Stationary coherent states of atoms in resonant interaction with elliptically polarized light. Coherent trapping of populations (general theory)

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(Submitted 3 July 1989)
Zh. Eksp. Teor. Fiz. **96**, 1613–1628 (November 1989)

The problem of determining the kernel of the interaction operator for a resonant, elliptically polarized field, $\hat{V}\Psi = 0$ (stationary coherent states), is completely solved for arbitrary types of transitions $J_n \to J_m$. It is proven that it is precisely these states that are responsible for the coherent population trapping effect in the ground state of systems with optical pumping, the effect being responsible for the clearing (increase in transparency) of the medium. The possible types of transitions are classified on the basis of this effect. The feasibility of deep cooling of atomic gases in resonant polarized electromagnetic fields on clearing transitions is discussed.

INTRODUCTION

The problem of the interaction of resonant radiation with atoms is one of the central problems of quantum mechanics. The simplest case of a two-level system without degeneracy has been well investigated.[1] However, when the atomic levels are degenerate in the projections of the angular momentum, the problem becomes significantly more complicated and acquires a number of new qualitative features. In elliptically polarized light, for example, effects of the Zeeman coherence of the atomic sublevels become important and new frequencies appear in the dynamic Stark effect. As a consequence of the high rank of the system of coupled equations for the Zeeman sublevels, a general solution of the problem of the interaction of the atoms with elliptically polarized light has been found only for a few transitions with small angular momenta ($0 \leftrightarrow 1$, $1/2 \leftrightarrow 1/2$, $1/2 \leftrightarrow 3/2$, $1 \leftrightarrow 1$).[2–5]

For the particular cases of interaction with linearly or circularly polarized light, the exact solution is known and in essence consists of a set of solutions for independent two-level systems with their own Stark frequencies, as is clear from the example of the transitions $2 \to 1$ and $2 \to 2$ (Figs. 1 and 2). In addition, in these cases there exist special states (marked in Figs. 1 and 2 by asterisks) with are unaffected by the interaction with the external field and are, naturally, free of Stark splitting. The existence of such states in the interaction with elliptically polarized light is in general not obvious. For example, if the momentum quantization axis (the z axis) is chosen in the direction of propagation of the electromagnetic wave **k**, then all of the Zeeman sublevels interact with the external field. However, for some simple transitions ($1 \leftrightarrow 0$, $1 \leftrightarrow 1$, $3/2 \leftrightarrow 1/2$), where the configuration of the interacting sublevels has the form of a Λ- or V-system, such solutions have been found.[5] Therefore it is reasonable to pose the problem of finding the states that are free of Stark splitting for the transitions $J_n \to J_m$ with arbitrary values of the total angular momenta J_n and J_m in elliptically polarized light (see also Ref. 6). In the present article this problem is completely solved. In addition, it is shown that it is specifically these found states that are responsible for the so-called effect of coherent capture (trapping) of populations in systems with optical pumping.[7–9] Applications of this effect to problems of clearing and cooling of atomic gases are also discussed.[4,10,11]

1. STATIONARY COHERENT STATES (THE PROBLEM OF FINDING THE KERNEL OF THE INTERACTION OPERATOR)

Before going on to the formulation of the problem, it should be noted that the results of this part of the paper are equivalent in part to the results obtained in Ref. 6, where the treatment is based on some different methodological assumptions and is of a more abstract, group-theoretic nature. However, in our opinion, it is meaningful to consider individually the actual situation which arises in practice. This allows us to classify the atomic transitions in a specific way and to carry out an interpretation and formulation of the various experiments on the basis of the same assumptions. In addition, a number of new results are obtained, which are not to be found in Ref. 6, about which we will speak in more detail at the conclusion of each section (see "Properties of the SCS").

Formulation of the problem

Let there be a two-level system degenerate in the magnetic sublevels with energies $E_n(J_n)$ and $E_m(J_m)$ ($E_n < E_m$; J_n and J_m are the total angular momenta of the corresponding levels) and with a complete set of orthogonal wave functions $\{\exp(-iE_n t/\hbar)\psi_k^n\}$ ($|k| \leqslant J_n$) and $\{\exp(iE_n t/\hbar)\psi_j^m\}$ ($|j| \leqslant J_m$]). Interacting with the field is a harmonic external field

$$\mathbf{E} = E_0 \mathbf{e} e^{-i\omega t} + \text{c.c.}, \qquad (1)$$

where **e** is an arbitrary complex unit vector of elliptical polarization. We choose the quantization axis (z) to be orthogonal to **e** (along the wave vector **k**). Then we have the following expansion in cyclic unit vectors:

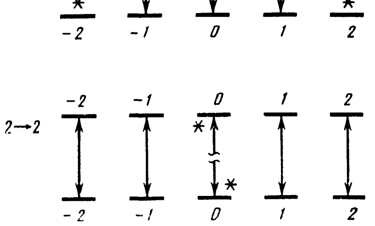

FIG. 1. Diagram of the stimulated transitions in linearly polarized light.

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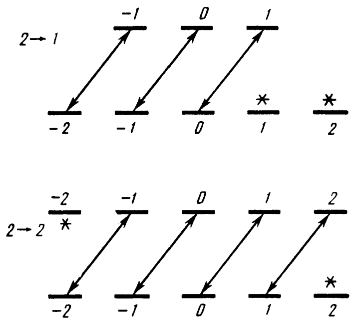

FIG. 2. Diagram of the stimulated transitions in circularly polarized light.

$$\mathbf{E}=E_0(q_+\mathbf{e}_+ + q_-\mathbf{e}_-)e^{-i\omega t}+\text{c.c.} \tag{2}$$

In what follows, unless otherwise specified, we will always work in this coordinate system. In the dipole approximation of the interaction $\hat{V}= -\mathbf{E}\cdot\mathbf{d}$ the following form of the operator $\hat{V}$ follows from Eq. (2):

$$\hat{V}=E_0(q_+\hat{d}_- + q_-\hat{d}_+)e^{-i\omega t}+\text{h.c.} \tag{3}$$

The components $q_\pm$ are connected with the elliptical light ε in the following way:

$$q_+=e^{-i\varphi}\sin(\varepsilon+\pi/4), \quad q_-=e^{i\varphi}\cos(\varepsilon+\pi/4), \tag{4}$$

$\tan|\varepsilon|$ $(-\pi/4\leqslant\varepsilon\leqslant\pi/4)$ is equal to the ratio of the minor and major axes of the ellipse, the sign of ε is determined by the direction of rotation, and φ is the angle between the major axis of the ellipse and the positive direction of the x axis.

