Real limitations on the closure hypothesis, and classical paradoxes of kinetic theory

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The behavior of a system of elastic interacting spheres under the action of a thermal electromagnetic field is considered. It is established that the reversible-behavior time, during which the system can be regarded as closed, depends logarithmically weakly on the level of the fluctuational fields. Estimates have shown that the reversible-behavior time is exceptionally small: It amounts to a few or a few tens of mean free times, however small the external fields may be. Allowance for weak fluctuational fields helps to resolve the classical paradoxes of kinetic theory (the reversibility paradox of Loschmidt and the return paradox of Zermelo) in a natural way, and indicates that in questions concerning the justification of kinetic theory the closure hypothesis is an example of an over-idealized physical model that inadequately reflects important features of the behavior of real physical systems.

INTRODUCTION

Many questions in statistical physics and physical kinetics, including questions concerning the derivation of the Boltzmann kinetic equation and the reconciliation of its properties with the fundamental laws of physics, are usually examined in the framework of the assumption that the system being studied is closed. At the same time, the closure hypothesis gives rise to substantial difficulties in a number of problems of kinetic theory, especially in the problem of time-reversal invariance. In our opinion, many of the difficulties of kinetic theory and statistical physics can be radically eliminated by rejecting the assumption of strict closure. The point is that the weak fluctuational fields that always penetrate an incompletely isolated system substantially alter the order of collisions that is characteristic of the closed system, and, after extremely short intervals of time, violate the time-reversal invariance.

In this paper we analyze the real levels of applicability of the closure hypothesis for the example of colliding bodies (absolutely elastic spheres) in conditions when an external fluctuational electromagnetic field is acting. For this physical system we estimate the reversible-behavior time $\tau_{\rm rev}$ and show that it depends logarithmically on the intensity of the fluctuational field. Of course, the connection between the nonclosure of real systems and their fundamental irreversibility is familiar to all who have thought about this problem, and so, in a certain sense, this connection is trivial. Not trivial are the results of the calculations: For a system of elastic spheres the reversibility time amounts to only a few or a few tens (in the extreme case, hundreds) of mean free times, however small the external field might be. Having such estimates of $\tau_{\rm rev}$ at our disposal, we can make a new approach to the resolution of the classical paradoxes of kinetic theory and to the problems of the justification of kinetic theory.

1. ESTIMATE OF THE REVERSIBILITY TIME UNDER THE ACTION OF A THERMAL ELECTROMAGNETIC FIELD

An estimate of the reversibility time $\tau_{\rm rev}$ for a system of hard spheres can be obtained from the following considerations. Let δr_1 be the root-mean-square lateral displacement of the trajectory of an individual particle that arises during the mean free time under the action of weak fluctuational fields. If a is the radius of the spheres, τ_0 is the mean free time, l_0 is the mean free path, and $h = \tau_0^{-1}\ln(l_0/a)$ is the K-entropy of the system, then for $\tau > \tau_0$ the root-mean-square lateral displacement δr of the particles will increase by an exponential law:

$$\delta r \sim \delta r_1 (l_0/a)^{\tau/\tau_0 - 1} = (a\delta r_1/l_0)\, e^{h\tau}. \qquad (1)$$

After a time

$$\tau = \tau_0 \frac{\ln(l_0/\delta r_1)}{\ln(l_0/a)} \equiv \tau_{\rm rev} \qquad (2)$$

the lateral displacement δr becomes comparable to the radius a of the spheres, after which the collision sequence prescribed (by the initial values of the coordinates and velocities) at the time $t = t_*$ at which reversal of all the velocities is performed (in strictly closed systems, this is equivalent to reversal of the sign of the time) is violated, and exchanges of partners will occur. Therefore, the time interval (2) must be regarded as the reversibility time of the system: For $\tau > \tau_{\rm rev}$ the collisions in the reversed system will occur in a different order from those in the initial system. Thus, at $\tau = \tau_{\rm rev} + \tau_0$, i.e., after one mean free time after $\tau_{\rm rev}$, the fluctuational uncertainty δr will reach one mean free path: $\delta r(\tau_{\rm rev} + \tau_0) \sim l_0$. The angular uncertainty $\delta\theta = \delta r/l_0$ at $\tau = \tau_{\rm rev}$ amounts to a/l_0, while at $\tau = \tau_{\rm rev} + \tau_0$ it is comparable to unity: $\delta\theta \sim 1$; i.e., after $n = \tau_{\rm rev}/\tau_0 + 1$ collisions the particle loses all memory of its direction in the absence of fluctuations.

