Structural anisotropy in a chaotic inflationary universe

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In the context of the chaotic inflation scenario, we study the effects of inhomogeneity of an inflationary universe on nascent pregalactic perturbations. We predict anisotropy in the spectrum of matter-density inhomogeneities in the observable part of the universe. Assuming the most natural physical conditions in the early universe, the magnitude of the anisotropy is of the order of several percent. A detection of the predicted anisotropy could be counted as evidence favoring the chaotic inflation scenario.

1. INTRODUCTION

In its present form, the inflationary universe scenario enables one to solve any number of important problems spanning the gamut from cosmology to elementary particle physics (the field is reviewed in Ref. 1). It encompasses such issues as the horizon problem, the problem of the homogeneity and flatness of the observable part of the universe, the lack of monopoles and other heavy stable particles predicted by grand unified models, and the origin of structure in the universe.

The principal means by which it deals with these and other problems is by postulating that in the early stages of evolution of the universe, there was an episode of quasiexponential expansion (the inflationary stage) during which all physical dimensions were magnified by an enormous factor. The region of the universe that is observable at the present epoch is then the result of the expansion (in the distant past—i.e., inflation) of some primordial, causally-connected region that may be considered in a natural way to have been both homogeneous and isotropic. The problem of the uniformity of the observable universe is thereby solved. The density of heavy particles and other exotic objects also decreases during inflation, so that if they are not produced afterwards, one can account for their present undetectability.

The inflationary universe scenario makes it possible to explain the origin of the observed structure of the universe in a completely natural manner (see the review in Ref. 2). Pregalactic perturbations (the precursors of galaxies and clusters of galaxies) are formed during inflation as a result of quantum fluctuations of both matter fields and of the metric as the scales of these fluctuations are effectively "distended." The pregalactic perturbations then acquire the spectrum necessary for the formation of the observable structure.

The many advantages of the inflationary universe scenario qualify it as practically the only candidate theory that can describe the evolution of the very early universe. An extremely important question in this regard, then, is how one might conduct a direct test of the inflationary scenario; in other words, are there any physical consequences that might be detectable by present-day astrophysical observations? Clearly, the theoretical prediction of such physical consequences together with their subsequent observational verification would not only serve to affirm the general idea of inflation, but would provide essential help in discriminating among the different versions of the inflationary scenario and elementary particle theories upon which the inflationary scenario is based.

For example, one of the most important predictions of the inflationary universe scenario is that the cosmological constant $\Omega = \rho/\rho_c$ is equal to unity to high precision. This is the case in any version of the inflationary scenario, and in principle can be utilized as an observational test of the latter. Unfortunately, the sizable uncertainties in current observational results prevent us from drawing any meaningful conclusions about whether Ω is indeed close to unity.[3]

Perhaps one of the most impressive achievements of the inflationary universe scenario is its explanation of the origin of the observed structure. The pregalactic perturbations that come into being during inflation may contain information about physical conditions in the very early universe, and this information may well be preserved in the properties of presently observable structure. In the present paper, we wish to demonstrate that these properties can serve as a test of the inflationary universe scenario.

One of the most prevalent trends in constructing a believable inflationary scenario is the tendency to derive inflation as a consequence of those physical conditions that might seem the most natural and likely in the early universe. In our opinion, the most attractive approach along those lines is provided by the chaotic inflation scenario.[4] In that scenario, inflation in the early universe is not at all exotic or exceptional, but on the contrary is entirely likely and perhaps even inevitable.[5] The chaotic inflation scenario radically alters our concept of the global structure of the universe,[6] allowing as it does for significant inhomogeneity on scales much larger than the observable horizon.

This last point plays a key role in our considerations. We wish to show that inhomogeneities on the largest scales can affect the spectrum of pregalactic perturbations that form during inflation. The mechanism involved is quite simple, and can be elucidated as follows. As high-frequency perturbations propagate against a nonuniform background, there is an effective "refraction" of the corresponding waves due mainly to inhomogeneities of the metric. In that sense, the effect is largely analogous to the well-known result obtained by Sachs and Wolfe.[7] In the language of quantum theory, we are dealing with polarization of the vacuum by background inhomogeneities. The net result of this effect is that the pregalactic perturbations that derive from vacuum fluctuations possess an anisotropic spectrum, and this anisotropy should be preserved in the spectrum of observable structure on scales at which the effects of nonlinear processes associated with the formation of galaxies and clusters of galaxies are negligible (that is, on scales of dozens of megaparsecs or more).

