\varphi^2, & \chi < \chi_0, \\ d\rho^2 + b^2\rho^2\, d\varphi^2, & \rho > \rho_0. \end{cases} \tag{2.6}$$

where $\chi = \chi_0$ at $\rho = \rho_0$, and

$$\chi_0 = (\pi/2 - q)R, \quad \rho_0 = R \operatorname{ctg} q, \quad \sin q = b; \tag{2.7}$$

we also have

$$b = 1 - 4G\mu. \tag{2.8}$$

In the limit $\rho_0 \to 0$, which yields a delta-function distribution for the matter,

$$T_t{}^t = T_z{}^z = \frac{\mu}{2\pi\rho}\delta(\rho), \tag{2.9}$$

the exterior solution is valid over all space (this is the model of an infinitely thin string).

This approximation will suffice for our purposes. As a background field, then, we shall take the gravitational field due to an infinitely thin string having the characteristic energy-momentum tensor (2.9) and generating a locally flat spacetime having the metric[13,14]

$$ds^2 = dt^2 - dz^2 - d\rho^2 - \rho^2 b^2 d\phi^2. \tag{2.10}$$

If we represent the stretching of the azimuthal angle in the form $\varphi = b\phi$, Eq. (10) reduces to the interval measured in cylindrical coordinates in a locally flat spacetime:

$$ds^2 = dt^2 - dz^2 - d\rho^2 - \rho^2 d\varphi^2, \tag{2.11}$$

although the domain over which the angle φ varies is not the usual one:

$$0 \leqslant \varphi \leqslant 2\pi b. \tag{2.12}$$

For a string with a positive energy density, it is clear from Eq. (2.8) that b is less than unity (in a typical realistic case, $1 = b \leqslant 10^{-5}$). The length of the unit circle in a plane of constant z will then be less than 2π.

Let us now imagine that a cosmic string encircles some agglomeration of matter whose gravitational field in the vicinity of the string may be assumed to be weak. The resultant gravitational field is then most conveniently described using a perturbation approach in the rest frame of the string. Accordingly, we must now deal with the problem of gravitational perturbations of a conical spacetime; in particular, such perturbations include a wave component that may be interpreted as gravitational radiation emitted by the system.

3. SMALL PERTURBATIONS OF THE METRIC IN A CONICAL SPACETIME

We wish to represent the total gravitational field of a cosmic string and some system of matter described by an energy-momentum tensor $\delta T_{\mu\nu}$ in the form

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}, \tag{3.1}$$

where $g^{(0)}_{\mu\nu}$ is the metric of the conical spacetime (2.11) in coordinates t, z, ρ, φ, and the $h_{\mu\nu}$ are small perturbations. As we noted above, since in terms of the azimuthal variable φ the metric (2.11) is locally flat, we are concerned here with the description of the linearized gravitational perturbations in cylindrical coordinates. The fact that the limits on φ change will lead to the appearance of terms proportional to $\delta(\rho)$ in the equations for the perturbations, but for our purposes, we may restrict our considerations to perturbations with $\rho\neq0$.

Working in the deDonder-Fock gauge,

$$\Psi_{\nu}^{\mu\nu}{}_{;}=0,\quad \Psi_{\mu\nu}=h_{\mu\nu}-{}^1/_2 g_{\mu\nu}^{(0)}h,\quad h=h_\mu^\mu, \tag{3.2}$$

and representing the covariant derivative in the metric (2.11) by a semicolon, we have

$$\Psi_{\mu\nu;\lambda}{}^{;\lambda}=16\pi G\delta T_{\mu\nu} \tag{3.3}$$

The calculation of the covariant second derivative of the tensor $\Psi_{\mu\nu}$ leads to a system of equations that is diagonal in the subscripts t and z. The components ψ_{tt}, ψ_{tz}, ψ_{zz} satisfy the "scalar" d'Alembertian equation

$$\Box\psi_{ab}=-\tau_{ab},\quad \tau_{ab}=16\pi G\delta T_{ab},\quad a,\ b\equiv t,\ z, \tag{3.4}$$

where the operator is

$$\Box=\frac{1}{\rho}\,\partial/\partial\rho(\rho\partial/\partial\rho)+\frac{1}{\rho^2}\,\partial^2/\partial\varphi^2+\partial^2/\partial z^2-\partial^2/\partial t^2. \tag{3.5}$$