A feature of the expression (2) is that the number $n_{\rm rev} = \tau_{\rm rev}/\tau_0$ of collisions corresponding to reversible behavior depends exceptionally weakly on the level of the fluctuational fields. We shall consider, e.g., the action of an electromagnetic field with temperature 300 K on particles with a mass of the order of 30 proton masses and with the same mean free path as in air under normal conditions. Assuming that the particles are acted upon by a force $\mathbf{F} = (\mathbf{d}\cdot\nabla)\mathbf{E}$, where $\mathbf{d} = \alpha\mathbf{E}$ is the dipole moment induced by the fluctua-

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tional field and α is the polarizability of the particles it is not difficult to write down the perturbation of the trajectory:

$$\delta \mathbf{r}(t) = \frac{1}{m} \int_0^t (t-t')\mathbf{F}(t')\,dt',$$

and then estimate the mean-square deviation from rectilinear motion during the mean free time τ_0:

$$(\delta r_1)^2 \equiv \langle \delta \mathbf{r}^2(\tau_0) \rangle \sim \langle \mathbf{F}^2 \rangle \tau_0^3 \tau_{min}/m^2, \quad (3)$$

where τ_{min} is the smaller of the mean free time τ_0 and the correlation time $\tau_T \sim \hbar/kT$ of the thermal field. In its turn, $\langle \mathbf{F}^2 \rangle$ is estimated as $\alpha^2 \langle E^4 \rangle / \lambda_T^2 \sim 3\langle E^2 \rangle^2 \alpha^2 / \lambda_T^2$, where $\lambda_T = c\tau_T = c\hbar/kT$ is the characteristic length scale of the thermal field.

To summarize, for the root-mean-square displacement of a particle at $T = 300$ K we obtain the estimate $\delta r_1 \sim 10^{14}$ cm. Taking $l_0 \sim 10^{-4}$ cm and $a \sim 10^{-8}$ cm, we obtain $l_0/a = 10^4$ and $l_0/\delta r_1 \sim 10^{10}$, so that from (2) it follows that $\tau_{rev} \sim 2.5\tau_0$ and $n_{rev} = 2.5$. It would also be possible to estimate δr_1 for particles possessing an intrinsic quadrupole moment, but we shall not do this, bearing in mind that, because of the logarithmic dependence of τ_{rev} on the strength of the fluctuational force, there will only be a slight change in the result.

The logarithmically weak dependence of the time τ_{rev} on the intensity of the fluctuational field is manifested in the fact that, when we go over from room temperature ~ 300 K to the relict-background temperature ~ 1 K, the time τ_{rev} increases only to $4.5\tau_0$.

The fact that the loss of reversibility over times equal to a few mean free times τ_0 remains true for any other conceivable agent, including the action of neutrino and graviton fluxes, which are practically impossible to screen, is extremely important. Without performing numerical estimates of the mean-square displacement for such influences, we note that if the displacement δr_1 turns out to be even 1000 orders smaller (not 10^3 times, but 10^{1000} times smaller!) than under the action of a 300-Kelvin electromagnetic field, then, for the parameter values used above, the reversibility time increases only to $250\tau_0$ (in air at normal pressure this time scarcely reaches 10^{-7} sec). In other words, however small the forces of some or other physical origin may be, a gas of hard spheres senses them after extremely short time intervals, comparable to τ_0; i.e., the time τ_{rev} actually remains microscopic under any outside influence.

2. IRREVERSIBLITY AS A CONSEQUENCE OF NONCLOSURE. A NEW LOOK AT THE LOSCHMIDT PARADOX

The extraordinarily small values of the reversibility time τ_{rev} give the possibility of throwing new light on the Loschmidt reversibility paradox. If we start from a simplified model of hard spheres, completely isolated from the external world, according to Loschmidt we should expect that the entropy increase that took place before the reversal time t_* will be replaced by an entropy decrease for $t > t_*$, i.e., $S^{rev}_{closed}(t_* + \tau) = S(t_* - \tau)$ (the behavior of the entropy S^{rev}_{closed} of the reversed, perfectly closed system is shown by line 1 in Fig. 1). At the same time, as follows from what has been said above, completely reversible behavior occurs only for a limited time $\tau \lesssim \tau_{rev}$, after which, under the action of fluctuational fields, the "real" entropy S^{rev}_{real} will again display a tendency to increase (curve 2 in the figure). The decrease of the entropy upon reversal (in comparison with the unperturbed behavior $S(t)$ indicated by the dashed curve in the figure) is estimated as