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An important remark is in order with regard to the way in which we derive our basic result. We shall be investigating the evolution of vacuum fluctuations, all the while remaining within the confines of field theory. The main contribution to the anisotropy of the spectrum of fluctuations comes from the evolutionary phase in which fluctuations exist on a scale much smaller than the Planck length. There exist serious reservations about the applicability of field theory at such distances. The theory that ultimately describes interactions beyond the Planck length may turn out to be substantially different; at present, the most popular alternative is superstring theory.[8] It would therefore be exceedingly desirable to study the problem that we attack in this paper, for example, from the vantage point of superstring theory. The anisotropy of vacuum fluctuations might then be due to polarization of the string vacuum induced by background inhomogeneities. It is to be hoped that considerations based on a more realistic theory will not qualitatively modify our results.

Bearing in mind the foregoing remarks, we now proceed with our calculations. In Sec. 2, we examine the principal results relevant to the evolution of a homogeneous and isotropic model, and in Sec. 3 we consider the origin of pregalactic perturbations in the context of the linear theory. Section 4 is devoted to an investigation of the development of small-scale fluctuations in an inhomogeneous background in the quasilinear approximation, and in Sec. 5 we study both qualitatively and quantitatively the onset of anisotropy in the spectrum of the pregalactic perturbations. In Sec. 6 we analyze the anisotropy of the observable structure, and we close in Sec. 7 by formulating our main conclusions.

2. HOMOGENEOUS COSMOLOGICAL SOLUTION

For simplicity, we consider a model containing a single scalar field φ driving inflation. The Lagrangian for this model takes the form

$$\mathscr{L}=\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi-V(\varphi)-\frac{M_p^2}{16\pi}R, \tag{1}$$

where $M_p = G^{-1/2}\approx 1.22\cdot 10^{19}$ GeV is the Planck mass and G is the gravitational constant (throughout this paper, we employ a system of units in which $\hbar = c = 1$). In order for the chaotic inflation scenario to be feasible, the potential $V(\varphi)$ of the scalar field must have a finite slope, as would a power function, for example. From here on, then, in explicit expressions we employ the potential

$$V(\varphi)=\lambda\varphi^4/4. \tag{2}$$

The equations of motion implied by the Lagrangian (1) are

$$R_{\mu\nu}=\frac{8\pi}{M_p^2}\left(\partial_\mu\varphi\,\partial_\nu\varphi-g_{\mu\nu}V(\varphi)\right), \tag{3}$$

$$\Box\varphi+\frac{dV(\varphi)}{d\varphi}=0, \tag{4}$$

and in the present case, Eq. (4) follows from Eq. (3).

Let us briefly review some results bearing upon the evolution of a homogeneous and isotropic region filled with a uniform field φ. (The reader can find a detailed analysis in Ref. 9.) In addition, we restrict ourselves to a spatially flat model. The metric is then

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu=dt^2-a^2(t)\,d\mathbf{x}^2 \tag{5}$$

or, in conformal coordinates,

$$ds^2=a^2(\eta)\,(d\eta^2-d\mathbf{x}^2); \tag{6}$$

$a(t)$ here is the scale factor. The relationship between physical time t and conformal time η is given by

$$dt=a\,d\eta. \tag{7}$$

The equations describing homogeneous evolution may be derived from (3) and (4), and they take the form

$$H^2=\frac{8\pi}{3M_p^2}\left(\frac{1}{2}\dot\varphi^2+V(\varphi)\right), \tag{8}$$

$$\ddot\varphi+3H\dot\varphi+\frac{dV(\varphi)}{d\varphi}=0, \tag{9}$$

where $H\equiv\dot a/a$ is the Hubble parameter. Henceforth, a dot over a function denotes a derivative with respect to time t, while a prime denotes a derivative with respect to conformal time η.

From (9), we find that

$$\dot\varphi=\dot\varphi_0\left(\frac{a_0}{a}\right)^3-\frac{1}{a^3}\int a^3\frac{dV}{d\varphi}dt, \tag{10}$$

where a_0 and φ_0 are constants that describe both the rapid evolution of the scalar field ($\dot\varphi\propto a^{-3}$) and the "slow rolling" regime ($\dot\varphi\approx-(1/3H)dV/d\varphi$). In the region under consideration, inflation begins when $\dot\varphi^2/2$ becomes less than $V(\varphi)$. The scale factor then starts to vary quasi-exponentially,

$$a(t)=a_0\exp\left(\int H\,dt\right)$$

with a slowly varying function $H(t)$. After inflation begins, the scalar field φ rapidly leaves the slow rolling regime, as is evident from Eq. (10).

3. FLUCTUATIONS. LINEAR THEORY

Let us now go on to consider the process whereby pregalactic perturbation are formed. Scalar-type perturbations play a fundamental part in this process. For scalar-type perturbations of the metric it is most convenient to use the relativistic potential gauge described in Ref. 10, in which the metric that describes perturbations of the homogeneous background considered in the previous section is of the form

$$ds^2=a^2[(1+2\Phi)d\eta^2-(1-2\Psi)d\mathbf{x}^2]. \tag{11}$$

Here Φ and Ψ are the relativistic potentials. Perturbations of the metric correspond to disturbances of the scalar field $\chi\equiv\delta\varphi$.

