Anomalous temperature dependences of the London penetration depth and of the lower critical field in superconducting superlattices

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The temperature dependences of the London penetration depth λ and of the lower critical field H_{c1} are calculated for superconducting superlattices. It is shown that the $\lambda^{-2}(T)$ and $H_{c1}(T)$ dependences are substantially nonlinear when the layer thickness d satisfies the condition $\xi_0 < d < \xi_{1,2}(T_c)$, where ξ_0 is the coherence length and $\xi_{1,2}(T)$ are the correlation lengths of the layer materials. The nature of this linearity differs from the cause of the nonlinear temperature dependence of the upper critical field H_{c2} in superlattices. Superlattices consisting of two different superconductors are considered, as well as superlattices in which superconducting and normal-metal layers alternate. The temperature dependences of the heat capacity are also calculated.

INTRODUCTION

Progress in vacuum deposition technology makes it possible at present to obtain highly perfected superlattices made up of alternating layers of different elements, with layer thicknesses constant to atomic accuracy.[1] The properties of superconducting superlattices can differ radically from those of the bulk superconductors from which the layers are prepared. In fact, new types of superconductors can be constructed by superlattice deposition.

A complicated nonlinear dependence of the upper critical field H_{c2} was recently observed experimentally in an $Nb/Nb_{0.6}Ti_{0.4}$ superlattice.[2] Deviations from a linear $H_{c2}(T)$ dependence were observed also in a number of other sublattices (see, e.g., Ref. 3). A theoretical treatment of H_{c2} in superlattices is contained in Refs. 4–7.

The temperature dependences of the London penetration depth $\lambda(T)$ and also of the lower critical field H_{c1} have been much less investigated. The usual linear $\lambda^{-2}(T)$ dependence was recorded in Ref. 8 for an Nb/Cu superlattice with layer thickness $d = 54$ Å. At the same time, a positive curvature of the $\lambda^{-2}(T)$ plot was observed near the critical temperature for a V/Ag superlattice with vanadium layer thickness 240 Å.[9]

The analysis in the present article will show that substantially nonlinear $\lambda^{-2}(T)$ and $H_{c1}(T)$ dependences can appear in superconducting superlattices with a layer thickness d satisfying the condition $\xi_0 < d < \xi_{1,2}(T_c)$, where $\xi_{1,2}(T)$ are the superconducting correlation lengths of the layer materials and T_c is the critical temperature of the superlattices. (We assume also that the superlattice period is much smaller than λ. For type-II superconductors this condition is not burdensome.) It is important to note that these anomalies can have an nature entirely different from that in the behavior of $H_{c2}(T)$.

Let us clarify the cause of the nonlinear $\lambda^{-2}(T)$ dependence, using as an example a superlattice consisting of two different type-II superconductors. At low temperatures, when the correlation lengths of the layer materials are small, i.e.

$$\xi_{1,2}(T) \sim \xi_0 \ll d,$$

an equilibrium order parameter of value specific for each layer is reached. The London penetration depth is essentially the same as in the corresponding bulk superconductor. The condition $\lambda_{1,2}(T) \gg d$ means effective screening of the field only on many periods of the superlattice, and by carrying out the corresponding averaging in the London equations[2)] we find immediately that the London screening depth for the superlattice is given by

$$\lambda^{-2} = \lambda_1^{-2} \frac{d_1}{d_1+d_2} + \lambda_2^{-2} \frac{d_2}{d_1+d_2}, \qquad (1)$$

where d_1 and d_2 are the layer thicknesses. We assume henceforth for simplicity that $d_1 = d_2 = d$. The required generalizations to include the case $d_1 \neq d_2$ are obvious.

The situation changes in principle near the temperature T_c, when the condition $d < \xi_{1,2}(T)$ is met. In this case a single value of the superconducting order parameter is established for the entire superlattice, and is not at all equal to the values of the order parameters in the bulk materials of the layers. The London penetration depth is in this no longer directly connected with λ_1 and λ_2, and it is this which leads to the appearance of nonlinearity in the $\lambda^{-2}(T)$ dependence.

