On the calculation of the Kolmogorov constant in a description of turbulence by means of the renormalization group method

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A field-theoretic formulation of the renormalization-group method is used for an investigation of turbulent viscosity and of the energy spectrum of turbulent velocity pulsations. The application of the field-theoretic method makes it possible to take account of the effect of frequency renormalization and its influence on the theoretically calculated value of the Kolmogorov constant.

1. INTRODUCTION

Renormalization group (RG) methods, originally developed in quantum field theory[1] and later successfully used in the analysis of critical phenomena,[2] also find applications in the description of fully developed turbulence. According to Wilson[3] the RG method is a method of describing multimode systems with a wide range of characteristic scales and strong mode-mode coupling. According to the hypothesis of Kuz'min and Patashinskiĭ[4] such systems exhibit tendencies towards localization of the interactions in the wave-number sapce (interaction between modes of approximately equal scales) an a cascade mechanism of the interaction between modes of substnatially different scales. The spatio-temporal properties of modes of different scales are functionally similar, i.e., they differ in a set of numerical parameters (functional scaling, Ref. 5). Fully developed turbulence belongs to this class of systems.[6]

The earliest attempts to use the RG method for calculation of the exponents of power-law behavior (scaling indices) of the statistical moments of turbulent pulsations of a velocity field were undertaken in Ref. 7 on the basis of the Kadanoff procedure of partial iterative averaging, and in Ref. 8 on the basis of the field-theory formulation of the RG method. The use of formulations of the RG method appropriated from the theory of critical phenomena gives one the possibility to determine the scaling indices only in the infrared limit $k \to 0$, but does not yield the amplitude coefficients. However, the scaling indices can be determined on the basis of dimensional considerations, and in this sense, as noted in Ref. 9, the RG method does not yield any new results.

However, in the field-theory formulation, the RG method can be considered as a method of summation of an infinite subsequence of terms the perturbation theory series[1] yielding the possiblity of describing an infinite cascade chain, which consists of separate acts of mode-mode interactions.[10] At the same time, there arises the possibility to find the amplitude coefficients too (this was first pointed out in Ref. 11), as well as the functional dependences in the region of incomplete scaling (Refs. 10, 12).

In particular, in a series of papers by Yakhot and Orzsag (e.g, Ref. 13) the RG in the Wilson formulation was used to calculate a set of universal constnats which characterize fully developed turbulence (the Kolmogorov, Batchelor, and von Karman constants, the turbulent Reynolds number, and others), the values of which were in good agreement with the experimental data. The results for the turbulent viscosity and for the Kolmogorov constant are based on perturbation-theory calculations of the variation of viscosity with a change of the cutoff in wave numbers, with a subsequent improvement of this dependence by solving the RG equations. The effect of renormalization of the amplitude of the response function (which reduces to renormalization of the frequencies) is not taken into account; the frequency was renormalized only when scaling transformations were carried out in the course of construction of the RG-transformation operator. We show below that the results of Ref. 13, obtained on the basis of asymptotic methods valid in the infrared limit, can be derived by means of the field-theory formulation of the RG method. We determine the behavior of the efffective viscosity in the region of incomplete scaling, including the region of wave numbers immediately adjacent to the dissipative interval. We take into account the effect of renormalization of the Green's function, and determine, in particular, a related correction to the Kolmogorov constant calculated in Ref. 13.

