Theory of surface waves in ferromagnets

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We investigate theoretically the effect of spatial dispersion (i.e., the exchange interaction) on the dispersion relation for surface waves in a ferromagnetic insulator. We study surface magnetic polaritons caused by the interaction of electromagnetic waves with Damon-Eshbach waves.

1. The magnetic permeability tensor $\mu_{ik} = \mu_{ik}(\omega,\mathbf{k})$ contains all the essential information about the high-frequency properties of magnets; the specific form of this tensor is in turn determined by the equation of motion of the magnetic moment[1] (or of the magnetic sublattices in the case of ferrites and antiferromagnets). The temporal dispersion of the magnetic permeability (i.e., its dependence on the frequency ω) is a consequence of the precession of the magnetic moments, while the spatial dispersion (i.e., the dependence on wave vector $\mathbf{k}$) is a consequence of the propagation of spin waves in the magnets. The characteristic frequencies are the precession frequency $\omega_0 = gH_0$ (where H_0 is the magnetic field within the magnet, which can also include the anisotropy field) and the frequency $\omega_M = 4\pi gM$ (where M is the magnetic moment per unit volume), which essentially plays the role of an oscillator strength (see below). The characteristic length for the spatial dispersion is $1/k_{\rm exc} = (J/\beta M)^{1/2}a$, where a is the interatomic spacing, $\beta \approx g\hbar$ is the Bohr magneton, and J is the exchange integral, which is the same order of magnitude as the Curie temperature T_c. As a rule we have $J \gg \beta M \sim 1$ K.

In order for spatial dispersion to be neglected, a condition must be fulfilled which is more rigorous than the usual condition for macroscopic fields, i.e., $ak < 1$ (see Ref. 2). Namely, the inequality

$$ak \ll (\beta M/J)^{1/2} \ll 1 \qquad (1)$$

must be satisfied. Neglect of dissipative processes implies that the magnetic permeability tensor is Hermitian ($\mu_{ik} = \mu_{ki}^*$) and ensures that the inequality

$$\gamma_d \ll k, \qquad (2)$$

holds, where γ_d is the logarithmic attenuation rate of the waves caused by dissipation.

For the simplest assumptions about the structure of the ferromagnet (i.e., free precession of the magnetic moment $\mathbf{M}$ around the magnetic field $\mathbf{H}$) and neglecting spatial dispersion along with dissipative processes, we have

$$\mu_{ik} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mu & i\mu' \\ 0 & -i\mu' & \mu \end{pmatrix}, \quad \mu = 1 + \frac{\omega_M\omega_0}{\omega_0^2 - \omega^2},$$

$$\mu' = \frac{\omega_M\omega}{\omega_0^2 - \omega^2}. \qquad (3)$$

Inclusion of spatial dispersion corresponds to the replacement

$$\omega_0 \to \omega_0 + \alpha\omega_M(k_x^2 + k_y^2 + k_z^2), \quad \alpha = (J/\beta M)a^2. \qquad (4)$$

The waves we are interested in propagate along the surface $y = 0$ of the ferromagnet, which occupies the half-space $y > 0$. We introduce the two-dimensional wave vector $\mathbf{k}$ of the surface waves:

$$\mathbf{k} = (k_x = k\cos\theta,\ k_z = k\sin\theta).$$

The dependence on the coordinate y is determined in the process of solving the problem (see below).

2. It is well-known that temporal dispersion of the magnetic susceptibility leads to the existence of peculiar quasistatic oscillations (the Walker modes[3]), whose dispersion relation can be obtained by solving the linearized spatially-inhomogeneous problem within the magnet:

$$\mathrm{rot}\,\mathbf{h} = 0, \quad \mathrm{div}\,\mathbf{b} = 0, \quad b_i = \mu_{ik}(\omega)h_k, \qquad (5)$$

and outside the magnet:

$$\mathrm{rot}\,\mathbf{h} = 0, \quad \mathrm{div}\,\mathbf{h} = 0, \qquad (5')$$

$\mathbf{h}$ and $\mathbf{b}$ are the time-varying components of the magnetic field and magnetic induction. Of course, Eqs. (5) and (5′) must be supplemented by corresponding boundary conditions, i.e., taking into account continuity of the tangential components of the vector $\mathbf{h}$ and the normal component of the vector $\mathbf{b}$ at the boundary of the magnet.

According to (5), (5′), and (3), a peculiar nonreciprocal surface wave (see Damon and Eshbach, Ref. 4) propagates along the positive direction of the z-axis ($k_z > 0$) whose frequency is

$$\omega = \omega_0 + \omega_M/2 = \omega_{DE}, \qquad (6)$$

the penetration depth γ^{-1} both into the bulk of the magnet ($y > 0$) and into the vacuum ($y < 0$) is 2π times smaller than the wavelength along the z-axis ($\gamma = k$). The absence of dispersion (i.e., ω does not depend on k) shows that in this approximation the Damon-Eshbach (DE) wave carries no energy: its group velocity satisfies $v_{\rm gr} = \partial\omega/\partial k = 0$.

There exist several mechanisms for making the Walker modes dispersive: retardation (i.e., finiteness of the velocity of light c converts the magnetostatic waves into magnetic polaritons; see Ref. 5); interference between magnetostatic waves propagating along the two sides of a magnetic film (see Ref. 1, ch. III, § 11)), and, finally, spatial dispersion of the magnetic susceptibility due, as we mentioned above, to spin waves. Of course, under real conditions all these mechanisms play a role. However, because they are described by parameters whose magnitudes differ considerably from one

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another, we can choose conditions which, when fulfilled, cause one of the dispersion mechanisms to dominate. In this paper, we will be interested principally in the role of spatial dispersion (see § 3).

3. Let us first discuss the propagation (along the z-axis) of a surface magnetic polariton. In addition to its intrinsic interest, this discussion allows us to establish the conditions for applicability of the expressions derived in Sec. 4, which describe quasistatic waves.