Neglecting relaxation processes, the problem of the interaction of the electromagnetic field with the atom reduces to the solution of the nonstationary Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi=(\hat{H}_0+\hat{V})\Psi \tag{5}$$

by a perturbation theory method. In our case the solution is sought in the form

$$\Psi=\exp\left(-i\frac{E_m}{\hbar}t\right)\times\sum_{|j|\leqslant J_m}a_j(t)\psi_j^m+\exp\left(-i\frac{E_n}{\hbar}t\right)\sum_{|k|\leqslant J_n}b_k(t)\psi_k^n, \tag{6}$$

where the amplitude $a_j(t)$ corresponds to the upper level (E_m), and the amplitude $b_k(t)$ corresponds to the lower level (E_n). Then Eq. (5) reduces in the resonant approximation, taking Eq. (6) into account, to the following system with initial conditions $\{a_j(0), b_k(0)\}$:

$$i\hbar\dot{a}_j(t)=E_0e^{-i\delta t}[q_-V^{mn}_{j,j-1}b_{j-1}(t)+q_+V^{mn}_{j,j+1}b_{j+1}(t)], \quad |j|\leqslant J_m,$$

$$i\hbar\dot{b}_k(t)=\overset{*}{E}_0e^{i\delta t}[\overset{*}{q}_+V^{nm}_{k,k-1}a_{k-1}(t)+\overset{*}{q}_-V^{nm}_{k,k+1}a_{k+1}(t)], \quad |k|\leqslant J_n, \tag{7}$$

$$V^{mn}_{j,j-1}=(-1)^{J_m-j}\begin{pmatrix}J_m & 1 & J_n\\ -j & 1 & j-1\end{pmatrix}\langle J_m\|\hat{\mathbf{d}}\|J_n\rangle,$$

$$V^{mn}_{j,j+1}=(-1)^{J_m-j}\begin{pmatrix}J_m & 1 & J_n\\ -j & -1 & j+1\end{pmatrix}\langle J_m\|\hat{\mathbf{d}}\|J_n\rangle,$$

$$V^{nm}_{k,j}=\overset{*}{V}{}^{mn}_{j,k} \quad (\forall j,k), \quad \delta=\omega-(E_m-E_n)/\hbar,$$

where $(a/d, b/e, c/f)$ is the $3j$-symbol.[12] The general solution of Eq. (7) can be written in the form

$$a_j(t)=\sum_p\exp[i(\lambda_p-\delta)t]a_j^p, \quad b_k(t)=\sum_p\exp(i\lambda_pt)\,b_k^p. \tag{8}$$

Substituting expressions (8) into Eqs. (7), we easily obtain the characteristic equation, whose roots are λ_p. It is clear that the number of roots is determined by the rank of system (7), i.e., it is equal to $2(J_m+J_n+1)$. In general, finding all the solutions in analytic form is impossible. However, it is possible to exactly solve the problem of finding the kernel of the interaction operator[6]:

$$\operatorname{Ker}\hat{V}:\ \hat{V}\Psi=0 \tag{9}$$

(we are talking here, of course, of the resonant approximation). Vector equation (9), as applied to system of differential equations (7), is transformed into a system of linear homogeneous equations with constant coefficients

$$q_-V^{mn}_{j,j-1}b^0_{j-1}+q_+V^{mn}_{j,j+1}b^0_{j+1}=0, \quad |j|\leqslant J_m,$$

$$\overset{*}{q}_+V^{nm}_{k,k-1}a^0_{k-1}+\overset{*}{q}_-V^{nm}_{k,k+1}a^0_{k+1}=0, \quad |k|\leqslant J_n, \tag{10}$$

$$\partial b_k^0/\partial t=\partial a_j^0/\partial t=0 \quad (\forall k,j).$$

[the superscript 0 in a_j^0 and b_k^0 refers to Eq. (9)].

We will call nontrivial solutions of Eq. (9) stationary coherent states (SCS) [since $\partial b_k^0/\partial t=\partial a_j^0/\partial t=0$ in Eq. (10)], i.e., $\hat{V}\Psi_{\text{SCS}}=0$. In fact, the Stark-unbroadened states mentioned in the Introduction are in fact solutions of Eq. (9). The remainder of our analysis will be directed precisely to the search for such SCS. It will be advantageous to carry out this analysis for each of the types of transitions individually.

1. The transition $J\to J-1$ ($J_n=J$, $J_m=J-1$)

As can be clearly seen from Fig. 3 in the instance of the transitions $2\to1$ and $5/2\to3/2$, there are two independent systems of interacting sublevels (SIS), each of which consists of Λ-links. One of them begins from the sublevel ψ_J^n (solid line), the other, from the sublevel ψ^n_{-J+1} (dashed line). The number of equations for the amplitudes b_k^0 in Eq.

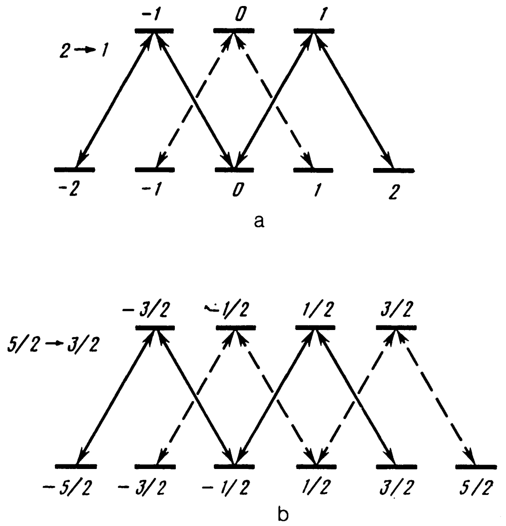

FIG. 3.

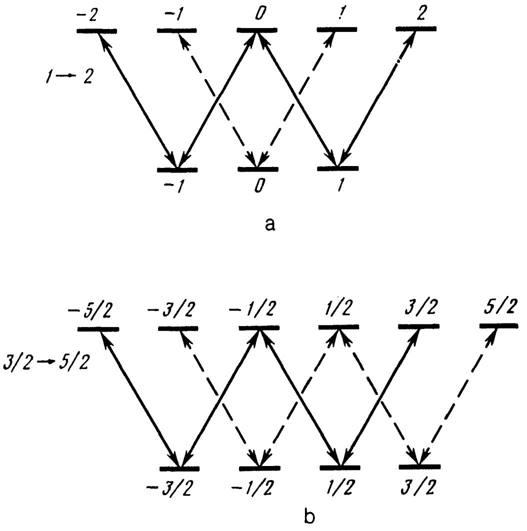

FIG. 4.