The trace of the tensor satisfies an analogous equation:

$$\Box\psi=-\tau,\quad \tau=16\pi G\delta T_\mu^\mu. \tag{3.6}$$

The components $\psi_{t\rho}$ and $\psi_{t\varphi}$ ($\psi_{z\rho}$ and $\psi_{z\varphi}$) satisfy a set of coupled equations, which when diagonalized in terms of the quantities

$$\psi_{as}=\psi_{a\rho}+\frac{is}{\rho}\psi_{a\varphi},\quad s=\pm1$$

leads to the separated equations

$$\Box_s\psi_{as}=-\tau_{as},\quad \tau_{as}=16\pi G\left(\delta T_{a\rho}+\frac{is}{\rho}\,\delta T_{a\varphi}\right). \tag{3.7}$$

Finally, diagonalizing the system for

$$\psi_s=\psi_{\rho\rho}-\frac{1}{\rho^2}\psi_{\varphi\varphi}+\frac{is}{\rho}\psi_{\rho\varphi},\quad s=\pm2$$

in the (ρ,φ) sector yields

$$\Box_s\psi_s=-\tau_s,\quad \tau_s=16\pi G\left(\delta T_{\rho\rho}-\frac{1}{\rho^2}\,\delta T_{\varphi\varphi}+\frac{is}{\rho}\,\delta T_{\rho\varphi}\right). \tag{3.8}$$

The operator $\Box_s$, in Eqs. (3.7) and (3.8) is given by

$$\Box_s=\Box+\frac{2is}{\rho^2}\,\partial/\partial\varphi-s^2/\rho^2=\Delta_s-\partial_t^2,\quad s=\pm1,\pm2. \tag{3.9}$$

Let us now construct the Green's function for this operator:

$$\Box_s G_s(x,x')=-\delta^4(x,x') \tag{3.10}$$

where on the right-hand side we have the invariant delta function

$$\delta^4(x,x')=(\rho\rho')^{-1/2}\delta(t-t')\delta(z-z')\delta(\rho-\rho')\delta(\varphi-\varphi'). \tag{3.11}$$

In order to be able to solve the radiation problem, we require the retarded and advanced Green's functions, which satisfy

$$G_s^{ret}(t<t')=0,\quad G_s^{adv}(t>t')=0, \tag{3.12}$$

as well as the radiation Green's function

$$G_s^{rad}={}^1/_2(G_s^{ret}-G_s^{adv}), \tag{3.13}$$

which satisfies the homogeneous equation

$$\Box_s G_s^{rad}=0. \tag{3.14}$$

We first construct a complete set of eigenfunctions for the following three commuting operators: the generalized Laplacian Δ_s, as defined by (3.9), the z-component of the angular momentum $\hat{L}_z=-i\partial/\partial\varphi$, and the z-component of the linear momentum $\hat{P}_z=-i\partial/\partial z$:

$$\hat{L}_z u_{smkp}=\tilde{m}u_{smkp}, \tag{3.15}$$

$$\hat{P}_z u_{smkp}=ku_{smkp}, \tag{3.16}$$

$$\Delta_s u_{smkp}=-(k^2+p^2)u_{smkp}. \tag{3.17}$$

Since the azimuthal angle φ varies within the limits set down by (2.12), the requirement that (3.15) have a unique solution yields a condition on the eigenvalues:

$$\tilde{m}=m/b,\quad m=0,\pm1,\pm2\dots. \tag{3.18}$$

The eigenfunctions u_{smkp} satisfying (3.15)–(3.17) and the normalization condition

$$\int_{-\infty}^{\infty}dz\int_0^{2\pi b}d\varphi\int_0^\infty d\rho\,\rho u^*_{sm'k'p'}u_{smkp}=\delta_{mm'}\delta(k-k')\delta(p-p'), \tag{3.19}$$

then take the form

$$u_{smkp}=(2\pi b^{1/2})^{-1}\exp[i((m/b)\varphi+kz)]J_{|m/b+s|}(p\rho), \tag{3.20}$$

where J is a Bessel function.

