Manifestation of insulating correlations in tunneling characteristics of superconductors

A. M. Ismagilov and Yu. V. Kopaev

P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, Moscow
(Submitted 18 May 1989)
Zh. Eksp. Teor. Fiz. **96**, 1492–1504 (October 1989)

The tunneling current-voltage characteristics of a system with insulating correlations are shown to depend strongly on the value of the insulating order parameter Σ. A transition near (below) T_c from a phase with a relatively large insulating gap to one with a relatively small one, caused by the onset of superconductivity in the system, gives rise to a structural feature of the superconducting type on the tunneling current-voltage characteristics. The gap Δ_{tunn} found from this structural feature might be erroneously interpreted as a superconducting gap Δ ($\Delta < \Delta_{\text{tunn}}$). The spread in the values of $2\Delta_{\text{tunn}}/T_c$ ($2\Delta_{\text{tunn}}/T_c > 3.5$) is attributed to a scatter in the values of Σ in a surface region (with a size on the order of the length scale of insulating correlations near the barrier), which shapes the structural features on the current-voltage characteristics associated with the conversion of the spectrum to an insulating nature. The spread in the values of Σ in this region results from the inhomogeneous composition of the system, which, while having a strong effect on Σ, may have only a slight effect on Δ because of the larger superconducting correlation length. It has been found that the current-voltage characteristics of *SIN* and asymmetric *SIS* junctions are asymmetric with respect to the voltage, while those symmetric *SIS* contacts are symmetric.

High-T_c superconductors undergo a metal-insulator phase transition as their composition is varied (as they are doped). Insulating correlations apparently play an important role even in superconducting compositons. These correlations are described either in the Hubbard model, which is valid in the limit $W/U \ll 1$ (W is the width of the band, and U is the repulsive energy at one center), or in the band limit ($W/U \gg 1$), in which the conversion to an insulating state gives rise to topological features on the Fermi surface. It is very probable that an intermediate situation ($W/U \sim 1$) prevails in the high-T_c superconductors. Such superconductors can be described qualitatively in the band limit through the use of the mean-field approximation to describe the insulating and superconducting correlations. In the limit of many "colors," the mean-field approximation is also valid in the Hubbard model.[1] There is the hope that the limit of an infinite number of colors will be applicable for a qualitative description of states with only two colors, with two indices corresponding to two spin projections. Affleck and Marston[1] use the mean-field approximation to describe the conversion to an insulating state, Ruckenstein *et al.*[2] use it to describe superconductivity, and Inui *et al.*[3] use it to describe the coexistence of superconducting and antiferromagnetic orders in a system describable by the Hubbard model. Since the mean-field Hamiltonians are equivalent in the band and Hubbard limits, the simultaneous incorporation of insulating and superconducting correlations in each limit will presumably lead to identical results, which will evidently also be applicable in the intermediate situation $W/U \sim 1$.

It follows from band calculations[4–6] and several experiments[7,8] that a "nesting" of the Fermi surface occurs in the metallic phase of the high-T_c superconductors. To determine the effect of the metal-insulator phase transition on the superconductivity it is thus convenient to use the model of a single-band metal with a nesting of the Fermi surface which is unstable with respect to insulating and superconducting correlations (see Fig. 1, where **Q** is the nesting vector).

The problem of the coexistence of insulating and superconducting pairings in the model of a single-band metal with nesting of the Fermi surface can be reformulated in the terms appropriate to the corresponding problem in the model of a doped semimetal with nearly coincident electron and hole Fermi surfaces, if the regions (Fig. 1) near *ABC* and *DEF* of the Fermi surface are designated as band 1, while those near *CMD* and *AKF* are designated as band 2. The problem of the coexistence of insulating and superconducting pairings in the model of a doped semimetal was solved in Ref. 9. We will use that model as a starting point for calculating the current-voltage characteristics of tunnel junctions. The original Hamiltonian in the model of the semimetal is written in the form

$$H=\sum_i \int \psi_i^+(\mathbf{x})\varepsilon_i(p)\psi_i(\mathbf{x})\,d\mathbf{x}+\frac{1}{2}\sum_{i,j}\lambda_{ij}\int(\psi_i^+(\mathbf{x})\psi_i(\mathbf{x}))\times(\psi_j^+(\mathbf{x})\psi_j(\mathbf{x}))\,d\mathbf{x}+\gamma\int\psi_1^+(\mathbf{x})u(\mathbf{x})\psi_2(\mathbf{x})\,d\mathbf{x}+\gamma^*\times\int\psi_2^+(\mathbf{x})u(\mathbf{x})\psi_1(\mathbf{x})\,d\mathbf{x}, \quad (1)$$

where λ_{ij} are interaction constants, and i and j are the band indices ($i,j = 1,2$).

The last two terms in the Hamiltonian (1) describe the interaction of electrons with a static deformation of a lattice

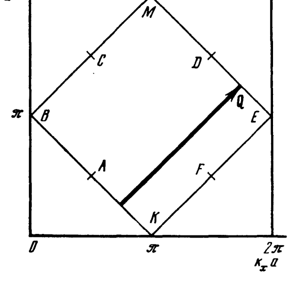

FIG. 1.

 0038-5646/89/100846-07$04.00

with a coupling constant γ (for brevity, we have omitted the elastic strain energy).

The seed spectrum of the system is assumed to be isotropic:

$$\varepsilon_{1,2}(\mathbf{p})=\delta\mu+\varepsilon(\mathbf{p}),\ \varepsilon(\mathbf{p})=\mathbf{p}^2/2m-\varepsilon_F,$$

where $\delta\mu$ is the shift of the Fermi level in each of the bands, caused by doping, for example.

The insulating order parameter $\Sigma=\langle\psi_1^+(\mathbf{x})\psi_2(\mathbf{x})\rangle$ generally has four components. It can be written in the following form if the z axis is chosen as the spin quantization axis:

$$\Sigma=\Sigma_{\mathrm{Re}}{}^s+\sigma^z\Sigma_{\mathrm{Re}}{}^t+i(\Sigma_{\mathrm{Im}}{}^s+\sigma^z\Sigma_{\mathrm{Im}}{}^t),$$

where s and t mean singlet and triplet, Re and Im are the real and imaginary parts of the order parameter, and σ^z is the Pauli matrix. Each component of the order parameter corresponds to different physical states of the system[10]: Σ^s_{Re} corresponds to a charge density wave and, if the band extrema coincide, to ferroelectricity; Σ^t_{Re} corresponds to a spin density wave, i.e., to a spin antiferromagnetism; Σ^s_{Im} corresponds to a current density wave, i.e., to orbital antiferromagnetism; and Σ^t_{Im} corresponds to a spin-current-density wave, i.e., to a nonmagnetic spinor material. This classification assumes that the Bloch factors $u_k(\mathbf{x})$ of the wave functions of the regular phase are chosen to be real.