The geometry of the problem was described above. The nonzero components of the magnetic and electric fields depend on coordinates in the following way:

$$h_y, h_z, e_x \propto e^{ik}\begin{cases} e^{-\gamma y}, & y>0, \\ e^{\gamma_0 y}, & y<0. \end{cases} \tag{7}$$

According to the Maxwell equations and Eqs. (3),

$$\gamma_0=(k^2-\omega^2/c^2)^{1/2}, \quad \gamma=(k^2-\varepsilon\,\mu_{\text{eff}}\,\omega^2/c^2)^{1/2}, \quad \mu_{\text{eff}}=(\mu^2-\mu'^2)/\mu. \tag{8}$$

The frequencies ω_0, ω_M are small compared to atomic frequencies. Therefore in the case of a ferromagnetic insulator the dielectric permeability ε is naturally taken to be the static dielectric permeability (in this case we have $\varepsilon > 1$).

From the Maxwell equations and natural boundary conditions (continuity of the tangential components of the magnetic and electric fields, i.e., h_z and e_x) we determine the dependence of the wave frequency ω on wave vector $\mathbf{k}$. The dispersion relation of the magnetostatic surface polariton has the form

$$(\mu'/\mu)k-\mu_{\text{eff}}(k^2-\omega^2/c^2)^{1/2}=(k^2-\varepsilon\,\mu_{\text{eff}}\,\omega^2/c^2)^{1/2},$$
$$\frac{\mu'}{\mu}=\frac{\omega\omega_M}{\omega_0(\omega_0+\omega_M)-\omega^2}, \tag{9}$$
$$\mu_{\text{eff}}=\frac{(\omega_0+\omega_M)^2-\omega^2}{\omega_0(\omega_0+\omega_M)-\omega^2}.$$

The presence in Eq. (9) of a term containing the wave vector k to the first power indicates that the wave is nonreciprocal [that is, $\omega(-k)\neq\omega(k)$]. This is a consequence of two facts: first of all, there is a vector $[\mathbf{H}_0, \mathbf{y}_0]$ in the problem, where $\mathbf{y}_0$ is the normal to the sample surface, which breaks the equivalence of the two directions along the z-axis; secondly, there is no invariance under the interchange $t\to -t$, because such an interchange reverses the signs of the magnetic field $\mathbf{H}$ and magnetic moment $\mathbf{M}$.

We will not write down the solution to Eq. (9), because of its complexity. Schematically, the function $\omega=\omega(k)$ looks as shown in Fig. 1 (compare Ref. 5). Let us note certain features of this function.

Let $k>0$. The curve $\omega=\omega(k)$ is monotonic and lies in the interval between the frequencies $[\omega_0(\omega_0+\omega_M)]^{1/2}$ and ω_{DE} given in (6);

$$\omega\approx\begin{cases}[(\omega_0+\omega_M)\omega_0]^{1/2}+\dfrac{\omega_M^2}{2\varepsilon\omega_0[\omega_0/(\omega_0+\omega_M)]^{1/2}}\dfrac{(k-k_{\text{gr}})^2}{k_{\text{gr}}^2}, & 0<k-k_{\text{gr}}\ll k_{\text{gr}}, \\ \omega_{DE}-\omega_M\omega_{DE}^2(1+\varepsilon)/8c^2k^2, & k\gg\omega_{DE}/c \end{cases} \tag{10}$$

where

$$ck_{\text{gr}}=(\omega_0+\omega_M)(\omega_0/\omega_M)^{1/2}.$$

For $k=k_{\text{gr}}$ the penetration depth γ^{-1} of the surface wave into the ferromagnet reduces to zero, since

$$\gamma=[k^2-(\omega^2/c^2)\,\mu_{\text{eff}}\,\varepsilon]^{1/2}\to\infty$$

as the frequency ω approaches $[\omega_0(\omega_0+\omega_M)]^{1/2}$ from above.

The divergence of γ implies that the macroscopic approach is inapplicable; however, it can be salvaged if we take into account the spatial dispersion of the magnetic susceptibility (see Sec. 5).

For $k<0$ the curve $\omega=\omega(k)$ is also monotonic. The limiting frequencies (points ω_A and ω_B in Fig. 1) are those values of frequency for which γ_0 (the point ω_A) and γ (the point ω_B) reduce to zero:

$$\omega_A^2=\omega_0(\omega_0+\omega_M)+\omega_0\omega_M/(\varepsilon-1), \quad k_A=-\omega_A/c,$$
$$2(\varepsilon-1)\omega_B^2=\varepsilon[2(\omega_0+\omega_M)^2+\omega_M^2]-(\omega_0+\omega_M)(2\omega_0+\omega_M)+\omega_M D^{1/2},$$
$$D=\varepsilon^2[4(\omega_0+\omega_M)^2+\omega_M^2]-2\varepsilon(\omega_0+\omega_M)(2\omega_0+\omega_M)+(\omega_0+\omega_M)^2,$$
$$k_B=-(\omega_B/c)(\varepsilon\mu_{\text{eff}})^{1/2}.$$

