Phonon absorption by two-dimensional electrons in a strong magnetic field

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The probability of phonon absorption by two-dimensional electrons located in a Gaussian random potential is calculated. It is assumed that the potential varies weakly along the magnetic length. The absorption is determined by both the mean square fluctuation of the potential and the second derivative of its pair correlation function.

The phonon thermal conductivity in layered heterostructures such as GaAs/Al/GaAs was measured in a recent work.[1] It was shown that there are quantum oscillations of the thermal conductivity with the magnetic field at sufficiently low temperatures, when all the charge carriers are concentrated in two-dimensional layers. The problem of phonon absorption by two-dimensional electrons and its effect on the phonon thermal conductivity has not been considered previously, and in the present work we shall calculate the corresponding lifetimes of the phonons. This problem differs significantly from the well-known problem of gigantic oscillations of sound absorption in a three-dimensional metal in a strong magnetic field[2] since the magnetic field in the two-dimensional system leads to a discrete spectrum of the free electrons, while in the three-dimensional system this spectrum remains continuous. The effect of two-dimensional electrons on the propagation of Rayleigh waves was considered in Ref. 3, but in the approximation of a phenomenological relaxation time. In the absence of impurities, phonon absorption in the two-dimensional case is possible only with a transition of the electrons between the Landau levels, and is exponentially small at low temperatures $T \ll \hbar\omega_c$, where $\omega_c = eH/mc$ is the cyclotron frequency, m the effective mass and H the magnetic field. Thus, the absorption depends essentially on the properties of the random potential that acts on the electrons in the two-dimensional layer.

At the present time, there exist a number of proofs of the fact that the electron density of states in MIS transistors and heterostructures (see Refs. 4, 5 and also Ref. 6) is determined by the large-scale charge fluctuations far from the layer containing the electrons. In this connection, we shall assume the presence of a smooth random potential with a Gaussian distribution and with a certain inhomogenous and isotropic correlation function, $\langle U(\mathbf{r})U(\mathbf{r}')\rangle = R(\mathbf{r}-\mathbf{r}')$ of scale $L \gg l_H$, where $l_H^2 = c\hbar/eH$ is the magnetic length.

1. LIFETIME OF THE PHONON

We shall start out from the well-known perturbation-theory formula for the probability of a transition with absorption of a single phonon (see, for example, Ref. 7) with momentum **k**, polarization s, and frequency ω:

$$\frac{1}{\tau(\mathbf{k},s)} = \frac{2\pi}{\hbar}\sum_{i,f}\delta(\varepsilon_f-\varepsilon_i-\hbar\omega)\,|\langle 0,\varphi_f|H_{c,ph}|1,\varphi_i\rangle|^2(n_i-n_f), \tag{1.1}$$

where

$$n_i = \left(\exp\frac{\varepsilon_i-\mu}{T}+1\right)^{-1}$$

is the Fermi distribution function, of the electrons, μ is their chemical potential, and $H_{e,\mathrm{ph}}$ is the Hamiltonian of electron-phonon interaction. We assume the electrons to be noninteracting, and the phonons to be acoustical, i.e., we consider sufficiently low temperatures.

In correspondence with the experimental data, we assume that the two-dimensional electrons add a small correction to the scattering of the acoustical phonons, connected mainly with the presence of boundaries on the sample. Therefore, the problem of the acoustic scattering and the amplitude of the sound wave in the two-dimensional channel, which in fact determines the value of $H_{c,\mathrm{ph}}$, must first be solved.

The quantity $H_{e,\mathrm{ph}}$ contains (see, e.g., Ref. 8) the deformation potential $H^{(1)}_{e,\mathrm{ph}} = \Xi_{nm}u_{nm}$, where Ξ_{nm} is the tensor of the deformation potential of the two-dimensional electrons, and u_{nm} is the strain tensor. Moreover, since GaAs is a weak piezoelectric, the dipole moment per unit volume is connected with the deformation $P_n = \beta_{nmk}u_{mk}$, where β_{nmk} are the piezomoduli.

The presence of free carriers in the layer can lead to the presence of a screening potential. However, taking into account the substantial spatial dispersion and the comparatively short wavelength λ of the thermal phones (assuming $\lambda \sim l_H$), we shall neglect the screening potential produced by the phonons.