We will be discussing the case $\Sigma=\Sigma^s_{\mathrm{Re}}$, but where necessary we will also stipulate features on the tunneling characteristics which are associated with a realization of other components of the order parameter ($\Sigma^t_{\mathrm{Re}},\Sigma^t_{\mathrm{Im}}$, and Σ^s_{Im}).

As will be seen in the discussion below, the current-voltage characteristics of tunnel junctions depend strongly on the sign of the order parameter Σ^s_{Re}, so we introduce the auxiliary notation $\Sigma^s_{\mathrm{Re}}\equiv\Sigma_1$ for a positive order parameter and $\Sigma^s_{\mathrm{Re}}\equiv\Sigma_2$ for a negative one.

For the superconducting order parameters $\Delta_{ij}=\langle\psi_i(\mathbf{x})\psi_j(\mathbf{x})\rangle$ (which can be rendered real through an appropriate gauge transformation of Δ_{ij}), two solutions are possible: a symmetric solution $\Delta_{11}=\Delta_{22}=\Delta$ and an antisymmetric solution $\Delta_{11}=-\Delta_{22}=\Delta$. In the model of a one-band metal with Fermi-surface nesting, these two cases correspond in the standard terminology to an s wave [there are no nodes of the parameter $\Delta(\mathbf{k})$ on the regions of the Fermi surface] and a d wave [the parameter $\Delta(\mathbf{k})$ has nodes at points A, C, D, and F of the Fermi surface).

We are interested in the case of a symmetric solution for Δ (the case of the s wave in the single-band model), since it is this case which corresponds to an increase in T_c in the insulating phase. For antiferromagnetism (a spin density wave), we have the opposite situation: An increase in T_c in the insulating phase corresponds to an antisymmetric solution for Δ (the d-wave case in the single-band model).

In our weak-coupling limit ($\lambda_{21}\ll 1$) the parameter Δ_{21} is small ($\Delta_{21}\ll\Delta$) over a wide region of values of the coupling constants, so we will set $\Delta_{21}=0$.

The spectrum of elementary excitations in the SI phase (in which insulating and superconducting order parameters coexist) was derived in Ref. 9; it is

$$\omega_\pm{}^2=(\delta\mu\pm E)^2+\Delta^2,\ E^2=\varepsilon^2+\Sigma^2.$$

For definiteness we assume $\delta\mu>0$. This assumption corresponds to an acceptor dopant in the system. We have $\delta\mu^2=n^2+\Sigma^2$, where $n=\delta n/4N(0)$ is the shift of the Fermi level in the normal phase ($\Sigma=\Delta=0$). Here δn is the difference between the densities of electrons and holes due to the doping (this difference is assumed to be given), and $N(0)$ is the state density at the Fermi level in one (either) of the bands in the semimetal. The spectrum of elementary excitations is shown in Fig. 2, where $B=[(\delta\mu+|\Sigma|)^2+\Delta^2]^{1/2}$, and ω is the energy of the excitations. The branches shown by dashed lines in Fig. 2 exist only in the superconducting phase.

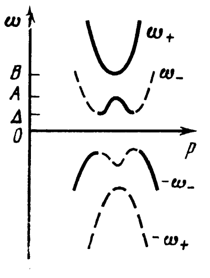

FIG. 2.

The Fourier components of the Green's functions

$$G_{ij}{}^{\alpha\beta}(x,x')=\langle T\psi_{i\alpha}(x)\psi_{j\beta}{}^+(x')\rangle=G_{ij}\delta_{\alpha\beta},$$
$$F_{ij}{}^{\alpha\beta}(x,x')=\langle T\psi_{i\alpha}(x)\psi_{j\beta}(x')\rangle=i\sigma_{\alpha\beta}{}^yF_{ij}$$

(α,β are spin indices; i,j are band indices; and $x=\mathbf{r},t$), which we will need in order to calculate the currents which flow through the SIN and SIS tunnel junctions, can be found from a system of Gor'kov equations which was derived in Ref. 9:

$$\left\{\begin{aligned}
&G_{11}=\{(\omega+\varepsilon_1)(\omega+\varepsilon_2)(\omega-\varepsilon_2)-\Sigma^2(\omega-\varepsilon_2)+\Delta^2(\omega+\varepsilon_1)\}/D,\\
&G_{12}=G_{21}=-\{-\Sigma(\omega+\varepsilon_1)(\omega+\varepsilon_2)+\Sigma\Delta^2+\Sigma^3\}/D,\\
&G_{22}=\{(\omega-\varepsilon_1)(\omega+\varepsilon_1)(\omega+\varepsilon_2)-\Sigma^2(\omega-\varepsilon_1)+\Delta^2(\omega+\varepsilon_2)\}/D,
\end{aligned}\right.\tag{2}$$

$$\left\{\begin{aligned}
&F_{11}=F_{11}{}^+=\{\Delta^3+\Delta\Sigma^2-\Delta(\omega-\varepsilon_2)(\omega+\varepsilon_2)\}/D,\\
&F_{12}=F_{21}{}^+=-\{-\Delta\Sigma(\omega-\varepsilon_2)+\Delta\Sigma(\omega+\varepsilon_1)\}/D,\\
&F_{21}=F_{12}{}^+=-\{-\Delta\Sigma(\omega-\varepsilon_1)+\Delta\Sigma(\omega+\varepsilon_2)\}/D,\\
&F_{22}=F_{22}{}^+=\{\Delta^3+\Delta\Sigma^2-\Delta(\omega-\varepsilon_1)(\omega+\varepsilon_1)\}/D.
\end{aligned}\right.\tag{3}$$

Here

$$D=(\omega-\omega_++i0)(\omega+\omega_+-i0)(\omega-\omega_-+i0)(\omega+\omega_--i0).$$

To calculate the currents which flow through the SIS and SIN tunnel junctions, we use the tunneling-Hamiltonian method, for which a detailed theory is given in Refs. 11 and 12, among other places.

We will first derive expressions for the currents which flow through an SIS junction, and then we will correct these expressions for the case of an SIN junction.

The tunneling Hamiltonian is

$$H_T=\sum_{\mathbf{p},\mathbf{q},\alpha,i}T_{\mathbf{pq}}a^+_{\mathbf{p}i\alpha}c_{\mathbf{q}\alpha}+\mathrm{H.a.}\tag{4}$$

It describes an entanglement of the states of "left-handed" and "right-handed" superconductors (a "left-handed" superconductor is a system which can be described by the SD model, while a "right-handed" one is an ordinary superconductor.

In Hamiltonian (4), $a^{+}_{\mathbf{p}i\alpha}$ and $c_{\mathbf{q}\alpha}$ are operators which create electrons with momenta **p** and **q** and spins α in respectively left-handed and right-handed superconductors, and i is the band index of a left-handed superconductor.