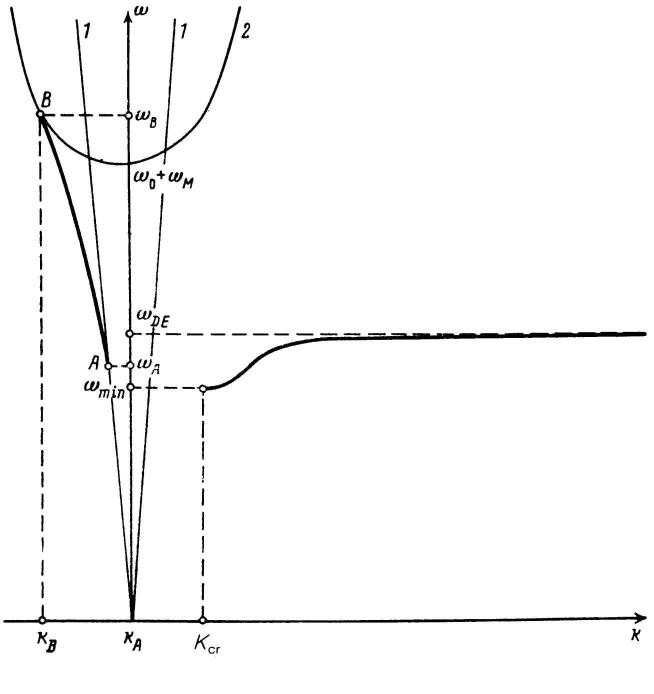

FIG. 1. Dispersion relation for surface magnetic polaritons (boldface curves); 1—$\omega=ck$; 2—$\omega=ck\,[\varepsilon\mu_{\text{eff}}]^{1/2}$ for $\omega>\omega_0+\omega_M$. See the text for the values of k_{gr}, k_A, k_B, ω_A, ω_B.

In the limits of large and near-unity values of the dielectric permittivity we have

$$\omega_B \approx \begin{cases} \omega_M/2+[(\omega_0+\omega_M)^2+\omega_M^2/4]^{1/2}, & \varepsilon \gg 1, \\ [\omega_M(\omega_0+2\omega_M)/(\varepsilon-1)]^{1/2}, & \varepsilon-1 \ll 1. \end{cases}$$

The endpoints (ω_A and ω_B) of the left branch of the dispersion curve $\omega = \omega(k)$ manifest themselves in the quadratic dependence of the phase velocity $v = \omega/k$ on frequency ($\Delta v \propto (\omega - \omega_{A,B})^2$, where $\Delta v = v - v_{A,B}$).

Comparing the left and right branches of the dispersion law for the surface magnetic polariton shows that magnetostatic waves ($kc \to \infty$) exist only for $k > 0$. Taking retardation into account in this limit [see the second expression of (10)] yields group velocities which are nonzero but small compared with the velocity of light c:

$$v_{gr} = \frac{d\omega}{dk} = 2^{5/2} \frac{(\omega_{DE}-\omega)^{3/2}}{\omega_M^{1/2}\omega_{DE}(1+\varepsilon)^{1/2}}, \quad \omega \lessdot \omega_{DE}. \quad (12)$$

The group velocity of the wave for $k < 0$ is on the order of the velocity of light over the entire frequency range.

Finally, we note that DE waves can exist not only in insulators but also in metals.[6] Assuming that the magnetic field has no effect on the motion of an electron (implying that $\omega_c\tau \ll 1$, where $\omega_c = eB/m^*c$, $B = H + 4\pi M$, m^* is the effective mass of a conduction electron, and τ is its collision time), we should replace the dielectric permittivity in (10) by its value $4\pi i\sigma/\omega$ in the metal. Then

$$\omega \approx \omega_{DE}(1-i/4\delta_M^2k^2), \quad \delta_M = c/(2\pi\sigma\omega_M)^{1/2}. \quad (13)$$

The condition for existence of DE waves in a metal is more restrictive than in the insulator: $\delta_M k \gg 1$ in place of $k \gg \omega_{DE}/c$.

4. DE waves can also propagate for $\theta \neq \pi/2$ (see §1). From (5) and (5') supplemented by the boundary conditions, it follows that

$$\omega_{DE}(\theta) = \frac{\omega_0+(\omega_0+\omega_M)\sin^2\theta}{2\sin\theta},$$

$$\omega_{DE}\left(\frac{\pi}{2}\right) = \omega_{DE}, \quad (14)$$

$$\gamma = k\frac{\omega_M\sin^2\theta+\omega_0\cos^2\theta}{\omega_M\sin^2\theta-\omega_0\cos^2\theta}. \quad (15)$$

It is clear that for $\theta = \theta_c = \arcsin[\omega_0/(\omega_0+\omega_M)]^{1/2}$ the penetration depth γ^{-1} of the DE wave decreases to zero, while for $\theta < \theta_c$ it becomes negative. Thus, θ_c serves as a critical angle for the existence of DE waves (Figs. 2 and 3).

The bulk oscillations in a ferromagnet with magnetic permeability (3) (i.e., the Walker modes) have eigenfrequencies equal to

$$\omega_W = [\omega_0(\omega_0+\omega_M\sin^2\varphi_k)]^{1/2},$$

where φ_k is the angle between the three-dimensional wave vector $\mathbf{k}$ and the magnetic moment $\mathbf{M}$. Consequently, the bulk oscillations occupy the frequency interval (ω_0, $[\omega_0(\omega_0+\omega_M)]^{1/2}$). The smallest (critical) frequency of the DE waves ω_c equals $\omega_{DE}(\theta_c) = [\omega_0(\omega_0+\omega_M)]^{1/2}$. It coincides with the upper limit of the frequency interval for bulk oscillations (Fig. 2). The passage to infinity of the quantity γ (at $\theta = \theta_c$; see Fig. 3) shows that in the neighborhood of $\theta = \theta_c$ we again have violated the condition for applicability of the macroscopic description, and in particular the condition (1), in which we must replace k by γ, that spatial dispersion can be neglected.

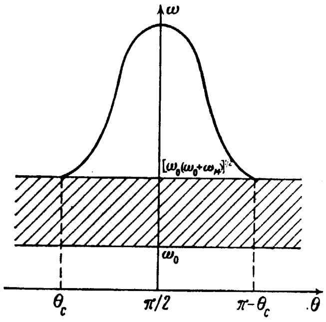

FIG. 2. Dependence of the DE wave frequency on the angle θ between the two-dimensional wave vector and the magnetic field. The crosshatched area is the region where internal Walker oscillations exist.