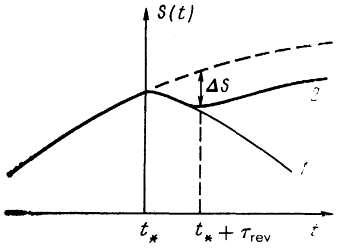

FIG. 1. Temporal behavior of the entropy after the reversal time t_*: 1) for a perfectly closed system; 2) for a weakly nonisolated system subjected to the action of fluctuational fields.

$$\Delta S \cong \frac{\partial S(t_*)}{\partial t} 2\tau_{rev}. \quad (4)$$

In the numerical experiment of Ref. 1 on the reversibility of a two-dimensional gas (a system of 100 disks on a plane) fluctuational forces were not specially introduced, but their role was taken in practice by the errors due to the finite accuracy of the computer. Under the influence of these errors the reversibility time amounted to only $(50\text{–}100)\tau_0$. With this interpretation, the experiment described in Ref. 1 can serve as a good illustration of the point of view developed here.

3. ESTIMATE OF THE ENERGY EXCHANGE WITH AN EXTERNAL FIELD DURING THE TIME OF REVERSIBLE BEHAVIOR

In the above arguments we have essentially identified the reversibility time τ_{rev} with the time during which the system can be regarded as closed:

$$\tau_{closed} \sim \tau_{rev}. \quad (5)$$

We should, however, emphasize the difference between this time and another, "energy" closure interval $\tau_{\Delta E}$, which corresponds to the exchange of an appreciable (say, one-percent) energy fraction ΔE with the surroundings and characterizes the rate of occurrence of macroscopic processes. These two quantities have different physical meanings, and characterize closure from different standpoints. Usually, the quantity $\tau_{\Delta E}$ is many orders of magnitude greater than τ_{rev}. The point is that the fluctuational velocity $\delta v_1 \sim \delta r_1/\tau_0$ imparted by a weak external force is a fraction $\delta r_1/l_0$ of the typical particle velocity $v_0 \sim l_0/\tau_0$, so that the kinetic energy acquired under the action of the field is smaller by a factor of $(\delta r_1/l_0)^2$ than the stored kinetic energy $mv_0^2/2$. In the above example of a 300-Kelvin electromagnetic field this ratio amounts to 10^{-20}, for a 1-Kelvin field it is equal to 10^{-36}, and for an external field 10^3 orders weaker than a 300-Kelvin field it is only 10^{-1020}. Thus, to pass into an irreversible state one needs to impart an entirely negligible amount of energy to the system of particles.

4. CONDITION FOR APPLICABILITY OF THE POINCARÉ RETURN THEOREM. ZERMELO'S PARADOX

The inclusion of fluctuational fields also makes it possible to resolve another paradox of kinetic theory—Zermelo's reversibility paradox. This paradox is related to Poincaré's return theorem and looks at the contradiction between the nondecrease of the entropy in a closed system and the inevitable return of a closed system to a small neighborhood of the initial state after a return time τ_{ret}. The standard arguments that appeal to the fact that the return time is usually very long do not so much resolve the paradox as "put it off" in time.

At the same time, the Poincaré theorem, which pertains to closed Hamiltonian systems, loses its validity after rather short times of the order of τ_{rev}. This time is usually immeasurably shorter than the return time τ_{ret}, so that the closure of any realistic system is violated long before the return time. The situation is made worse if fluctuational fields are also included in the system being studied. The return time τ_{ret} of the extended "particles plus field" system is substantially longer than the previous time τ_{ret}, since not only the particles but also the photons must return to a small neighborhood of the original state. To summarize, the application of the Poincaré theorem to the extended system will be even more dubious than its application to the particle system.

Bearing this in mind, we can conjecture that the only object that could claim the right to obey Poincaré's theorem is the universe as a whole. In this case it would be natural to interpret the return time as the lifetime of the closed universe.

CONCLUSION

The above analysis has shown that, in questions of reversibility and of the justification of kinetic theory, the closure hypothesis is an example of an over-idealized physical model that inadequately reflects important features of the behavior of real physical systems.

Rejection of the hypothesis of perfect closure and allowance for even exceptionally weak fluctuational fields of any origin make it possible to eliminate in a radical way difficulties that have remained present in kinetic theory up to now. We have in mind not only the resolution of the classical paradoxes discussed above but also external fluctuations as the cause of the noninvariance of the kinetic equation under time reversal and as a universal cause of the increase of entropy in real systems.

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[1]A. Bellemans and J. Orban, Phys. Lett. **24A**, 620 (1967).

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