Following the procedure of Refs. 11 and 12, and working to lowest order in $T_{\mathbf{pq}}$ in the adiabatic approximation [i.e., under the assumption that the time evolution of the voltage across the barrier, $V = V(t)$, is slow], we find for the total current I, the sum of the Josephson component I_d and the one-particle component I_0 flowing through the *SIS* tunnel junction,

$$I=I_d+I_0. \tag{5}$$

The Josephson current is

$$I_d(V(t))=\mathrm{Im}[I_c(V(t)\exp(2i\chi(t))], \tag{6}$$

where

$$\chi(t)=\frac{e}{\hbar}\int_{t_0}^{t}V(t')dt'+\delta\chi,$$

and $\delta\chi$ is the initial phase difference between the superconducting order parameters of the left-handed and right-handed superconductors.

The amplitude of the Josephson current is

$$I_c(V(t))=-\frac{1}{8\pi^3eR}\int_{-\infty}^{\infty}d\omega_1\int_{-\infty}^{\infty}d\omega_2$$

$$\times\left\{\frac{\sum_{i,j}F_{ij}^{+}(\omega_1)F(\omega_2)}{\omega_1+\omega_2+eV-i0}-\frac{\sum_{i,j}F^{*}_{i,j}(\omega_1)(F^{+}(\omega_2))^{*}}{\omega_1+\omega_2-eV+i0}\right\}, \tag{7}$$

where

$$F_{ij}(\omega_1)=\int_{-\infty}^{\infty}d\varepsilon_{\mathbf{p}}F_{ij}(\mathbf{p},\omega_1),\qquad F(\omega_2)=\int_{-\infty}^{\infty}d\varepsilon_{\mathbf{q}}F(\mathbf{q},\omega_2),$$

and $F_{ij}(\mathbf{p},\omega_1)$ and $F(\mathbf{q},\omega_2)$ are Fourier components of the anomalous causal Green's functions $F_{ij}(\mathbf{p},t-t')$ and $F(\mathbf{q},t-t')$ of the left-handed and right-handed superconductors. They do not interact with each other, and they are in a state of thermal equilibrium at a temperature T.

The one-particle tunneling current is

$$I_0(V(t))=\frac{1}{4\pi^2eR}\int_{-\infty}^{\infty}d\varepsilon_{\mathbf{p}}\int_{-\infty}^{\infty}d\varepsilon_{\mathbf{q}}\int_{-\infty}^{\infty}d\omega\left(\mathrm{th}\frac{\omega}{2T}-\mathrm{th}\frac{\omega-eV(t)}{2T}\right)$$

$$\times\mathrm{Im}\sum_{i,j}G_{ij}^{R}(\mathbf{p},\omega)\,\mathrm{Im}\,G^{R}(\mathbf{q},\omega-eV(t)), \tag{8}$$

where $G^{R}_{ij}(\mathbf{p},\omega), G^{R}(\mathbf{q},\omega-eV(t))$ are Fourier components of the retarded Green's functions of left-handed and right-handed superconductors. The quantity R in (7) and (8) is the resistance of the junction in its normal state. Only a one-particle current I_0 flows through an *SIN* junction; the Josephson current is zero, $I_d = 0$ [since there are no anomalous functions $F(\mathbf{q},\omega)$ in a normal metal].

We calculate the current $I = I_0(V(t))$ in an *SIN* junction from the same expression, (8), after we replace the Green's function of the superconductor $(G^{R}\mathbf{q},\omega-eV(t))$

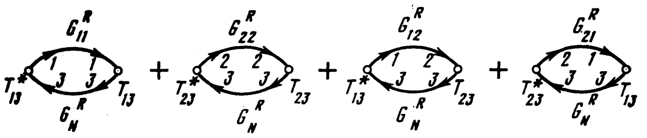

FIG. 3.

by that of the normal metal, $G^{R}_{N}(\mathbf{q},\omega-eV(t))$:

$$I(V)=\frac{1}{4\pi^2eR}\int_{-\infty}^{\infty}d\varepsilon_{\mathbf{p}}\int_{-\infty}^{\infty}d\varepsilon_{\mathbf{q}}\int_{-\infty}^{\infty}d\omega\left(\mathrm{th}\frac{\omega}{2T}-\mathrm{th}\frac{\omega-eV}{2T}\right)$$

$$\times\mathrm{Im}\sum_{i,j}G_{ij}^{R}(\mathbf{p},\omega)\,\mathrm{Im}\,G_{N}^{R}(\mathbf{q},\omega-eV). \tag{9}$$

In terms of polarization loops, the diagrams in Fig. 3 contribute to the current (9). The indices 1 and 2 are the indices of the bands of the semimetal; index 3 refers to the band of the normal (right-handed) metal.

In the case under study, $\Sigma = \Sigma^{s}_{\mathrm{Re}}$, the last two diagrams in Fig. 3 generate a current component which is linear in Σ, i.e., which depends on the sign of Σ. When the components Σ^{t}_{Re} and Σ^{t}_{Im} are nonzero these diagrams are identically zero by virtue of the spin structure of tunneling Hamiltonian (4) and because a normal metal (right-handed) has no magnetic expectation values $\langle T\psi_{3\uparrow}(x)\psi^{+}_{3\uparrow}(x')\rangle$. In the case $\Sigma = \Sigma^{s}_{\mathrm{Im}}$, the last two diagrams in Fig. 3 cancel out, since we have $G^{R}_{12}\sim i\Sigma^{s}_{\mathrm{Im}}, G^{R}_{21}\sim(i\Sigma^{s}_{\mathrm{Im}})^{*} = -i\Sigma^{s}_{\mathrm{Im}}$.

In the cases in which the states Σ^{t}_{Re}, Σ^{t}_{Im}, and Σ^{s}_{Im} are realized, there is thus no current component which is linear in Σ, and the entire one-particle tunneling current is determined by the first two diagrams in Fig. 3.

Anomalous functions $F_{ij}(\mathbf{p},\omega)$ can not be present in the expression for the one-particle tunneling current since the current is expressed in terms of expectation values of products of four operators, which include paired creation and annihilation operators. We convolve two of these four operators into the Green's function of the normal metal:

$$G_N^{\alpha\beta}(x,x')=\langle T\psi_{3\alpha}(x)\psi_{3\beta}^{+}(x')\rangle=G_N(x,x')\delta_{\alpha\beta}.$$

Using expressions (2) and the expression for the Green's function of a normal metal,

$$G_N^{R}(\mathbf{q},\omega-eV)=1/(\omega-eV-\varepsilon_{\mathbf{q}}+i0),$$

we can easily put expression (9) in the form

$$I(V)=\frac{1}{4eR}\int_{-\infty}^{\infty}d\varepsilon_{\mathbf{p}}\int_{-\infty}^{\infty}d\omega\left(\mathrm{th}\frac{\omega}{2T}-\mathrm{th}\frac{\omega-eV}{2T}\right)$$

$$\times[A_{\mathbf{p}}\delta(\omega-\omega_+)+B_{\mathbf{p}}\delta(\omega+\omega_+)+C_{\mathbf{p}}\delta(\omega-\omega_-)+D_{\mathbf{p}}\delta(\omega+\omega_-)], \tag{10}$$

where

$$\left.\begin{matrix}A_{\mathbf{p}}\\B_{\mathbf{p}}\end{matrix}\right\}=\frac{1}{2}\left(1+\frac{\Sigma}{E}\right)\left(1\pm\frac{\delta\mu+E}{\omega_+}\right),$$
$$\left.\begin{matrix}C_{\mathbf{p}}\\D_{\mathbf{p}}\end{matrix}\right\}=\frac{1}{2}\left(1-\frac{\Sigma}{E}\right)\left(1\pm\frac{\delta\mu-E}{\omega_-}\right). \tag{11}$$