2. THE INITIAL EQUATIONS

We consider a model of an incompressible fluid described by the system of Navier-Stokes equations in the presence of an external force representing a Gaussian stochastic process of the "white noise" type. The quantities characteristic of the hydrodynamic field—the pressure p and the components v_i of the velocity at the point $1 = \{\mathbf{r}_1, t_1\}$ will be considered in a space of d dimensions, as components of a $d+1$-dimensional vector according to the definition (Ref. 14):

$$\psi_\alpha(1) = \{\psi_0(1), \psi_i(1)\} = \{p(\mathbf{r}_1, t_1), v_i(\mathbf{r}_1, t_1)\},$$
$$\alpha = 0, 1, \ldots d, \quad i = 1, 2, \ldots d.$$

In the formalism of "field doubling" (Refs. 15, 16) the system under consideration is defined by an action $S[\psi,\hat{\psi}] = S_0[\psi,\hat{\psi}] + \lambda_0 S_1[\psi,\hat{\psi}]$. The part $S_0[\psi,\hat{\psi}]$ which is quadratic in the fields $\psi,\hat{\psi}$ describes the linear processes and $S_1[\psi,\hat{\psi}]$ corresponds to the nonlinear interaction between the modes:

$$S_0[\psi, \hat{\psi}] = -\hat{\psi}_\alpha(1) L_{\alpha\beta}(12) \psi_\beta(2) + (i/2) \hat{\psi}_\alpha(1) D_{\alpha\beta}(12) \hat{\psi}_\beta(2), \tag{1}$$

The linear part of the Navier-Stokes operator, $L_{\alpha\beta}(12)$, the correlation function $D_{\alpha\beta}(12)$ of the external forces, and the coefficient $V_{\alpha\beta\gamma}$ are defined by the relations

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$$L_{\alpha\beta}(12)=\begin{bmatrix}0 & \partial_j^{(1)}\\ \partial_i^{(1)} & (\partial_t^{(1)}-\nu_0\Delta)\,\delta_{ij}\end{bmatrix}\delta(1-2),$$
$$D_{ij}(12)=\delta_{ij}D(|\mathbf{r}_1-\mathbf{r}_2|)\,\delta(t_1-t_2), \qquad (2)$$
$$V_{ijk}(123)=-[\delta_{ij}\partial_k^{(2)}+\delta_{ik}\partial_j^{(3)}]\,\delta(1-2)\,\delta(1-3),$$

where ν_0 is the molecular-viscosity coefficient, λ_0 is a formal expansion parameter, which in the final result has to be put equal to one. The object of our discussion will be the Green's function $G_{ij}(12)=i\langle\psi_i(1)\hat{\psi}_j(2)\rangle$, describing the averaged linear response of the velocity field to an external disturbance, and the pair correlation function $C_{ij}(12)=\langle\psi_i(1)\psi_j(2)\rangle$.

3. THE RENORMALIZATION PROCEDURE

Perturbation theory corresponds to a representation of the characteristic functional of the system

$$\Psi[\eta,\hat{\eta}]=\int d[\psi]\,d[\hat{\psi}]\times\exp\{i(S_0[\psi,\hat{\psi}]+\lambda_0 S_1[\psi,\hat{\psi}]+\eta\psi+\hat{\eta}\hat{\psi})\}$$

as an expansion in powers of $\lambda_0 S_1$. However, the decomposition of the action into an unperturbed and a perturbation is not unique, since some of the terms can be shifted from S_0 into S_1, which is equivalent to a finite renormalization of the field amplitudes and of the physical parameters, with the addition to S_1 of compensating counterterms (see Ref. 1). The independence of the result of the choice of the renormalization constants is reflected in a requirement that the perturbation-theory series be invariant to renormalizations, a property of the complete series which does not hold for a sum of a finite number of its terms.

In the system (1) under discussion we effect a renormalization of the field amplitude $\hat{\psi}$ and the viscosity coefficient (on account of a Ward identity which follows from Galilean invariance, the field amplitude ψ does not get renormalized). This renormalization reduces to the substitutions

$$\hat{\psi}\to\hat{\psi}^R=z\hat{\psi},\quad \lambda_0\to\lambda=z^{-1}\lambda_0,\quad D\to D^R=z^{-2}D,\quad \nu_0\to\nu \qquad (3)$$

and to an addition to S_1 of counterterms of the form

$$\delta S_1=-(z^{-1}-1)\hat{\psi}_i\partial_t\psi_i+(\nu_0 z^{-1}-\nu)\hat{\psi}_i\Delta\psi_i. \qquad (4)$$

The renormalization parameters z and ν/ν_0 are arbitrary quantities.