(10) is one less for those same b_k^0 for each of the indicated SIS, wherefore the recursion system (10) always has two nontrivial solutions:

$$b^0_{-J}=C_1,$$

$$b^0_{-J+2l}=C_1(-1)^l\left(\frac{q_-}{q_+}\right)^l$$

$$\times\prod_{f=1}^{l}\frac{V^{mn}_{-J-1+2f,\,-J-2+2f}}{V^{mn}_{-J-1+2f,\,-J+2f}},\quad a^0_{-J+1+2l}=0\quad(l\geqslant 0),\tag{11a}$$

$$b^0_{-J+1}=C_2,$$

$$b^0_{-J+1+2l}=C_2(-1)^l\left(\frac{q_-}{q_+}\right)^l$$

$$\times\prod_{f=1}^{l}\frac{V^{mn}_{-J+2f,\,-J-1+2f}}{V^{mn}_{-J+2f,\,-J+1+2f}},\quad a^0_{-J+2+2l}=0\quad(l\geqslant 0),\tag{11b}$$

$$\prod_{1}^{0}\ldots\equiv 1.$$

The solutions of Eq. (9) then have the form

$$\Psi^{(1)}_{\rm SCS}=\exp\left(-i\frac{E_n}{\hbar}t\right)\sum_{l=0}^{N_1}b^0_{-J+2l}\psi^n_{-J+2l},\tag{12a}$$

$$\Psi^{(2)}_{\rm SCS}=\exp\left(-i\frac{E_n}{\hbar}t\right)\sum_{l=0}^{N_2}b^0_{-J+1+2l}\psi^n_{-J+1+2l},\tag{12b}$$

i.e., $\Psi^{(1)}_{\rm SCS}$ and $\Psi^{(2)}_{\rm SCS}$ are in the lower state (E_n). The numbers N_1 and N_2 are found from the condition that the sublevels ψ''_{-J+2N_1} and $\psi''_{-J+1+2N_2}$ are the last of the corresponding SIS. We find then that

$$N_1=\begin{cases}J, & J\text{ — integer}\\ J-{}^1/_2, & J\text{ — half-integer;}\end{cases}$$

$$N_2=\begin{cases}J-1, & J\text{ — integer}\\ J-{}^1/_2, & J\text{ — half-integer.}\end{cases}\tag{13}$$

The constants C_1 and C_2 in Eqs. (11a) and (11b) are determined from the initial conditions. It would seem that to find them it is necessary to know the complete set of all other (nonstationary) solutions of the system (7). However, this is not so. Let us consider, for example, the SIS associated with the sublevel ψ''_{-J} (the solid line in Fig. 3). As was already noted, the number of upper sublevels in it is one less than the number of lower sublevels, wherefore there exists a definite linear combination of equations in system (7) for $b_k(t)$ in which we set the right side equal to zero:

$$i\hbar\frac{\partial}{\partial t}\left[\sum_{l=0}^{N_1}K^{(1)}_{-J+2l}b_{-J+2l}(t)\right]=0,\tag{14}$$

i.e., system (7) has the integral of motion

$$\sum_{l=0}^{N_1}K^{(1)}_{-J+2l}b_{-J+2l}(t)=\text{inv}=I_1.\tag{15}$$

The value of the invariant is determined from the initial conditions:

$$I_1=\sum_{l=0}^{N_1}K^{(1)}_{-J+2l}b_{-J+2l}(0).\tag{16}$$

The coefficients $K^{(1)}_{-J+2l}$ can be easily found by successively setting the coefficients of $a_j(t)$ equal to zero on the right-hand side of the equations of system (7):

$$K^{(1)}_{-J+2l}=(-1)^l(\overset{*}{q}_-)^l(\overset{*}{q}_+)^{N_1-l}$$

$$\times\left(\prod_{u=0}^{l-1}V^{nm}_{-J+2u,\,-J+1+2u}\right)\left(\prod_{g=l+1}^{N_1}V^{nm}_{-J+2g,\,-J-1+2g}\right)\tag{17}$$

(here it is implicit that $\Pi_{u=0}^{l-1}\ldots=\Pi_{g=l+1}^{N_1}\ldots\equiv 1$). Substituting the general form of solution (8) into expression (15), we have

$$\sum_{l=0}^{N_1}K^{(1)}_{-J+2l}b_{-J+2l}(t)=\sum_{l=0}^{N_1}K^{(1)}_{-J+2l}\left(\sum_p\exp(i\lambda_pt)\,b^p_{-J+2l}\right)$$

$$=\sum_p\exp(i\lambda_pt)\left(\sum_{l=0}^{N_1}K^{(1)}_{-J+2l}b^p_{-J+2l}\right)=I_1=\text{inv}.\tag{18}$$

Since I_1 is in general not equal to zero, we can draw the following conclusions from Eq. (18): a) for $\lambda_p\neq 0$ it follows that

$$\sum_{l=0}^{N_1}K^{(1)}_{-J+2l}b^p_{-J+2l}=0;\tag{19}$$

b) the root $\lambda_0=0$, i.e., a stationary solution of system (7), necessarily exists (in the opposite case it is impossible to have $I_1\neq 0$).

It is perfectly obvious that the above solution (11a) corresponds to this value $\lambda_0=0$. Thus we obtain

$$\sum_{l=0}^{N_1}K^{(1)}_{-J+2l}b^0_{-J+2l}=I_1.\tag{20}$$

Substituting solution (11a) here, taking into account the fact that $V^{nm}_{k,j}=V^{mn}_{j,k}$, after some uncomplicated transformations we find

$$C_1=I_1(q_+)^{N_1}\prod_{f=1}^{N_1}V^{mn}_{-J-1+2f,-J+2f}\Big/\sum_{l=0}^{N_1}|q_-|^{2l}|q_+|^{2(N_1-l)}\left(\prod_{u=0}^{l-1}|V^{mn}_{-J+1+2u,-J+2u}|^2\right)\times\left(\prod_{g=l+1}^{N_1}|V^{mn}_{-J-1+2g,-J+2g}|^2\right),\tag{21}$$

and the total population associated with the state $\Psi^{(1)}_{\text{SCS}}$ (12a) is

$$\rho^{(1)}_{\text{SCS}}=\sum_{l=0}^{N_1}|b^0_{-J+2l}|^2=|I_1|^2\Big/\sum_{l=0}^{N_1}|q_-|^{2l}\times|q_+|^{2(N_1-l)}\left(\prod_{u=0}^{l-1}|V^{mn}_{-J+1+2u,-J+2u}|^2\right)\times\left(\prod_{g=l+1}^{N_1}|V^{mn}_{-J-1+2g,-J+2g}|^2\right).\tag{22}$$

After carrying out a completely analogous procedure for the SIS associated with the sublevel ψ^n_{-J+1} (the dashed line in Fig. 3), we find

$$\sum_{l=0}^{N_2}K^{(2)}_{-J+1+2l}b_{-J+1+2l}(t)=\text{inv}=I_2=\sum_{l=0}^{N_2}K^{(2)}_{-J+1+2l}b_{-J+1+2l}(0),\tag{23}$$

where

$$K^{(2)}_{-J+1+2l}=(-1)^l(\overset{*}{q}_-)^l(\overset{*}{q}_+)^{N_2-l}\times\left(\prod_{u=0}^{l-1}V^{nm}_{-J+1+2u,-J+2+2u}\right)\left(\prod_{g=l+1}^{N_2}V^{nm}_{-J+1+2g,-J+2g}\right).\tag{24}$$