5. In this section (following Refs. 5–7) we will investigate the effect of spatial dispersion (spin waves) on the DE-waves. The replacement (4) leads to a complicated dependence on the y coordinate [see (7)], because of the increased number of solutions (in terms of γ) to the bulk dispersion relation

$$(k_z^2-\gamma^2)\mu(\omega, k_z, k_x, \gamma)+k_x^2=0. \quad (16)$$

According to Eqs. (3) and (4), Eq. (16) is a bicubic. Those surface waves with $\mathrm{Re}\,\gamma > 0$ must be discarded. From this we see that the structure of the magnetic potential for $y > 0$ is as follows:

$$\varphi \propto e^{c r}(Ae^{-\gamma_1 y}+Be^{-\gamma_2 y}+Ce^{-\gamma_3 y}), \quad \mathrm{Re}\,\gamma_i > 0, \quad i=1, 2, 3, \quad (17)$$

where $\boldsymbol{\rho}$ is a two-dimensional vector with coordinates x and z. The increase in the number of solutions requires additional boundary conditions (in order to determine the coefficient A, B, and C). For their derivation it is necessary to investigate the motion of the magnetic moment at the sample boundary, taking into account the difference between the

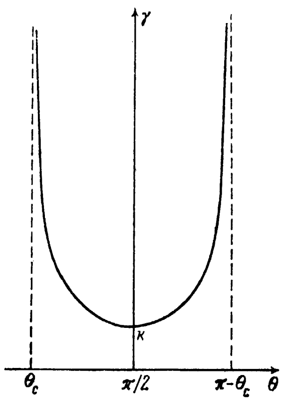

FIG. 3. Dependence of γ on θ for the DE waves (γ^{-1} is the attenuation depth).

surface and volume anisotropy energy, and also the absence of magnetic atoms for $y<0$ (Ref. 10). However, the Landau-Lifshits equation, which describes the motion of the magnetic moment in the long-wavelength limit, is a second-order equation; therefore the following phenomenological boundary condition is sufficient, in which we introduce a single constant d with the dimensions of length:

$$\mathbf{m}_{y=0}+d(d\mathbf{m}/dy)_{y=0}=0. \tag{18}$$

In the two limiting cases, condition (18) has an especially simple physical limit: in the limit $d\to\infty$ the surface anisotropy plays no role (i.e., there is no free magnetic moment at the boundary), while for $d\to 0$ the opposite case holds: the surface anisotropy fixes the direction of the magnetic moment at the boundary. Of course, the anisotropy of the surface can lead to more complicated versions of the boundary condition (18), e.g., to the replacement of the scalar d by a second-rank tensor $d_{\beta\beta'}$, where $\beta,\beta'=x,z$. Note that the sign of d and of the components $d_{\beta\beta'}$ are not determined—they can be both positive and negative; furthermore, the principal directions of the tensor $d_{\beta\beta'}$ need not coincide with the basic crystallographic directions of the sample.

To begin with, let us investigate the simplest case ($\theta=\pi/2$, i.e., $k_x=0$), which admits an exact solution to Eq. (16):

$$\gamma_1=k,\qquad \gamma_{2,3}^2=k^2+\frac{\omega_{DE}\pm[\omega_M^2/4+\omega^2]^{1/2}}{\omega_M\alpha}$$
$$\alpha=\frac{J}{\beta M}a^2 \tag{19}$$

[see (3) and (4)]. The absence of spatial dispersion implies $\alpha=0$. It is clear from (19) that $\gamma_{2,3}\to\infty$ as $\alpha\to 0$. The existence of a difference between $|\gamma_2|$, $|\gamma_3|$ and $|\gamma_1|$ allows us to analyze the dispersion relation and to obtain a fairly compact expression for the dispersion relation of the DE-waves, taking into account spatial dispersion.

Using the continuity of the corresponding components of the vectors $\mathbf{h}$ and $\mathbf{b}$, and also the additional boundary condition (18), we obtain an equation that determines the dispersion of DE waves in the form of a vanishing determinant:

$$0=\begin{vmatrix} 2\dfrac{\omega_{DE}-\omega}{\omega_M(kd-1)} & k\left(\dfrac{\omega}{\omega_+}+1\right) & k\left(\dfrac{\omega}{\omega_-}+1\right) \\ 1 & \left(\dfrac{k\omega}{\omega_+}-\gamma_2\right)(d\gamma_2-1) & \left(\dfrac{k\omega}{\omega}-\gamma_3\right)(d\gamma_3-1) \\ 1 & \left(\dfrac{\gamma_2\omega}{\omega_+}-k\right)(d\gamma_2-1) & \left(\dfrac{\gamma_3\omega}{\omega}-k\right)(d\gamma_3-1) \end{vmatrix}, \tag{20}$$

where we have written $\omega_\pm=\omega_M\pm(\omega_M^2/4+\omega^2)^{1/2}$. From (20) it is easy to obtain

$$\omega=\omega_{DE}+\frac{\omega_M^2k(1-dk)[(1-d\gamma_2)(k-\gamma_2)-(1-d\gamma_3)(k-\gamma_3)]}{4(\alpha\gamma_2-1)(\alpha\gamma_3-1)[k\omega(\gamma_2-\gamma_3)+(\omega^2+\omega_M^2/4)^{1/2}(\gamma_2\gamma_3-k^2)]}. \tag{21}$$

Expressions (19)–(20) are limited only by the weak condition ($ak\ll 1$) for macroscopic waves. However, it is clear from Eq. (19) that the DE wave preserves its meaning as a macroscopic surface excitation (i.e., a type of Walker mode) only under the stronger condition (1). In this case the exchange interaction either changes the structure of the field in the immediate vicinity of the surface (γ_2 and γ_3 are real quantities, and γ_2, $\gamma_3\gg k$), or, if Im $\gamma_{2,3}\neq 0$, the exchange interaction manifests itself in an oscillatory dependence of the field on position; the wavelength of the "wave" along y is considerably smaller than the wavelength of the DE wave.