In what follows it will be convenient to carry out a Fourier transform in the space and time variables, making use of the stationarity and spatial homogeneity of the system under consideration. We choose the renormalization parameters in such a manner that in the renormalized field theory the total Green's function

$$G_{ij}^R(\mathbf{k},\omega)=P_{ij}(\mathbf{k})[-i\omega+\nu k^2-\Sigma^R(k^2,\omega)]^{-1},$$
$$(P_{ij}(\mathbf{k})=\delta_{ij}-k_ik_jk^{-2})$$

should coincide with the Green's function at the renormalization point $\omega=0$, $k^2=\mu^2$; in other words, we impose the following two conditions:

$$\Sigma^R(\mu^2,0)=0,\qquad \frac{\partial}{\partial\omega}\Sigma^R(\mu^2,\omega)\,|_{\omega=0}=0. \qquad (5)$$

The requirement of invariance with respect to renormalizations reduces to independence of the resultant physical quantities of the choice of the normalization point μ.

4. RENORMALIZATION-GROUP ANALYSIS

We choose the function $D(\mathbf{k})$ defined in Eq. (2) in the form

$$D(\mathbf{k})=D_0(k^2)^{-d/2+2-\varepsilon}.$$

For $\varepsilon=2$ the dimension of the parameter D_0 coincides with the dimension of the rate of energy dissipation, which in the Kolmogorov theory[6] is the only essential dimensional parameter determining the turbulent pulsations of the velocity in the inertial range. In this sense the value $\varepsilon=2$ corresponds to the "real theory" (Ref. 16). The value $\varepsilon=0$ leads to a theory with a logarithmic divergence of the self-energy operator. According to Wilson[3] such a "logarithmic theory" is characterized by the absence of a distinguished scale and by the ensuing locality of the interaction in the space of wave numbers, thus making the RG approach effective. Near a logarithmic theory the *de facto* expansion parameter of perturbation theory turns out to be proportional to ε. The method of ε-expansion in the theory of turbulence consists in analytically continuing, with respect to ε, from a logarithmic theory ($\varepsilon=0$) to the real theory ($\varepsilon=2$). In doing this, the perturbatively calculated numerical coefficients are taken at $\varepsilon=0$ (Refs. 13, 17).

We introduce the effective viscosity $\tilde{\nu}$ by means of the relation

$$G_{ij}^{-1}(\mathbf{k},\omega)=[-i\omega+\tilde{\nu}(k^2,\omega)k^2]\delta_{ij}. \qquad (6)$$

The effective viscosity describes momentum transport both by the molecular and by the turbulent vortex motion.

On account of renormalization invariance, the renormalized Green's functions corresponding to two different normalization points μ and μ_1 are related as follows,

$$z^{-1}(\mu^2)G^R(\mathbf{k},\omega;\mu^2)=z^{-1}(\mu_1^2)G^R(\mathbf{k},\omega;\mu_1^2),$$

a relation which can be represented in the form

$$z(\nu,\lambda,D;\mu^2)[-i\omega+\tilde{\nu}(k^2,\omega,\nu,\lambda,D;\mu^2)k^2]$$
$$=z(\nu_1,\lambda_1,D_1;\mu_1^2)[-i\omega+\tilde{\nu}(k^2,\omega,\nu_1,\lambda_1,D_1;\mu_1^2)k^2]. \qquad (7)$$

In the sequel we shall only consider the static effective viscosity

$$\tilde{\nu}(k^2,\nu,\lambda,D;\mu^2)=\tilde{\nu}(k^2,\omega,\nu,\lambda,D;\mu^2)\,|_{\omega=0},$$

which in Edwards' theory (Ref. 18) describes the time dependence of the response function and of the velocity correlations.

