Galaxy mass and momentum distribution formed by merging, and the problem of nuclear activity

A. V. Kats and V. M. Kontorovich

Kharkov Scientific-Research Institute of Metrology; Radio-Astronomy Institute of the Ukrainian Academy of Sciences

(Submitted 30 June 1989)
Zh. Eksp. Teor. Fiz. **97**, 3–19 (January 1990)

We consider an exactly solvable merging problem under mass and momentum conservation conditions in a statically homogeneous system of galaxies. Rather simple analytic expressions are obtained for the asymptotes for large masses, momenta, and times. These solutions are valid for a constant merging coefficient. If this coefficient is not constant, it is possible to formulate a self-similar combination of variables which describes the propagation of a stationary front towards larger masses. The distribution asymptotes obtained in this manner take in the simplest case the form of the Schechter functions used to describe the observation data, with parameters determined by the scales and by the action time of the "source" that simulates the initial galaxy distribution. We consider also the asymptote of the initial distribution, with account taken of the cosmological expansion. The possible connection between the mass and momentum distributions and the activity of galaxies is discussed briefly.

1. INTRODUCTION

A problem closely related to the origin of galaxies [1] is that of the activity of galactic cores [2], which leads to the phenomenon of Seyfert galaxies, radiogalaxies, quasars, and their dependence on the activity scale.

Most researchers agree that the direct cause of the activity is accretion of matter on a central compact object in the galactic core. One of the causes of appreciable accretion to the center is tidal interaction and, ultimately, merging of galaxies. In the latter case the main cause of the accretion can be the cancelation of the angular momentum of the disk component following the merging of gas-rich spiral systems. This circumstance was noted long ago by Toomre,[3] and also in a number of recent papers (see, e.g., Refs. 4 and 5). Komberg[6] cited arguments favoring the idea that quasars are the second generation resulting from merging of thicker and less massive objects. The advances in optical astronomy, which made it possible to observe galaxies surrounding the nearest quasars (their cores) have directly confirmed a similar point of view. According to the data in Hutchings's review[7] about 30% of the galaxies belonging to quasars are in a state of interaction (collision) with the galaxy. The images of the brightest IR sources from the IRAS catalog also demonstrate a substantial peculiarity of all the objects (amounting to about 30% of the total list), namely double tails, double cores, rings, and jets, all of which attest to a merging phase.[8]

Additional arguments are the dependence of the morphology of the galaxies on the density of their surroundings, which influences in particular the luminosity function, i.e., the distribution of the galaxies in luminosity, and ultimately in mass.[9]

The inevitability (or possibility) of a merging phase arises also in theoretical models of galaxy formation,[1,10] for example in the theory of entropy perturbations.

Long ago, in the context of the solution of coagulation equations that describe merging of "particles,[11]" their mass distribution was examined both analytically[12,13] and numerically.[14,15] We consider below the statistical consequences of merging processes that lead to formation of compatible galaxy-mass and angular momentum distributions. One can attempt to compare them with the observed luminosity functions for galaxies of various morphological types.[16–19] It is important, however, that by using compatible distribution functions it is possible to formulate an activity problem[1)] in the context of kinetics (see the last section of this paper).

Galaxy-collision dynamics is a complicated problem still far from solution. Of importance to us is that analysis[20] shows that inelastic galaxy collisions as well as collisions in which the total mass and angular momentum are preserved may be perfectly acceptable premises, which we shall adopt hereafter. In addition, the spheroidal subsystem, which has both large size and large mass (including the hidden one) should be responsible for the merging. Although the merging probability depends, of course, on the relative orientation of the proper ("spin") angular momenta, this dependence will likewise be neglected for relatively low velocities (for only then is mixing possible). We ignore also the role of the orbital momentum (which, generally speaking, can be substantial, see Ref. 21) of the pair of colliding galaxies. Some justification for this neglect may be the relative smallness of the impact parameter (the momentum lever-arm) in collisions that lead to mergings.

This allows us to formulate in closed form the problem, solvable under the above assumptions, of deriving the kinetic equation (KE) that describes mergings with allowance for angular-momentum and mass conservation.