The equations describing the evolution of perturbations to first order may be obtained by linearizing Eqs. (3) and (4) in Φ, Ψ, and χ:

$$\Phi=\Psi, \tag{12}$$

$$\frac{4\pi}{M_p^2}\dot\varphi\chi=\dot\Phi+H\Phi, \tag{13}$$

$$\left(\frac{\dot H}{H}\partial_t\frac{H^2}{a\dot H}\partial_t\frac{a}{H}-\frac{\Delta}{a^2}\right)\Phi=0. \tag{14}$$

We emphasize here that Eq. (14) holds in the case of a spatially flat background solution. The relativistic potentials Φ and Ψ in the present instance are identical to the gauge invariant introduced by Bardeen.[11]

In quantum field theory, Φ, Ψ, and χ are operators, which can be represented in the linear theory by

$$\Phi=\int \frac{d^3k}{(2\pi)^3}\Phi_{\mathbf{k}}e^{i\mathbf{kx}}a_{\mathbf{k}}+\text{h.c.}, \tag{15}$$

$$\chi=\int \frac{d^3k}{(2\pi)^3}\chi_{\mathbf{k}}e^{i\mathbf{kx}}a_{\mathbf{k}}+\text{h.c.}, \tag{16}$$

where a_k^+ and a_k are creation and annihilation operators, respectively, and h.c. denotes the hermitian conjugate.

At high frequencies ($k\gg aH$), self-gravitation is general, and the normalized solutions for the eigenmodes χ_k in this frequency domain take the form (see Ref. 12)

$$\chi_{\mathbf{k}}=\frac{1}{a}\frac{1}{(2k)^{1/2}}e^{-ik\eta}, \quad k\gg aH. \tag{17}$$

Equation (13) then implies that in this same frequency domain,

$$\Phi_{\mathbf{k}}=\frac{4\pi i\dot{\varphi}}{2^{1/2}M_p^2}\frac{1}{k^{3/2}}e^{-ik\eta}, \quad k\gg aH. \tag{18}$$

At low frequencies (distances larger than the Hubble scale, $k\lesssim aH$), we find by solving Eq. (14) that

$$\Phi_{\mathbf{k}}=A_{\mathbf{k}}\frac{H}{a}\int\frac{a\dot{H}}{H^2}dt+B_{\mathbf{k}}\frac{H}{a}, \tag{19}$$

where A_k and B_k are slowly varying functions of time.

Notice now that when $k\lesssim aH$, we can neglect the term containing $\Delta\Phi$ in (14). In this approximation, A_k and B_k in (19) become constants. Furthermore, the second term in (19) describes an evanescent mode, which rapidly becomes negligible. The first term in (19) can also be transformed if one recognizes that $\dot{H}/H^2$ varies in time much more slowly than the scale factor. To high accuracy, then, the result is

$$\Phi_{\mathbf{k}}=A_{\mathbf{k}}\frac{\dot{H}}{H^2}, \quad A_{\mathbf{k}}=\text{const}, \quad k\lesssim aH. \tag{20}$$

Merging Eqs. (18) and (20) at the instant the horizon is crossed (when $k=aH$), we obtain

$$|A_{\mathbf{k}}|=\frac{H_k^2}{|\dot{\varphi}_k|}\frac{1}{2^{1/2}k^{3/2}}, \tag{21}$$

where the subscript k refers to homogeneous quantities corresponding to the wave number k at the moment the horizon is crossed.

For the potential (2), we obtain

$$|A_{\mathbf{k}}|^2=\frac{\lambda}{k^3}\left(\ln\frac{a_1\lambda^{1/2}M_p}{k}\right)^3, \tag{22}$$

up to a factor of order unity. Here a_1 is the value of the scale factor right at the end of inflation [when $\dot{\varphi}^2/2$ becomes equal to $V(\varphi)$].

If we choose as the quantum state the vacuum state defined by the conditions $a_k|0\rangle=0$, then (22) will characterize the spectrum of pregalactic perturbations. The aggregate of observational limits on this spectrum then yields a value for the self-action constant of the scalar field φ,

$$\lambda\sim10^{-12} \tag{23}$$

(see Ref. 2).

4. FLUCTUATIONS. QUASILINEAR APPROXIMATION

In the preceding section, we examined the behavior of fluctuations in a homogeneous background. We now wish to study the effects of large-scale background inhomogeneities on small-scale fluctuations.