At $T=0$, the expression for the current simplifies:

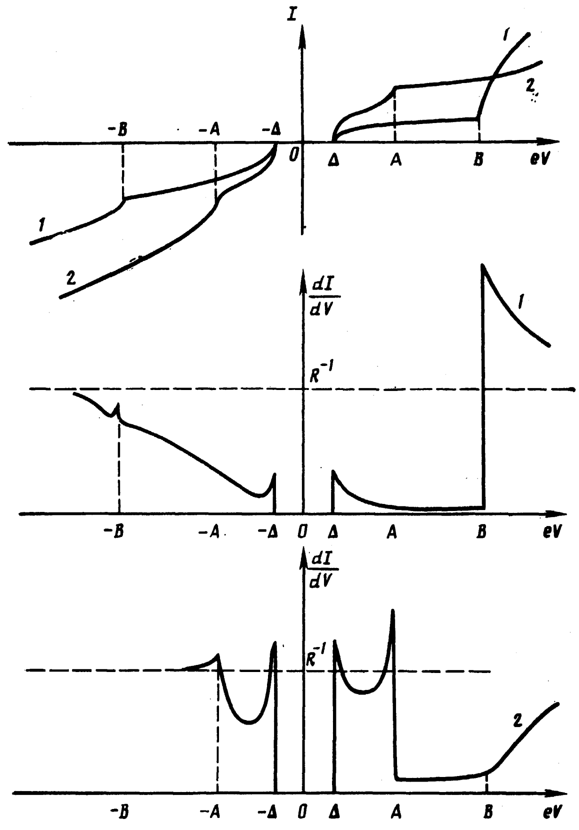

FIG. 4.

$$I(V)=-\frac{1}{2eR}\int\limits_{-\infty}^{\infty}d\varepsilon_p\int\limits_0^{eV}d\omega[A_p\delta(\omega-\omega_+)+B_p\delta(\omega+\omega_+)+C_p\delta(\omega-\omega_-)+D_p\delta(\omega+\omega_-)].$$

This characteristic is shown along with the differential conductivity $\sigma(V)=dI/dV$ in Fig. 4, where the curves 1 correspond to the case $\Sigma^s_{\rm Re}>0$, and the curves 2 to $\Sigma^s_{\rm Re}<0$.

We will also write analytic expressions for the differential conductivity $\sigma(V)$ near its singular points.

1. $\Sigma=\Sigma_1>0$.

a) $e|V|\to\Delta$:

$$\sigma(V)=\frac{1}{2R}\left(\frac{\delta\mu-\Sigma_1}{\delta\mu+\Sigma_1}\right)^{1/2}\left(\frac{2\Delta}{e|V|-\Delta}\right)^{1/2},$$

$$\left(\frac{e|V|}{\Delta}-1\right)^{1/2}\ll\frac{\delta\mu-\Sigma_1}{2\Delta}.$$

b) $eV\to\pm B$:

$$\sigma(V)=\frac{1}{2R}\left\{\left(\frac{B}{\delta\mu+\Sigma_1}\right)^{1/2}\pm\left(\frac{\delta\mu+\Sigma_1}{B}\right)^{1/2}\right\}\times\left(\frac{2\Sigma_1}{e|V|-B}\right)^{1/2},\quad e|V|-B\ll B.$$

2. $\Sigma=\Sigma_2<0$.

a) $e|V|\to\Delta$:

$$\sigma(V)=\frac{1}{2R}\left(\frac{\delta\mu+|\Sigma_2|}{\delta\mu-|\Sigma_2|}\right)^{1/2}\left(\frac{2\Delta}{e|V|-\Delta}\right)^{1/2},$$

$$\times\left(\frac{e|V|}{\Delta}-1\right)^{1/2}\ll\frac{\delta\mu-|\Sigma_2|}{2\Delta}.$$

b) $eV\to\pm A$:

$$\sigma(V)=\frac{1}{2R}\left[\left(\frac{A}{\delta\mu-|\Sigma_2|}\right)^{1/2}\pm\left(\frac{\delta\mu-|\Sigma_2|}{A}\right)^{1/2}\right]\left(\frac{2|\Sigma_2|}{A-e|V|}\right)^{1/2},\quad A-e|V|\ll A.$$

The current-voltage characteristics thus behave in different ways, depending on the sign of the insulating order parameter $\Sigma^s_{\rm Re}$. The physical reason for this result lies in the different structures of the wave functions of the elementary excitations for the different signs of Σ. This difference in the structure of the wave functions arises even in the insulating phase of the system, i.e., for $T>T_c$. To demonstrate the point, we note that the wave functions of the elementary excitations of an excitonic (or Peierls) insulator are

$$\psi_{1p}=u_p\varphi_{1p}+v_p\varphi_{2p},\quad \psi_{2p}=u_p\varphi_{2p}-v_p\varphi_{1p},$$

where $i=1,2$ are the indices of the band φ_{1p}, φ_{2p} are the wave functions of the elementary excitations of the regular phase (the semimetal), and u_p, v_p are the coefficients of a canonical Bogolyubov transformation. Here we have $\operatorname{sign}v_p=\operatorname{sign}\Sigma$, and we find that the wave functions of the elementary excitations have different structures for different signs of the order parameter Σ, with consequences for the magnitude of the tunneling current:

$$I(t)=-4e\,{\rm Re}\int\limits_{-\infty}^{t}dt'\sum_{p,q}\{|T_{pq}|^2(u_p+v_p)^2\times[\langle\alpha_{1p}^+(t)\alpha_{1p}(t')\rangle\langle c_q(t)c_q^+(t')\rangle-\langle c_q^+(t')c_q(t)\rangle\langle\alpha_{1p}(t')\alpha_{1p}^+(t)\rangle]+|T_{pq}|^2(u_p-v_p)^2[\langle\alpha_{2p}^+(t)\alpha_{2p}(t')\rangle\langle c_q(t)c_q^+(t')\rangle-\langle c_q^+(t')c_q(t)\rangle\langle\alpha_{2p}(t')\alpha_{2p}^+(t)\rangle]\}.$$

Here the operator α^+_{ip} creates an electron in the state ψ_{ip} in an excitonic insulator, and c^+_q does the same in a metal. The coherence factors $(u_p+v_p)^2,(u_p-v_p)^2$ are seen to cause different renormalizations of the transition matrix elements T_{pq} for the transition from a semimetal to a metal, depending on the sign of Σ. The further result is a difference between the current-voltage characteristics in the cases $\Sigma=\Sigma_1>0$ and $\Sigma=\Sigma_2<0$.