The deformation created by a phonon has the form

$$u_{jn} = e^{ik_zz}e^{i\mathbf{qx}}(\hbar/\rho c_s k\mathscr{L}^3)^{1/2}\,i(k_jd_n^s+k_nd_j^s)/2,$$

where **q** is the component of the wave vector in the plane of the layer, c_s is the velocity of sound, $\mathbf{d}^s(\mathbf{k})$ is the polarization vector, ρ is the density of GaAs, $\mathscr{L}$ is the dimension of the sample. Solving the Poisson equation with account of the piezocharges, it is not difficult to find the electric potential in the plane of the layer.[8] We obtain the expression

$$H_{e,ph} = \left(\frac{\hbar}{\mathscr{L}^3}\right)^{1/2}\left\{\sum_{ks}V_s(\mathbf{k})\,a_{\mathbf{k}s}\int e^{i\mathbf{kr}}\Psi^+(\mathbf{r})\,\Psi(\mathbf{r})\,d^3r + \text{c.c.}\right\}, \tag{1.2}$$

where the vertex is

$$V_s(\mathbf{k}) = \frac{i}{(\rho c_s k)^{1/2}}\,\frac{k_nd_m^s+k_md_n^s}{2}\left[\Xi_{nm}-\frac{4\pi i}{\chi k^2}k_j\beta_{jnm}\right], \tag{1.3}$$

$a_{\mathbf{k},s}$ is the phonon annihilation operator, Ψ^+ is the electron creation operator χ is the dielectric susceptibility of GaAs. The wave functions of the electron have the form $\varphi_n(\mathbf{r}) = f_0(z)\psi_n(x)$, where $f_0(z)$ describes the wave function at the first level of size quantization transverse to the two-dimensional level, $\psi_n(\mathbf{x})$ are the wave functions describing the states of the electron in the layer. Upon substitution of this expression in (1.2), we obtain the factor

$$J(k_z)=\int \exp(ik_z z) f_0(z) f_0^*(z)\,dz,$$

which can be included in the effective electron-phonon quantity

$$\tilde{V}_s(\mathbf{k})=J(k_z)V_s(\mathbf{k}).$$

The thickness of the two-dimensional layer, where the electrons are located, amounts to tens of lattice constants; therefore, with sufficient accuracy, the quantitites $\Xi_{n,m}$ and β_{jnm} are equal to their bulk values.

Introducing the retarded and advanced Green's functions of the electrons:

$$G^{R,A}(\varepsilon,r_1,r_2)=\hbar\sum_n \frac{\psi_n^*(r_1)\psi_n(r_2)}{\varepsilon-E_n\pm i\delta},\quad \delta>0,$$

$$\tilde{G}(\varepsilon,\mathbf{r}_1,\mathbf{r}_2)=G^R-G^A=-2\pi i\hbar\sum_n \delta(\varepsilon-E_n)\psi_n^*(\mathbf{r}_1)\psi_n(\mathbf{r}_2),$$

we can write down the expression for the lifetime of the phonon in the form

$$\frac{1}{\tau}=\frac{|\tilde{V}_s(\mathbf{k})|^2}{2\pi\hbar^2\mathscr{L}^3}\int d\varepsilon_1\,d\varepsilon_2\,d^2r_1\,d^2r_2\,(n_1-n_2)$$
$$\times\,\delta(\varepsilon_1-\varepsilon_2+\hbar\omega)\tilde{G}(\varepsilon_1,\mathbf{r}_1,\mathbf{r}_2)\tilde{G}(\varepsilon_2,\mathbf{r}_2,\mathbf{r}_1)\cdot$$
$$\times\,\exp[i\mathbf{q}(\mathbf{r}_1-\mathbf{r}_2)].$$

This expression should be averaged over the random potential of the impurities, on which depend the wave functions of the electron. The behavior of the Green's function will then depend only on the difference $\mathbf{r}_1-\mathbf{r}_2$, so that the second integration over d^2r gives the area of the heterojunction.

Multiplying by the number N of heterojunctions in the sample, we obtain

$$\frac{1}{\tau}=\frac{|\tilde{V}_s(\mathbf{k})|^2}{2\pi\hbar^2}\frac{N}{\mathscr{L}}\int d\varepsilon_1\,d\varepsilon_2\,d^2r\,e^{i\mathbf{q}\mathbf{r}}\langle\tilde{G}(\varepsilon_1,0,\mathbf{r})\tilde{G}(\varepsilon_2,\mathbf{r},0)\rangle$$
$$\times\,(n_1-n_2)\,\delta(\varepsilon_1-\varepsilon_2+\hbar\omega),\tag{1.4}$$

where the angular brackets denote averaging over the random potential

Thus, for calculation of the corrections to the lifetime of the phonons it is necessary to find the imaginary part of the polarization operator

$$\Pi(\mathbf{q},\omega)=\frac{1}{2\pi}\int d\varepsilon_1\,d\varepsilon_2\,d^2r\,e^{i\mathbf{q}\mathbf{r}}\langle\tilde{G}(\varepsilon_1,0,\mathbf{r})\tilde{G}(\varepsilon_2,\mathbf{r},0)\rangle$$
$$\times\,(n_1-n_2)\delta(\varepsilon_1-\varepsilon_2+\hbar\omega).\tag{1.5}$$