2. KINETIC EQUATION DESCRIBING THE MERGING WITH MASSES AND ANGULAR MOMENTA CONSERVED

Consider, for the distribution function $f(M,\mathbf{S};t)$, a KE that generalizes the Smoluchowski coagulation equation[11] for the case when the "particles" (in our case, galaxies) that collide and merge have a conserved mass M and a conserved angular momentum $\mathbf{S}$ (classical "spin"). This KE has the form

$$\partial f/\partial t = I_{\text{ст}}\{f\}, \quad f \equiv f(M,\mathbf{S};t),$$
$$I_{\text{col}}\{f\} = \int dM_1\, dM_2\, d\mathbf{S}_1\, d\mathbf{S}_2 \left[U \delta_M \delta_{\mathbf{S}} f_1 f_2 - \curvearrowright - \curvearrowleft \curvearrowright\right]. \tag{2.1}$$

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The transition probability $U\delta_M\delta_S$ contains here all the delta-functions that express the conservation laws for merging:

$$\delta_M = \delta(M - M_1 - M_2), \quad \delta_S = \delta(\mathbf{S} - \mathbf{S}_1 - \mathbf{S}_2), \tag{2.2}$$

we omit the arguments of the coefficient U:

$$U \equiv U_{M\mathbf{S}|M_1\mathbf{S}_1M_2\mathbf{S}_2}, \tag{2.3}$$

and use the abbreviated notation

$$f_1 \equiv f(M_1, \mathbf{S}_1;\, t), \quad f_2 \equiv f(M_2, \mathbf{S}_2;\, t), \tag{2.4}$$

while the arrows in (2.1) denote two successive cyclic permutations of the three subscripts M, M_1, M_2, and $\mathbf{S}$, $\mathbf{S}_1$, $\mathbf{S}_2$. The first term corresponds to arrival of the particles in the phase-space element near $M\mathbf{S}$ (Fig. 1a), and the second and third to departure from it (Figs. 1b,c). We consider first a constant merging coefficient

$$U = \text{const.} \tag{2.5}$$

Physical arguments favoring this far-reaching assumption are given in the Introduction and in the Appendix. (We shall forego (2.5) partially in Sec. 8.) The KE (2.1) can be solved exactly under condition (2.5) also in the presence of a source (cf. Ref. 22).

We take a Laplace transform with respect to mass and a Fourier transform with respect to spin (henceforth simply "Fourier" for short):

$$F(p, \mathbf{q}, t) = \int_0^\infty dM e^{-pM} \int d\mathbf{S} e^{-i\mathbf{q}\mathbf{S}} f(M, \mathbf{S};\, t). \tag{2.6}$$

The equation for F takes the form

$$\frac{\partial F(p, \mathbf{q};\, t)}{\partial t} = \int d\tau_M\, d\tau_S \exp(-pM - i\mathbf{q}\mathbf{S})\,[U\delta_M\delta_S f_1 f_2 - \circlearrowleft - \circlearrowleft\circlearrowleft], \tag{2.7}$$

where

$$d\tau_M = dM dM_1 dM_2, \quad d\tau_S = d\mathbf{S} d\mathbf{S}_1 d\mathbf{S}_2.$$

Redesignating the integration variables in the second and third terms: $M \leftrightarrow M_2$ and $M \leftrightarrow M_1$, we transform the expression in the square brackets in the integral of (2.7) into

$$\int dM_1\, dM_2\, d\mathbf{S}_1\, d\mathbf{S}_2 U f_1 f_2 \{\exp[-p(M_1 + M_2) - i\mathbf{q}(\mathbf{S}_1 + \mathbf{S}_2)] - \exp(-pM_2 - i\mathbf{q}\mathbf{S}_2) - \exp(-pM_1 - i\mathbf{q}\mathbf{S}_1)\}.$$

Using the condition (2.5) and adding the source D, we obtain a KE in Riccati form:

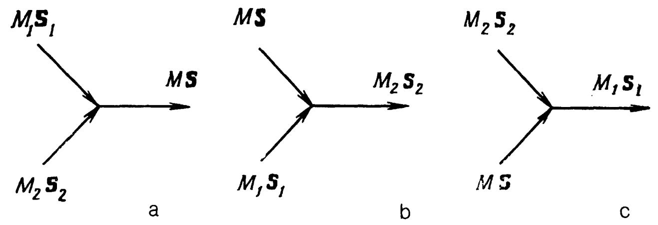

FIG. 1.