We also find that

$$\sum_{l=0}^{N_2}K^{(2)}_{-J+1+2l}b^0_{-J+1+2l}=I_2.\tag{25}$$

Substituting expressions (11b) here, we obtain

$$C_2=I_2(q_+)^{N_2}\prod_{f=1}^{N_2}V^{mn}_{-J+2f,-J+1+2f}\Big/\sum_{l=0}^{N_2}|q_-|^{2l}|q_+|^{2(N_2-l)}\times\left(\prod_{u=0}^{l-1}|V^{mn}_{-J+2+2u,-J+1+2u}|^2\right)\left(\prod_{g=l+1}^{N_2}|V^{mn}_{-J+2g,-J+1+2g}|^2\right),\tag{26}$$

and the population associated with the state $\Psi^{(2)}_{\text{SCS}}$ (12b) is

$$\rho^{(2)}_{\text{SCS}}=\sum_{l=0}^{N_2}|b^0_{-J+1+2l}|^2=|I_2|^2\Big/\sum_{l=0}^{N_2}|q_-|^{2l}|q_+|^{2(N_2-l)}\times\left(\prod_{u=0}^{l-1}|V^{mn}_{-J+2+2u,-J+1+2u}|^2\right)\left(\prod_{g=l+1}^{N_2}|V^{mn}_{-J+2g,-J+1+2g}|^2\right).\tag{27}$$

2. The transition $J-1\to J$ ($J_n=J-1$, $J_m=J$)

This case is completely equivalent to the above-considered case ($J\to J-1$). Only here the SIS consists of V-links (Fig. 4). Therefore the results obtained in the previous section can be carried over here by making the necessary transformations. Thus, for the SIS associated with ψ^m_{-J} (the solid line in Fig. 4), we have the solution of system (10):

$$a^0_{-J}=C_1,\quad a^0_{-J+2l}=C_1(-1)^l\left(\frac{\overset{*}{q}_+}{\overset{*}{q}_-}\right)^l\prod_{f=1}^{l}\frac{V^{nm}_{-J-1+2f,\,-J-2+2f}}{V^{nm}_{-J-1+2f,\,-J+2f}},$$
$$b^0_{-J+1+2l}=0\quad(l\geqslant 0),\tag{28}$$

$$\Psi^{(1)}_{\text{SCS}}=\exp\left(-i\frac{E_m}{\hbar}t\right)\sum_{l=0}^{N_1}a^0_{-J+2l}\psi^m_{-J+2l},$$

and for the SIS associated with ψ^m_{-J+1} (the dashed line in Fig. 4), we find

$$a^0_{-J+1}=C_2,$$
$$a^0_{-J+1+2l}=C_2(-1)^l\left(\frac{\overset{*}{q}_+}{\overset{*}{q}_-}\right)^l\prod_{f=1}^{l}\frac{V^{nm}_{-J+2f,\,-J-1+2f}}{V^{nm}_{-J+2f,\,-J+1+2f}},$$
$$b^0_{-J+2+2l}=0\quad(l\geqslant 0),\tag{29}$$

$$\Psi^{(2)}_{\text{SCS}}=\exp\left(-i\frac{E_m}{\hbar}t\right)\sum_{l=0}^{N_2}a^0_{-J+1+2l}\psi^m_{-J+1+2l}.$$

Here $\Psi^{(1)}_{\text{SCS}}$ and $\Psi^{(2)}_{\text{SCS}}$ are found in the upper state (E_m). For N_1 and N_2, see Eqs. (13).

The determination of the constants C_1 and C_2 from the initial conditions with the help of the corresponding invariants is completely analogous to the case of the transition $J\to J-1$, which was analyzed in detail above.

3. The transition $J\to J$ (J-integer)

In this case, as is clearly obvious from Fig. 5 in the example of the transition $2\to 2$, there are two SIS. One of them, associated with the lower sublevel ψ^n_{-J} (the solid line in Fig. 5), consists of Λ-links, which is equivalent to the situation with the transition $J\to J-1$. For this SIS it is effortless to find nontrivial solutions of system (10):

$$b^0_{-J}=C_1,$$
$$b^0_{-J+2l}=C_1(-1)^l\left(\frac{q_-}{q_+}\right)^l\prod_{f=1}^{l}\frac{V^{mn}_{-J-1+2f,\,-J-2+2f}}{V^{mn}_{-J-1+2f,\,-J+2f}},$$
$$a^0_{-J+1+2l}=0\quad(l\geqslant 0),$$
$$\Psi^{(1)}_{\text{SCS}}=\exp\left(-i\frac{E_n}{\hbar}t\right)\sum_{l=0}^{J}b^0_{-J+2l}\psi^n_{-J+2l}.\tag{30}$$

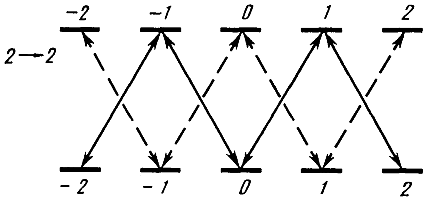

FIG. 5.

For the SIS associated with the upper sublevel ψ^m_{-J} (the dashed line in Fig. 5) and consisting of V-links, in analogy with the transition $J-1\rightarrow J$ we find

$$a^0_{-J}=C_2,$$

$$a^0_{-J+2l}=C_2(-1)^l\left(\frac{\overset{*}{q}_+}{\overset{*}{q}_-}\right)^l\prod_{f=1}^{l}\frac{V^{nm}_{-J-1+2f,\,-J-2+2f}}{V^{nm}_{-J-1+2f,\,-J+2f}},\qquad(31)$$

$$b^0_{-J+2l}=0\quad(l\geqslant 0),$$

$$\Psi^{(2)}_{\rm SCS}=\exp\left(-i\frac{E_m}{\hbar}t\right)\sum_{l=0}^{J}a^0_{-J+2l}\psi^m_{-J+2l}.$$

Thus we have that $\Psi^{(1)}_{\rm SCS}$ is in the lower state (E_n), and $\Psi^{(2)}_{\rm SCS}$ is in the upper state (E_m). The determination of C_1 and C_2 from the initial conditions is carried out in analogy with the above-considered cases.

4. The transition $J\rightarrow J$ (J-half-integer)

As can be seen from Fig. 6a (the transition $5/2\rightarrow 5/2$), the SIS in this case do not have the configurations of either Λ- or V-link chains, as was the case with the above-considered transitions. For each SIS the number of upper sublevels coincides with the number of lower sublevels, i.e., the matrices both for a^0_j and for b^0_k in the system of homogeneous equations (10) are quadratic, wherefore in general system (10) does not have any nontrivial solutions. The one exception is the case of circularly polarized light (Fig. 6b). Here we have

$$\Psi^{(1)}_{\rm SCS}=C_1\exp(-iE_nt/\hbar)\psi_J{}^n,\qquad \Psi^{(2)}_{\rm SCS}=C_2\exp(-iE_mt/\hbar)\psi_{-J}{}^m,$$

or (for the opposite polarization)

$$\Psi^{(1)}_{\rm SCS}=C_1\exp(-iE_nt/\hbar)\psi_{-J}{}^n,\qquad \Psi^{(2)}_{\rm SCS}=C_2\exp(-iE_mt/\hbar)\psi_J{}^m.$$

Properties of the SCS

The above analysis allows us to unambiguously assert that we have completely solved the problem of finding the kernel of the interaction operator (9) for degenerate (in the projections of their angular momenta) atoms with arbitrary values of J_n and J_m in an elliptically polarized resonant field. The above-found SCS possess a number of remarkable properties which fundamentally distinguish them from the remaining set of perturbed wave functions of the "dressed" atom.[13] We list these properties.