2. TEMPERATURE DEPENDENCES OF λ^{-2}, OF THE LOWER CRITICAL FIELD, AND OF THE HEAT CAPACITY OF THE SUPERLATTICE

We consider in the context of the Ginzburg–Landau theory the question of the temperature dependences of λ, H_{c1}, and the heat capacity for a superlattice made up by layer-by-layer deposition of two different type-II superconductors. To describe the superconductivity in each of the layers we employ the usual Ginzburg–Landau functional (see, e.g., Ref. 10):

$$F_j = a_j|\psi|^2 + \frac{1}{4m_j}\left|\left(\nabla - \frac{2ie}{c}\mathbf{A}\right)\psi\right|^2 + \frac{b_j}{2}|\psi|^4, \qquad (2)$$

where $\mathbf{A}$ is the vector potential, and the subscript $j = 1, 2$ labels the type of layer. We choose an order parameter ψ that is continuous on the layer boundaries, and formulate the boundary condition for the derivative in the form of continuity of $m_j^{-1}\psi'$. We note especially that the actual form of the

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boundary conditions is immaterial to us. The coefficients in (2) are of the form $a_j = \alpha_j(T - T_{cj})$, where T_{c1} and T_{c2} are the critical temperatures of the bulk superconductors from which layers 1 and 2 are made, respectively, and put for the sake of argument $T_{c2} > T_{c1}$.

A simple connection between the coefficients of the functional (2), the density of the electronic states, and the electron–phonon interaction constant exists only for the BCS model. At the same time, allowance for the tight-binding effects or, for example, the influence of magnetic impurities, upsets this simple connection, and in real superlattice the corresponding coefficients are expected to vary in layers 1 and 2 in a wide range. To keep inessential details out of our analysis, we assume that $\alpha_1 = \alpha_2 = \alpha$.

Near the temperature T_c, under the condition $\xi_{1,2}(T_c) > d$, the order parameter is practically constant in the entire superlattice, and in the absence of a magnetic field we can neglect the gradient terms in (1) and express the free energy in the form

$$F=F_1+F_2=(a_1+a_2)|\psi|^2+{}^1/_2(b_1+b_2)|\psi|^4. \tag{3}$$

In this case the critical temperature is determined from the conditions that the coefficients of $|\psi|^2$ vanish, i.e.,

$$T_c={}^1/_2(T_{c1}+T_{c2}). \tag{4}$$

We assume hereafter that in the absence of a magnetic field the order parameter is real. At temperatures not much lower than T_c, such that $\xi_{1,2}(T) > d$, we obtain from (3)

$$\psi^2=-(a_1+a_2)/(b_1+b_2), \tag{5}$$

and the screening depth is given by

$$\lambda^{-2}=-\frac{4\pi e^2}{c^2}\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\frac{a_1+a_2}{b_1+b_2}. \tag{6}$$

At lower temperatures, when the condition $\xi_{1,2}(T) < d$ is met, the screening depth is given by the expression

$$\lambda^{-2}=-\frac{4\pi e^2}{c^2}\left(\frac{a_1}{m_1 b_1}+\frac{a_2}{m_2 b_2}\right). \tag{7}$$

The $\lambda^{-2}(T)$ dependences given by Eqs. (6) and (7) are different; it is this which illustrates the onset of the nonlinear temperature dependences of λ^{-2} and of the lower critical field

$$H_{c1}=(\Phi_0/4\pi\lambda^2)\ln(\lambda/\xi)$$

(Φ_0 is the flux quantum). The character of the $\lambda^{-2}(T)$ dependence changes at the temperature at which the correlation length in the superlattice becomes of the order of the layer thickness.

Equations (6) and (7) give the limiting cases of the $\lambda^{-2}(T)$ dependence. In the intermediate region this dependence can be determined by calculating the mean value $\langle\psi^2/m\rangle$ with the aid of the Ginzburg–Landau equation for the order parameter. In the absence of a magnetic field the functional (2) leads, as usual, to the equations

$$-\frac{1}{4m_j}\frac{d^2\psi}{dx^2}+b_j\psi^3+a_j\psi=0,\quad j=1,2, \tag{8}$$

if the x axis is chosen perpendicular to the layer. We change in these equations to the dimensionless quantities

$$x'=(4m_1\alpha T_{c1})^{1/2}x,\quad t=(T-T_{c1})/T_{c1},\quad f=(b_1/|a_1|)^{1/2}\psi.$$

We introduce also the notation

$$t_{c2}=(T_{c2}-T_{c1})/T_{c1},\quad b=b_2/b_1,$$

$$m=m_2/m_1,\quad d'=(4m_1\alpha T_{c1})^{1/2}d.$$

We shall omit the primes hereafter. We choose the origin at the center of layer 2. To be specific, we consider the case $T<c_1$, i.e., $t<0$. In the new notation, Eqs. (8) take the form

$$\frac{d^2f}{dx^2}-tf+tf^3=0 \tag{9}$$

(for layer 1) and

$$\frac{1}{m}\frac{d^2f}{dx^2}+(t_{c2}-t)f+btf^3=0 \tag{9'}$$

(for layer 2). Using the first integrals of Eqs. (9) and (9'), we easily write their solutions in quadratures, and in the case $b>1$, $t<t_{c2}/(1-b)$ we obtain the relations:

$$\frac{d}{2}(-t)^{1/2}=\int_{f_3}^{f_1}df\{(f_1^2-f^2)[1-{}^1/_2(f_1^2+f^2)]\}^{-1/2}, \tag{10}$$

$$\frac{d}{2}[m(t_{c2}-t)]^{1/2}$$

$$=\int_{f_2}^{f_3}df\left\{(f^2-f_2^2)\left[\frac{bt}{2(t-t_{c2})}(f^2+f_2^2)-1\right]\right\}^{-1/2}, \tag{10'}$$

where f_1 is the value of the order parameter at the center of layer 1, f_2 at the center of layer 2, and f_3 on the layer boundary (in our case $f_1>f_3>f_2$). Inverting Eqs. (10) and (10'), we can express the ratios f_3/f_1 and f_3/f_2 in terms of Jacobi elliptic functions.[11] The boundary condition for the derivative of the order parameter leads to the equation

$$mt[f_1^2-f_3^2+{}^1/_2(f_3^4-f_1^4)]+(t_{c2}-t)(f_2^2-f_3^2)+{}^1/_2bt(f_2^4-f_3^4)=0. \tag{11}$$

Solving the system (10), (10'), and (11) we obtain the rms order parameter

$$\langle f^2\rangle=\frac{1}{d}\int_{f_2}^{f_1}f^2\left|\frac{dx}{df}\right|df,$$

and consequently the temperature dependence of λ^{-2}:

$$\lambda^{-2}=\frac{\lambda_1^{-2}}{2}\left[\langle f^2\rangle_1+\frac{1}{m}\langle f^2\rangle_2\right], \tag{12}$$

where $\langle f^2\rangle_1$ and $\langle f^2\rangle_2$ are the rms values of the order parameter in layers 1 and 2, respectively.

If the factor $\ln(\lambda/\xi)$ that depends little on the temperature is neglected, the $H_{c1}(T)$ dependence is similar. Note that when H_{c1} is calculated it is necessary, generally speaking, to take into account the finite transparency (which we consider to be ideal) of the boundaries between the layers, and the anisotropy of λ. If, however the currents are parallel to the layers, finite-transparency effects can be disregarded. The results of a numerical calculation of $\lambda^{-2}(T)/\lambda_1^{-2}(0)$, meaning also $H_{c1}(T)/H_{c1}^{(1)}(0)$, are shown in Figs. 1 and 2. Here $\lambda_1(0)$ and $H_{c1}^{(1)}(0)$ are respectively the London screening depth and the lower critical field of the bulk materials of

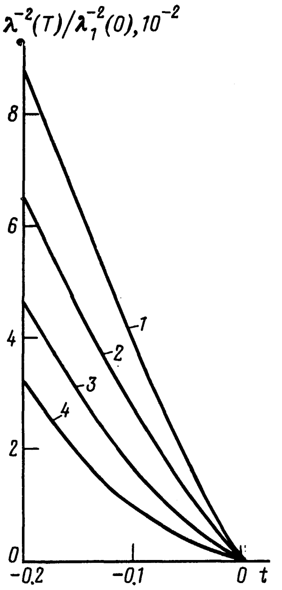

FIG. 1. Temperature dependences of $\lambda^{-2}(T)/\lambda_1^{-2}(0)$, where $\lambda_1(0)=(8\pi e^2\alpha T_{c1}/m_1c^2b_1)^{-1/2}$, for a superlattice with the following parameters; $m=1$, $t_{c2}=0.01$ and $b=10$, $d=10$ (curve 1); $b=10$, $d=5$, (2); $b=20$, $d=5$ (3); $b=40$, $d=5$ (4).

layer 1, both extrapolated to $T=0$. Note that, depending on the values of the parameters, we get either a positive (Fig. 1) or, e.g., a weak negative (Fig. 2) curvature of the $\lambda^{-2}(T)$ and $H_{c1}(T)$ plots. These forms of $\lambda^{-2}(T)$ and $H_{c1}(T)$ differ from the temperature dependences of the parallel ($H_{c2\parallel}$) and perpendicular ($H_{c2\perp}$) upper critical fields. In particular, the $H_{c1}(T)$ and $H_{c2}(T)$ dependences in a superlattice can have in principle curvatures of equal sign.