$$\frac{\partial F(p, \mathbf{q};\, t)}{\partial t} = UF^2(p, \mathbf{q};\, t) - 2UF(p, \mathbf{q};\, t)F(0, 0;\, t) + D(p, \mathbf{q};\, t). \tag{2.8}$$

By definition,

$$F(0, 0, t) = \int dM\, d\mathbf{S} f(M, \mathbf{S};\, t) \equiv n(t), \tag{2.9}$$

where $n(t)$ is the "density" of the galaxies. We have for it from (2.8) the equation

$$\partial n(t)/\partial t = -Un^2(t) + D(0, 0;\, t), \tag{2.10}$$

which makes it convenient to rewrite the KE for the quantity

$$X(p, \mathbf{q};\, t) \equiv n(t) - F(p, \mathbf{q};\, t) \tag{2.11}$$

in the form

$$\frac{\partial X(p, \mathbf{q};\, t)}{\partial t} + UX^2(p, \mathbf{q};\, t) = U\Delta^2(p, \mathbf{q};\, t), \tag{2.12}$$

where the right-hand side is connected with the source:

$$U\Delta^2(p, \mathbf{q};\, t) \equiv D(0. 0:\, t) - D(p, \mathbf{q};\, t). \tag{2.13}$$

The condition (2.5) permits thus the Smoluchowski generalized KE to be reduced to a Riccati equation for the $p\mathbf{q}$ transform of the distribution function and obtain exact solutions in a number of interesting cases, which we shall write down and analyze below. These solutions are still quite complicated and we shall therefore consider their asymptotes for sufficiently large masses, angular momenta, and times.

3. STATIONARY SOLUTION

Consider a steady-state source and introduce the notation

$$n_\infty^2 \equiv D(0,\, 0)/U. \tag{3.1}$$

We rewrite the equation for the density in the form

$$\frac{1}{U}\frac{\partial n}{\partial t} = n_\infty^2 - n^2. \tag{3.2}$$

Its solution is

$$n(t) = n_\infty \frac{n_0 + n_\infty \operatorname{th}\tau}{n_\infty + n_0 \operatorname{th}\tau}, \quad \tau = n_\infty Ut. \tag{3.3}$$

We solve similarly the solution of KE (2.12), where $\Delta = \Delta(p,\mathbf{q})$ is independent of t:

$$X(t) = X_\infty \frac{X_0 + X_\infty \operatorname{th}(UX_\infty t)}{X_\infty + X_0 \operatorname{th}(UX_\infty t)}, \tag{3.4}$$

$$X_0 \equiv X(p, \mathbf{q};\, 0), \quad X_\infty \equiv \Delta(p, \mathbf{q}).$$

We obtain from this the stationary solution by letting $t \to \infty$:

$$F_\infty = n_\infty - X_\infty \equiv \left[\frac{D(0.0)}{U}\right]^{1/2} - \left[\frac{D(0.0) - D(p, \mathbf{q})}{U}\right]^{1/2} \tag{3.5}$$

Inversion yields a stationary distribution in mass and angular momentum:

$$f(M, \mathbf{S}) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dp e^{pM} \int \frac{d\mathbf{q}}{(2\pi)^3} e^{i\mathbf{q}\mathbf{S}} F_\infty(p, \mathbf{q}). \tag{3.6}$$

The asymptote for large M and S corresponds to a contribution of small p and $|\mathbf{q}|$ to the integral in (3.6). For an isotropic source we assume

$$D(p, \mathbf{q}) \approx D(0, 0) + pD_{10} + {}^1/_2 q^2 D_{02}, \tag{3.7}$$

meaning a finite rate of injection of mass and squared angular momentum, and hence localization of the source at masses and angular momenta that are small compared with M and S. Note that as p, $|\mathbf{q}| \to \infty$ we have $D(p,\mathbf{q}) \to 0$ and, according to (3.5), $F_\infty(p,\mathbf{q}) \to 0$. This ensures convergence of (3.6) and analyticity of the solution.