The eigenmodes u_{smkp} so constructed form a complete set in three-dimensional conical space, a condition which may be expressed in terms of the completeness relation

$$\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty}dk\int_0^\infty dp\,pu^*_{smkp}(z',\rho',\varphi')u_{smkp}(z,\rho,\varphi)$$
$$=(\rho\rho')^{-1/2}\delta(\rho-\rho')\delta(\varphi-\varphi')\delta(z-z'). \tag{3.21}$$

Furthermore, following the usual procedure for constructing Green's functions by integrating over the complex frequency plane, we find

$$G_s^{ret(adv)}(x,x')$$
$$=\frac{1}{2\pi}\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty}d\omega\int_{-\infty}^{\infty}dk\int_0^\infty dp\,p\,\frac{u^*_{smkp}(x')u_{smkp}(x)}{p^2+k^2-\omega^2\mp i\varepsilon\omega}e^{-i\omega(t-t')}, \tag{3.22}$$

where $\varepsilon>0$ is an infinitesimal that determines the direction in which the integration contour goes around the pole. The radiation Green's function may be obtained by substituting (3.22) into (3.13) and integrating over ω:

$$G_s^{rad}(x,x')$$
$$=\frac{i}{2}\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty}d\omega\frac{|\omega|}{\omega}\int_{-\infty}^{\infty}dk\int_0^\infty dp\,pu^*_{smkp}(x')$$
$$\times u_{smkp}(x)e^{-i\omega(t-t')}\delta(k^2+p^2-\omega^2)=\frac{1}{2}$$
$$\times\sum_{m=-\infty}^{\infty}\int_{-\infty}^{\infty}dk\int_0^\infty dp\,pu^*_{smkp}(x')u_{smkp}(x)\omega_{pk}^{-1}$$
$$\cdot\sin\omega_{pk}(t-t'), \tag{3.23}$$

where $\omega_{pk} = (p^2 + k^2)^{1/2}$.

The simplest approach calculating the gravitational radiation emitted by a source $\delta T_{\mu\nu}$ in the field of a cosmic string is to calculate the work done by the radiative damping force. In the linearized theory and with the present curved background, this is given by[15,16]

$$\delta\mathscr{E} = -\frac{1}{2}\int \delta T^{\mu\nu}(x)\frac{\partial h^{rad}_{\mu\nu}(x)}{\partial t}(-g)^{1/2}d^4x, \qquad (3.24)$$

where the sign has been chosen so that the expression yields the total energy (over all space and time) emitted in the form of gravitational radiation.

Transforming from $h^{\rm rad}_{\mu\nu}(x)$ to $\psi^{\rm rad}_{\mu\nu}(x)$ and making use of the results derived in the preceding section, this can be represented in the following computationally convenient form:

$$\delta\mathscr{E} = -\frac{1}{32\pi G}\int d^4x\, d^4x'(-g)^{1/2}(-g')^{1/2}\Big\{\Big[\tau^{ab}(x)\tau_{ab}(x') + \frac{1}{2}\tilde{\tau}(x)\tilde{\tau}(x') - \frac{1}{2}\tau(x)\tau(x')\Big]\frac{\partial}{\partial t}G_0^{rad}(x,x') - \sum_{s=\pm1}\tau_s{}^a(x)\tau_{as}{}^*(x')\,\partial/\partial t G_s^{rad}(x,x') + \frac{1}{4}\sum_{s=\pm2}\tau_s(x)\tau_s^*(x')\,\partial/\partial t G_s^{rad}(x,x')\Big\}, \qquad (3.25)$$

where

$$\tilde{\tau}(x) = 16\pi G(\delta T_{\rho\rho} + \rho^{-2}\delta T_{\varphi\varphi}), \qquad (3.26)$$

the subscripts $(a,b) \equiv (t,z)$ are summed over, and the indices are raised and lowered in accordance with the metric (2.11).

4. GRAVITATIONAL RADIATION FROM A POINT MASS

Let a test mass M move along a geodesic of the spacetime (2.10) in a plane orthogonal to the z-axis (by virtue of the symmetry of the background under Lorentz transformations along the z-axis, this does not restrict the generality of our discussion). The laws of motion in cylindrical coordinates may be written as

$$\rho^2(t) = d^2 + v^2t^2, \quad \phi = \frac{1}{b}\,\mathrm{arctg}\,\frac{vt}{d} \qquad (4.1)$$

where v is the constant velocity of the body in the rest frame of the string, and d is its impact parameter.