We introduce a new function which describes the change of normalization when going from one normalization point to another:

$$Z(\nu,\lambda,D;\mu^2|\nu_1,\lambda_1,D_1;\mu_1^2)$$
$$=z(\nu_1,\lambda_1,D_1;\mu_1^2)z^{-1}(\nu,\lambda,D;\mu^2). \qquad (8)$$

It follows from the normalization condition (5), the definition (8), and dimensional considerations that Z is a function only of the dimensionless parameter $h=\lambda^2 D\nu^{-3}(\mu^2)^{-\varepsilon}$ and of the ratio μ_1^2/μ^2. It satisfies the normalization condition $Z(\mu_1^2/\mu^2,h)_{\mu_1^2=\mu^2}=1$ and the following group composition law:

$$Z(\mu^2/\mu_1^2,\ h_1)=Z(\mu^2/\mu_2^2,\ h_2)Z^{-1}(\mu_1^2/\mu_2^2,\ h_2). \tag{9}$$

According to Eq. (7) the static effective viscosities at different normalizations are related by

$$\tilde{\nu}(k^2/\mu^2,\ \nu,\ h)=Z(\mu_1^2/\mu^2,\ h)\tilde{\nu}(k^2/\mu_1^2,\ \nu_1,\ h_1). \tag{10}$$

We introduce a new dimensionless function

$$\tilde{h}\left(\frac{k^2}{\mu^2},h\right)=\frac{\lambda^2 DZ^{-1}(k^2/\mu^2,h)}{\tilde{\nu}^3(k^2/\mu^2,\nu,h)\,(k^2)^{\varepsilon}}. \tag{11}$$

By means of (9), (10), and the relations

$$\lambda_1=Z^{-1}\left(\frac{\mu_1^2}{\mu^2},h\right)\lambda,\qquad D_1=Z^{-2}\left(\frac{\mu_1^2}{\mu^2},h\right)D$$

one can show that $\tilde{h}(k^2/\mu^2,h)$ is an invariant under the renormalization transformation $\mu\to\mu_1$, $h\to h_1$, i.e.,

$$\tilde{h}(k^2/\mu^2,\ h)=\tilde{h}(k^2/\mu_1^2,\ h_1). \tag{12}$$

The function $\tilde{h}$, which is in fact the expansion parameter of perturbation theory, represents an analog of the invariant charge in quantum field theory, and satisfies the functional equation of the renormalization group (Ref. 1):

$$\tilde{h}(x,\ h)=\tilde{h}(x/t,\ \tilde{h}(t,\ h)), \tag{13}$$

as well as the differential (Callan-Symanzik) equation following from it:

$$\left\{-x\frac{\partial}{\partial x}+\beta(h)\frac{\partial}{\partial h}\right\}\tilde{h}(x,h)=0,\qquad \beta(h)=\left.\frac{\partial\tilde{h}(x,h)}{\partial x}\right|_{x=1}. \tag{14}$$

5. THE RENORMALIZATION-GROUP METHOD

According to Ref. 1 the RG is a method for improving perturbation theory by taking into account the invariance of the complete perturbation theory series with respect to renormalizations. It consists in computing the RG function $\beta(h)$ by means of perturbation theory followed by finding a solution of the differential equation (14). Applied to our case, the problem reduces to a determination of the self-energy operator in the lowest order of renormalized perturbation theory

$$\Sigma_{ij}^R(\mathbf{k},\ \omega)=\Sigma_{ij}(\mathbf{k},\ \omega)+i\omega(z^{-1}-1)\delta_{ij}-k^2(\nu_0 z^{-1}-\nu)\delta_{ij}. \tag{15}$$

