Magnetic field generation by sound waves in the solar atmosphere

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We show that sound waves which are generated in the convective zone of the Sun excite an electric current (and a magnetic field) in the transition region from the chromosphere to the corona. The excitation of the current is connected with the absorption of part of the momentum of the waves by electrons as a result of the electron thermal conductivity. When sound waves propagate in the direction of decreasing density their leading front becomes steeper. This leads to the formation of weak shock waves and—thanks to the fast energy dissipation at the front—to a stronger magnetic field generation.

The plasma of the solar atmosphere is continuously subjected to the action of waves which are generated in the convective zone. These waves make an important contribution to the energy balance of the upper chromosphere and the corona (see, e.g., Ref. 1). In the present communication we shall show that the absorption of waves can also play a significant role in the excitation of currents and in the generation of magnetic fields in the plasma.

The fact that the absorption of waves can be accompanied by the excitation of a current is well known in the physics of high-temperature plasmas and is used to maintain the current in toroidal thermonuclear devices (see, e.g., Ref. 2). However, the calculations which are carried out in that connection are aimed at a situation where the collision frequency of the particles in the plasma is very low and where the absorption of the waves is caused by "collisionless" resonance effects (such as Landau damping). On the other hand, in the case of the solar atmosphere, in which we are interested, the collision frequency is many orders higher than the frequency of the oscillations generated in the convective zone, and the absorption of these oscillations is caused by the classical dissipation mechanisms of thermal conductivity, viscosity, and electric conductivity which lead to a number of peculiar features in the current excitation mechanism.

In the present paper we restrict ourselves to effects caused by ordinary sound waves (we assume the external magnetic field to be small and we consider waves moving along the direction of the field). We note that the problem of the generation of a current by such waves in a plasma with frequent collisions can be of interest not only for a study of astrophysical objects, but also in connection with problems of studying a laboratory a gas-discharge plasma where the corresponding effects can be applied for diagnostic purposes. We also note that an effect related to the one considered by us is known in solid-state physics where it is called the "acousto-electric" effect.[3] A new feature in our problem is the fact that when sound waves propagate in a gas (in particular, when they propagate in the direction of decreasing density) they can easily "break" and be transformed into a sequence of shock waves—just in that strongly non-linear stage where there is an extremely efficient current generation (see below).

We give numerical estimates pertaining to a characteristic point in the transition region between the chromosphere and the corona. We set the plasma density n and temperature T, respectively, equal to 10^{10} cm^{-3} and 10 eV. Following Ref. 4 we then find that the mean free path l of the plasma particles is equal to 300 m. We take the magnitude of the characteristic wavelength $\lambdabar$ of the sound waves to be equal to 10^5 m (i.e., $\lambda \equiv 2\pi\lambdabar = 600$ km)[1)] We see that the mean free path l satisfies with a large margin not only the

$$l \ll \lambdabar (m_e/m_i)^{1/2}, \tag{1}$$

where m_i and m_e are the ion and electron masses. One easily checks that inequality (1) automatically guarantees that the time needed to equalize the electron and ion temperatures is small compared to the period of the sound wave. In other words, the perturbations of the electron and ion temperatures are the same under the conditions (1), i.e., the sound speed s can be evaluated using the standard formula for a monatomic gas:

$$s = (5p/3\rho)^{1/2}, \tag{2}$$

where $p = 2nT$ is the pressure and $\rho = m_i n$ the mass density of the plasma.

The absorption of long-wavelength sound is caused by viscosity and thermal conductivity processes. The damping rate of linear oscillations which is given by the formula

$$\Gamma = -d \ln W/dt, \tag{3}$$

where W is the energy density of the oscillations is, for instance, evaluated in Ref. 5[2)]:

$$\Gamma = \frac{\omega^2}{s^2}\left[\frac{1}{\rho}\left(\frac{4}{3}\eta + \zeta\right) + \frac{2\varkappa}{15n}\right], \tag{4}$$

where η and ζ are the hydrodynamic viscosity coefficients and $\varkappa$ is the thermal conductivity coefficient. In our case the viscosity is determined by the ions and the thermal conductivity by the electrons; the corresponding estimates are

$$\eta \sim \zeta \sim m_i n l v_{Ti}, \quad \varkappa \approx \varkappa_e \sim n l v_{Te}.$$

It is then clear from (4) that the main contribution to the damping comes from the electron thermal conductivity, i.e.,

$$\Gamma \approx \frac{2\omega^2 \varkappa_e}{15 s^2 n}, \tag{5}$$

and the contribution from the viscosity and the thermal conductivity determined by the ions is smaller by a factor $(m_i/m_e)^{1/2}$. The dissipation caused by radiative effects is

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negligibly small in the conditions considered.