Since we do not take into consideration the interactions of the electrons, the Green's functions that enter into (1.5) are single-electron Green's functions, for which we can use the usual Feynman representation[9] in the form of an electron path integral:

$$G^R(\varepsilon,r,0)=-i\int_0^\infty dt\int D\mathbf{q}(\tau)\exp\left[i\frac{S(t,\mathbf{r},0)+\varepsilon t}{\hbar}\right],$$

$$G^A(\varepsilon,r,0)=i\int_{-\infty}^0 dt\int D\mathbf{q}(\tau)\exp\left[i\frac{S(t,\mathbf{r},0)+\varepsilon t}{\hbar}\right],$$

where the action is

$$S(t,\mathbf{r},0)=\int_0^t d\tau\left[\frac{m\dot{q}^2}{2}+\frac{e}{c}\mathbf{A}\dot{\mathbf{q}}-U(\mathbf{q})\right],$$

$\mathbf{q}(0)=0$, $\mathbf{q}(t)=\mathbf{r}$, A is the vector potential of the constant magnetic field, $U(\mathbf{q})$ is the potential energy of the electron in the random potential of the impurities, and integration over t, as usual, implies convergence of the integral at $t\to\pm\infty$ on account of the imaginary contributions to ε. Introducing an averaging over the random (Gaussian) potential U and carrying out Gaussian integration over U, we obtain

$$\Pi(\mathbf{q},\omega)=\frac{1}{2\pi}\int d\varepsilon_1\,d\varepsilon_2\,dt_1\,dt_2\,d^2r\exp[i(\mathbf{q}\mathbf{r}+\varepsilon_1t_1+\varepsilon_2t_2)]$$
$$\times\,\delta(\varepsilon_1-\varepsilon_2+\hbar\omega)(n_1-n_2)\int D\mathbf{q}_1(\tau)D\mathbf{q}_2(\tau)\exp\left[i\frac{S_{eff}(t_1,t_2,\mathbf{r})}{\hbar}\right],\tag{1.6}$$

where

$$S_{eff}=\int_0^{t_1}L_0(\mathbf{q}_1,\dot{\mathbf{q}}_1)\,d\tau+\int_0^{t_2}L_0(\mathbf{q}_2,\dot{\mathbf{q}}_2)\,d\tau$$
$$+\frac{i}{\hbar}\int_0^{t_1}d\tau_1\int_0^{t_1}d\tau_2\,R(\mathbf{q}_1(\tau_1)-\mathbf{q}_1(\tau_2))\cdot$$
$$+2\int_0^{t_1}d\tau_1\int_0^{t_2}d\tau_2\,R(\mathbf{q}_1(\tau_1)-\mathbf{q}_2(\tau_2))$$
$$+\int_0^{t_2}d\tau_1\int_0^{t_2}d\tau_2\,R(\mathbf{q}_2(\tau_1)-\mathbf{q}_2(\tau_2)),\tag{1.7}$$

and where $R(|\mathbf{r}|)$ is the pair correlation function of the random potential, and $L_0(\mathbf{q},\dot{\mathbf{q}})$ is the Lagrangian function of free electrons in a magnetic field. The integrals over t_1 and t_2 converge as $t_{1,2}\to\pm\infty$ because of the presence of the function R, even without the addition of imaginary parts to ε_1 and ε_2. Therefore, the integrand is the same with respect to t_1 as to t_2, and the contour can be displaced from the real axis into the complex plane.

2. CALCULATION OF THE POLARIZATION OPERATOR

We shall carry out the calculation of the polarization operator under the assumption that the momentum of the phonon is sufficiently large, $ql_H\sim 1$, while the correlator of the random potential changes little over the magnetic length, meaning a quasi-classical situation, i.e., the applicability of the method of steepest descent for the functional integral (1.6).

Varying the effective action (1.7), we obtain the equation of motion

$$m\frac{d^2\mathbf{q}_1}{d\tau^2}=\frac{e}{c}[\dot{\mathbf{q}},\mathbf{H}]+\frac{i}{\hbar}\frac{\partial}{\partial\mathbf{q}_1(\tau)}\left\{\int_0^{t_1}d\tau_1 R(\mathbf{q}_1(\tau)-\mathbf{q}_1(\tau_1))\right.$$

$$\left.+\int_0^{t_2}d\tau_1 R(\mathbf{q}_1(\tau)-\mathbf{q}_2(\tau_1))\right\},$$

$$m\frac{d^2\mathbf{q}_2}{d\tau_2}=\frac{e}{c}[\dot{\mathbf{q}}_2\mathbf{H}]+\frac{i}{\hbar}\frac{\partial}{\partial\mathbf{q}_2(\tau)}\left\{\int_0^{t_1}d\tau_1 R(\mathbf{q}_2(\tau)-\mathbf{q}_1(\tau_1))\right.$$

$$\left.+\int^{t_2}d\tau_1 R(\mathbf{q}_2(\tau)-\mathbf{q}_2(\tau_1))\right\} \tag{2.1}$$

with boundary conditions

$$\mathbf{q}_1(0)=\mathbf{q}_2(t_2)=0,\quad \mathbf{q}_1(t_1)=\mathbf{q}_2(0)=\mathbf{r}.$$