The corresponding components of the energy-momentum tensor are given by

$$\delta T^{\mu\nu} = Mu^\mu u^\nu(\rho u^0)^{-1}\delta(\rho-\rho(t))\delta(\varphi-\varphi(t))\delta(z), \qquad (4.2)$$

where $u^\mu = dx^\mu/ds$ is the body's velocity. Transforming to variables τ_{ab}, τ_{as}, τ_s, τ, $\tilde{\tau}$ and substituting these into Eq. (3.25), a rather involved series of calculations yields the expression for the total gravitational energy radiated:

$$\delta\mathscr{E} = (\pi b)^{-1}G[2Mu^0\sin(\pi/b)]^2\int_0^\infty d\omega\,\omega\int_0^\omega \frac{dp\,p}{(\omega^2-p^2)^{1/2}}\frac{e^{-2\varkappa d}}{\varkappa^2}\times\Big\{\Big[2\,\mathrm{ch}\,2\alpha - \frac{1}{2v^2}(1+2v^2+v^4\,\mathrm{ch}\,4\alpha)\Big]\times\frac{\mathrm{cth}(\alpha/b)}{\mathrm{ch}(2\alpha/b)-\cos(2\pi/b)} + v^2\,\mathrm{sh}\Big[2\alpha\Big(2-\frac{1}{b}\Big)\Big]\Big\} \qquad (4.3)$$

where

$$\varkappa = (\omega^2/v^2-p^2)^{1/2}, \quad e^{-\alpha} = p(\omega/v+\varkappa)^{-1}. \qquad (4.4)$$

This expression is valid for any value of the parameter b. We see from Eq. (4.3) that as might be expected, $\delta\mathscr{E}$ goes to zero when $b = 1$ (i.e., in Minkowski space). Likewise, there is no radiation when $b = 1/n$, where n is an integer. In a realistic case, the parameter $\beta = (1-b)/b$ is much less than unity, and Eq. (4.3) can be simplified somewhat. If we take $p = y\omega$, we obtain for the radiation spectrum

$$\frac{d}{d\omega}\delta\mathscr{E} = \pi G(\beta v M)^2\int_0^1\frac{dy\,y}{(1-y^2)^{1/2}}\frac{8-(1+7v^2)y^2}{(1-v^2y^2)^{5/2}}\times\exp\Big[-\frac{2\omega d}{v}(1-v^2y^2)^{1/2}\Big]. \qquad (4.5)$$

Clearly, then, the frequencies present in the spectrum are those that satisfy

$$0 \leqslant \omega \leqslant v\gamma/d. \qquad (4.6)$$

With increasing particle energy, the maximum frequency is directly proportional to the Lorentz factor $\gamma = (1-v^2)^{1/2}$.

Integrating (4.5) first over ω and then over y, we obtain an expression for the total energy lost to gravitational radiation:

$$\delta\mathscr{E} = \frac{\pi(\beta\gamma M)^2}{16d}G[v(30v^2-1)+\gamma(28v^2+1)\,\mathrm{arctg}(v\gamma)]. \qquad (4.7)$$

In the nonrelativistic limit, this yields

$$\delta\mathscr{E} = {}^{11}/_3 G\pi v^3(\beta M)^2/d. \qquad (4.8)$$

For ultrarelativistic motion, the total energy liberated in the form of gravitational radiation is proportional to the cube of the Lorentz factor:

$$\delta\mathscr{E} = {}^{29}/_{32}G\gamma^3(\pi\beta M)^2/d. \qquad (4.9)$$

A similar energy dependence holds for gravitational bremsstrahlung in the collision of two relativistic masses.[17] but in that case there is an additional factor $\ln\gamma$.

The expressions obtained for the spectral distribution and total gravitational energy radiated by a point mass moving on a straight line in the field of a cosmic string are qualitatively similar to the corresponding quantities describing the electromagnetic radiation due to a charge in flight past a string.[18] The ratio of the energy lost to gravitational radiation when a mass M collides with a cosmic string to the energy liberated in the collision of that same body with another mass m is approximately[17]

$$\delta\mathscr{E}/\delta\mathscr{E}_m \sim (d\beta/r_m)^2 \qquad (4.10)$$

where r_m is the gravitational radius of the mass. Note that in the present treatment, the field of the string (the conical space) is taken to be fixed, and the mass is considered a test object. In reality, the string will be deformed by the passage of the mass, and the corresponding world line of that mass will be distorted. Furthermore, the impact parameter is subject to the constraint $d \gg r_g$, where r_g is the gravitational radius of the mass M.