What we have in mind when we speak of large-scale inhomogeneities is the long-wavelength part of the fluctuation spectrum, with $k\lesssim aH$. During inflation, fluctuations on such scales can be treated as classical random inhomogeneities.[6] As is clear from the solution (20), the spatial form of these inhomogeneities is frozen in, so to speak, while their amplitude varies slowly with time. With respect to small-scale quantum fluctuations ($k\gg aH$), such inhomogeneities act as if they were background inhomogeneities. We shall concern ourselves here with the question of how extensively these background inhomogeneities affect the spectrum of nascent pregalactic perturbations.

We employ the subscript c to designate field quantities describing large-scale background inhomogeneities. For example, Φ_c will denote an inhomogeneity in the relativistic potential, and χ_c an inhomogeneity in the scalar field on scales exceeding the Hubble distance.

In the high-frequency domain ($k\gg aH$), we assume self-gravitation to be negligible, and examine the behavior of solutions for perturbations of the scalar field χ in the background with metric

$$ds^2=a^2[(1+2\Phi_c)d\eta^2-(1-2\Phi_c)d\mathbf{x}^2], \tag{24}$$

which contains large-scale inhomogeneities Φ_c. We write the equation for perturbations of the scalar field

$$\Box\chi+m^2\chi=0, \tag{25}$$

where

$$m^2(\varphi+\chi_c)=\frac{d^2V(\varphi+\chi_c)}{d\varphi^2},$$

in the quasilinear approximation, i.e., one which is linear in both the fluctuations χ and the large-scale inhomogeneities Φ_c and χ_c. We then obtain

$$\chi''+2\alpha\chi'-\Delta\chi+m_0^2a^2\chi-4\Phi_c'\chi'-4\Phi_c\Delta\chi$$
$$+2m_0^2a^2\Phi_c\chi+m_1^2a^2\chi=0, \tag{26}$$

where

$$\alpha=\frac{a'}{a}, \quad m_0^2=\frac{d^2V(\varphi)}{d\varphi^2}, \quad m_1^2=\frac{d^3V(\varphi)}{d\varphi^3}\chi_c.$$

We may expand the solution of (26) in creation and annihilation operators, as we did in (16):

$$\chi=\int\frac{d^3k}{(2\pi)^3}\chi_{\mathbf{k}}e^{i\mathbf{kx}}a_{\mathbf{k}}+\text{h.c.} \tag{27}$$

Now however, the solution for the eigenmodes χ_k will differ from (17) on account of the presence of large-scale fluctuations Φ_c and χ_c. Recall that these can be treated as classical (albeit random) inhomogeneities, which is what makes the expansion of (27) feasible.

We seek a solution for the eigenmodes χ_k in the form

$$\chi_{\mathbf{k}}=\frac{1}{a}\frac{1}{(2k)^{1/2}}(1+\Phi_c)\exp(-ik\eta-iS_{\mathbf{k}}), \tag{28}$$

where $S_{\mathbf{k}}(\eta,\mathbf{x})$ is an unknown phase induced by the background inhomogeneities. We have separated out the factor $(1+\Phi_c)$ in (28) because of its connection with the choice of fluctuation spectrum on the initial hypersurface: in locally inertial coordinates, the high-frequency region of this spectrum should be the same as the vacuum fluctuation spectrum. On the initial hypersurface $\eta=\eta_0$, the necessary assumption for the phase $S_{\mathbf{k}}(\eta,\mathbf{x})$ is

$$S_{\mathbf{k}}|_{\eta=\eta_0}=0. \tag{29}$$

As with all the equations that we consider in the quasi-linear approximation, Eq. (28) is valid up to terms linear in Φ_c, and in the same approximation the phase $S_{\mathbf{k}}$ is also linear in Φ_c. In Eq. (28), we have left the phase $S_{\mathbf{k}}$ in the exponent for clarity and to facilitate subsequent calculations.

Substituting (27) and (28) into (26), we obtain the equation for the phase $S_{\mathbf{k}}$ in the approximation linear in the background inhomogeneities. At high frequencies ($k\gg aH$), we seek a solution for $S_{\mathbf{k}}$ in the form of a WKB expansion in powers of k^{-1}, retaining only the first two terms.