Equations (2.1), in spite of their integral character, possess an energy integral for each of the particles in correspondence with the elastic character of the scattering by the random potential; moreover, the interacting particles exchange momenta, owing to the homogeneity of the function R.

The problem of finding for (2.1) solutions satisfying the boundary conditions is very complicated, even under the assumption of slowness of change in R. We limit ourselves to the case in which the distances traversed along the saddle trajectories within the times t_1 and t_2 (also subject to determination) are small in comparison with the correlation length L. In this case the function R, which enters into (2.1), can be expanded in terms of its arguments with accuracy to second order, and the solutions can be found analytically. Here we shall make use of the fact that at a very large correlation length the corrections to the free motion of the electrons will be small and we can limit ourselves to first order-perturbation theory in the action, i.e., calculate the action S_{eff} directly on a trajectory of the free electrons in the magnetic field; this trajectory satisfies the imposed boundary conditions

$$z_1\equiv q_{1x}+iq_{1y}=z_{01}+A_1\exp(i\omega_c\tau), \tag{2.2}$$

$$z_1^*\equiv q_{1x}-iq_{1y}=z_{01}^*+B_1\exp(-i\omega_c\tau),$$

where

$$z_{01}=-A_1=-z[\exp(i\omega_c t_1)-1]^{-1}, \tag{j}$$

$$z_{01}^*=-z^*[\exp(-i\omega_c t_1)-1]=-B_1,$$

$$z=x+iy,\ z^*=x-iy.$$

Similar formulas are obtained for the solution of the second equation in (2.1):

$$z_2\equiv q_{2x}+iq_{2y},\quad z_2^*\equiv q_{2x}-iq_{2y},$$

$$z_{02}=-A_2=-z[\exp(-i\omega_c t_2)-1]^{-1},$$

$$z_{02}^*=-B_2=-z^*[\exp(i\omega_c t_2)-1]^{-1}. \tag{2.3}$$

For perturbation theory to be applicable it is necessary that the characteristic stiffness in the random potential $tR''(0)/\hbar$ be small in comparison with $m\omega_c^2$. Moreover, at different centers of the orbits, the particle 1 will have the mean drift velocity

$$v_1=(ct_2/eH)\langle q_{01}-q_{02}\rangle R''(0),$$

which leads to a displacement $\delta q_1=v_1\tau$ for the first particle and similarly (with the replacement 1→2) for the second. In order that such a perturbation-theory calculation be correct, it is necessary that the difference $\delta q_1-\delta q_2$ be small in comparison with $\langle q_{01}-q_{02}\rangle$. Finally, we obtain the conditions

$$\frac{|t_1+t_2|\,|t_{1,2}|}{\hbar^2}|R''(0)|\,l_H{}^2\ll 1,\quad \frac{|t_{1,2}R''(0)|}{\hbar^2\omega_c}l_H{}^2\ll 1, \tag{2.4}$$

which can also be obtained by substitution of (2.2) and (2.3) in the terms containing $R''(0)$ in Eqs. (2.1), and comparison of them with those retained. Substitution of (2.2) and (2.3) in the action (1.7) gives the extremal action with accuracy to first order in $R''(0)$, since (2.2) and (2.3) are extremal for the action with $R''(0)=0$, which is easily calculated:

$$S_{eff}(t_1,t_2,\mathbf{r})=\frac{m\omega_c r^2}{2}\left(\operatorname{ctg}\frac{\omega_c t_1}{2}+\operatorname{ctg}\frac{\omega_c t_2}{2}\right)+\frac{iR(0)}{2\hbar}(t_1+t_2)^2$$

$$-i\frac{R''(0)r^2}{8\hbar}\left\{\frac{t_1(t_1+2t_2)}{\sin^2(\omega_c t/2)}+\frac{t_2(t_2+2t_1)}{\sin^2(\omega_c t_2/2)}\right.$$

$$\left.+2t_1t_2\frac{\cos[\omega_c(t_1+t_2)/2]}{\sin(\omega_c t_1/2)\sin(\omega_c t_2/2)}\right\}$$

$$-i\frac{R''(0)r^2}{2\hbar\omega_c}(t_1+t_2)\left(\operatorname{ctg}\frac{\omega_c t_1}{2}+\operatorname{ctg}\frac{\omega_c t_2}{2}\right)^2. \tag{2.5}$$