We turn particularly to the current excitation problem. When we consider such a problem in a collisionless plasma the question of to which kind of particle the momentum of the absorbed wave is transferred is answered automatically: the answer is uniquely determined by which resonance causes the absorption (e.g., if it is is electron resonance, transfer is to electrons). In the case considered by us, of a plasma with very frequent collisions, the answer to this problem is not so simple. At first sight, it seems that the momentum must be transferred to the electrons, since the absorption of the wave is connected just with the electron thermal conductivity. However, the electron mass is very small and therefore the opposite answer is suggested, that all the momentum is absorbed by the ions (as they are very heavy). We shall show tht the correct answer lies—somewhat paradoxically—in the middle (of course, we must realize that the momentum which the wave transfers to the electrons is transferred ultimately, through friction between electrons and ions, to the ions).

We write down the equations of motion of a two-component plasma (see Ref. 4) taking the z axis along the direction of the wave propagation:

$$m_i n\left(\frac{\partial v_i}{\partial t}+v_i\frac{\partial v_i}{\partial z}\right)=-\frac{\partial p_i}{\partial z}+enE+0{,}71n\frac{\partial T}{\partial z}-\frac{enj}{\sigma}, \quad (6)$$

$$0=-\frac{\partial p_e}{\partial z}-enE-0{,}71n\frac{\partial T}{\partial z}+\frac{enj}{\sigma}, \quad (7)$$

where p_i and p_e are the ion and electron pressures, E is the electric field, j the current density, and σ the conductivity. The third term on the right-hand side describes the thermoforce and the last one the friction between the electrons and the ions. We neglect the ion viscosity in Eq. (6) as it contributes little to the absorption of the wave. The appearance of a current is connected with the absorption of the wave (the perturbation of the current in the oscillations is negligible small by virtue of the quasi-neutrality of the plasma).

We divide Eqs. (6) and (7) by n and average them over a spatial period of the wave without linearizing:

$$m_i\frac{\partial}{\partial t}\langle v_i\rangle=-\left\langle\frac{1}{n}\frac{\partial p_i}{\partial z}\right\rangle+e\langle E\rangle-e\left\langle\frac{j}{\sigma}\right\rangle, \quad (8)$$

$$0=-\left\langle\frac{1}{n}\frac{\partial p_e}{\partial z}\right\rangle-e\langle E\rangle+e\left\langle\frac{j}{\sigma}\right\rangle \quad (9)$$

(we assume that there exists an average electric current in the plasma). Adding these equations we get

$$m\frac{\partial}{\partial t}\langle v_i\rangle=-\left\langle\frac{1}{n}\frac{\partial p}{\partial z}\right\rangle, \quad (10)$$

where $p=p_e+p_i$. This equation expresses the fact that momentum is conserved in the system where the quantity on the right-hand side can be treated as the momentum lost per unit time by the sound wave (per ion). Equation (10) shows clearly that the electron thermal conductivity leads to the absorption of the wave momentum: the fact is that when thermal conductivity is taken into account the pressure is no longer a function of the density n (as would happen in the isentropic case) and the quantity $(1/n)(\partial p/\partial z)$ is no longer a total derivative with respect to the coordinate. For a small-amplitude travelling periodic sound wave (which is not necessarily harmonic!) the momentum lost per unit volume is equal to $(-1/s)(\partial W/\partial t)$, whence we find that

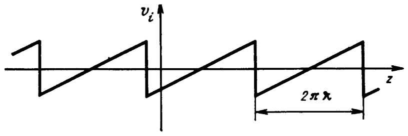

FIG. 1. Velocity profile of a shock wave after weak shock waves have been formed.

$$\langle n\rangle\left\langle\frac{1}{n}\frac{\partial p}{\partial z}\right\rangle=\frac{1}{s}\frac{\partial W}{\partial t}.$$

In the case of the strongly collisional case considered by us, which is defined by condition (1), $p_e=p_i$ and we easily find the mean force on the electron gas in which we are interested. This force per electron is equal to

$$F_e\equiv-\left\langle\frac{1}{n}\frac{\partial p_e}{\partial z}\right\rangle=-\frac{1}{2s\langle n\rangle}\frac{\partial W}{\partial t}. \quad (11)$$

As we have already noted above, half the momentum of the wave is transferred to the ions and half to the electrons [this follows immediately from the condition $p_e=p_i$ and Eqs. (8) and (9)]. We emphasize also that in deriving Eq. (11) we have nowhere used the fact that the perturbations depended harmonically on the coordinate: we can apply Eq. (11) to any traveling periodic wave, including a wave with a profile which under the influence of non-linear effects deviates from a sinusoidal one.

The fact that the momentum lost by the wave is equally divided between the electrons and the ions can also be explained on the basis of a quantum analogy. Indeed, when condition (1) is satisfied the electron and ion temperatures are the same so that automatically half of the energy absorbed by the plasma goes to electrons and half to ions. However, for sound waves the energy of a quantum ($\hbar\omega$) is proportional to its momentum ($\hbar k$) so that the absorbed momentum must be evenly divided between the plasma components.