1. $\Psi^{(i)}_{\rm SCS}$ ($i=1,2$) has no dependence on the detuning δ or the intensity of the external field $|E_0|^2$, i.e., these states are not Stark broadened, and an atom in such SCS possesses an unperturbed energy (E_n and E_m).

2. The stationary coherent states are linearly independent and are coherent mixtures of unperturbed Zeeman states with coefficients a^0_j and b^0_k which depend only on the ellipticity of the external field ε and the angular characteristics of the two-level system J_n and J_m.

3. The problem of finding the SCS is completely decoupled from the problem of finding the remaining nonstationary solutions of system (7) even when determining the corresponding constants from the initial conditions. At the same time, the already known solutions $\Psi^{(1)}_{\rm SCS}$ and $\Psi^{(2)}_{\rm SCS}$ lower the rank of the system by 2, which can be useful in finding nonstationary solutions of system (7) (e.g., by using Eq. (19), which is valid for all $\lambda_p\neq 0$).

As is well known, the usual nonstationary solutions of the problem determine the oscillations (with the Rabi frequencies) in the probabilities of finding the atom in the interacting levels. In contrast with this, the physical meaning of the SCS is connected with the "capture" of the population in a certain level. The other level in this process plays the role of a virtual level. The existence of SCS leads to an effective decrease of the interaction of light with the atoms, and the found values of $p^{(i)}_{\rm SCS}$ [see, e.g., Eqs. (22) and (27)] also determine that part of the total population that is excluded from the interaction. In this regard, it would appear that a reconsideration of the concept of the "dressed" atom in the field of the resonant polarized radiation is required since in general we can only talk of a "partially dressed" atom.

It should also be noted that we have focused our attention on the problem of the interaction of polarized radiation with a two-level, degenerate (in the magnetic sublevels) system only for definiteness. In principle, such SCS can arise for a wider class of problems in the case of field-induced coherence and for independent levels[6–9] (e.g., for irradiation of the atoms by a multifrequency field, when coherence is induced between the fine and hyperfine components of the atomic levels). But in all these cases the recipe for finding the SCS remains the same—the search for stationary solutions of the nonstationary Schrödinger equation (5).

The above analysis, as was already noted at the beginning of this paper, leads to results which are equivalent to those obtained in Ref. 6, where the treatment proceeds from a quite general position. However, in Ref. 6, no determination was made of the invariants, such as (15) and (23), which allow the problem of finding the kernel of the interaction operator (9) to become completely autonomous, and, starting from the initial conditions, to calculate that part of the total population that is excluded from the interaction (22), (27). For the nonstationary solutions the general inequality (19) is valid. In addition, the concrete approach makes possible a customized classification of actual atomic transitions in terms of the nontrivial solutions of Eq. (9), which can be extremely useful in the interpretation of various experiments.

The above-found SCS play a still more fundamental role in systems with optical pumping associated with pro-

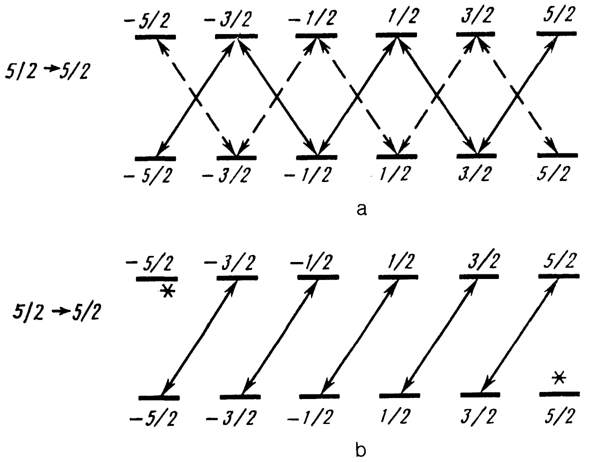

FIG. 6.

cesses of radiative relaxation in an atomic gas. The next section of this paper is dedicated to a consideration of this question.

2. COHERENT TRAPPING OF POPULATIONS IN SYSTEMS WITH OPTICAL PUMPING

Formulation of problem

In the preceding section we considered the interaction of polarized radiation with the atomic structure, neglecting processes of radiative relaxation. Such an approach is valid, for example, in the case of propagation of short polarized pulses. However, in problems of optical pumping, where the interaction time of the light with the atoms is quite large, the role of relaxation becomes fundamental.

Let us consider the interaction of elliptically polarized resonant radiation (1) with a two-level system when the lower level E_n is the ground state ($\gamma_n = 0$). Neglecting all collisions (the case of a rarefied gas), we take as the main relaxation mechanism the radiative mechanism that leads to spontaneous emission from the excited state E_m. In contrast with the previous section, in the description of the interaction we will use the density-matrix formalism, for which we have the formal operator equation

$$i\hbar \frac{\partial}{\partial t}\hat{\rho} + \hat{\Gamma}\hat{\rho} = [\hat{H}_0 + \hat{V}, \hat{\rho}], \qquad (32)$$

where the operator $\hat{\Gamma}$ describes the relaxation processes. The density matrix formalism has been quite well developed in various articles.[14-16] In our case of the two-level system $E_n(J)$, $E_m(J)$ in the basis of the wave functions of the unperturbed Hamiltonian $\hat{H}_0$ $\{\exp(-iE_n t/\hbar)\psi_k^n\}$ ($|k| \leqslant J_n$) and $\{\exp(-iE_m t/\hbar)\psi_j^m\}$ ($|j| \leqslant J_m$), taking into account expansions (2) and (3), Eq. (32) for the components of the density matrix takes the following form:

$$\begin{aligned}
&(\partial/\partial t - i\delta + \gamma_m/2)\rho_{j,k}^{mn}\\
&\quad = -iE_0(q_- V_{j,j-1}^{mn}\rho_{j-1,k}^{nn} + q_+ V_{j,j+1}^{mn}\rho_{j+1,k}^{nn} - \rho_{j,k-1}^{mm} q_+ V_{k-1,k}^{mn}\\
&\quad - \rho_{j,k+1}^{mm} q_- V_{k+1,k}^{mn}),\\
&(\partial/\partial t + \gamma_m)\rho_{j,j'}^{mm}\\
&\quad = -iE_0(q_- V_{j,j-1}^{mn}\rho_{j-1,j'}^{nm} + q_+ V_{j,j+1}^{mn}\rho_{j+1,j'}^{nm}) + iE_0(\rho_{j,j'-1}^{mn}\dot{q}_- V_{j'-1,j'}^{nm}\\
&\quad + \rho_{j,j'+1}^{mn}\dot{q}_+ V_{j'+1,j'}^{nm}),\\
&\partial/\partial t\rho_{k,k'}^{nn} - A\langle\hat{d}\hat{\rho}^m\hat{d}\rangle_{k,k'} = -i\dot{E}_0(\dot{q}_+ V_{k,k-1}^{nm}\rho_{k-1,k'}^{nm} + \dot{q}_- V_{k,k+1}^{nm}\rho_{k+1,k'}^{mn})\\
&\quad + iE_0(\rho_{k,k'-1}^{nm} q_+ V_{k'-1,k'}^{mn} + \rho_{k,k'+1}^{nm} q_- V_{k'+1,k'}^{mn}),\\
&\overset{*}{\rho}{}_{j,k}^{mn} = \rho_{k,j}^{nm},\ \overset{*}{\rho}{}_{j,j'}^{mm} = \rho_{j',j}^{mm},\ \overset{*}{\rho}{}_{k,k'}^{nn} = \rho_{k'k}^{nn},\ (|k|, |k'| \leqslant J_n),\\
&\quad (|j|, |j'| \leqslant J_m),\ \gamma_m > 0
\end{aligned} \qquad (33)$$

with normalization condition

$$\sum_{|k|\leqslant J_n} \rho_{k,k}^{nn} + \sum_{|j|\leqslant J_m} \rho_{j,j}^{mm} = 1.$$