It must be remembered that the action S_{eff} is calculated at the point of the extremum also and with variables t_1, t_2 which are complex, so that the singularities associated with the poles of (2.5) are insignificant. As we shall see, at low temperatures, the extremal values of t_1, t_2 are such that $|t_1+t_2|\ll|t_{1,2}|, \omega_c|t_{1,2}\gg 1$, which allows us to simplify substantially the expression for the action:

$$S_{eff}\approx r^2\left[\frac{m\omega_c}{4}\left(\operatorname{ctg}\frac{\omega_c t_1}{2}+\operatorname{ctg}\frac{\omega_c t_2}{2}\right)\right.$$

$$\left.+\frac{iR''(0)t_1t_2}{8\hbar}(\operatorname{ctg}\omega_c t_1+\operatorname{ctg}\omega_c t_2)^2\right]$$

$$+\frac{i}{2\hbar}R(0)(t_1+t_2)^2. \tag{2.6}$$

The corrections to the extremal action (2.6) should be quadratic in the deviations from the extremal trajectory, so that to obtain the polarization operator (1.6), it is necessary to carry out the Gaussian integration over periodic trajectories that begin and end at $q_1=q_2=0$. The corrections connected with the second derivatives R'' are small and can be neglected as having the additional smallness $1/\omega_c t$ in comparison with the other terms.

Integration over r is Gaussian and, keeping in the experiment only those terms that are linear in $R''(0)$, we obtain

$$\Pi(\mathbf{q},\omega)=\frac{1}{2\pi}\int dt_1\,dt_2\,d\varepsilon_1\,d\varepsilon_2\exp\left(\frac{i\varepsilon_1t_1}{\hbar}+\frac{i\varepsilon_2t_2}{\hbar}\right)$$

$$\times[n(\varepsilon_1)-n(\varepsilon_2)]\delta(\varepsilon_1-\varepsilon_2+\omega)$$

$$\times\exp\left[-\frac{R(0)(t_1+t_2)^2}{2\hbar^2}-\frac{R''(0)t_1t_2}{2\hbar^2}q^2l_H{}^4\right]\int D\mathbf{q}_1(\tau)D\mathbf{q}_2(\tau)$$

$$\times\exp\left[\frac{iS_1{}^0[t,r,0)}{\hbar}+\frac{iS_2{}^0(t,\mathbf{r},0)}{\hbar}\right]e^{i\mathbf{qr}}\,d^2r, \tag{2.7}$$

where we have combined the factor corresponding to the free particles in the extremal action with the integral over the periodic trajectories. The result is a product of two Green's functions of free particles

$$\int D\mathbf{q}_1(\tau)\exp\left[\frac{iS_1{}^0(t,\mathbf{r},0)}{\hbar}\right]=\frac{1}{\mathscr{L}l_H}\sum_{n,\varkappa}\exp[-i\omega_c t(n+{}^1/_2)]$$

$$\times\exp(i\varkappa y)\,\Phi_n\left(\frac{x}{l_H}+\varkappa l_H\right)\Phi_n(\varkappa l_H),$$

where n denumerates the Landau levels, the Landau gauge is chosen (Π itself does not depend on the gauge), and $\Phi_n(\xi)$ are the normalized harmonic oscillator functions. Upon substition of this expression in the polarization operator, we get a sum of terms corresponding to transitions between the different Landau levels. The integral over t_1 and t_2 corresponding to one such term is also Gaussian:

$$I=\int dt_1\,dt_2\exp\left[i\frac{\tilde\varepsilon_1t_1+\tilde\varepsilon_2t_2}{\hbar}-\frac{1}{2}\frac{R(0)\,(t_1+t_2)^2}{\hbar^2}\right.$$

$$\left.-\frac{R''(0)\,t_1t_2}{2\hbar^2}q^2l_H{}^4\right],\tag{2.8}$$

where

$$\tilde\varepsilon_{1,2}=\varepsilon_{1,2}-\hbar\omega_c(n+{}^1/_2),$$

and is determined by the extremal points of the exponential:

$$t_1{}^0-t_2{}^0=-2i(\tilde\varepsilon_i-\tilde\varepsilon_2)\hbar/R''(0)\,q^2l_H{}^4.$$

$$t_1{}^0+t_2{}^0=i(\tilde\varepsilon_1+\tilde\varepsilon_2)\hbar/2R(0)\tag{2.9}$$