Using (3.7), we rewrite (3.5) in the form

$$F_\infty(p, \mathbf{q})|_{p,\,\mathbf{q}\to 0} = n_\infty - (ap+bq^2)^{1/2}, \tag{3.8}$$

$$a = -D_{10}/U > 0, \quad b = -D_{02}/2U > 0. \tag{3.9}$$

The inequalities (3.9) follow from $D(M,S) > 0$:

$$\begin{aligned} D_{10} &= -\int dM\, d\mathbf{S} D(M,\mathbf{S}), \\ q^2 D_{02} &= -\int dM\, d\mathbf{S}(\mathbf{q}\mathbf{S})^2 D(M,\mathbf{S}). \end{aligned} \tag{3.10}$$

Inversion of (3.8) in terms of p is determined by the branching point, after which the inversion in terms of q reduces to calculation of a Gaussian integral. We obtain ultimately

$$f(M,\mathbf{S})|_{\substack{M\to\infty \\ \mathbf{S}\to\infty}} = \frac{1}{16\pi^2}\frac{a^2}{b^{3/2}}\frac{1}{M^3}\exp\left(-\frac{a}{4b}\frac{S^2}{M}\right). \tag{3.11}$$

We see that, as a consequence of the assumed isotropy, the distribution contains the angular momentum as part of the combination S^2/M. For constant S, the distribution as a function of M first increases exponentially at small masses, reaches a maximum at $M \sim S^2$ (in the corresponding mass and angular-momentum units governed by the source), and falls off as a power law ($\propto M^{-3}$) for large M. The distribution in the angular momenta, however, is monotonic at fixed mass [constant for small masses and decreases exponentially for $S^2 \gg M$ (see Fig. 2)].

Integrating (3.12) over the angular momenta we obtain a mass function

$$f(M) = \int d\mathbf{S}\, f(M,\mathbf{S}) = \frac{1}{2}\left(\frac{a}{\pi M^3}\right)^{1/2}, \quad M\to\infty, \tag{3.12}$$

corresponding to a distribution with a constant flux over the spectrum (distribution for a constant source localized in the small-mass region).

Integrating (3.11) over the masses, we obtain the asymptote of the distribution function in the angular momenta:

$$f(\mathbf{S}) = \int_0^\infty dM f(M,\mathbf{S})$$

$$= \frac{1}{16\pi^2}\frac{a^2}{b^{3/2}}\int_0^\infty \frac{dM}{M^3}\exp\left(-\frac{a}{4b}\frac{S^2}{M}\right) = \frac{1}{\pi^2}\frac{b^{1/2}}{S^4}. \tag{3.13}$$

4. STATIONARY SOLUTION (ANISOTROPIC SOURCE)

We see that owing to the rapid decrease of the distribution with increase of S [cf. (3.11)] there is no angular momentum spread over the spectrum in the isotropic case. We forego the isotropy assumption, but for simplicity we consider initially in place of (3.8) an expansion in the form

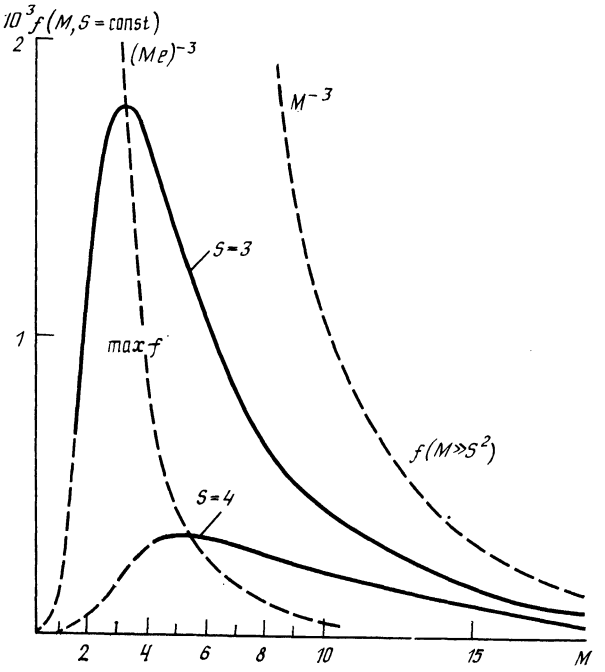

FIG. 2. Mass distribution at fixed angular momentum (3.11) in terms of the dimensionless variables $M = M/M_0$ and $S^2 = S^2/4S_2^2$.