5. GRAVITATIONAL RADIATION FROM THE COLLISION OF STRAIGHT COSMIC STRINGS

As our second interesting example of the generation of gravitational waves in the field of a cosmic string (conical spacetime), we examine collisions between two strings. As we noted above, collisions between straight segments of cosmic strings are important soon after they have been formed in a phase transition, and according to the current scenario it is precisely these string collisions that result in the hierarchy of closed loops required for the subsequent formation of structure in the universe. Gravitational radiation exerts a critical influence on the further evolution of the loops, being the principal energy-loss mechanism, and ultimately leading to the disappearance of the loops. This radiation derives from oscillations of the loops, and is therefore of the usual dynamical variety. On the other hand, the gravitational radiation resulting from the collision of straight strings moving a constant relative velocity is due to the global conical structure of the spacetime governed by the cosmic string, and can thus be ascribed to the effects of topology.

Consider now the case of two strings in parallel motion, passing one another with some impact parameter d. We shall assume that one of these strings is the source of a gravitational field [described by the conical spacetime metric (2.10)], while the second is a test body—the presumption is that the azimuth angle defect $2\pi(1-b)$ is small. Because of the translational symmetry along the common direction of motion, there can be no mutual deformation of the strings in this case, and a model along these same lines should also hold merely if the impact parameter is large compared with the actual radius of the string.

One can construct the energy-momentum tensor of a moving string either by varying the Nambu action for an infinitely thin string or by invoking a Lorentz transformation out of the rest frame of the string, in which that tensor has the form

$$\delta T_{\mu'\nu'}=\mu'\delta(x')\delta(y')(\delta_{\mu'}{}^{t'}\delta_{\nu'}{}^{t'}-\delta_{\mu'}{}^{z'}\delta_{\nu'}{}^{z'}). \tag{5.1}$$

In the rest frame of the test string, the components of $\delta T_{\mu'\nu'}$ of most interest to us are

$$\begin{aligned}
&\delta T_{tt}=-\gamma^2\delta \dot{T}_{zz}=\mu'\gamma(\rho)^{-1}\delta(\rho-\rho(t))\delta(\varphi-\varphi(t)),\\
&\tilde{T}=\delta T_{\rho\rho}+\rho^{-2}\delta T_{\varphi\varphi}=v^2\delta T_{tt},\\
&T_t{}^s=\delta T_{t\rho}+is\rho^{-1}\delta T_{t\varphi}=isve^{-is\varphi}\delta T_{tt}, \qquad s=\pm 1,\\
&T^s=\delta T_{\rho\rho}-\rho^{-2}\delta T_{\varphi\varphi}+is\rho^{-1}\delta T_{\rho\varphi}=-v^2e^{-is\varphi}\delta T_{tt},\quad s=\pm 2,
\end{aligned} \tag{5.2}$$

where ν is the velocity of the test string with linear mass density μ'. In that event, the projection of the string onto a plane orthogonal to its moves in the same way as the point test mass in Sec. 4. But in contrast to the point mass (4.2), the energy-momentum tensor of the string is independent of z, so the total energy emitted in gravitational radiation by an infinite, rectilinear string will be infinite. A unit segment of the string, however, will radiate a finite amount of energy; using Eq. (3.25), we obtain

$$\begin{aligned}
\delta\mathscr{E}'=&8G(\mu'\gamma)^2b^{-1}\sin^2\frac{\pi}{b}\int_0^\infty dp\, p\frac{e^{-2\varkappa' d}}{\varkappa'^2}\Big\{\Big[\operatorname{ch}2\alpha'-1\\
&-\frac{v^2}{4}(\operatorname{ch}4\alpha'-1\Big]\Big(\operatorname{ch}\frac{2\alpha'}{b}-\cos\frac{2\pi}{b}\Big)^{-1}\\
&\times\operatorname{cth}\frac{\alpha'}{b}+\frac{v^2}{2}\operatorname{sh}\Big[2\alpha'\Big(2-\frac{1}{b}\Big)\Big]\Big\},
\end{aligned} \tag{5.3}$$

in which

$$\varkappa'=p/v\gamma, \alpha'=\ln[(1+\gamma^{-1})v^{-1}]. \tag{5.4}$$

If $\mu'/\mu \ll 1$, Eq. (5.3) will hold for any value of angle defect $2\pi(1-b)$, and it is clear from this equation that there will be no radiation if $b=1/n$, where n is an integer. In particular as might be expected, a free, straight string in uniform motion does not radiate ($n=1$). In the realistic case of a small angle defect $2\pi(1-b)$, Eq. (5.3) reduces to the simple form

$$\delta\mathscr{E}'=8G(\pi\mu'\beta v)^2\gamma^3\int^\infty dp\, p^{-1}\exp(-2pd/v\gamma). \tag{5.5}$$