[in the derivation of this formula it was assumed that$|R''(0)|q^2l_H^4\ll R(0)$]. It is seen then that $|\mathrm{t}_1^0-\mathrm{t}_2^0|\gg|t_1^0-t_2^0|$ at sufficiently low temperatures, since $\tilde\varepsilon_2-\tilde\varepsilon_1\gg\hbar\omega$. The integral under consideration is easily calculated:

$$I=\frac{\pi\hbar}{ql_H{}^2(R(0)\,|R''(0)|)^{1/2}}\exp\left[-\frac{(\tilde\varepsilon_1+\tilde\varepsilon_2)^2}{8R(0)}+\frac{(\tilde\varepsilon_1-\tilde\varepsilon_2)^2}{2R''(0)\,q^2l_H{}^4}\right].$$

It is seen then that only transitions within the limits of a single Landau level ($n_1=n_2$) are significant under the condition $|R''(0)|q^2l_H^4\ll(\hbar\omega_c)^2$ that is valid by virtue of the large correlation length of the random potential.

In the derivation of the formula (2.7) it was assumed that

$$|t_1|=\hbar^2\omega/|R''(0)|\,q^2l_H{}^4\gg\omega_c{}^{-1},$$

which, under the condition $ql_H\sim1$, imposes the limitation

$$L^2\hbar^2\omega_c\omega/l_H{}^2R(0)\gg1,$$

which can also be satisfied at sufficiently large L. This is not a limitation in principle, since the answer can also be obtained by use of the complete formula (2.5). However, the latter is extremely cumbersome.

In addition, we have assumed that the arguments of the function R in Eq. (2.1), which are of the order of ql_H^2, are small in comparison with L, a condition certainly satisfied in the entire region $ql_H\sim1$. The conditions (2.4) of applicability of perturbation theory in the case of large imaginary times (2.9) correspond to the inequalities

$$\hbar\omega(\varepsilon_1+\varepsilon_2)/R(0)\,q^2l_H{}^2\ll1,\quad \omega/\omega_cq^2l_H{}^2\ll1,$$

and, in the case $\omega\ll\omega_c[R(0)/\hbar^2]^{1/2}$, both inequalities will be satisfied at not too small ql_H.

To obtain the final expression for the polarization operator, it is necessary to calculate an integral with oscillator functions

$$\int\Phi_n\left(\frac{x}{l_H}+\varkappa l_H\right)\Phi_n(\varkappa l_H)\,e^{i\varkappa y}\frac{d\varkappa}{2\pi}\int\Phi_n\left(\frac{x}{l_H}+\varkappa' l_H\right)$$

$$\times\Phi_n(\varkappa' l_H)\frac{d\varkappa'}{2\pi}e^{i\mathbf{qr}}\,d^2r$$

$$=\mathscr{L}_n{}^2(ql_H),$$

where

$$\mathscr{L}_n(r)=\int\Phi_n\left(x-\frac{\xi}{2}\right)\Phi_n\left(x+\frac{\xi}{2}\right)e^{i\eta x}\,dx=e^{-r^2/4}L_n\left(\frac{r^2}{2}\right)$$

are the Laguerre polynomials and $r^2=\xi^2+\eta^2$ (see, for example, Ref. 10).

In sum, we obtain from (2.7) a formula for the polarization operator

$$\Pi(\mathbf{q},\omega)=\frac{\hbar^2}{l_H{}^2}(R(0)\,|R''(0)|\,q^2l_H{}^4)^{-1/2}$$

$$\times\sum_{m,\sigma}\mathscr{L}_m{}^2(ql_H)\int d\varepsilon_1\,d\varepsilon_2[n(\varepsilon_1)-n(\varepsilon_2)]$$

$$\times\delta(\varepsilon_1-\varepsilon_2+\hbar\omega)\exp\left[-\frac{(\varepsilon_{1m\sigma}+\varepsilon_{2m\sigma})^2}{8R(0)}+\frac{(\varepsilon_{1m\sigma}-\varepsilon_{2m\sigma})^2}{2R''(0)\,q^2l_H{}^2}\right],$$

$$\varepsilon_{im\sigma}=\varepsilon_i-E_{m\sigma},\quad E_{m\sigma}=\hbar\omega_c(m+{}^1/_2)+g\mu_B\sigma,\quad i=1,2,$$

where we have taken the spin splitting into account by introducing the g-factor (μ_B is the Bohr magneton). In summing over m, σ, only such m, σ are important for which $|\varepsilon_{im\sigma}-\mu|<\hbar\omega_c$. Integration over ε_1, ε_2 is not difficult to achieve, by expanding $n(\varepsilon_1)-n(\varepsilon_2)$ in terms of $\varepsilon_2-\varepsilon_1$ and, as usual, replacing the derivative $\partial n/\partial e$ by $\delta(\varepsilon-\mu)$. We finally obtain

$$\Pi(\mathbf{q},\omega)=\frac{\hbar^3\omega}{l_H{}^2[R(0)\,|R''(0)|\,q^2l_H{}^4]^{1/2}}$$

$$\times\sum_{m,\sigma}\exp\left[-\frac{(\mu-E_{m\sigma})^2}{2R(0)}+\frac{\hbar^2\omega^2}{2R''(0)\,q^2l_H{}^4}\right]\mathscr{L}_m{}^2(ql_H).\tag{2.10}$$