Here $A\langle\hat{d}\hat{p}^m\hat{d}\rangle_{k,k'}$ is the term of arrival at the ground state as a result of spontaneous dipole emission, which according to Refs. 14 and 15 has the form

$$A\langle\hat{d}\hat{\rho}^m\hat{d}\rangle_{k,k'} = \gamma_m(2J_m+1)\sum_{j,j',p}(-1)^p(-1)^{J_n-k} \times \begin{pmatrix} J_n & 1 & J_m \\ -k & p & j \end{pmatrix} \rho_{j,j'}^{mm}(-1)^{J_m-j'} \begin{pmatrix} J_m & 1 & J_n \\ -j' & -p & k' \end{pmatrix}. \qquad (34)$$

We are interested only in the asymptotic solution of system (33) as $t \to \infty$, which is found from the stationarity condition

$$\frac{\partial}{\partial t}\rho_{j,k}^{mn} = \frac{\partial}{\partial t}\rho_{j,j'}^{mm} = \frac{\partial}{\partial t}\rho_{k,k'}^{nn} = 0 \quad (\forall j, j', k, k'). \qquad (35)$$

This corresponds, for example, to the case of a plane light wave.

For some of the simplest transitions ($1\to 0$, $1\to 1$), where the SIS form a Λ-configuration, the stationary solutions of system (33) are well-known[4,17] and possess the remarkable property that the atoms are excluded fr[illegible]n the interaction and the medium becomes transparer[illegible] [illegible]clears). This phenomenon is known as the coherent trapping of populations in a three-level Λ-system,[7-9] and a general theory for it for arbitrary transitions is still lacking.

From the analysis carried out in the first part of this paper, the almost obvious thought arises that the SCS found by us, located in the ground state (E_n), lie at the basis of such effects since in these SCS the atom does not interact with the field ($\hat{V}\Psi_{\text{SCS}} = 0$), does not relax ($\gamma_n = 0$), and in the process of optical pumping it should go over entirely to the so very "convenient" unperturbed states. We will now prove this.

We seek solutions that correspond to coherent trapping of populations (clearing of the atom) from the condition of vanishing of the currents (off-diagonal elements):

$$\rho_{j,k}^{mn} = \rho_{k,j}^{nm} = 0 \qquad (\forall j, k), \qquad (36)$$

which along with the stationarity condition (35) leads, as follows from Eqs. (33), to the system

$$\begin{aligned}
&\rho_{j,j'}^{mm} = \rho_{j',j}^{mm} = \rho_{j,k}^{mn} = \rho_{k,j}^{nm} = 0, \quad \rho_{k',k}^{nn} = \overset{*}{\rho}{}_{k,k'}^{nn}, \quad (\partial/\partial t)\rho_{k,k'}^{nn} = 0,\\
&q_- V_{j,j-1}^{mn}\rho_{j-1,k}^{nn} + q_+ V_{j,j+1}^{mn}\rho_{j+1,k}^{nn} = 0,\\
&(|k|, |k'| \leqslant J_n), (|j|, |j'| \leqslant J_m),\\
&\sum_{|k|\leqslant J_n}\rho_{k,k}^{nn} = 1.
\end{aligned} \qquad (37)$$

We will carry out a further analysis, as in the first part of the paper, for each type of transition individually.

1. The transition $J \to J-1$ ($J_n = J$, $J_m = J-1$)

The system can be solved for each SIS individually, i.e., when the indices j and k belong to the same SIS whose configuration is a chain of Λ-links (Figs. 3a and b). It is clear that in this case the number of homogeneous equations in system (37) is less than the number of terms $\rho_{k,k'}^{nn}$ themselves, and the solution of system (37) is easily found in the form

$$\rho^{nn}_{-J+2l_1,-J+2l_2}=\overset{*}{b}{}^0_{-J+2l_1}b^0_{-J+2l_2},$$

$$b^0_{-J}=P_1,\ b^0_{-J+2l}=P_1(-1)^l\left(\frac{q_-}{q_+}\right)^l$$

$$\times\prod_{f=1}^{l}\frac{V^{mn}_{-J-1+2f,\,-J-2+2f}}{V^{mn}_{-J-1+2f,\,J+2f}}\quad(0\leqslant l\leqslant N_1),$$

$$\rho^{nn}_{-J+1+2l_1,\,-J+1+2l_2}=\overset{*}{b}{}^0_{-J+2l_1+1}b^0_{-J+1+2l_2}, \qquad (38)$$

$$b^0_{-J+1}=P_2,\ b^0_{-J+1+2l}=P_2(-1)^l\left(\frac{q_-}{q_+}\right)$$

$$\times\prod_{f=1}^{l}\frac{V^{mn}_{-J+2f,\,-J-1+2f}}{V^{mn}_{-J+2f,\,-J+1+2f}}\quad(0\leqslant l\leqslant N_2),$$

with the normalization condition

$$|P_1|^2\left(\sum_{l=0}^{N_1}\left|\frac{q_-}{q_+}\right|^2\prod_{f=1}^{l}\left|\frac{V^{mn}_{-J-1+2f,\,-J-2+2f}}{V^{mn}_{-J-1+2f,\,-J+2f}}\right|^2\right)$$
$$+|P_2|^2\left(\sum_{l=0}^{N_2}\left|\frac{q_-}{q_+}\right|^2\prod_{f=1}^{l}\left|\frac{V^{mn}_{-J+2f,\,-J-1+2f}}{V^{mn}_{-J+2f,\,-J+1+2f}}\right|^2\right)=1.$$

Thus, coherent trapping of populations is present, and we have not expressed the result in the form of products of coefficients b^0_k merely by chance, but rather in order to explicitly reflect the connection with the above-found SCS located in the ground state (11a), (11b), which are realized in the process of optical pumping.

From Eqs. (38) it can be seen that the normalization condition does not uniquely determine the numbers $|P_1|^2$ and $|P_2|^2$, i.e., the solution contains some degree of arbitrariness. But this means that the evolution of the system and its asymptotic state ($t\to\infty$) depend significantly on the initial conditions before the interaction.