The second and third terms of the right-hand side of Eq. (15) take into account the contribution of the counterterms (4), and the first term is determined by the expression

$$\Sigma_{ij}(\mathbf{k},\omega)=\lambda^2 V_{imn}(\mathbf{k})\times\int\frac{d\mathbf{q}}{(2\pi)^d}\frac{d\Omega}{2\pi}G_{mm'}(\mathbf{q},\Omega)C_{nn'}(\mathbf{k}-\mathbf{q},\omega-\Omega)V_{m'n'j}(\mathbf{q}), \tag{16}$$

where

$$V_{imn}(\mathbf{k})=i(\delta_{in}k_n+\delta_{in}k_m),\ G_{ij}(\mathbf{k},\ \omega)=P_{ij}(\mathbf{k})[-i\omega+\nu k^2]^{-1},$$
$$C_{ij}(\mathbf{k},\ \omega)=P_{ij}(\mathbf{k})D(\mathbf{k})[\omega^2+\nu^2k^4]^{-1}.$$

The isotropy condition implies that the expression for Σ_{ij} must have the form

$$\Sigma_{ij}(\mathbf{k},\ \omega)=\Sigma(k^2,\ \omega)\delta_{ij}+\Sigma'(k^2,\ \omega)k_ik_j,$$

however, one may leave out terms proportional to $\Sigma'(k^2,\omega)$ since Σ_{ij} is always contracted with a Green's function or a pair correlation function, each containig the transverse projection operator P_{ij}. As a result of this, one can use for the calculation of Σ the relation

$$\Sigma(k^2,\ \omega)=(d-1)^{-1}P_{ij}(\mathbf{k})\Sigma_{ji}(\mathbf{k},\ \omega).$$

The connection between the renormalization constant and the self-energy operator follows from the condition (5) and has the form

$$z^{-1}=1+i\partial\,\Sigma(\mu^2,\ \omega)/\partial\omega|_{\omega=0}. \tag{17}$$

Calculating Σ to lowest order of perturbation theory, and making use of Eqs. (16) and (17), we find near the point $\varepsilon=0$:

$$\tilde{\nu}=\nu[1+A_d h(x^{-\varepsilon}-1)/\varepsilon],\quad z^{-1}=1+B_d h,\quad x=k^2/\mu^2, \tag{18}$$

where

$$A_d=\frac{1}{8}\frac{d-1}{d+2}\frac{s_d}{(2\pi)^d},$$

$$B_d=\frac{1}{8}\left\{\frac{d-1}{2}\left[\psi\left(\frac{d}{4}+\frac{1}{2}\right)-\psi\left(\frac{d}{4}\right)\right]+\frac{2}{d}\right\}\frac{s_d}{(2\pi)^d},$$

$s_d=2\pi^{d/2}/\Gamma(d/2)$ is the area of the d-dimensional unit sphere, and $\psi(x)=d\ \ln\Gamma(x)/dx$ (Ref. 19).

From Eqs. (11) and (18) we determine the RG beta function

$$\beta(h)=h(-\varepsilon+3A_d h)(1+B_d h) \tag{19}$$

and by means of the well known methods (Ref. 1) we write out an implicit solution of Eq. (14)

$$\frac{\varepsilon/3A_d-\tilde{h}}{\tilde{h}}\left(\frac{1/B_d+\tilde{h}}{\varepsilon/3A_d-\tilde{h}}\right)^{\varepsilon B_d/(\varepsilon B_d+3A_d)}=Cx^{\varepsilon}. \tag{20}$$

The constant C can be determined from the supplementary requirement that for large k the effective viscosity $\tilde{\nu}$ should go over into the molecular viscosity ν_0. As a result of this we obtain:

$$C=\left(\frac{\mu^2}{k_d^2}\right)^{\varepsilon}\frac{\varepsilon}{3A_d}\left(\frac{3A_d}{\varepsilon B_d}\right)^{\varepsilon B_d/(\varepsilon B_d+3A_d)}, \tag{21}$$

where for $\varepsilon=2$ the quantity $k_d^2=(D_0\nu_0^{-3})^{1/\varepsilon}$ is the analog of the internal scale of the turbulence.