Formula (2.10) corresponds to the following physical picture: the electrons with a given energy ε are localized near the level lines of the random potential $U(\mathbf{r})=\varepsilon$. Because of the large correlation length L we can neglect the curvature of this line, so that the wave functions can be represented locally in the form $\psi=\Phi(\eta/l_H-q_sl_H)\exp iq_ss$, where s is the coordinate along the level line, η is the coordinate along the normal to it, and q_s must be so chosen that $U(l_H^2q_s)=\varepsilon$. Substituting the wave functions and the energies in (1.1), and averaging over the potential, we obtain the same answer (2.10).

In order that such a local consideration be correct in the calculation of the matrix element entering into (1.1), it is necessary in addition that the wavelength of the phonon be small in comparison with the correlation radius L. The reason is that as $qL\to0$, the matrix element of $e^{iq'\cdot r}$ between the states with ε_1 and ε_2 also tends to zero because of the orthogonality of the corresponding wave functions. The number of levels with $\varepsilon_1-\varepsilon_2=\hbar\omega$ will still be large enough, since in the quasiclassical limit $\varepsilon_1-\varepsilon_2\sim\Delta\eta\partial U/\partial\eta$, where the minimal distance between the level lines representing the wave function and corresponding to the given energy is determined from the flux-quantization condition $\Delta\eta L\sim l_H^2$. By virtue of the not very large difference between l_H and L in the experiment, we shall assume that the increase of Γ ceases with decrease of q at ql_H, which we shall take into consideration in what follows by the replacement $q^2 \to q^2 + l_H^{-2}$ in the denominator of the factor in front of the summation sign.

3. CALCULATION OF CORRECTIONS TO THE THERMAL CONDUCTIVITY

The lifetime of the phonon is obtained by substitution of (2.10) in (1.4), which gives

$$\frac{1}{\tau}=\frac{\hbar\omega(\mathbf{k},s)}{l_H^2}\frac{N}{\mathscr{L}}\frac{|\tilde{V}_s(k)|^2}{[R(0)|R''(0)|(q^2l_H^4+l_H^2)]^{1/2}}$$
$$\times\exp\left[\frac{(\hbar\omega(\mathbf{k},s))^2}{2R''(0)q^2l_H^4}\right]\sum_{m,\sigma}\mathscr{L}_m^2(ql_H^2)\exp\left[-\frac{(\mu-E_{m\sigma})^2}{2R(0)}\right]. \qquad (3.1)$$

The thermal conductivity $\varkappa = C_{\rm ph}c^2\tau_{\rm ph}/3$, where $C_{\rm ph}$ is the phonon heat capacity and c is the mean velocity of sound, contains the total lifetime, including the effects of scattering on boundaries; as usual, the reciprocal lifetimes are additive (independent transition processes):

$$\tau_{ph}^{-1}=\langle\tau_0^{-1}\rangle+\langle\tau^{-1}\rangle$$

(the angular brackets denote averaging over the thermal distribution of the phonons). The quantity τ corresponds to the residual scattering not connected with the two-dimensional electrons and with the magnetic field. In the experiment[1] the path length of the phonons is comparable with the length of the sample and with the distance between the thermometers. Therefore, the interpretation of the results of the experiment, made in Ref. 1 in terms of the equation of thermal conductivity, can be correct only in order of magnitude. If we adhere to this point of view, they measured in the experiment of Ref. 1 the difference in temperatures for fixed heat flow, i.e., the quantity

$$\Delta\varkappa^{-1}\sim(C_{ph}c^2)^{-1}\langle\tau^{-1}\rangle \qquad (3.2)$$

was measured to within a constant coefficient. The temperature dependence of $\Delta\varkappa^{-1}$, according to the experiment, has the form T^{-3} (Ref. 1), so that $\langle\tau^{-1}\rangle$ should not depend on the temperature. Using formulas (3.1), (1.3) we see that this agrees best with a predominant effect of bulk piezoelectricity:

$$\left\langle\frac{1}{\tau}\right\rangle=\left\{\sum_s\int\frac{2\pi k^2\,dk\sin\vartheta\,d\vartheta}{\exp[\hbar\omega(k,s)/T]-1}\right.$$
$$\times\frac{N}{\mathscr{L}\rho l_H^4[R(0)|R''(0)|(q^2+l_H^{-2})]^{1/2}}$$
$$\times\exp\left[\frac{(\hbar c_s)^2}{2R''(0)l_H^4\sin^2\vartheta}\right]\left(\frac{4\pi}{\chi}\beta_{jpr}\frac{k_pk_j}{k^2}d_r^s\right)^2$$
$$\times J(k\cos\vartheta)\sum_{n,\sigma}\mathscr{L}_n^2(ql_H)$$
$$\left.\times\exp\left[-\frac{(\mu-E_{m\sigma})^2}{2R(0)}\right]\right\}\left[\int\frac{d^3\mathbf{k}}{\exp[\hbar\omega(k,s)/T]-1}\right]^{-1}.$$