For *SIN* junctions the corresponding coherence factors A_p, B_p, C_p, D_p are given by (11); their dependence on the sign of Σ is obvious. The state density in the spectrum of elementary excitations of the *SD* phase (Fig. 2) diverges as $(|E-E_c|)^{1/2}$ at the points $E_c=\pm B$ (the branches $\pm\omega_+$) and $E_c=\pm A,\pm\Delta$ (the branches $\pm\omega_-$). The structure of the coherence factors (11) is such that the singularities in $\sigma(V)$ reproduce those in the state density of the spectrum of elementary excitations at the points $eV=\pm\Delta$, $\pm A$ in the case $\Sigma=\Sigma_2<0$ and at the points $eV=\pm\Delta$, $\pm B$ in the case $\Sigma=\Sigma_1>0$. The singularities at the spectral points $E_c=\pm B$ in the case $\Sigma=\Sigma_2<0$ and the points $E_c=\pm A$ in the case $\Sigma=\Sigma_1>0$ are suppressed by the coherence factors.

We recall that in states with nonzero $\Sigma^t_{\rm Re}$, $\Sigma^t_{\rm Im}$, and $\Sigma^s_{\rm Im}$, in contrast with the case $\Sigma=\Sigma^s_{\rm Re}$, the tunneling current (9) is determined exclusively by the first two diagrams in Fig. 3. The coherence factors do not have a linear dependence on Σ [they are found from (11) by eliminating the terms linear in Σ], and the characteristic of the differential tunneling conductivity $\sigma(V)$ immediately reproduces all the singularities in the state density of the spectrum (Fig. 2) at the points $e|V| = \Delta, A, B$, regardless of the sign of $\Sigma^{l}_{\rm Re}, \Sigma^{l}_{\rm Im}$, and $\Sigma^{s}_{\rm Im}$.

In the insulating phase of the system (i.e., for $T > T_c$), $\sigma(V)$ has singularities at the points $eV = \delta\mu - |\Sigma|, \delta\mu + |\Sigma|$, regardless of the sign of $\Sigma^{l}_{\rm Re}, \Sigma^{l}_{\rm Im}$, and $\Sigma^{s}_{\rm Im}$, in accordance with the spectrum of elementary excitations in the insulating phase (the solid lines in Fig. 2). In the case $\Sigma = \Sigma^{s}_{\rm Re}$ in the insulating phase, a singularity of $\sigma(V)$ exists only at the point $eV = \delta\mu - |\Sigma|$ in the case $\Sigma^{s}_{\rm Re} < 0$ or only at the point $eV = \delta\mu + |\Sigma|$ in the case $\Sigma^{s}_{\rm Re} > 0$.

To avoid any misunderstanding, however we should state that in using the tunneling Hamiltonian (4) we have assumed that the matrix elements $T_{\rm pq}$ are independent of the band indices i, j, and we have assumed $T_{{\rm pq}i} T^*_{{\rm pq}j} = |T_{\rm pq}|^2$ (in the single-band model, we have $T_{\rm pq} T^*_{\rm p+Q,q} = |T_{\rm pq}|^2$, correspondingly, where $\mathbf{Q}$ is a nesting vector). Actually, however, selection rules are imposed on the matrix elements $T_{\rm pq}$. They may cause the quantity $T_{\rm pq1} T^*_{\rm pq2}$ to vanish (or $T_{\rm pq} T^*_{\rm p+Q,q}$ in the single-band model) and thus cause the last two diagrams in Fig. 3 to vanish in the case $\Sigma = \Sigma^{s}_{\rm Re}$. In this case the tunneling current will be independent of the sign of $\Sigma^{s}_{\rm Re}$. This fact can be demonstrated most simply in the single-band model in the example of a tunnel junction between two identical crystal lattices with a $z = 0$ contact plane. In the expression

$$T_{\rm pq} = \langle \varphi_{\rm p}(\mathbf{r}) | H_{\rm tunn}(\mathbf{r} - \mathbf{r}') | \psi_{\rm q}(\mathbf{r}') \rangle$$

we expand the Wannier wave functions $\varphi_{\rm p}(\mathbf{r}), \psi_{\rm q}(\mathbf{r}')$ of the lattices on the right and left, and we take into account the overlap integrals only between nearest neighbors in the right and left lattices. We then find

$$T_{\rm pq} \sim T_0 \sum_{mn} \sum_{m'n'} \exp\{i(p_x an + p_y bm - q_x an' - q_y bm')\} \delta_{nn'} \delta_{mm'} = T_0 \delta_{p_x p_y} \delta_{q_x, q_y},$$

where T_0 is the matrix element of $H_{\rm tunn}(\mathbf{r} - \mathbf{r}')$ between Wannier functions of the nearest neighbors. Only transitions which conserve the longitudinal component of the quasimomentum are allowed. We thus have $T_{\rm pq} T^*_{\rm p+Q,q} \neq 0$ only for the nesting vector $Q = (0, 0, Q_z)$ which corresponds to a doubling in the direction perpendicular to the boundary of the tunnel junction. In the case of doubling along the boundary, the cases $\Sigma^{s}_{\rm Re} > 0$ and $\Sigma^{s}_{\rm Re} < 0$ are physically equivalent (a shift of half a period along the boundary causes one of these solutions to go over to the other). We thus have $T_{\rm pq} T^*_{\rm p+Q,q} = 0$; i.e., in this situation the tunneling current does not depend on the sign of $\Sigma^{s}_{\rm Re}$.

For the copper-oxide high-T_c superconductors, the structure of the wave functions sensitivity to the sign of $\Sigma^{s}_{\rm Re}$ may be manifested in, for example, a difference in the behavior of the electron density near the oxygen atoms O(2) and O(3), which lie in CuO_2 planes along respectively the a and b lattice axes. For one sign of Σ, there will be an increase in the electron density near O(2) atoms, while there will be a decrease near O(3) atoms; for the other sign of Σ, the O(2) and O(3) atoms trade roles.