Similar anomalies are possible also in the temperature dependence of the thermodynamic field H_c. Since the heat capacity per unit volume of the superlattice is equal to $-Td^2(H_c^2/8\pi)/dT^2$, the unusual behavior of $H_c(T)$ should cause also singularities in the temperature dependence of the heat capacity. Using the fact that the solution of Eqs. (8) satisfies the condition $\delta F/\delta\psi=0$, we can write the expression for the heat capacity of an inhomogeneous superconducting system in the form

$$\frac{C}{\Delta C_1}=\frac{T}{T_{c1}d}\frac{d}{dt}\left(t\int_{f_2}^{f_1} f^2\left|\frac{dx}{df}\right| df\right), \qquad (13)$$

where $\Delta C_1=T_{c1}\alpha^2/b_1$ is the heat-capacity jump on going to the superconducting state at $T=T_{c1}$ in the bulk material of layer 1 (just as before, we consider the case $b>1$, $t<t_{c2}/(1-b)$, in which $f_2<f_3<f_1$). The heat capacity is thus expressed in terms of the derivative of the rms order parameter with respect to temperature, and below T_c the heat capacity should increase in the region where $\lambda^{-2}(T)$ is nonlinear.

In the measurements known to us[8,9] of the temperature dependence of λ^{-2} they used superlattices made up of alternating layers of a superconductor and a normal metal. We consider therefore a superlattice of this type, assuming for simplicity that the superconductor and normal-metal layers have equal thickness d. In the region where the Ginzburg–Landau approach is valid we can consider only sufficiently thick layers (whose thickness should be large compared with ξ_0). Under these conditions the normal-metal layers exert a strong destructive action on the Cooper pairs, and it can be assumed that $\psi=0$ on the layer interface.[12] Retaining the previous notation and assuming that layer 1 is the superconducting one, we obtain from (10) the relation

$$\frac{d}{2}(-t)^{1/2}=\int_0^{f_1} df\{(f_1^2-f^2)[1-{}^1/_2(f^2+f_1^2)]\}^{-1/2}, \qquad (14)$$

which can also be expressed in terms of a complete elliptic integral of the first kind $K(k)$:

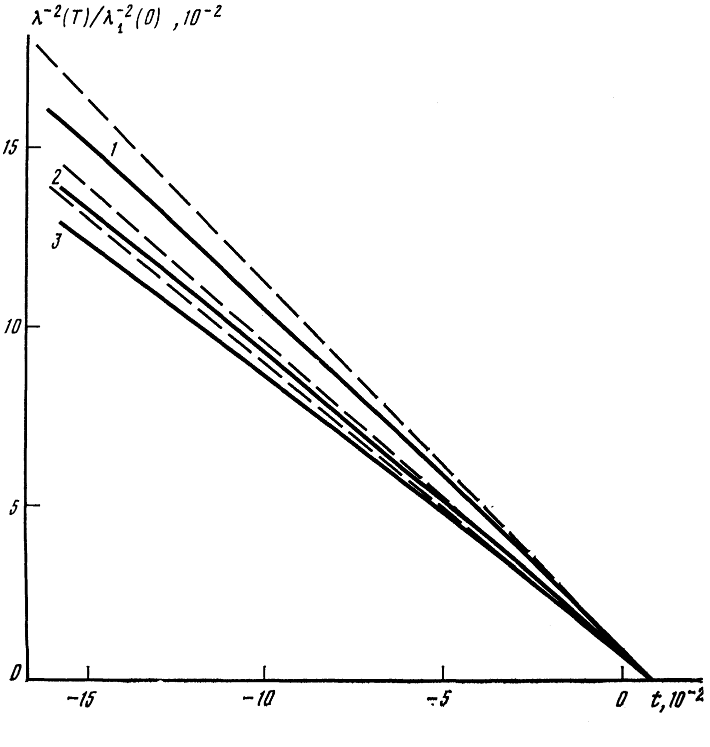

FIG. 2. Temperature dependences of $\lambda^{-2}(T)/\lambda_1^{-2}(0)$ for another set of parameters: $t_{c2}=0.01$ and $m=15$, $b=0.1$, $d=5$ (curve 1); $m=10$, $b=0.2$, $d=5$ (2); $m=10$, $b=0.2$, $d=10$ (3). The dashed straight lines are tangent to curves 1–3 at $T=T_c$.

$$K\left(\frac{f_1}{(2-f_1^2)^{1/2}}\right)=\frac{d}{2}\left[t\left(\frac{f_1^2}{2}-1\right)\right]^{1/2}. \qquad (15)$$