By virtue of the smallness of $R''(0)$, the integral over the angles builds up in the vicinity of $\vartheta = \pi/2$ and it can be computed by expanding an exponential containing $1/\sin^2\vartheta$ about this point. Moreover, since we have used a rough gas-kinetic formula for $\varkappa$, we can then, with the same degree of accuracy, replace k by the thermal momentum of the phonon $k_T = T/\hbar c_s$, which gives

$$\Delta\varkappa^{-1}=\frac{N}{\rho l_H^4\mathscr{L}}\sum_s(C_{ph}^s(T)c_s^2)^{-1}$$
$$\times\{R(0)|R''(0)|[(k_T^s)^2+l_H^{-2}]\}^{-1/2}$$
$$\times\exp\left[\frac{\hbar c_s^2}{2R''(0)l_H^2}\right]\overline{\left(\frac{4\pi}{\chi}\beta_{jpr}\frac{k_pk_j}{k^2}d_r^s\right)}$$
$$\times\sum_{n,\sigma}\mathscr{L}_n^2(k_T^sl_H)\exp\left[-\frac{(\mu-E_{n\sigma})^2}{2R(0)}\right],$$

where the bar indicates averaging over the angles in the plane of the two-dimensional structure and we have taken it into account that $J(0) = 1$ in view of the normalizability of the wave functions of the transverse motion. Thus, the fundamental temperature dependence is given by the phonon heat capacity $C_{\rm ph}$ and the remaining factors either do not depend on the temperature, or depend weakly.

As is known from experiments, the mean square value of the random potential oscillates with the occupation n_s of the Landau levels. Here the smallest value corresponds to the half-integer occupation, when there are free charges and the screening is a maximum; at the same time, the largest value of the random potential is observed close to full occupation, when the screening is weak. At present, there exists no detailed theory that is quantitatively confirmed by experiment. For orientation, we use the results of Ref. 11, where the thermodynamic density of states has been measured, and where it is shown that the experimental data are satisfactorily explained by a mean-square fluctuation of the random potential $R(0) = 0.25H$ meV/T.

The results of the calculation of $\Delta\varkappa^{-1}$ from Eq. (3.3) are shown in Fig. 1 (solid curve) at a temperature 0.6 K, with $R''(0) = -(3/2)d^{-2}R(0)$, where $d = 300$ Å is the size of the spacer. It turned out that under the conditions of the experiment[1] the contribution from the correlation of the potential at different points is insignificant, so that the specific coefficient in the expression for $R''(0)$ plays no role. The proportionality coefficient and the part of the temperature difference that does not depend on the magnetic field were determined from the conditions of best agreement with experiment,[1] the results of which were indicated by the

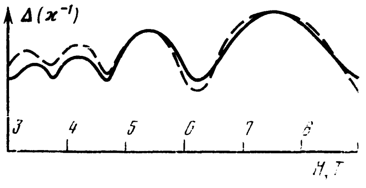

FIG. 1. Temperature difference, at fixed heat flow, proportional to $\Delta(\varkappa^{-1})$, as a function of the magnetic field. The solid curve is the result of calculation from Eq. (3.3). The coefficient of proportionality and the part of the temperature difference that is not dependent on the magnetic field were determined from the condition of best agreement with the experiment of Ref. 1. The dashed curve represents the data of Ref. 1.

dashed curve. The quantity $\mu(n_s)$ was taken from the experiments of Ref. 11.

In conclusion, we note that the use of the Feynman representation for the single Green's function of noninteracting electrons allowed us to avoid the necessity of using a replica or a supersymmetric approach for the calculation of the kinetic quantities, as was first noted in Ref. 12. The calculation of the phonon lifetime greatly simplifies the finiteness of the value of the phonon momentum, thanks to which it is possible to obtain an answer within the limit of a slowly changing random potential.

One can attempt to apply a similar approach to the calculation of the ohmic conductivity of two-dimensional electrons. In this case, however, $q \rightarrow 0$ and the time on the saddle trajectory becomes large, so that the approximations made in the current paper become invalid, since the electron manages to drift to significant distances.

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