We evaluate F_e in two limiting cases, for a purely sinusoidal wave and for a wave in the final stage of non-linear evolution (see Ref. 5) when its profile has already acquired a "sawtooth" character (Fig. 1). In the first case $\partial W/\partial t=-\Gamma W$, where Γ is given by Eq. (5) and we find for the force F_e

$$F_e=\frac{m_i s\varkappa_e}{15\lambda^2 n}\xi, \quad (12)$$

where we have introduced the notation $\xi=W/\rho s^2$.

In the second case the energy dissipation proceeds on the front of weak shock waves and no longer depends on $\varkappa_e$; according to Ref. 5 we have

$$\frac{\partial W}{\partial t}=-\frac{8\sqrt{3}}{\pi}\frac{\rho s^2}{\lambda}\xi^{3/2}. \quad (13)$$

Using (11) we now find that

$$F_e=\frac{4\sqrt{3}}{\pi}\frac{m_i s^2}{\lambda}\xi^{3/2}. \quad (14)$$

We understand by W in Eqs. (13), (14) the energy density averaged over a wave period.

We emphasize that when $\xi \ll 1$ the width of the front of the weak shock waves, which as to order of magnitude is equal to $\varkappa_e/ns\xi^{1/2}$, is such that within the limits of the front equilibrium between electrons and ions (which is the condition for the applicability of the results obtained) can be established.

To find the spatial and temporal distributions of the currents and the magnetic fields we must take into account the vector nature of the force $\mathbf{F}_e$. Using the expression

$$\mathbf{j}=\sigma(\mathbf{E}-\mathbf{F}_e/e)$$

and the Maxwell equations we get an equation which determines the evolution of the magnetic field (we restrict ourselves to the case of a stationary plasma):

$$\frac{\partial \mathbf{B}}{\partial t} = -\operatorname{rot}\left(\frac{c^2}{4\pi\sigma}\operatorname{rot}\mathbf{B}\right) - \frac{c}{e}\operatorname{rot}\mathbf{F}_e. \quad (15)$$

As should be the case, current and field generation is possible only if the force $\mathbf{F}_e$ is not purely potential—otherwise an electric field would arise in the plasma which would exactly cancel the action of the force $\mathbf{F}_e$ on the electron. Under the conditions in the solar atmosphere the vector $\mathbf{F}_e$ will necessarily have a large ("of the order of unity") solenoidal component due to the spatial variations in the intensity (and direction) of the sound flux and also of the plasma density and temperature. For illustrative purposes we show in Fig. 2 a picture of current lines for an imaginary case when the sound flux is generated solely on a small aea of an underlying surface.

A stationary state in which the current is determined by the estimate $j \sim \sigma \mathbf{F}_e/e$ is established after a time of the order of the skin-effect time. When $T \sim 10$ eV this time for scales ~ 300 km is very large—of the order of 30 years and is considerably larger than the lifetime of the various non-stationary structures (such as sun spots) on the surface of the Sun. For smaller times we must get an estimate for the magnetic field (and the current) by dropping the first term on the right-hand side of Eq. (15).

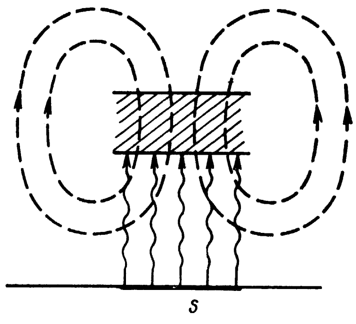

FIG. 2. Current generation by sound waves emitted from a limited area S. The region of strong absorption connected with the production of discontinuities is hatched. The dashed lines show current lines.

$$\frac{\partial \mathbf{B}}{\partial t} = -\frac{c}{e}\operatorname{rot}\mathbf{F}_e. \quad (16)$$

Bearing in mind the conditions in the transition layer between the chromosphere and the corona, where, apparently, breaking of sound waves takes place (due to the increase in their amplitude because of the drop in density) and the wave profile becomes a "sawtooth", we use Eq. (14) for an estimate of F_e. Putting the spatial scale of changes in the force F_e equal to 300 km, $T \sim 10$ eV, and $\lambda \sim 100$ km we find from (16) for $\xi \sim 1$ the rate of growth of the magnetic field $\dot{B} \sim 10^{-5}$ G/s, i.e., the field reaches 1 G only after a day.

To some extent related to the effect considered here is the effect of magnetic field generation under the action of a thermo-emf which appears when the vectors ∇n and ∇T are not collinear (see, e.g., Ref. 6). Our effect can, generally speaking, act also in the case when $\nabla T \| \nabla n$, and even in a uniform plasma.

If the magnetic field in the plasma becomes rather large it starts to affect both the dispersion characteristics of the oscillations and also the dissipative processes. The problem about current generation under such conditions becomes a separate one and it must be considered separately.

[1)]Motions in the convective zone excite a broad spectrum of sound oscillations with frequencies $f > f_{\min} = 3\times 10^{-3}$ Hz. The value of λ chosen by us corresponds to a frequency $f \sim 10^{-1}$ Hz.

[2)]We are dealing here with a small amplitude harmonic wave. A number of the formulae in what follows below have a wider range of applicability.

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