It is clear from Eqs. (20) and (21) that $\tilde{h}$ is a function only of k^2/k_d^2 and is an invariant under RG trnasfrmations, i.e., is independent of the choice of normalization point μ.

If one does not consider the renormalization of the field amplitide $\hat{\psi}$, i.e., if one sets $B_d=0$, the equation (20) for $\tilde{h}=\lambda_0^2 D_0\tilde{\nu}^{-3}(k^2)^{-\varepsilon}$ is easily solved and the effective viscosity takes the form

$$\tilde{\nu}(k)=\nu_0\left[1+\frac{3A_d}{\varepsilon}\left(\frac{k_d^2}{k^2}\right)^{\varepsilon}\right]^{1/3}=\left[\nu_0^3+\frac{3A_dD_0}{\varepsilon(k^2)^{\varepsilon}}\right]^{1/3}. \tag{22}$$

For $\varepsilon=2$ Eq. (22) yields the result which was already known (Refs. 12, 14).

6. CALCULATION OF THE KOLMOGOROV CONSTANT

In order to find the form of the energy spectrum $E(k)$ of turbulent pulsations of the velocity we make use of the definition

$$E(\mathbf{k})=\frac{1}{2}\frac{s_d}{(2\pi)^d}k^{d-1}\int\frac{d\omega}{2\pi}C_{ii}(\mathbf{k},\omega) \tag{23}$$

and the representation for the pair correlation function

$$C_{ij}(\mathbf{k},\omega)=G_{in}(\mathbf{k},\omega)\times D_{nm}(\mathbf{k})\,G_{jm}(-\mathbf{k},-\omega)=\frac{P_{ij}(\mathbf{k})\,D_0(k^2)^{-d/2+2-\varepsilon}}{\omega^2+\tilde{\nu}^2(k)\,k^4}. \tag{24}$$

Substitution of (24) into (23) and integration yields

$$E(k)=\frac{d-1}{4}\frac{s_d}{(2\pi)^d}D_0\frac{k^{1-2\varepsilon}}{\tilde{\nu}(k)}. \tag{25}$$

In the infrared limit $k^2/k_d^2\to 0$ which, according to Ref. 10 corresponds to the inertial interval, the substitution (22), account being taken of Eq. (18), yields

$$E(k)=\frac{d-1}{4}\left[\frac{3}{8\varepsilon}\frac{d-1}{d+2}\right]^{-1/3}\left[\frac{s_d}{(2\pi)^d}D_0\right]^{2/3}k^{1-4\varepsilon/3}. \tag{26}$$

In order to calculate the Kolmogorov constant C_E it is necessary to establish a connection between the rate of dissipation $\bar{\varepsilon}$ and the parameter D_0. According to Ref. 13 this can be achieved by making use of the constant $\varkappa=C_\nu/C_E^2$ introduced in Ref. 20, where C_ν and C_E are defined by the relations

$$\tilde{\nu}(k)=C_\nu\bar{\varepsilon}^{1/3}k^{-4/3},\quad E(k)=C_E\bar{\varepsilon}^{2/3}k^{-5/3}.$$

As a result of comparing Eqs. (22) and (26) for $\varepsilon=2$ we find

$$\frac{s_d}{(2\pi)^d}D_0=\frac{3\varepsilon}{(d-1)(d+2)\varkappa},\quad C_E=\left[\frac{3}{4(d+2)\varkappa^2}\right]^{1/3}. \tag{27}$$

The quantity $\varkappa$ can be calculated in the framework of some theoretical model by imposing the condition of a local balance of the turbulent energy in the inertial range of the spectrum. For $d=3$ the calculation carried out by Kraichnan[21] yields $\varkappa=0.1904$, leading to a value of the Kolmogorov constant $C_E=1.605$ (Ref. 13).