We assume that condition (1) is fulfilled, and in subsequent transformations we will retain in (21) only the first nonvanishing terms with respect to $(J/\beta M)(ak)^2$. Specifically, in this case we should replace ω by ω_{DE} in the right side of Eq. (21), i.e., in Eqs. (19). In analyzing the expressions we obtain in this way, we will start with the following principle: every ferromagnet is characterized by a set of parameters, among them the parameter d which describes the clamping the magnetic moment at the surface. We will find that d is one of the most important of these parameters, and that it is necessary to distinguish ferromagnets (more precisely, ferromagnetic boundaries) with $d\gg(J/\beta M)^{1/2}a$ and $d\ll(J/\beta M)^{1/2}a$. The attenuation and dispersion of the DE waves depends on the "position" of $1/k$ (i.e., the wavelength of the DE wave) relative to the parameter d.

	$dk\gg 1$	$dk\ll 1$
$\mathrm{Re}(\omega-\omega_{DE})$	$2\dfrac{J}{\beta M}(ak)^2\omega_M$	$\dfrac{J}{\beta M}\dfrac{a}{d}ak\omega_M$
$-\mathrm{Im}\,\omega$	$\left(\dfrac{J}{\beta M}\right)^{3/2}(ak)^3\omega_M S_2$	$\left(\dfrac{J}{\beta M}\right)^{3/2}\left(\dfrac{a}{d}\right)^2 ak\omega_M S_1$

For the case $d\gg(J/\beta M)^{1/2}a$, our results can be reduced to a table, in which the following notations

$$S_1\left(\frac{\omega_{DE}}{\omega_M}\right)=2\left\{\left[\left(\frac{\omega_{DE}}{\omega_M}\right)^2+\frac14\right]^{1/2}+\frac{\omega_{DE}}{\omega_M}\right\}^{1/2}$$
$$-\frac12\left\{\left(\left[\left(\frac{\omega_{DE}}{\omega_M}\right)^2+\frac14\right]^{1/2}-\frac{\omega_{DE}}{\omega_M}\right)\Big/\left[\left(\frac{\omega_{DE}}{\omega_M}\right)^2+\frac14\right]\right\}^{1/2},$$

$$S_2\left(\frac{\omega_{DE}}{\omega_M}\right)=\left\{\frac{\omega_{DE}}{\omega_M}\left(\left[\left(\frac{\omega_{DE}}{\omega_M}\right)^2+\frac14\right]^{1/2}+\frac{\omega_{DE}}{\omega_M}\right)^{1/2}\right.$$
$$\left.-\frac12\left(\left[\left(\frac{\omega_{DE}}{\omega_M}\right)^2+\frac14\right]^{1/2}-\frac{\omega_{DE}}{\omega_M}\right)^{1/2}\right\}\Big/\left[\left(\frac{\omega_{DE}}{\omega_M}\right)^2+\frac14\right]^{1/2} \tag{22}$$

are introduced. In the limit of small ω_M ($\omega_M \ll \omega_0$) we have

$$S_1 \approx S_2 = 2(2\omega_0/\omega_M)^{1/2}, \tag{23}$$

while in the limit of large ω_M ($\omega_M \gg \omega_0$),

$$S_1 = [2(2^{1/2}+1)]^{1/2} - {}^1/_2(2^{1/2}-1)^{1/2} \approx 1.9,$$
$$S_2 = {}^1/_2[(2^{1/2}+1)^{1/2} - (2^{1/2}-1)^{1/2}] \approx 0.5. \tag{24}$$

According to the table, as k increases the linear dependence of Re $\omega(k)$ becomes quadratic, while the attenuation is much smaller than the dispersion.

In the case $d \ll (J/\beta M)^{1/2} a$, the condition $dk \ll 1$ is always fulfilled due to condition (1), while from (21) and (19) we have

$$\text{Re}\,\omega(k) - \omega_{DE} = (J/\beta M)^{1/2}\omega_M akS_-,$$

$$\text{Im}\,\omega(k) = -(J/\beta M)^{1/2}\omega_M akS_+, \tag{25}$$

where

$$S_\mp\left(\frac{\omega_{DE}}{\omega_M}\right) = \frac{1}{4}\left\{\left[\left(\frac{\omega_{DE}}{\omega_M}\right)^2 + \frac{1}{4}\right]^{1/2} \mp \frac{\omega_{DE}}{\omega_M}\right\}^{1/2} \Big/ \left[\left(\frac{\omega_{DE}}{\omega_M}\right)^2 + \frac{1}{4}\right]^{1/2}.$$

In the case under discussion here ($dk \ll 1$) the dispersion is "washed out" by the attenuation: the quantities Im ω and Re $(\omega - \omega_D)$ are of the same order. Their dependence on k is linear. This was pointed out in Refs. 7–9, while Ref. 7 confirms the smallness of the bulk wave component for $dk \gg 1$.

We have mentioned several times that dissipative processes were neglected in deriving the dependence of the dispersion law of DE waves. In particular, it should be understood that this implies the inequality Im $\omega \gg 1/\tau$, where τ is the dissipative attenuation time of the DE waves.

The appearance of attenuation [see the table and Eq. (25)] is a consequence of the diversion of energy from the surface by spin waves, in our case by waves with $\gamma = \gamma_3$ [with a minus sign in front of the root in (19)]. Condition (1) implies that $\gamma_3^2 < 0$, so that γ_3 is a pure imaginary quantity. In order to choose the sign of γ_3 correctly, it is necessary to introduce a small dissipative attenuation, requiring as always that the quantity Re γ_3 be positive, and then to reduce this dissipative attenuation to zero. The correctness of this procedure is confirmed by the correct sign of Im ω in the table and in (25).