The tilting of the oxygen octahedra in the La_2CuO_4-based high T_c superconductors, and also in the $YBa_2Cu_3O_{7-x}$ system, because of the presence of Cu–O chains running along the b axis, makes the O(2) and O(3) atoms nonequivalent even in the absence of insulating correlations. The result is a difference in the electron densities near the O(2) and O(3) atoms, due to an orthorhombic seed distortion of the crystal lattice, which gives rise to a seed insulating gap in the spectrum of elementary excitations.[13] This gap corresponds to an order parameter h which is unrelated to insulating correlations. The size of the resulting insulating gap in the excitation spectrum of the system, $|\Sigma + h|$, and thus the magnitude of the thermodynamic potential will differ for the different signs of Σ: The phase with the larger gap $|\Sigma + h|$ will correspond to a greater imbalance in the electron density near the O(2) and O(3) atoms and thus greater orthorhombic distortions. At temperatures $T < T_c$, the phase with the gap $|\Sigma + h|$ of greater magnitude will be realized, since it is preferable from the energy standpoint. For temperatures $T < T_c$, however, under the condition $|h| \ll |\Sigma|$, in a certain range of the doping n $(n_1 < n < n_2)$, the phase with the smaller insulating gap may be more favorable from the energy standpoint, since this phase is more effective in promoting superconductivity. Figure 5 shows the functional dependence $\Delta(n)$, where we are using the following notation:

$$n_2 = \Sigma_1{}^0 \Sigma_2{}^0 \frac{\beta_1 \beta_2}{\Sigma_2{}^0 \beta_2 - \Sigma_1{}^0 \beta_1}, \quad \beta_{1,2} = \ln \frac{\Sigma^0_{1,2}}{\Delta^0},$$

Δ^0 is the superconducting gap in the absence of a conversion to an insulating situation, $\tilde{\Sigma}^0_{1,2} = |\Sigma^0_{1,2} + h|$ is the insulating gap in the absence of superconductivity, λ_s is the Cooper interaction constant, $(n_1/n_2)^2 \ll 1$, and for definiteness we are assuming $|\Sigma_1 + h| < |\Sigma_2 + h|$. The benefit in terms of the "insulating" energy in the transition to the phase with the smaller insulating gap $|\Sigma + h|$ must be smaller than the benefit in terms of the "superconducting" energy, because the phase with the smaller value of $|\Sigma + h|$ corresponds to a larger value of Δ.

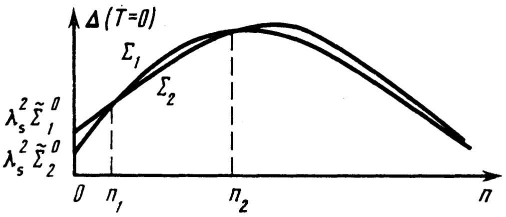

FIG. 5.

Because of the relation $|h| \ll |\Sigma|$, the transition from the phase with the larger insulating gap $|\Sigma + h|$ to that with the smaller one occurs near (below) T_c. We can make use of the fact to explain the origin of the large value of the tunneling gap in measurements on high T_c superconductors.

Specifically, if there is a phase $\Sigma_2 < 0$ (i.e., $|\Sigma_2 + h| > |\Sigma_1 + h|$) above T_c, and if a reduction of the temperature below T_c results in a transition to a phase $\Sigma_1 > 0$, then a peak will appear on the characteristic of the differential conductivity near (below) T_c at a finite voltage $V = \pm B/e$. The value $eV = B$ might be erroneously interpreted as the size of a superconducting gap, especially since the peak at the point $eV = B$ is significantly higher than that at the point $eV=\Delta$, which is the peak associated with the actual superconducting gap. At $T=0$, for example, we have

$$\frac{\sigma(B+e\delta V)}{\sigma(\Delta+e\delta V)} =\left(\frac{\Sigma}{\Delta}\right)^{1/2}\left\{\frac{\delta\mu+\Sigma_1}{\delta\mu-\Sigma_1}\right)^{1/2}\left\{\left(\frac{B}{\delta\mu+\Sigma_1}\right)^{1/2}+\left(\frac{\delta\mu+\Sigma_1}{B}\right)^{1/2}\right\}$$

$$\gg 1,\ \delta V\to 0,$$

as follows from the behavior of $\sigma(V)$ which we have already seen near its singular points at $T=0$.

The value of $2B/T_c$ may be significantly greater than $2\Delta/T_c$, which is 3.5 in the *SD* model under the condition $\lambda_s \ll 1$ (Ref. 9). Essentially all tunneling experiments[14–21] on $YBa_2Cu_3O_{7-x}$ have revealed values $2\Delta_{\text{tunn}}/T_c=4$–$13$; for $Bi_2Sr_2Cu_2CaO_x$, this value is 7 (Ref. 23).

A temperature dependence $\Delta_{\text{tunn}}(T)$ of the tunneling gap was found for $YBa_2Cu_3O_7$-oxide-In junctions in Ref. 18 with a transition temperature $T_c=87$ K in the interior of the $YBa_2Cu_3O_7$. A structural feature (a peak) on the characteristic of the differential conductivity, which Geerk *et al.*[18] associated with a superconducting gap, arises for $T<T_c$ at a finite voltage and undergoes essentially no shift along the voltage scale, increasing only in height, with a further lowering of the temperature.

The large scatter in the values of $2\Delta_{\text{tunn}}/T_c$ (from 4 to 13) in tunneling measurements on $YBa_2Cu_3O_{7-x}$ can be attributed to the scatter in the values of the insulating order parameter Σ (and thus in the values of A and B) in the barrier region of the junctions. This scatter in Σ is caused by a deficiency of oxygen in the separation region between the tunnel barrier and the interior of the $YBa_2Cu_3O_{7-x}$. This deficiency was not controllable in the experiments. Observation of the $\sigma(V)$ structural features associated with the conversion of the spectrum of excitations of the system to an insulating nature can provide information about the value of Σ in a surface region, with a size on the order of the insulating correlation length near the barrier. The insulating correlation length at $\Sigma>\Delta$ is smaller than the superconducting correlation length, which determines the region near the barrier. Information about it is contained in the structural features on $\sigma(V)$ associated with the superconducting gap. Consequently, variations in the composition of the system near the surface of the junction affect the structural features in $\sigma(V)$ which are associated with the conversion of the spectrum to an insulating nature much more strongly than they affect the structural features due to the superconducting gap. Here we have the reason for the strong dependence of the tunneling gap and thus the value of $2\Delta_{\text{tunn}}/T_c$ on the oxygen deficiency in the surface region of $YBa_2Cu_3O_{7-x}$.

The possibility of a transition near (below) T_c from a phase with a larger (in magnitude) insulating gap to one with a smaller one can also explain the nonmonotonic temperature dependence observed experimentally for the orthorhombic distortions near T_c in $YBa_2Cu_3O_{7-x}$ (Ref. 23). Specifically, the phase with the larger insulating gap should correspond to greter orthorhombic distortions, since in this phase, as was mentioned above, the disbalance in the electron densities near the O(2) and O(3) atoms is greater. The transition to the phase with the smaller insulating gap which results from the onset of superconductivity in the system will be accompanied by a decrease in the orthorhombic distortions.