As to the components of the type $\rho^{nn}_{-J+2l_1,\,-J+1+2l_2}$ and $\rho^{nn}_{-J+1+2l_1,\,-J+2l_2}$ "joining" the various SIS, here we have no unique answer and everything depends on the initial conditions before the interaction. Thus, if there was an isotropic gas of atoms before the interaction, i.e., $\rho^{nn}_{k,k'}\sim\delta_{k,k'}$, then in the stationary solution, obviously, we will have $\rho^{nn}_{-J+2l_1,\,-J+1+2l_2}=\rho^{nn}_{-J+1+2l_1,\,-J+2l_2}=0$. However, in general this is not so.

2. The transition $J-1\to J$ ($J_n=J-1$, $J_m=J$)

For each SIS consisting of V-links, the number of homogeneous equations in system (37) is greater than the number of elements of $\rho^{nn}_{k,k'}$, i.e., system (37) is inconsistent and there is no coherent trapping of populations. The SCS (28), (29) found in the first part of the paper are in the excited state E_m and in the process of optical pumping are not realized (they relax, since $\gamma_m>0$).

3. The transition $J\to J$ (J-integer)

For an SIS consisting of V-links (dashed line, Fig. 5), the system of homogeneous equations in system (37) is inconsistent (as in the case of the $J-1\to J$-transition), while for an SIS consisting of Λ-links (solid line, Fig. 5), system (37) is consistent (as in the case of the $J\to J-1$-transition), and thus the total solution has the form

$$\rho^{nn}_{-J+2l_1,\,-J+2l_2}=\overset{*}{b}{}^0_{-J+2l_1}b^0_{-J+2l_2},\ \rho^{nn}_{-J+1+2l,\,k}=\rho^{nn}_{k,\,-J+1+2l}=0\quad(\forall l,k),$$
$$b^0_{-J}=P,\ b^0_{-J+2l}=P(-1)^l\left(\frac{q_-}{q_+}\right)^l\prod_{f=1}^{l}\frac{V^{mn}_{-J-1+2f,\,-J-2+2f}}{V^{mn}_{-J-1+2f,\,-J+2f}} \qquad (39)$$

with normalization

$$\sum_{l=0}^{J}|b^0_{-J+2l}|^2=|P|^2\sum_{l=0}^{J}\left|\frac{q_-}{q_+}\right|^{2l}\prod_{f=1}^{l}\left|\frac{V^{mn}_{-J-1+2f,\,-J-2+2f}}{V^{mn}_{-J-1+2f,\,-J+2f}}\right|^2=1.$$

Coherent trapping of the populations takes place. In this case the SCS (30) in the ground state E_n is realized. In contrast with the transition $J\to J-1$, solution (39) is uniquely determined by the normalization condition.

4. The transition $J\to J$ (J-half-integer)

For each SIS (see Fig. 6a) the number of homogeneous equations in system (37) coincides with the number of terms $\rho^{nn}_{k,k'}$, and in general the inhomogeneous system (37) has no solutions. The exception consists of the case of circularly polarized light. In this case $\rho^{nn}_{k,k'}=\delta_{J,k}\delta_{J,k'}$ or $\rho^{nn}_{k,k'}=\delta_{-J,k}\delta_{-J,k'}$ (depending on the direction of rotation).

Some conclusions and applications

The above analysis makes it possible to formulate a number of assertions of a general nature:

1. The effect of coherent trapping of populations is equivalent to clearing of the atoms in a field of resonant polarized radiation and is linked in the most immediate way with the SCS that are located in the ground state. More precisely: if there are SCS in the ground state, then the effect of coherent trapping of populations also takes place. This is valid not only for two-level degenerate systems. Such SCS can also arise in the case of multifrequency irradiation of an atom, when the ground state has several energy levels[18,19] (e.g., splitting in a magnetic field or fine and hyperfine splitting in atoms), in which case the balance between the detunings δ_i of the interacting fields becomes important.

2. The transitions $J\to J$ ($J_n=J$, $J_m=J-1$) and $J'\to J'$ (J'-integer) are clearing in the field of a polarized wave with arbitrary ellipticity ε, and solutions (38) and (39) do not depend on the detuning δ or the field intensity $|E_0|^2$, but depends only on ε. In addition, for the transition $J\to J-1$ the asymptotic solution significantly depends on the initial conditions before the interaction. The transition $J''\to J''$ (J''-half-integer) is cleared only for a circularly polarized field. The transition $J-1\to J$ ($J_n=J-1$, $J_m=J$) never clears.

Recently, a number of remarkable experiments have been carried out on the deep cooling of atomic gases,[10,11] where the effect of conherent trapping of populations plays a fundamental role. Let us consider the situation in somewhat more detail.

Let the atomic gas be located in the field of two counterpropagating single-frequency waves:

$$\mathbf{E}=\mathbf{E}_1\exp[-i(\omega t-\mathbf{kr})]+\mathbf{E}_2\exp[-i(\omega t+\mathbf{kr})]+\text{c.c.},\qquad(40)$$

resonant with the transition $J_n\to J_m$ (J_n is the ground state), where the vectors $\mathbf{E}_1$ and $\mathbf{E}_2$ are in general arbitrary. Such a formulation of the problem arises, for example, in experiments on one-dimensional cooling of gases.[11] We will show that when the transition $J_n \to J_m$ is clearing, the situation is fundamentally different from the case of unclearing transitions.

Let the quantization axis be aligned with the vector $\mathbf{k}$. We consider an atom moving with velocity $\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$. Transforming to the moving coordinate system of the atom, we find the field acting it:

$$\mathbf{E}_a = \mathbf{E}_1' \exp[-i(\omega - kv_z)t] + \mathbf{E}_2' \exp[-i(\omega + kv_z)t] + \text{c.c.} \quad (41)$$

If $v_z = 0$, then a single-frequency harmonic perturbation acts on the atom, which in the given coordinate system has the form (2), (3). Therefore all the results which have been obtained in this paper are valid, and for such atoms ($v_z = 0$) in the case of the transitions $J \to J - 1$ and $J' \to J'$ (J'–integer) there exist clearing solutions. Consequently, if in the dynamic treatment of the problem there acts upon the moving atom ($v_z \neq 0$) a braking force (along the z axis), then in the limit $t \to \infty$ all the atoms are brought to a halt ($v_z = 0$) and cleared. Moreover, the result is valid that in the process of clearing, spontaneous emission [$\rho_{jj'}^{mm} = 0 \forall j, j'$, see system (37)], which is one of the basic mechanisms of heating of an atomic gas and prevents complete cooling in the case of the transitions $J - 1 \to J$ and $J'' \to J''$ (J''–half-integer), disappears. Note that in the process of clearing the atom ceases to interact with the light ($\hat{V}\Psi_{\rm SCS} = 0$) and the depth of its potential well, formed by the fields, degenerates to zero. For unclearing transitions the situation is completely different.