Just as above, we find the London screening depth

$$\lambda^{-2}=\frac{\lambda_1^{-2}}{d}\int_0^{f_1} f^2\left|\frac{dx}{df}\right|df=\lambda_1^{-2}\left\{1-\frac{f_1^2}{2}\right.$$
$$\left.-\frac{2}{d(-t)^{1/2}}\left[1-\frac{f_1^2}{2}\right]^{1/2}E\left(\frac{f_1}{(2-f_1^2)^{1/2}}\right)\right\} \qquad (16)$$

[$E(k)$ is a complete elliptic integral of the second kind], and also the heat capacity

$$\frac{C}{\Delta C_1}=\frac{T}{T_{c1}}\frac{d}{dt}\left\{t\left(1-\frac{f_1^2}{2}\right)+\frac{2}{d}\left[t\left(\frac{f_1^2}{2}-1\right)\right]^{1/2}\right.$$
$$\left.\times E\left(\frac{f_1}{(2-f_1^2)^{1/2}}\right)\right\}. \qquad (17)$$

From (15) we can easily determine the superlattice critical temperature $t_c=(T_c-T_{c1})/T_{c1}$ which corresponds to $f_1\to 0$:

$$t_c=-(\pi/d)^2. \qquad (18)$$

Expanding in (15) and (17), near t_c, all the integrals with respect to the parameter $f_1(2-f_1^2)^{-1/2}\ll 1$, we get

$$f_1^2={}^4/_3[(t-t_c)/t_c], \qquad (19)$$

$$C(t=t_c)/T_c={}^1/_3\Delta C_1/T_{c1}. \qquad (20)$$

The temperature dependences of λ^{-2}, calculated from Eq. (16) for different d, are shown in Fig. 3. The $\lambda^{-2}(T)$ dependence has a positive curvature near T_c, in accord with the experimental results.[9] Note that under the experimental conditions of Ref. 9 the layer thicknesses were of order ξ_0, and therefore our analysis based on the Ginzburg–Landau equations is valid in this case only qualitatively.

In addition to the superlattice types considered above, it is also of interest to consider a superlattice with thin layers of a normal metal (of thickness less than ξ_0). In this case, to take into account the influence of the normal layer it is necessary to add to the superlattice free-energy functional the terms

$$\gamma\sum_n\delta(x-x_n)|\psi|^2, \qquad x_n=nd+d/2,$$

where x_n is the coordinate of the nth normal layer. The parameter γ is directly connected with the superlattice critical temperature t_c (see below) and can therefore be determined from experiment. Just as above, we change to dimensionless quantities. The order parameter in the middle of the superconducting layer will again be designated f_1. We introduce also the notation $f_0\equiv f(x_n)$. The equation for the order parameter is of the form

$$-f''+tf-tf^3+2\tilde{\gamma}\sum_n\delta(x-x_n)f=0, \qquad (21)$$

where $\tilde{\gamma}=\gamma(m_1/\alpha T_{c1})^{1/2}$. Using its first integral, we arrive at the relation

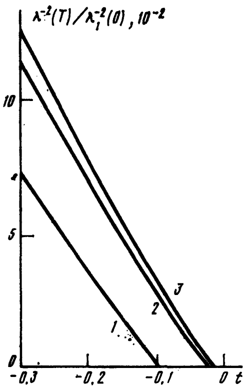

FIG. 3. Dependences of $\lambda^{-2}(T)/\lambda_1^{-2}(0)$ on the temperature for a superlattice with alternating normal and superconducting layers at $d=10$ (curve 1), $d=20$ (2), and $d=30$ (3).

$$f_0=f_1\,\mathrm{cd}\left(\frac{d}{2}\left[t\left(\frac{f_1^2}{2}-1\right)\right]^{1/2}\middle|\frac{f_1^2}{2-f_1^2}\right), \qquad (22)$$

where $\mathrm{cd}(xm)$ is the Jacobi elliptic function. The role of the δ function in (21) reduces to the boundary condition

$$f'|_{x=d/2\pm0}=\pm\tilde{\gamma}f_0, \qquad (23)$$

which leads to the equation

$$-\tilde{\gamma}^2f_0^2+tf_0^2-{}^1/_2tf_0^4=tf_1^2-{}^1/_2tf_1^4. \qquad (24)$$

Determining f_0 and f_1 from (22) and (24) we easily get

$$\lambda^{-2}=\lambda_1^{-2}\langle f^2\rangle, \qquad \frac{C}{\Delta C_1}=\frac{T}{T_{c1}}\frac{d}{dt}(t\langle f^2\rangle).$$