We will also report the basic results of a calculation of the currents which flow through an *SIS* tunnel junction at $T=0$. According to (5)–(8), in this case we need to substitute in (2), (3), and the expressions for the Green's functions of an ordinary superconductor with a superconducting gap Δ_3:

$$G^R(\mathbf{q},\omega)=\frac{1}{2}\frac{1+\varepsilon_q/(\Delta_3^2+\varepsilon_q^2)^{1/2}}{\omega-(\Delta_3^2+\varepsilon_q^2)^{1/2}+i0}+\frac{1}{2}\frac{1-\varepsilon_q/(\Delta_3^2+\varepsilon_q^2)^{1/2}}{\omega+(\Delta_3^2+\varepsilon_q^2)^{1/2}+i0},$$

$$F(\mathbf{q},\omega)=\Delta_3/(\omega-(\Delta_3^2+\varepsilon_q^2)^{1/2}+i0)\,(\omega+(\Delta_3^2+\varepsilon_q^2)^{1/2}-i0).$$

For $\Sigma^s_{\text{Re}}=\Sigma_1>0$, the one-particle current $I_0(V)$ will have jumps, and the amplitude of the Josephson current will have Riedel structural features [a logarithmic divergence in $\operatorname{Re}I_c(V)$, and jumps in $\operatorname{Im}I_c(V)$] at the points $e|V|=\Delta+\Delta_3, B+\Delta_3$:

$$I_0(\Delta+\Delta_3+0)-I_0(\Delta+\Delta_3-0)=I_0(-\Delta-\Delta_3+0)-I_0(-\Delta-\Delta_3-0)$$
$$=\frac{\pi}{2eR}\left(\frac{\delta\mu-\Sigma_1}{\delta\mu+\Sigma_1}\right)^{1/2}(\Delta\Delta_3)^{1/2},$$
$$\operatorname{Re}I_c(V)=-\frac{(\Delta\Delta_3)^{1/2}}{2eR}\left(\frac{\delta\mu-\Sigma_1}{\delta\mu+\Sigma_1}\right)^{1/2}\ln|e|V|-(\Delta+\Delta_3)|,$$

where $|e|V|-(\Delta+\Delta_3)|\ll 2\Delta_3$,

$$\operatorname{Im}[I_c(\Delta+\Delta_3+0)-I_c(\Delta+\Delta_3-0)]$$
$$=\operatorname{Im}\{I_c(-\Delta-\Delta_3+0)-I_c(-\Delta-\Delta_3-0)\}$$
$$=\frac{\pi}{2eR}\left(\frac{\delta\mu-\Sigma_1}{\delta\mu+\Sigma_1}\right)^{1/2}(\Delta\Delta_3)^{1/2},$$
$$I_0(B+\Delta_3+0)-I_0(B+\Delta_3-0)=\frac{\pi}{eR}(\Sigma_1\Delta_3)^{1/2}$$
$$\times\left[\left(\frac{B}{\delta\mu+\Sigma_1}\right)^{1/2}+\left(\frac{\delta\mu+\Sigma_1}{B}\right)^{1/2}\right],$$
$$I_0(-B-\Delta_3+0)-I_0(-B-\Delta_3-0)$$
$$=\frac{\pi}{eR}(\Sigma_1\Delta_3)^{1/2}\left[\left(\frac{B}{\delta\mu+\Sigma_1}\right)^{1/2}-\left(\frac{\delta\mu-\Sigma_1}{B}\right)^{1/2}\right],$$
$$\operatorname{Re}I_c(V)=-\frac{(\Sigma_1\Delta_3)^{1/2}}{2eR}\frac{\Delta}{[B(\delta\mu+\Sigma_1)]^{1/2}}\ln|e|V|-(B+\Delta_3)|,$$

where $|e|V|-(B+\Delta_3)|\ll 2\Delta_3$,

$$\operatorname{Im}[I_c(B+\Delta_3+0)-I_c(B+\Delta_3-0)]$$
$$=\operatorname{Im}[I_c(-B-\Delta_3+0)-I_c(-B-\Delta_3-0)]$$
$$=\frac{\pi}{eR}\frac{\Delta}{[B(\delta\mu+\Sigma_1)]^{1/2}}(\Sigma_1\Delta_3)^{1/2}.$$

Under the condition $\Sigma^s_{\text{Re}}=\Sigma_2<0$, the one-particle current $I_0(V)$ will have jumps at the points $e|V|=\Delta+\Delta_3, A+\Delta_3$, and the amplitude of the Josephson current I_c will have Riedel structural features at the points $e|V|=\Delta+\Delta_3, A-\Delta_3$:

$$I_0(\Delta+\Delta_3+0)-I_0(\Delta+\Delta_3-0)=I_0(-\Delta-\Delta_3+0)-I_0(-\Delta-\Delta_3-0)$$
$$=\frac{\pi}{2eR}\left(\frac{\delta\mu+|\Sigma_2|}{\delta\mu-|\Sigma_2|}\right)^{1/2}(\Delta\Delta_3)^{1/2},$$
$$\operatorname{Re}I_c(V)=-\frac{(\Delta\Delta_3)^{1/2}}{2eR}\left(\frac{\delta\mu+|\Sigma_2|}{\delta\mu-|\Sigma_2|}\right)^{1/2}\ln|e|V|-(\Delta+\Delta_3)|,$$

where $||e|V| - (\Delta + \Delta_3)| \ll 2\Delta_3$,

$$\mathrm{Im}[I_c(\Delta+\Delta_3+0)-I_c(\Delta+\Delta_3-0)]$$
$$=\mathrm{Im}[I_c(-\Delta-\Delta_3+0)-I_c(-\Delta-\Delta_3-0)]$$
$$=\frac{\pi}{2eR}\left(\frac{\delta\mu+|\Sigma_2|}{\delta\mu-|\Sigma_2|}\right)^{1/2}(\Delta\Delta_3)^{1/2},$$

$$I_0(A+\Delta_3+0)-I_0(A+\Delta_3-0)=\frac{\pi}{eR}(|\Sigma_2|\Delta_3)^{1/2}\times\left[\left(\frac{A}{\delta\mu-|\Sigma_2|}\right)^{1/2}+\left(\frac{\delta\mu-|\Sigma_2|}{A}\right)^{1/2}\right];$$

$$I_0(-A-\Delta_3+0)-I_0(-A-\Delta_3-0)=\frac{\pi}{eR}(|\Sigma_2|\Delta_3)^{1/2}\left[\left(\frac{A}{\delta\mu-|\Sigma_2|}\right)^{1/2}-\left(\frac{\delta\mu-|\Sigma_2|}{A}\right)^{1/2}\right],$$

$$\mathrm{Re}\, I_c(V)=\frac{(|\Sigma_2|\Delta_3)^{1/2}}{2eR}\frac{\Delta}{[A(\delta\mu-|\Sigma_2|)]^{1/2}}\ln|e|V|-(A-\Delta_3)|,$$

where $||e|V| - (A - \Delta_3)| \ll 2\Delta_3$,

$$\mathrm{Im}[I_c(A-\Delta_3+0)-I_c(A-\Delta_3-0)]$$
$$=\mathrm{Im}[I_c(-A+\Delta_3+0)-I_c(-A+\Delta_3-0)]$$
$$=\frac{\pi}{eR}\frac{\Delta}{[A(\delta\mu-|\Sigma_2|)]^{1/2}}(|\Sigma_2|\Delta_3)^{1/2}.$$