The attenuation of the surface waves even in the absence of dissipative terms in the equations of motion of the magnetic moment is a well-known fact (see, e.g., Ref. 11). Nevertheless, in §8 below we will discuss the limiting transition from a film of finite thickness (taking dissipation into account) to a half-space. This allows us to find criteria for the existence of weakly-damped DE waves, in whose description we will consistently neglect dissipative processes.

6. Now let us investigate the effect of the exchange interaction on surface waves which propagate at an angle $\theta \neq \pi/2$ to the direction of the magnetic field. The exact solution to (16) is too complicated; however, we can make use of the fact that for $\Delta\theta \gg (J/\beta M)(ak)^2$, where $\Delta\theta = \theta - \theta_c$, the roots of Eq. (16) differ significantly in magnitude [here, as previously, we limit ourselves to the case of weak spatial dispersion, i.e., (1)]. Even at $\theta = \pi/2$ we have $|\gamma_{2,3}| \gg \gamma_1$; to the required accuracy we can assume that for $\omega = \omega_{DE}, \gamma_1$ is determined by Eq. (15), while

$$\gamma_{2,3}^2|_{\omega=\omega_{DE}(\theta)} \approx \frac{1}{(J/\beta M)(ak)^2\omega_M}\left\{\omega_{DE} \pm \left[\omega_{DE}^2(\theta) + \frac{\omega_M^2}{4}\right]^{1/2}\right\},$$

$$\Delta\theta \gg \frac{J}{\beta M}(ak)^2, \tag{26}$$

and the function $\omega_{DE}(\theta)$ is given by Eq. (14). The values γ_1 in this form were used in Refs. 12 and 8.

Let us begin with the case of free magnetic moments at the boundary ($d = \infty$). We will not calculate the dispersion relation in the form of a vanishing determinant; we need only note that the determinant depends on the values of the components of the tensor μ_{ik}, in which we must substitute $-\gamma_i^2$ ($i = 1,2,3$) in place of k_v^2. Stopping at terms of first nonvanishing order in $(J/\beta M)^{1/2}(ak)$, we obtain

$$\omega = \omega_{DE}(\theta) + \omega_M S_\infty(\theta)(J/\beta M)(ak)^2,$$

$$S_\infty(\theta) = \frac{[\omega_0^2 - \omega_{DE}^2(\theta) + \omega_{DE}(\theta)\omega_M \sin\theta]\omega_{DE}(\theta)}{\sin\theta[\omega_0^2 - \omega_{DE}^2(\theta)][\omega_0(\omega_0+\omega_M) - \omega_{DE}^2(\theta)]}$$
$$\times\left\{\left(\frac{\omega_M \sin^2\theta + \omega_0\cos^2\theta}{\omega_M\sin^2\theta - \omega_0\cos^2\theta}\right)^2\right.$$
$$\times\left[\omega_0\sin\theta + \frac{\omega_M}{2}\sin\theta - \omega_{DE}(\theta) + \frac{\omega_0\omega_M}{2\omega_{DE}(\theta)}\right]$$
$$+\frac{\omega_M\sin^2\theta + \omega_0\cos^2\theta}{\omega_M\sin^2\theta - \omega_0\cos^2\theta}$$
$$\left.\times\left[\frac{\omega_M}{2} - \omega_0 + \omega_{DE}(\theta)\sin\theta + \frac{\omega_0\omega_M\sin\theta}{2\omega_{DE}(\theta)}\right]\right\},$$
$$\Delta\theta \gg (J/\beta M)(ak)^2. \tag{27}$$

The subscript ∞ indicates that have set $d = \infty$ in the boundary conditions (21). For $\theta = \pi/2$ we naturally obtain the expression in the table for $k \gg 1/d$. The value of Im ω (i.e., the attenuation) can be calculated by using the following approximation in $(J/\beta M)^{1/2}(ak)$:

$$\text{Im}\,\omega \propto (J/\beta M)^{3/2}(ak)^3. \tag{27'}$$

The reasons for the appearance of attenuation are the same as before.

If the magnetic moment is pinned at the boundary ($d = 0$), analogous calculations lead to the following results:

$$\omega = \omega_{DE}(\theta) + \omega_M(J/\beta M)^{1/2}akS_0(\theta), \tag{28}$$

where

$$S_0(\theta) = \frac{S_\infty(\theta)}{2}\left[\frac{\omega_{DE}^2(\theta) - \omega_0(\omega_0+\omega_M)}{\omega_{DE}^2(\theta) + \omega_M^2/4}\right]^{1/2}$$
$$\times\frac{\omega_M\sin^2\theta - \omega_0\cos^2\theta}{\omega_M\sin^2\theta + \omega_0\cos^2\theta} \underset{\theta\to\theta_c}{\to} 0,$$
$$\Delta\theta \gg \frac{J}{\beta M}(ak)^2.$$

As for $\theta = \pi/2$, attenuation here is of the same order as dispersion:

$$\text{Re}\,[\omega-\omega_{DE}(\theta)], \quad \text{Im}\,\omega \propto (J/\beta M)^{1/2} ak. \qquad (28')$$