The integral on the right-hand side of Eq. (5.5) diverges at the lower limit, a behavior that can be traced to the inadmissibility of treating radiation from the string in an infinite conical space. In the real world, the region of space in which the metric takes the form (2.10) is bounded by some radius R, beyond which the geometry of spacetime depends on the distribution of other matter. If we introduce a cutoff $p_{\min}$ for the lower limit of the integral in (5.5) and assume that $p_{\min}=R^{-1}$, then to logarithmic accuracy,

$$\delta\mathscr{E}'=8\pi^2(G\mu')^2\beta^2v^2\gamma^3\ln(v\gamma R/2d). \tag{5.6}$$

The total energy lost to gravitational radiation may conveniently be characterized by the dimensionless ratio of (5.6) to the geometric mean of the linear mass densities of the colliding strings. This ratio can be written in a form that is symmetric in the parameters describing the two strings:

$$\delta\mathscr{E}'=2\pi^2v^2\gamma^3(1-b)^{3/2}(1-b')^{3/2}\ln(v\gamma R/2d), \tag{5.7}$$

where $b'=1-4G\mu'$ is the conical parameter of the second string. For large enough γ, this quantity will be of order unity ("catastrophic" collision).

CONCLUSION

Gravitational radiation from a body moving in the field of a straight cosmic string provides an example of a radiation process that is a nontrivial consequence of the topology of spacetime. The spacetime of an infinitely thin straight string is in fact locally flat, and a point mass moving within it experiences no forces attributable to the string. But the intrinsic gravitational field of a test mass, which is nonvanishing over all space, will feel the effects of the varying spatial topology, and therefore even in uniform rectilinear motion, some part of the intrinsic field will become "detached" from the source, or in other words the source will radiate. Linet[19] has given another example of the nonlocal influence of a cosmic string on the intrinsic field of a point source; deformation of the Coulomb field of an electric charge in a conical space will result in an effective repulsive force between the charge and the string that is proportional to the square of the charge.

It has been estimated that bodies moving in the field of a cosmic string can emit a great deal of gravitational radiation, and in principal can therefore serve to indicate the presence of the string. At the present epoch, the most likely such process would consist of the passage of a cosmic string through a galaxy, cluster of galaxies, or some other gravitating system. The resultant gravitational radiation can be estimated using the equations derived above.

Thus, for a string moving relativistically ($\sim 0.5c$) at a distance of about five gravitational radii ($d \approx 5r_g$) from a black hole of mass $10^9 M_\odot$ at the center of a galaxy, we find from Eq. (4.8) that the equivalent mass lost in the form of gravitational radiation would be $10^{-2} M_\odot$. Since the duration of the encounter would be of order 10^5 sec, the mean radiated power would be 10^{47} erg/sec. Naturally, the amount of "conventional" gravitational radiation emitted in the collision of two such black holes would be much greater, as is clear from (4.10) with $r_m \sim d$. This underscores the distinction between the dynamical and topological radiation mechanisms. Nevertheless, the overall energy liberated in passing a string is clearly quite large.

From a general theoretical standpoint, we find it particularly interesting that gravitational radiation is emitted in a collision between two straight, parallel strings. If we knew how to construct an appropriate exact solution of the Einstein equations, the latter would describe a spacetime with varying conical structure. Our calculations indicate that such a restructuring of the global characteristics of a manifold ought to be accompanied by the emission of gravitational waves. The topological genesis of such radiation is underscored by the fact that Eq. (5.7) for the energy loss depends on the product of the conical parameters of the two strings, and only weakly (logarithmically) on the impact parameter. On the other hand, there is a certain analogy between topological radiation stemming from a collision between strings and electromagnetic radiation from a medium whose permittivity varies with time.

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