We show now the connection between the parameter γ and the superlattice critical temperature. To determine the critical temperature in (21) it suffices to retain the terms linear in f:

$$-f''+tf+2\tilde{\gamma}\sum_n\delta(x-x_n)f=0. \qquad (25)$$

A solution of (25) for $-d/2<x<d/2$ and satisfying the condition $f'(0)=0$ is

$$f\propto\cos(|t|^{1/2}x). \qquad (26)$$

Taking the boundary condition (23) into account, we obtain a relation between γ and the critical temperature:

$$\gamma=\left(\frac{\alpha T_{c1}|t_c|}{m_1}\right)^{1/2}\mathrm{tg}\left(\frac{d}{2}|t_c|^{1/2}\right). \qquad (27)$$

If the distance between the normal layers is large ($d\gg 1$) it is easy to obtain an approximate analytic solution near t_c, by using $|t_c|$ as a small parameter. At a temperature not much lower than t_c the value of f_1 is small and we obtain from (24)

$$f_0^2=\frac{|t|f_1^2}{\tilde{\gamma}^2-t}\approx\frac{|t_c|}{\tilde{\gamma}^2}f_1^2. \qquad (28)$$

It can be assumed in first-order approximation that $f_0=0$. The problem reduces then to that of a superlattice whose normal and superconducting layers are of equal thickness, a problem already considered above. Equations (18) and (19) remain in force, but the heat-capacity jump at t_c is now twice as large (see, e.g., Ref. 13):

$$C(t=t_c)/T_c={}^2/_3\Delta C_1/T_{c1} \tag{29}$$

[cf. Eq. (20)]. The reason is that now there are no massive normal interlayers that make no contribution (within the framework of our analysis) to the heat capacity. The value of $\lambda^{-2}(T)$ is also double that given by Eq. (16).

3. TEMPERATURE DEPENDENCES OF UPPER CRITICAL FIELD

Using the Ginzburg–Landau functional we can also calculate the upper critical fields $H_{c2\perp}$ and $H_{c2\parallel}$. The $H_{c2}(T)$ dependence was investigated earlier[6,7] by more complicated methods, but we shall demonstrated that the main results of these references can be easily obtained by using the Ginzburg–Landau approach.

Consider a superlattice made up of the type-II superconductors. We place the z axis in the layer plane. As before, the x axis is perpendicular to the layers, and the layers are assumed to have equal thickness.

We consider first the case of a field parallel to the layers and directed along the z axis. Using the dimensionless variable (see Sec. 2)

$$\mathbf{r}\to\mathbf{r}(4m_1\alpha T_{c1})^{1/2}=\mathbf{r}/\xi_{eff},$$

we write down the linearized Ginzburg–Landau equation

$$\frac{1}{4m_j}\left(\nabla-\frac{2ie}{c}\xi_{eff}\mathbf{A}\right)^2\psi=a_j\xi_{eff}^2\psi,\quad j=1,2. \tag{30}$$

We choose the vector potential in the form $\mathbf{A}=(0,H\xi_{eff}x,0)$. Choosing the order parameter in (30) in the form

$$\psi(x,y)=f(x)e^{iky} \tag{31}$$

and introducing the new variables

$$\rho=\left(\frac{2eH}{c}\right)^{1/2}\xi_{eff}x,\qquad \rho_0=\left(\frac{c}{2eH}\right)^{1/2}\frac{k}{\xi_{eff}},\qquad h=\frac{eH}{2m_1\alpha T_{c1}c},$$

we obtain, as usual, an oscillatory equation for f:

$$-\frac{d^2f}{d\rho^2}+(\rho-\rho_0)^2f=g_jf,\quad j=1,2, \tag{32}$$

where

$$g_1=-\frac{t}{h},\qquad g_2=\frac{m}{h}(t_{c2}-t).$$

We write the solution of Eq. (32) in the nth pair of layers in the form

$$f=A_j^n\Delta_{g_j}(\rho-\rho_0)+B_j^n\Delta_{g_j}(\rho_0-\rho),\qquad j=1,2, \tag{33}$$

where $\Delta_{g_j}(\rho)$ is a Weber function.[14,15] Using the condition for f and f' on the layer boundaries, as well as the fact that $f\to 0$ as $\rho\to\pm\infty$, we can determine the coefficients A and B in (33), and also the temperature dependence of H_{c2}. The results of a numerical calculation of the field $h_{c2\parallel}=eH_{c2\parallel}/2m_1\alpha T_{c1}c$ are shown in Fig. 4. It can be seen from this figure that if the ratio $m=m_2/m_1$ is not too close