If we write

$$S_{\mathbf{k}}=k\left(S_{\mathbf{k}}^{(0)}+\frac{1}{k}S_{\mathbf{k}}^{(1)}\right)+O\left(\frac{1}{k}\right), \tag{30}$$

some straightforward rearrangement yields the equations for $S_{\mathbf{k}}^{0}$ and $S_{\mathbf{k}}^{1}$:

$$S_{\mathbf{k}}^{(0)\prime}+\mathbf{n}\nabla S_{\mathbf{k}}^{(0)}=2\Phi_c, \tag{31}$$

$$S_{\mathbf{k}}^{(1)\prime}+\mathbf{n}\nabla S_{\mathbf{k}}^{(1)}=-\frac{i}{2}(S_{\mathbf{k}}^{(0)\prime\prime}-\Delta S_{\mathbf{k}}^{(0)})-i\mathbf{n}\nabla\Phi_c+i\Phi_c', \tag{32}$$

where $\mathbf{n}=\mathbf{k}/k$ is the unit vector in the direction of $\mathbf{k}$. The solution of (31) and (32) with (29) as the initial condition is elementary:

$$S_{\mathbf{k}}^{(0)}(\eta,\mathbf{x})=2\int_{\eta_0}^{\eta}\Phi_c(\tilde\eta,\mathbf{x}-\mathbf{n}(\eta-\tilde\eta))\,d\tilde\eta, \tag{33}$$

$$S_{\mathbf{k}}^{(1)}(\eta,\mathbf{x})=i[\Delta-(\mathbf{n}\nabla)^2]\int_{\eta_0}^{\eta}\Phi_c(\tilde\eta,\mathbf{x}-\mathbf{n}(\eta-\tilde\eta))(\eta-\tilde\eta)\,d\tilde\eta. \tag{34}$$

Equations (27), (28), (30), (33), and (34) determine the evolution of high-frequency ($k\gg aH$) fluctuations of the field χ in an inhomogeneous background.

At low frequencies ($k\lesssim aH$), i.e., after the horizon has been crossed, the evolution of perturbations is described by the equations of the linear theory (12)–(14). In this domain of wave numbers, it is more convenient to work with perturbations of the relativistic potential Φ. To this end, we first make use of an equation that follows from (13) to (14),

$$\left(\Delta+\frac{4\pi\varphi'^2}{M_p^2}\right)\Phi=\frac{4\pi\varphi'^2}{M_p^2a}\left(\frac{a\chi}{\varphi'}\right)', \tag{35}$$

which relates perturbations of χ to perturbations of Φ in the high-frequency domain. We next write

$$\Phi_{\mathbf{k}}=\frac{4\pi i\varphi}{2M_p^2}\frac{1}{k^{3/2}}\exp[-ik\eta-iC_{\mathbf{k}}(\eta,\mathbf{x})],\qquad k\gg aH \tag{36}$$

[cf. Eq. (18)]. Furthermore, expanding the phase $C_{\mathbf{k}}(\eta,\mathbf{x})$ in powers of k^{-1}, as in (30), we have

$$C_{\mathbf{k}}=k\left(C_{\mathbf{k}}^{(0)}+\frac{1}{k}C_{\mathbf{k}}^{(1)}\right)+O\left(\frac{1}{k}\right) \tag{37}$$

and we thereby obtain the solution

$$C_{\mathbf{k}}^{(0)}\equiv S_{\mathbf{k}}^{(0)}, \tag{38}$$

$$C_{\mathbf{k}}^{(1)}\equiv S_{\mathbf{k}}^{(1)}+i\mathbf{n}\nabla S_{\mathbf{k}}^{(0)}+3i\Phi_c. \tag{39}$$

By analogy with (20), we find at low frequencies that

$$\Phi_{\mathbf{k}}=A_{\mathbf{k}}\frac{\dot H}{H^2}\exp(-iC_{\mathbf{k}}(\eta_k,\mathbf{x})),\qquad k\lesssim aH, \tag{40}$$

where the phase $C_{\mathbf{k}}$ in (40) has been taken at the instant when η_k crosses the horizon, and the magnitude of A_k is given by Eqs. (21) and (22).

5. ANISOTROPY IN THE SPECTRUM OF PREGALACTIC PERTURBATIONS

Let us now consider the spectrum of fluctuations Φ at the end of inflation. To do so, we must calculate the correlation function $1/2\langle\Phi(\mathbf{x})\Phi(\mathbf{x}')+\Phi(\mathbf{x}')\Phi(\mathbf{x})\rangle$ at the moment inflation ends—that is, at $\eta=\eta_1$. Note that at that time, the quantity $\dot H/H^2$ is of order unity. Up to a factor of order unity, then, and taking account of Eq. (40), we obtain

$$\frac{1}{2}\langle\Phi(\mathbf{x})\Phi(\mathbf{x}')+\Phi(\mathbf{x}')\Phi(\mathbf{x})\rangle=\int\frac{d^3k}{(2\pi)^3}|A_{\mathbf{k}}|^2\exp(i\mathbf{k}(\mathbf{x}-\mathbf{x}'))\times[\exp(-iC_{\mathbf{k}}(\mathbf{x})+iC_{\mathbf{k}}^*(\mathbf{x}'))+\exp(-iC_{-\mathbf{k}}(\mathbf{x}')+iC_{-\mathbf{k}}^*(\mathbf{x}))]. \tag{41}$$

The scale of inhomogeneity in the phase $C_{\mathbf{k}}(\mathbf{x})$ appearing in (41) is much greater than that corresponding to the wave number k. We may therefore expand the phase $C_{\mathbf{k}}(\mathbf{x})$ in (41) in powers of $\mathbf{x}$ in the vicinity of the observation point, which with no loss of generality we may take to be the origin ($\mathbf{x}=0$). For the real part of $C_{\mathbf{k}}(\mathbf{x})$ in this expansion, which is of higher order in k [see (37)–(39)], we retain the terms linear in $\mathbf{x}$ and keep only the zero-order term in the imaginary part.