Note that by generalizing the boundary conditions (e.g., passing from a scalar d to a tensor $d_{\beta\beta'}$) we can investigate the mixed boundary conditions:

$$(\partial m_y/\partial y)_{y=0}=0, \quad m_z|_{y=0}=0, \qquad (29)$$

or

$$(\partial m_z/\partial y)_{y=0}=0, \quad m_y|_{y=0}=0. \qquad (30)$$

It is found that pinning of even of a single component of the magnetic moment at the boundary is sufficient to ensure rapid growth of the attenuation; in both cases (29) and (30), $\text{Re}\,(\omega-\omega_{\text{DE}})$ and $\text{Im}\,\omega$ are of the same order of magnitude, and are proportional to $(J/\beta M)^{1/2}\,(ak)$. Note the very important dependence of the parameters of the DE wave on its direction of propagation: although the frequency $\omega_{\text{DE}}(\theta)$ varies between finite limits as the angle θ varies from $\pi/2$ to θ_c, the values of S_∞ and S_0 go to infinity and zero, respectively. Of course, our investigations limited by the inequality

$$|\theta-\theta_c| \gg (J/\beta M)\,(ak)^2.$$

7. As we have seen, $\gamma_1\to\infty$ as $\theta\to\theta_c$. Consequently, a basic property of the DE-waves—their characteristically large damping length compared to the same length for spin waves—is lost. However, when spatial attenuation is taken into account, all three values γ_i remain finite at $\theta=\theta_c$, although one of them turns out to be imaginary, i.e., the wave is, strictly speaking, not a surface wave; nevertheless, as we will see, it is weakly damped (even for $d=0$!).

Thus, for $\theta=\theta_c$ we have

$$\gamma_1^2 \approx \left(\frac{\beta M}{J}\right)\frac{k}{a}\left[\frac{\omega_0\omega_M}{2(\omega_0+\omega_M)\,\omega_{DE}}\right]^{1/2} \approx -\gamma_3^2, \qquad (31)$$

$$\gamma_2^2 \approx 2\frac{\omega_{DE}}{\omega_M}\frac{\beta M}{J}\frac{1}{a^2}. \qquad (32)$$

Note that

$$|\gamma_{1,3}^2/\gamma_2^2| \sim (J/\beta M)^{1/2} ak \ll 1$$

(for $\omega\sim\omega_M$), i.e., only the second wave is a pure spin wave, while the first and third are a result of "entangling" of the spin wave with the Walker mode.

In the case of free magnetic moments at the boundary ($d\to\infty$)

$$\omega=[\omega_0(\omega_0+\omega_M)]^{1/2}-(J/\beta M)\,\omega_M (ak)^2 S_R^\infty - i(J/\beta M)^{5/4}\omega_M (ak)^{5/2} S_I^\infty,$$

where

$$S_R^\infty=\omega_M\omega_0/2(\omega_0+\omega_M)^{3/2}\omega_0^{1/2},$$

$$S_I^\infty=\left[\frac{\omega_M\omega_0}{(\omega_0+\omega_M)(2\omega_0+\omega_M)}\right]^{3/4}\frac{\omega_M^2}{2(2\omega_0+\omega_M)[\omega_0(\omega_0+\omega_M)]^{1/2}}. \qquad (33)$$

For moments pinned at the boundary ($d\to 0$)

$$\omega=[\omega_0(\omega_0+\omega_M)]^{1/2}-(J/\beta M)^{1/2}\omega_M ak S_R^0 - i[ak(J/\beta M)^{1/2}]^{3/2}\omega_M S_I^0,$$

$$S_R^0=\frac{\omega_0}{2\omega_0+\omega_M}\left[\frac{\omega_M(2\omega_0+\omega_M)}{\omega_0(\omega_0+\omega_M)}\right]^{1/2},$$

$$S_I^0=\left[\frac{\omega_0\omega_M}{(\omega_0+\omega_M)(2\omega_0+\omega_M)}\right]^{1/4}\frac{\omega_M^2}{(2\omega_0+\omega_M)^2}. \qquad (34)$$

As we have said, even in this case the wave is weakly damped:

$$\text{Im}\,\omega \ll \text{Re}\,|\omega-\omega(k=0)|.$$

We emphasize again that in both cases ($d=\infty$ and $d=0$) the weakly-damped waves possess anomalous dispersion, i.e., $\partial\omega/\partial k<0$.

Comparing the expressions obtained here with those given in Sec. 6, we can easily see that the spectrum is considerably altered near $\theta=\theta_c$: in particular, the coefficient of k_2 (or k) in the dispersion law for surface waves changes sign in a narrow region of angles $|\theta-\theta_c|\lesssim (J/\beta M)^{1/2}(ak)$, while in the damping dependence on wave vector changes [compare Eqs. (27) and (33)].

8. A half-space occupied by a ferromagnet is, of course, an abstraction. Let us investigate the half-space limit for the example of a film whose thickness L will be assumed to be significantly larger than all other parameters in the problem with the dimensions of length. It will become clear that our investigation of this model (i.e., of independent weakly-attenuated surface waves) presupposes the existence of dissipation, although the dissipative characteristics (which we derived above) do not enter into the final answers.

Let us begin with the simplest case of Walker modes (we neglect spatial dispersion). There is a wave which is analogous to the surface wave with the following field structure:

$$\varphi=\exp[i(k_x x+k_z z)]\begin{cases} A\,\text{sh}(\gamma y)+B\,\text{ch}(\gamma y), & 0<y<L,\\ C_1 e^{-ky}, & y>L,\\ C_2 e^{ky}, & y<0.\end{cases} \qquad (35)$$

Using the boundary conditions, for $\theta\neq\pi/2$ we can write the dispersion relation with the help of the two equations:

$$\frac{\gamma^2}{k^2}=\frac{\omega_0(\omega_0+\omega_M\sin^2\theta)-\omega^2}{\omega_0(\omega_0+\omega_M)-\omega^2}, \qquad (36)$$

$$(\gamma^2/k^2)(\omega_M\sin^2\theta-\omega_0\cos^2\theta)-(2\gamma/k)\,\text{cth}(\gamma L)\,\omega_0\cos^2\theta - (\omega_M\sin^2\theta+\omega_0\cos^2\theta)=0. \qquad (37)$$

For $\theta=\pi/2$ both these equations reduce to an identity[1)], and the expression for the frequency of the waves can be obtained by making use of the fact that in this case $\gamma=k$:

$$\omega^2=\omega_0(\omega_0+\omega_M)+\omega_M^2/2[\text{cth}(|k|L)+1]. \qquad (38)$$