If the effects of a finite renormalization of the field amplitude $\hat{\psi}$ are taken into account ($B_d\neq 0$), the procedure for determining the spectrum according to Eq. (25) turns out to be more complicated, since it is impossible to obtain an explicit analytic form the expression for $\tilde{\nu}(k)$. Nevertheless, one can find a solution in the infrared limit $k\to 0$ corresponding to the inertial range.

In order to calculate $\tilde{\nu}(k)$ it is necesary, according to Eq. (11), to know $Z(x,h)$, i.e., to solve the functional equation (9). An apllication of the methods described in Ref. 1 shows that a solution compatible with perturbation theory has the form

$$Z(x,h)=(1+B_dh)/[1+B_d\tilde{h}(x,h)]. \tag{28}$$

As a result of this, taking it into account that $\lambda=z^{-1}\lambda_0=z^{-1}$ and $D=z^{-2}D_0$, we obtain, after passing to renormalized values of the function $\hat{\psi}$

$$\tilde{\nu}(k)=\left[\frac{1+B_d\tilde{h}(k^2/k_d^2)}{\tilde{h}(k^2/k_d^2)}D_0\right]^{1/3}(k^2)^{-\varepsilon/3}. \tag{29}$$

In the inertial range of the spectrum $k^2/k_d^2\to 0$ Eq. (20) implies $\tilde{h}(k^2/k_d^2)\to\varepsilon/3A_d$, which leads to the relations

$$\tilde{\nu}(k)=\left[\frac{3}{8\varepsilon}\frac{d-1}{d+2}\left(1+\frac{\varepsilon B_d}{3A_d}\right)\right]^{1/3}\left[\frac{s_d}{(2\pi)^d}D_0\right]^{1/3}(k^2)^{-\varepsilon/3},$$

$$E(k)=\frac{d-1}{4}\left[\frac{3}{8\varepsilon}\frac{d-1}{d+2}\left(1+\frac{\varepsilon B_d}{3A_d}\right)\right]^{-1/3}\left[\frac{s_d}{(2\pi)^d}D_0\right]^{2/3}k^{1-4\varepsilon/3}. \tag{30}$$

Similar to the derivation of the result (27) one can obtain from Eq. (30) a relation between the quantities D_0 and $\bar{\varepsilon}$, and an expression for the Kolmogorov constant:

$$\frac{s_d}{(2\pi)^d}D_0=\frac{3}{(d-1)(d+2)}\left(1+\frac{2B_d}{3A_d}\right)\frac{\varepsilon}{\varkappa},$$

$$C_E=\left[\frac{3}{4}\frac{1}{d+2}\left(1+\frac{2B_d}{3A_d}\right)\frac{1}{\varkappa^2}\right]^{1/3}. \tag{31}$$

For $d=3$ one obtains for the Kolmogorov constant the value $C_E=2.447$, i.e., a value approximately one and one half times larger than that of the authors of Ref. 13. This value of C_E is in worse agreement with the existing experimental data although it is not outside the confidence interval.

7. CONCLUSION

We have investigated the influence of a renormalization of the frequency on the Kolmogorov constant calculated by means of the RG. Since this effect is essential (it is related to the nonvanishing of the quantity $[\partial\Sigma(\mu^2,\omega)/\partial\omega]|_{\omega=0}$) it must be taken into account in carrying out the renormalization procedure (see also Ref. 23). It follows from our results that the good agreement with the experimental data found by the authors of Ref. 13 without taking it into account should, apparently, be considered coincidental, and their model requires refinement.

We also note that the Wilson formulation of the RG method used by the authors of Ref. 13, based on a reduction of the number of modes by means of the Kadanoff procedure, does not allow one to take into account the influence of renormalization of external random forces, since the elimination of a band of small-scale modes leads toe the appearance of new terms in the Navier-Stokes equation, terms which differ in structure from the original ones. In this sense the field-theoretic formulation of the RG method used here, where the role of the group parameter is played not by the ultraviolet cutoff constant in wave number space, but rather by the position of the normalization point (the Bogolyubov-Shirkov formulation[1]), seems preferable.

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