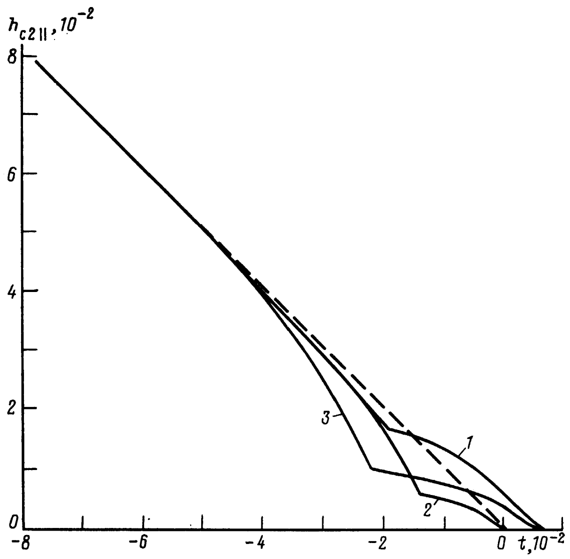

FIG. 4. Temperature dependences of the parallel upper critical field at $t_{c2}=0.01$, $d=20$, $m=0.5$ (curve 1); $t_{c2}=4.10^{-4}$, $d=25$, $m=0.1$ (2); $t_{c2}=0.01$, $d=20$ $m=0.1$ (3).

to unity and the $h_{c2\parallel}(t)$ curves have for sufficiently thick layers a kink corresponding to a transition of the superconducting seed from a layer with a larger diffusion coefficient to a layer with a smaller diffusion coefficient and a stronger field H_{c2}. It should be noted that in all the cases considered the center of the orbit (ρ_0) was localized at the middle of the layers and, as follows directly from the numerical computations, the kink on the $h_{c2\parallel}(t)$ curve corresponded to a displacement of the orbit center into the neighboring layer. In principle we cannot exclude a situation in which the most favorable is location of the orbit center near the layer boundary, in analogy with the surface-superconductivity situation (field H_{c3}). This is possible at a large mass ratio m_1/m_2 and an appreciable difference between T_{c1} and T_{c2}.

The $h_{c2\parallel}(t)$ acquires near t_c a small positive curvature corresponding to a change of the sublattice from two-dimensional in strong fields to three-dimensional in weak ones. In strong fields the order parameter is in the main different from zero in one layer, whereas in weak field the superconducting seed covers simultaneously many layers. As the field is decreased, the $h_{c2\parallel}(t)$ changes from an approximately square-root dependence into an approximately linear one, and this causes the appearance of the positive curvature. This character of the temperature dependence of a parallel upper critical field (the existence of a kink and of a positive curvature near t_c) agrees qualitatively with the experimental results.[2]

If the magnetic field is perpendicular to the layers, the case of greatest interest is $m<1$. We take the vector potential in the gauge $\mathbf{A}=(0,0,H\xi_{eff}y)$. Separating the variables in (30) and choosing the solution corresponding to the strongest field, we find for the nth pair of layers at $t<t^*=mt_{c2}/(m-1)$

$$\psi(x,y)=\exp\left(-\frac{eH}{c}\xi_{eff}^2y^2\right)\times\begin{cases}\beta_1\cos[k_1\xi_{eff}(x-d(2n+1))] & \text{in layer } 1,\\ \beta_2\,\mathrm{ch}[k_2\xi_{eff}(x-2dn)] & \text{in layer } 2\end{cases} \tag{34}$$

and at $t' < t < t_c$

$$\psi(x, y) = \exp\left(-\frac{eH}{c}\xi_{eff}^{2} y^2\right)$$
$$\times\begin{cases}\beta_1 \operatorname{ch}\left[k_1\xi_{eff}(x-d(2n+1))\right] & \text{in layer 1,}\\ \beta_2 \cos\left[k_2\xi_{eff}(x-2dn)\right] & \text{in layer 2.}\end{cases} \qquad (35)$$

Joining the solutions in the different layers with the aid of the boundary conditions, we obtain an equation for $h_{c2\perp} = eH_{c2\perp}/2m_1\alpha T_{c1}c$ at $t^* < t < t_c$:

$$m(t+h)^{1/2}\operatorname{th}\left[\frac{d}{2}(t+h)^{1/2}\right]$$
$$=[m(t_{c2}-t)-h]^{1/2}\operatorname{tg}\left\{\frac{d}{2}[m(t_{c2}-t)-h]^{1/2}\right\}, \qquad (36)$$

and at $t < t'$:

$$m(-t-h)^{1/2}\operatorname{tg}\left[\frac{d}{2}(-t-h)^{1/2}\right]$$
$$=[m(t-t_{c2})+h]^{1/2}\operatorname{th}\left\{\frac{d}{2}[m(t-t_{c2})+h]^{1/2}\right\}. \qquad (37)$$

The calculation results are shown in Fig. 5. The temperature dependence of $h_{c2\perp}$ has a characteristic anomaly: the $h_{c2\perp}(t)$ curves for different d but equal t_{c2} and $m < 1$ intersect at a point (t^*, h^*) where $h^* = -t^*$. This was to be expected, since both equations (30) for $j = 1$ and 2 coincide at $t = t^*$. In this case the order parameter is independent of x and the superlattice is a homogeneous superconductor whose upper critical field coincides with the field H_{c2} of each of the separate layers (see also Ref. 6).