We may then integrate (41) with respect to the new wave numbers $\mathbf{k}-\nabla\mathrm{Re}\,C_{\mathbf{k}}|_{\mathbf{x}=0}$. The expression for the correlation function then takes on the standard form

$$\frac{1}{2}\langle\Phi(\mathbf{x})\Phi(\mathbf{x}')+\Phi(\mathbf{x}')\Phi(\mathbf{x})\rangle=\int\frac{d^3k}{(2\pi)^3}|A_{\mathbf{k}}|^2(1+\tilde\nu_{\mathbf{k}})\cos\mathbf{k}(\mathbf{x}-\mathbf{x}'), \tag{42}$$

where

$$\tilde\nu_{\mathbf{k}}=-\frac{2d\ln(k^3|A_{\mathbf{k}}|^2)}{d\ln k}\mathbf{n}\nabla\int_{\eta_0}^{\eta_k}d\eta[\Phi_c(\eta,\mathbf{x}+\mathbf{n}(\eta_k-\eta))-\Phi_c(\eta,\mathbf{x}-\mathbf{n}(\eta_k-\eta))]|_{x=0}. \tag{43}$$

The change in the fluctuation spectrum due to background inhomogeneities shows up in (42) through the quantity $\tilde\nu_{\mathbf{k}}$, which is given by (43). It can be shown that the main contribution to $\tilde\nu_{\mathbf{k}}$ comes from the quadrupole term, and comprises at least 85% of the overall anisotropy. Restricting ourselves to the quadrupole part of $\tilde\nu_{\mathbf{k}}$, we may write

$$\tilde\nu_{\mathbf{k}}=-\frac{2d\ln(k^3|A_{\mathbf{k}}|^2)}{d\ln k}(\mathbf{n}\nabla)^2\int_{\eta_0}^{\eta_k}\Phi_c(\eta,\mathbf{x})(\eta_k-\eta)\,d\eta|_{x=0}. \tag{44}$$

In deriving Eqs. (43) and (44), we have assumed that $|A_{\mathbf{k}}|^2$ is independent of the direction of $\mathbf{k}$.

Before continuing our calculations, let us take a close look at this result. The anisotropy of the fluctuation spectrum is characterized by the quantity $\tilde\nu_{\mathbf{k}}$, which appears in (42) and is defined by (43). Clearly, anisotropy appears only in the long-wavelength part of the spectrum ($k\lesssim aH$); in that case, $|A_{\mathbf{k}}|^2$ is given by Eq. (22) and the logarithmic derivative in (43) is nonzero. For $k\gg aH$, we would have

$|A_k|^2 \sim k^{-3}$ and $\tilde{\nu}_k = 0$ according to (43). One may therefore say that in the present approximation there is no anisotropy in fluctuations of those waves at the adiabatic stage ($k \gg aH$), but only of those in a nonadiabatic evolutionary regime ($k \lesssim aH$).

It is worth noting that this anisotropy is due to inhomogeneities on a scale that is always far greater than that of the fluctuations we are considering here. Furthermore, anisotropy comes into play at a stage when the effects of large-scale inhomogeneities are already insignificant. One could perhaps interpret this result to mean that during the earliest phase of inflation, background inhomogeneities mainly polarize the vacuum, without any substantial alteration of the fluctuation spectrum at fairly short wavelengths. However, starting at a certain instant, under conditions of a non-stationary universe, quasiparticles are copiously produced from the polarized vacuum. This would then result in the onset of anisotropy in the fluctuation spectrum at the corresponding scale lengths.[1)]

As we noted above, anisotropy in the spectrum of pregalactic perturbations is basically quadrupole in nature. The quadrupole contribution to $\tilde{\nu}_k$ is given by Eq. (44), and can be written as

$$\tilde{\nu}_{\mathbf{k}} = n_\alpha n_\beta \tilde{\Lambda}_{\alpha\beta}(k), \tag{45}$$

or, with (22),

$$\tilde{\Lambda}_{\alpha\beta} = \frac{6}{\ln(a_1\lambda^{1/4}M_p/k)} \int_{\eta_0}^{\eta_k} \nabla_\alpha \nabla_\beta \Phi_c(\eta, \mathbf{x})\,|_{\mathbf{x}=0}(\eta_k - \eta)\,d\eta. \tag{46}$$

Notice that the magnitude of the anisotropy (the matrix $\tilde{\Lambda}$) is very slight, depending on the logarithm of the wave number k.