For realizations of the insulating order parameters $\Sigma^{t}_{\mathrm{Re}}, \Sigma^{t}_{\mathrm{Im}}$ and Σ^{s}_{Im}, in contrast with the case $\Sigma = \Sigma^{s}_{\mathrm{Re}}$, the one-particle tunneling current $I_0(V)$ will have jumps at the points $e|V| = \Delta + \Delta_3, A + \Delta_3, B + \Delta_3$, while the amplitude of the Josephson current, $I_c(V)$, will have Riedel structural features at the points $e|V| = \Delta + \Delta_3$, $A - \Delta_3$, $B + \Delta_3$, simultaneously, regardless of the sign of Σ. The reason is that, as in the case of an *SIN* junction, the currents through an *SIS* junction in realizations of $\Sigma^{t}_{\mathrm{Re}}, \Sigma^{t}_{\mathrm{Im}}$, and Σ^{s}_{Im} are expressed in terms of only those Green's functions $G_{ii}(\mathbf{p},\omega)$ and $F_{ii}(\mathbf{p},\omega)$ of the left-handed superconductor (describable by the *SI* model) which are diagonal in terms of the band indices and which do not depend on the sign of Σ.

It can be seen from these results that the current-voltage characteristics of the one-particle tunneling current I_0 which is flowing through *SIN* and *SIS* tunnel junctions are asymmetric with respect to the sign of the voltage. Structural features appear in the state density at the points A and $-A$, B and $-B$ of extrema of the branches of the spectrum of the *SD* system (Fig. 2) in structural features on the current-voltage characteristics with different intensities. The reason lies in the different values of the residues on the ω_- and $-\omega_-$, ω_+ and $-\omega_+$ branches of the spectrum.

In connection with the possible formation of weak links between superconducting grains, note that tunneling measurements may reveal structural features on the current-voltage characteristics of an embedded *SIS* junction (and the right-handed and left-handed superconductors will be described by the *SI* model). In particular, a one-particle tunneling current flowing through a symmetric *SIS* junction in the case $\Sigma^{s}_{\mathrm{Re}} = \Sigma_1 > 0$ will have jumps at the points $eV = \pm(\Delta + B)$, of magnitude

$$\frac{2\pi}{eR}\left(\frac{\delta\mu-\Sigma_1}{\delta\mu+\Sigma_1}\right)^{1/2}\left(\frac{B}{\delta\mu+\Sigma_1}\right)^{1/2}(\Delta\Sigma_1)^{1/2}.$$

The current-voltage characteristic of a symmetric *SIS* junction is symmetric with respect to the voltage.

We note in conclusion that the manifestation of structural features in the quasiparticle state density at the points $eV = \pm 2\Delta$, $\pm(B + \Delta)$ in the case $\Sigma^{s}_{\mathrm{Re}} > 0$ and at the points $eV = \pm 2\Delta$, $\pm(A \pm \Delta)$ in the case $\Sigma^{s}_{\mathrm{Re}} < 0$ on the tunneling current-voltage characteristics of symmetric *SIS* junctions is analogous to the manifestation (studied in Ref. 24) of structural features in the quasiparticle state density in the IR spectra of a system with insulating and superconducting correlations.

[1] I. Affleck and J. B. Marston, Phys. Rev. B **37**, 3774 (1988).
[2] A. E. Ruckenstein, P. J. Hirshfeld, and J. Appel, Phys. Rev. B **36**, 857 (1987).
[3] M. Inui, S. Doniach, P. J. Hirshfeld, and A. E. Ruckenstein, Phys. Rev. B **37**, 2320 (1988).
[4] F. Herman, R. V. Kasowski, and W. Y. Hsu, Phys. Rev. B **36**, 6904 (1987).
[5] S. Massidda, J. Yu, A. J. Freeman, and D. D. Koelling, Phys. Lett. A **122**, 198 (1987).
[6] J. Yu, S. Massidda, A. J. Freeman, and D. D. Koelling, Phys. Lett. A **122**, 203 (1987).
[7] A. A. Hoffman Manuel and M. Peter, Europhys. Lett. **6**, 61 (1988).
[8] L. S. Smedskjaer, J. L. Liu, R. Benedec, *et al.*, Physica C **156**, 269 (1988).
[9] A. I. Rusinov, Do Chan Kat, and Yu. V. Kat Kopaev, Zh. Eksp. Teor. Fiz. **65**, 1984 (1973) [Sov. Phys. JETP **38**, 991 (1973)].
[10] A. A. Gorbatsevich, Yu. V. Kopaev, and V. V. Tugushev, Zh. Eksp. Teor. Fiz. **85**, 1107 (1983) [Sov. Phys. JETP **58**, 643 (1983)].
[11] I. O. Kulik and I. K. Yanson, *The Josephson Effect in Superconducting Tunnel Structures*, (Veter, Jerusalem, 1971) [in Russian], Nauka, Moscow, 1970, §2, 3.
[12] A. V. Svidzinskiĭ, *Spatially Nonuniform Problems in Superconductivity* [in Russian], Nauka, Moscow, 1982, §25.
[13] D. C. Mattis and M. P. Mattis, Phys. Rev. Lett. **59**, 2780 (1987).
[14] J. R. Kirtley, R. I. Collins, L. Schlesinger, *et al.*, Phys. Rev. B **35**, 8846 (1987).
[15] M. F. Crommie, L. C. Bourne, A. Zettle, *et al.*, Phys. Rev. B **35**, 8853 (1987).
[16] M. D. Kirk, D. P. E. Smith, D. B. Mitzi, *et al.*, Phys. Rev. B **35**, 8850 (1987).
[17] P. J. M. van Bentum, L. E. C. van de Leemput, L. W. M. Schreurs, *et al.*, Phys. Rev. B **36**, 843 (1987).
[18] J. Geerk, X. X. Xi, and G. Linker, J. Phys. B **37**, 329 (1988).
[19] I. Igichi, S. Narumi, Y. Kasai, and A. Sagishita, Physica B **148**, 322 (1987).
[20] M. Sera, S. Shamoto and M. Sato, Solid State Commun. **65**, 997 (1988).
[21] M. D. Kirk, S. B. Eom, B. Oh, *et al.*, Appl. Phys. Lett. **52**, 2071 (1988).
[22] M. Lee, D. B. Mitzi, A. Kapitulnic, and M. R. Beasely, Phys. Rev. B **39**, 801 (1989).
[23] P. M. Horn, D. T. Keane, G. A. Held, *et al.*, Phys. Rev. Lett. **59**, 2772 (1987).
[24] E. A. Zhukovskiĭ and Yu. V. Kopaev, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 284 (1989) [JETP Lett. **49**, 323 (1989)].

Translated by Dave Parsons