4. CONCLUSION

As shown above, nonlinear temperature dependences of λ^{-2} and of the lower critical field can occur in superconducting superlattices under the conditions $\xi_0 < d < \xi_{1,2}(T_c)$ and $d \ll \lambda$. It should be noted that such temperature dependences can hold in a region wider than that of the applicability of the Ginzburg–Landau approach used by us. Similar anomalies are possible also in the temperature dependence of the thermodynamic field H_c. Nonlinear dependences of $\lambda^{-2}(T)$ and $H_{c1}(T)$ are possible not only for superlattices produced by layer-by-layer deposition of two different superconductors, but also for superlattices made up of alternating superconductor and normal-metal layers.

The positive curvature predicted in Sec. 2 for the $\lambda^{-2}(T)$ plot for a superlattice consisting of superconductor and normal-metal layers agrees with experimental data.[9] In Ref. 9 they used a V/Ag superlattice with vanadium-layer thickness 240 Å, which is approximately double the correlation length ξ_0 of bulk vanadium. At the same time, the absence of a curvature in the experiments of Ref. 8 can also be explained: The layer thickness (54 Å) in the Nb/Cu superlattice investigated there is much less than the superconducting correlation length of niobium, and the superlattice parameters prevent observation of a nonlinear $\lambda^{-2}(T)$ dependence.

The unusual behavior of $\lambda^{-2}(T)$ and $H_{c1}(T)$ is also the cause of the anomalies in the temperature dependence of the heat capacity of a superlattice.

We emphasize in conclusion that the cause of the anomalous dependences of $\lambda^{-2}(T)$ and of the lower critical field is entirely different from the cause of the onset of nonlinear dependences of the upper critical field. The considered mechanism of the onset of nonlinear $\lambda^{-2}(T)$ dependences and of the lower critical field is typical not only of superlattices but also of layered superconductors having at least two nonequivalent layers per unit cell. An example is a high-temperature superconductor such as Tl–Ba–Ca–Cu–O (Ref. 16).

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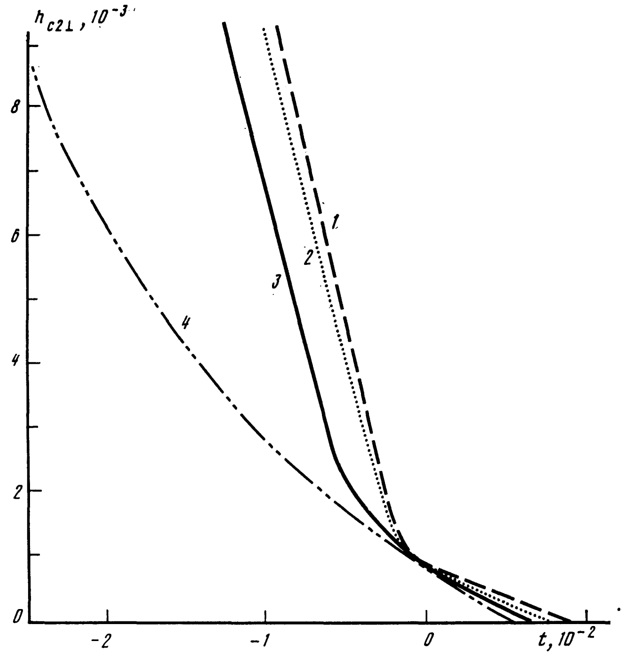

FIG. 5. Temperature dependences of the perpendicular upper critical field for the following parameter values: $t_{c2} = 0.01$ $m = 0.1$ and $d = 200$ (curve 1), $d = 100$ (2), $d = 50$ (3), $d = 20$ (4).

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[2)]It is assumed that the magnetic field and the layers are parallel to the surface. The screening currents flow in this case along the layers, and it is just this situation which corresponds to the conditions of the measurements of $\lambda^{-2}(T)$ in Refs. 8 and 9. In a geometry where the screening currents are perpendicular to the layers the quantity averaged is λ^2 rather than λ^{-2}.

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