Let us separate the isotropic part out of (45), since it does not contribute to the anisotropy. We denote the residual by ν_k, which we write in the form

$$\nu_{\mathbf{k}} = n_\alpha n_\beta \Lambda_{\alpha\beta}, \tag{47}$$

where

$$\Lambda_{\alpha\beta} = \tilde{\Lambda}_{\alpha\beta} - {}^1/_3\delta_{\alpha\beta}\mathrm{Sp}\tilde{\Lambda}. \tag{48}$$

The trace of the matrix Λ vanishes: averaging (47) over all directions of $\mathbf{n}$ yields zero.

To characterize the degree of anisotropy, we use the mean square of the trace of Λ^2:

$$\zeta^2 = \langle \mathrm{Sp}(\Lambda^2) \rangle. \tag{49}$$

The average in (49) is taken over the random large-scale inhomogeneities Φ_c.

Bearing in mind that the inhomogeneities Φ_c carry the contribution due to fluctuations on scales exceeding the Hubble distance, we finally obtain

$$\zeta^2 = f_k^2 \int_{\eta_0}^{\eta_k} d\eta\,(\eta_k - \eta) \times \frac{\dot{H}}{H^2} \int_{\eta_0}^{\eta} d\tilde{\eta}\,(\eta_k - \tilde{\eta}) \frac{\dot{H}}{H^2} \int_{p<aH(\tilde{\eta})} \frac{d^3p}{(2\pi)^3}\,|A_p|^2 p^4, \tag{50}$$

where

$$f_k \approx \frac{7}{\ln(a_1\lambda^{1/4}M_p/k)}. \tag{51}$$

If the initial spectrum is specified at the start of inflation, Eq. (50) yields

$$\zeta \approx \frac{\lambda^{1/4} f_k}{10} \ln\frac{a_1}{a_0}. \tag{52}$$

If then inflation starts at a density of the order of the Planck density, then $\ln(a_1/a_0) \sim 10^6$. Recalling also that $f_k \sim 10^{-1}$ for the wave numbers responsible for the structure that is actually observed, and that $\lambda \sim 10^{-12}$, we obtain

$$\zeta \sim 10^{-2} \tag{53}$$

in the present case; that is, the anisotropy is of the order of several percent.[2)]

If inflation begins at less than the Planck density, but the initial spectrum is specified at a time when the energy density is of the order of the Planck density, we again obtain the result (53).

Thus, by making eminently reasonable assumptions about the initial conditions involving pregalactic perturbations in the early universe, we obtain an anisotropy of several percent.

6. ANISOTROPY OF OBSERVED STRUCTURE

Anisotropy in the spectrum of pregalactic perturbations ought to be manifested in the spectrum of the distribution of matter in the observable part of the universe. The easiest way to detect the presence of anisotropy is perhaps by measuring the dependence of the correlation function

$$\xi(\mathbf{x}) = \langle \delta(0)\delta(\mathbf{x}) \rangle \tag{54}$$

on the directions of $\mathbf{x}$. Here $\delta(\mathbf{x}) = \delta\rho(\mathbf{x})/\rho$, where ρ is the mean density of matter in the observable part of the universe, and $\delta\rho(\mathbf{x})$ is the inhomogeneity of this density.

Naturally, anisotropy ought to show up in the correlation function (54) on those scales at which nonlinear processes associated with the formation of galaxies and clusters of galaxies have not come into play and destroyed information bearing on the spectrum of primordial inhomogeneities. We must therefore now concern ourselves with scales of the order of several dozen megaparsecs or more, up to the present size of the horizon. The lower limit on the scales considered, ~ 30 Mpc, also determines the scale over which $\delta(\mathbf{x})$ is averaged in obtaining the correlation function.

Since our concern here is with scales on which evolution has not emerged into the nonlinear regime, we have a linear relationship between $\delta(\mathbf{x})$ and $\Phi(\mathbf{x})$. In the present epoch, this relationship is quite simple:

$$\delta(\mathbf{x}) \propto \Delta\Phi(\mathbf{x}) \tag{55}$$

on scales less than the Hubble distance. As for the evolution of $\Phi(\mathbf{x})$ in the post-inflationary phase, most models predict the appearance of a multiplicative factor. This factor is of no interest to us here, as we wish to consider the anisotropy of the spectrum.