The analysis shows that Eq. (37) has only one real solution for an arbitrary thickness L for

$$\theta>\theta_c=\arcsin[\omega_0/(\omega_0+\omega_M)],$$

while for $\theta<\theta_c$ there is no solution at all. From this we see that the angle θ_c, even in the half space, is a critical angle for solutions of the type (35). Changing the sign of k does not change the value of the frequency, i.e., the wave is reciprocal. In the limit $L\to\infty$ it follows from (36) and (37) that there exist two solutions:

$$\omega_1=[\omega_0+(\omega_0+\omega_M)\sin^2\theta]/2\sin\theta,\quad \sin\theta>0,$$
$$\omega_2=-[\omega_0+(\omega_0+\omega_M)\sin^2\theta]/2\sin\theta,\quad \sin\theta<0, \tag{39}$$

the wave with frequency ω_1 keeps close to one side of the film, while the ω_2 stays close to the other.

Naturally, the inclusion of spatial dispersion greatly complicates the problem. Therefore, we will investigate only the case $\theta=\pm\pi/2$. The dispersion equation for the film is obtained, as always, in the form of a vanishing determinant D. In the case of a film this determinant is sixth order:

$$D\equiv\begin{vmatrix}\{11\} & \{12\}\\ \{21\} & \{22\}\end{vmatrix}=0. \tag{40}$$

We denote 3×3 blocks by the symbols $\{...\}$. If dissipation is omitted, of course, all solutions to Eq. (40) are real. Let us recall: the damping of DE waves is a consequence of the fact that one of the values of γ in (19) is imaginary. In the solution of the film problem, this implies that the wave field is a superposition not only of hyperbolic functions but also the trigonometric functions $\sin(k_y y)$ and $\cos(k_y y)$, where $k_y=i\gamma$ [see (19)], while in (40) not only the functions $\mathrm{sh}(\gamma L)$ and $\mathrm{ch}(\gamma L)$ but also $\sin(k_y L)$ and $\cos(k_y L)$ appear. The latter terms do not diverge as $L\to\infty$, which interferes with the passage to the half-space limit in Eq. (40). In order to carry out this limit, we introduce a relaxation term into the Landau-Lifshits equation (i.e., we include dissipation processes). As a result, all the γ and k_y acquire a rather small imaginary part. In this case of the hyperbolic functions this is unimportant, but for the trigonometric functions it plays a decisive role: as $\gamma L\to\infty$ and $\mathrm{Im}\,(k_y L)\to\infty$ the element in blocks $\{12\}$ and $\{21\}$ reduce to zero and the determinant (40) can be written as a product of two factors:

$$D=|\{11\}||\{22\}|,$$

while the dispersion relation is split into two parts:

$$|\{11\}|=0,\quad |\{22\}|=0, \tag{41}$$

each of which coincides with the dispersion relation for surface waves which keep close to one of the surfaces. Naturally, one equation has a solution for $k_z>0$, and the other for $k_z<0$. For $\mathrm{Im}\,k_y\ll\gamma_{1,2}$ the dissipative terms can be omitted in Eqs. (41).

This analysis allows us to trace the origin of the damping for the DE waves when the spatial dispersion is included and dissipation is neglected. Formally, the imaginary unit i "remains" when, by virtue of the fact that $|\mathrm{Im}\,(k_y L)|\gg1$, we neglect one of the exponentials in the expression for $\sin(k_y L)=[\exp(ik_y L)-\exp(-ik_y L)]/2i$.

Thus, the DE wave dispersion laws found above are valid for $\mathrm{Im}\,k_y L)\gg1$ and $\mathrm{Im}\,(k_y\ll\gamma_i\,(i=1,2)$.

9. A detailed analysis of the DE wave dispersion law, and also of the magnetic surface polariton, as we have defined them, can be useful in comparing the experimental results with theories (in particular, with respect to surface scattering; see, e.g., Ref. 13).

The surface waves naturally contribute to the surface energy of a magnet, and correspondingly to the surface magnetization. Because of the complicated dependence of the wave parameters on their propagation direction θ (see §6), an analytic calculation of the temperature dependence of the surface characteristics is extremely awkward to carry out, and possibly would not be of much interest, because isolating the surface magnetic moments from the bulk background is apparently a very complicated problem. In particular, because the frequency $\omega_{\rm DE}(\theta)$ is larger than the frequency of the Walker modes, as $T\to0$ the principal contribution to the temperature variation of the magnetization is provided by the bulk oscillations.

Of course, it is necessary to note the following circumstance. The presence of nonreciprocal waves confined near the surface of the magnet compels us to assume that it is possible for quasiparticle currents flowing around the surface of the magnet to exist in equilibrium (!) which correspond to the nonreciprocal waves. In fact, because of the nonreciprocal property,

$$\mathbf{j}=\int\frac{\mathbf{v}}{\exp(\hbar\omega/T)-1}\frac{d^2k}{(2\pi)^2}\neq0,$$
$$\mathbf{q}=\int\frac{\mathbf{v}\hbar\omega(k)}{\exp[\hbar\omega(k)/T]-1}\frac{d^2k}{(2\pi)^2}\neq0, \tag{42}$$
$$\mathbf{v}=\partial\omega/\partial\mathbf{k}.$$

This implies that on the two sides of the film there exist macroscopic currents of quasiparticles and energy, directed toward the opposite sides. A cylinder magnetized along its axis should be surrounded by a quasiparticle and energy current. How to observe this current remains unclear, as well as what role is played by dissipative processes in the phenomenon, although it would seem that their role (for $\mathrm{Re}\,\omega\gg\mathrm{Im}\,\omega$) cannot be too important.

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[1)] For $\theta=\pi/2$, i.e., for $k_x=0$, the derivation of Eqs. (36) and (37) is invalid, because it would require dividing by zero.

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