However, in the case of the clearing transitions $J_n \to J_m$ it is absolutely necessary to see to it that for the moving atoms ($v_z \neq 0$) clearing, which naturally makes complete cooling impossible, does not take place. For this it is necessary and sufficient that the condition be satisfied that can formally be written in the form

$$\operatorname{Ker} \hat{V}_1^{(n)} \cap \operatorname{Ker} \hat{V}_2^{(n)} = 0, \quad (42)$$

where $\hat{V}_j^{(n)} = -(\mathbf{E} \cdot \mathbf{d})e^{-i\omega t} + \text{h.c.}$ ($j = 1, 2$), and the index (n) on $\hat{V}_j^{(n)}$ denotes that we are only interested in those SCS that lie in the ground state E_n. Thus, it is necessary to solve the problem of finding $\operatorname{Ker} \hat{V}_j$ for each $\hat{V}_j$ individually, which was done in the first part of this paper, and determine the conditions of existence of a general vector in those subspaces that lies in the lower level E_n. For the transitions $J' \to J'$ (J'–integer) the space $\operatorname{Ker} \hat{V}_j^{(n)}$ is one-dimensional for each $\hat{V}_j$ [see solution (30)], wherefore to satisfy Eq. (42) it is necessary that $\mathbf{E}_1$ and $\mathbf{E}_2$ be linearly independent. For the transitions $J \to J - 1$ the dimensionality of $\operatorname{Ker} \hat{V}_j^{(n)}$ is equal to two for each $\hat{V}_j$ [see solution (12a), (12b)], wherefore a concrete analysis of each transition is required. Thus, for example, for the transition $1 \to 0$ (Fig. 7) we have $\psi_0^n \in \operatorname{Ker} \hat{V}_1^{(n)} \cap \operatorname{Ker} \hat{V}_2^{(n)} \neq 0$, i.e., any moving atom is cleared in the state ψ_0^n (in Fig. 7 it is marked by an asterisk) even in spite of the linear independence of the vectors $\mathbf{E}_1$ and $\mathbf{E}_2$. All the same, for all other types of transitions $J \to J - 1$ ($J > 1$) linear independence of $\mathbf{E}_1$ and $\mathbf{E}_2$ is sufficient to satisfy Eq. (42).

The above arguments are easily generalized to the case of any finite number N of arbitrarily directed and arbitrarily polarized single-frequency waves

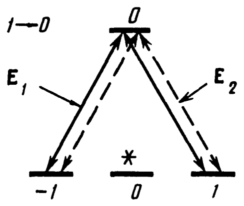

FIG. 7.

$$\mathbf{E} = \sum_{j=1}^{N} \mathbf{E}_j \exp[-i(\omega t + \mathbf{k}_j\mathbf{r})] + \text{c.c.}, \quad (43)$$

and it is also possible to realize two- and three-dimensional cooling of the gas. In fact, an immobile atom located at the point $\mathbf{r}_0$ is acted upon by the field

$$\mathbf{E}_a(\mathbf{r}_0) = \vec{\mathscr{E}}(\mathbf{r}_0)\exp(-i\omega t) + \text{c.c.}$$
$$\equiv \left(\sum_{j=1}^{N} \mathbf{E}_j \exp(-i\mathbf{k}_j\mathbf{r}_0)\right)\exp(-i\omega t) + \text{c.c.}, \quad (44)$$

where the vector $\mathscr{E}(\mathbf{r}_0)$ can in general be represented in the form

$$\vec{\mathscr{E}}(\mathbf{r}_0) = \vec{\mathscr{E}}_R(\mathbf{r}_0) + i\vec{\mathscr{E}}_I(\mathbf{r}_0), \quad (45)$$

where $\mathscr{E}_R(\mathbf{r}_0)$ and $\mathscr{E}_I(\mathbf{r}_0)$ are real. Then, choosing the quantization axis z of this atom to be orthogonal to the plane ($\mathscr{E}_R(\mathbf{r}_0)$, $\mathscr{E}_I(\mathbf{r}_0)$), we obtain an expansion of the field (2), i.e., for clearing transitions the immobile atoms clear independently of their location. The condition of the absence of clearing for the moving atoms can, in analogy with Eq. (42), be written in the form

$$\bigcap_{j=1}^{N} \operatorname{Ker} \hat{V}_j^{(n)} = 0. \quad (46)$$

Here by "moving" one must, of course, understand the velocity field that underlies the suppression and is determined by the directions of the vectors $\{\mathbf{k}_j\}$. It need not necessarily be three-dimensional, but can be two-dimensional or one-dimensional as in the case of the field (40).

In the still more general formulation of the problem in which several transitions take part in the interaction of the atoms with the resonant fields, it is necessary to add to what has already been said the requirement that at least one of the transitions participating in the interaction should be clearing. Thus, for example, from Ref. 10 it is quite clear (see Ref. 10, pp. 170–171, Fig. 2) that the best experimental results on the cooling of ^{23}Na atoms are observed when the frequency of the main field comes into resonance with the transition $F_n = 2 \to F_m = 2$, which is clearing. In this case, a weak mode which is resonant with the unclearing transition $F_n = 1 \to F_m = 2$ plays an auxiliary role by not permitting the atoms to go over to the lower hyperfine state with $F_n = 1$ and be excluded from the interaction. However, if the interaction goes through the upper hyperfine level with $F_m = 1$, then both polarized modes take part in the process of cooling indirectly since the transitions $1 \to 1$ and $2 \to 1$ are clearing (we do not take the influence of other transitions into account, assuming the magnitudes of the hyperfine splittings to be sufficiently large).

Of course, the above analysis is of a qualitative nature and does not in any way pretend to a completeness which is possible only in a concrete dynamic description of the problem. Nevertheless, we have demonstrated the fundamental possibility of superdeep cooling of atomic gases (see Ref. 11) in polarized electromagnetic fields resonant with the clearing transitions. In this case there exist simple experimental criteria of the validity of the above-presented approach. Indeed, if the cooling is accompanied by clearing, then there is a) an absence of spontaneous emission and b) an absence of absorption of the interacting fields.

In conclusion we note that the given method should be very sensitive to a magnetic field, which destroys the SCS and in fact promotes the "heating" of the atoms, for which reason it is necessary to screen the probe. But even a residual weak magnetic field **H** can have a substantial influence on the result. The fact is that for the ground state the concept of a "weak" magnetic field is highly relative and is usually determined by the dimensionless quantity $\Omega_n t_{\rm tr}$ (Ref. 20), where $\Omega_n = g_n H$ is the Larmor frequency in the ground state and $t_{\rm tr}$ is the transit time of the interaction of the atoms with the field. Therefore, for sufficiently slow atoms even a magentic field that is moderate in magnitude can play a noticeable role. In this case its effect on the dynamics and the result of the cooling process has a tensor character, i.e., it depends on the mutual directions of **H** and the vectors $\mathbf{E}_j$ and $\mathbf{k}_j$ of the polarized radiation. This can also possibly explain the differences in the experimental results[10] upon variation of the directions of the polarization vectors of the interacting fields. Even in the complete absence of a magnetic field ($\mathbf{H} = 0$) then, the dynamics of the cooling process should depend on the directions of the vectors $\mathbf{E}_j$ and $\mathbf{k}_j$ all the more although, of course, more detailed analysis is necessary.

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Translated by P. F. Schippnick