For the correlation function (54), then, with (42) and (55) taken into consideration, we may write

$$\xi(\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} \delta_k^2 (1 + \tilde{\nu}_{\mathbf{k}}) \cos \mathbf{kx}, \tag{56}$$

where $\tilde{\nu}_k$ has been defined in (43), and δ_k is the amplitude of the Fourier transform of $\delta(\mathbf{x})$ in the isotropic case.

If on the other hand we are interested in the quadrupole contribution to the anisotropy, which as we already pointed out comprises at least 85% of the total, it is necessary to substitute $\nu_{\mathbf{k}}$ (as defined by Eqs. (47) and (48) of the previous section) into (56) instead of $\tilde{\nu}_{\mathbf{k}}$. We then obtain

$$\xi(\mathbf{x})=\xi_0(x)+\xi_1(\mathbf{x}), \tag{57}$$

where

$$\xi_0(x)=\int\frac{d^3k}{(2\pi)^3}\delta_k^2\cos\mathbf{kx}, \tag{58}$$

$$\xi_1(\mathbf{x})=\int\frac{d^3k}{(2\pi)^3}\delta_k^2\nu_{\mathbf{k}}\cos\mathbf{kx}. \tag{59}$$

Since the functional dependence of the matrix Λ on the wave number k is very weak, we may neglect it entirely. We may then rewrite (59) in the form

$$\xi_1(\mathbf{x})=\Lambda_{\alpha\beta}\int\frac{d^3k}{(2\pi)^3}\delta_k^2 n_\alpha n_\beta\cos\mathbf{kx}. \tag{60}$$

By symmetry, we then immediately obtain

$$\xi_1(\mathbf{x})=-h(x)\Lambda_{\alpha\beta}\hat{x}_\alpha\hat{x}_\beta, \tag{61}$$

where $\hat{\mathbf{x}}=\mathbf{x}/x$ is the unit vector in the $\mathbf{x}$ direction.

As we mentioned above, we are justified in taking only those perturbations with long enough wavelengths into account, and if we do so by means of a weighting function $\exp(-kl)$, where l is the cutoff scale length, the functions $\xi_0(\mathbf{x})$ and $\xi_1(\mathbf{x})$ can be easily calculated. The former is positive at small distances $x<3^{1/2}l$ and is negative at large distances. Asymptotically, its absolute value decreases as x^{-4}. The function $h(x)$ in (61), which characterizes the anisotropic part of the correlation function, is everywhere positive, and has a maximum at $x=x_m=l/2^{1/2}$. The value $h(x_m)$ at the maximum is proportional to l^{-4}, and

$$h(x_m)/\xi_0(x_m)=4/5. \tag{62}$$

Thus, up to the factor given in (62), the magnitude of the anisotropy of the correlation function is determined by the matrix Λ, and its expectation value is given by Eq. (53).

7. CONCLUSIONS

According to the inflationary-universe scenario, the presently observable structure in the universe stems from the nonadiabatic enhancement of vacuum fluctuations during inflation. In this paper, we have chaotic inflation can induce anisotropy in the spectrum of pregalactic perturbations. On those scales at which subsequent nonlinear processes associated with the formation of galaxies and clusters of galaxies has not destroyed information about the spectrum of primordial inhomogeneities (meaning scales of dozens of megaparsecs or more), the distribution of matter in the observable part of the universe should be anisotropic. At the scales considered, one indicator of anisotropy is the magnitude of the scale invariant. The predicted anisotropy could perhaps most easily be detected by measuring the way in which the correlation function of density inhomogeneities, $\xi(\mathbf{x})=\langle\delta(0)\delta(\mathbf{x})\rangle$, depends on the direction of $\mathbf{x}$.

We have arrived at a predicted level of anisotropy of several percent by employing field methods in our calculations at scales much shorter than the Planck length. It is to be hoped that a more realistic calculation, perhaps making use of string theory, will not drastically alter our conclusions; this, however, is a question for the future.

If there were to be a successful direct observational test of the predicted effect, it seems to us that it would amount to a vote in favor of the chaotic inflation scenario, and it would also shed a certain amount of light on the nature of interactions in the realm of energies inaccessible to direct experimental investigation.

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[1)] The following analogy may be of some use. High-frequency oscillations tend to behave like particles. The spectrum of zero-point oscillations corresponds to a distribution function $f=$ const in momentum space (with the same spread k^4 in T_μ^ν). Obviously, $f=$ const is an exact solution of the collisionless kinetic equation in arbitrary external fields (the kinetic equation contains only derivatives of f). Thus, the actual shape of the spectrum is responsible for preserving isotropy.

[2)] The discrepancy between the structural anisotropy ($\zeta\sim10^{-2}$) and the anisotropy of the microwave background ($\Delta T/T\sim10^{-5}$) is due to the contribution to ζ from regions much larger than the observable part of the universe.

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