$$F(p, \mathbf{q}) \approx n_\infty - (ap + i\mathbf{c}\mathbf{q} + bq^2)^{1/2}, \tag{4.1}$$

and retain only the term linear in p and both the linear and quadratic terms in $\mathbf{q}$, in view of the difference between their symmetries. The expansion coefficients a and c are proportional to the mass and momentum fluxes J and $\mathbf{J}_S$:

$$a = J/U, \quad \mathbf{c} = \mathbf{J}_S/U, \tag{4.2}$$

the coefficient b has here the same meaning as before, but it must be remembered that for $c \neq 0$ we must, generally speaking, use for the expansion in $\mathbf{q}$ the more accurate $q_i q_k$ rather than q^2. Introducing quantities having the dimensions of mass, M_0 and angular momenta S_1 and S_2, and describing the source by

$$Ua = M_0 D(0,0), \quad Ub = S_2^2 D(0,0), \quad U\mathbf{c} = \mathbf{S}_1 D(0,0), \tag{4.2'}$$

we obtain the distribution asymptote of interest to us in the form

$$f(M,\mathbf{S})|_{\substack{M\to\infty \\ \mathbf{S}\to\infty}}$$

$$= \frac{(J/UM^3)^{1/2}}{16\pi^2}\left(\frac{M_0}{M}\right)^{3/2}\frac{1}{S_2^3}\exp\left[-\frac{M_0}{4M}\left(\frac{\mathbf{S}}{S_2} - \frac{\mathbf{S}_1}{S_2}\frac{M}{M_0}\right)^2\right] \tag{4.3}$$

In fact, the branching point that determines the form of the function now is

$$p(\mathbf{q}) = -\frac{b}{a}q^2 - \frac{i\mathbf{c}}{a}\mathbf{q}. \tag{4.4}$$

Drawing the cut and displacing the contour, we obtain the integral with respect to p:

$$\frac{1}{2}\left(\frac{a}{\pi}\right)^{1/2}\frac{1}{M^{3/2}}\exp[Mp(\mathbf{q})], \tag{4.5}$$

after which the integration over $\mathbf{q}$ is easy. Using

$$\int\frac{d\mathbf{q}}{(2\pi)^3}\exp(i\mathbf{q}\mathbf{S}'-\alpha q^2)=\frac{1}{4\pi iS'}\int_{-\infty}^{\infty}dq\,q\exp(-\alpha q^2+iqS'),$$

where $\mathbf{S}'=\mathbf{S}-c\mathbf{M}/a$, we arrive at (4.3).

If the source is anisotropic, a maximum appears not only in the dependence on the mass for a fixed angular momentum, as in the isotropic variant, but also to a maximum with respect to S and at constant M. The equations for the lines of the maxima in the (M,S) plane take for $\mathbf{S}\|\mathbf{S}_1$ the form

$$S^2=3M+S_1M^2,\quad S=MS_1 \tag{4.6}$$

(the angular momenta are measured here in units of S_2 and the masses in units of M_0).

The observed nonmonotonicity in the distribution of spiral galaxies of a selected morphological type is possibly of the same nature.

5. EVOLUTION OF INITIAL DISTRIBUTION

We consider now the problem in a different formulation, without a source but with an initial distribution assumed here to be unimodal in the masses (with a characteristic scale $\overline{M}$) and having a certain anisotropy of $\overline{\mathbf{S}}$ and a distribution width $\overline{(S^2)}^{1/2}$. This means that we assume, for example, that the formation of this initial distribution (the onset of the first generation of galaxies) was of considerably shorter duration than the considered evolution through merging.

The solution of the initial problem in the absence of a source takes for the Fourier transform of the distribution the form [see (3.4) as $X_\infty\to 0$]

$$F(p,\mathbf{q};t)=n(t)-1/Ut+1/Ut(1+X_0Ut), \tag{5.1}$$

where

$$n(t)=n_0/(1+n_0Ut),$$

and X_0 is expressed in terms of the initial distribution $F_0(p,\mathbf{q})\equiv F(p,\mathbf{q};t=0)$:

$$X_0(p,\mathbf{q})=n_0-F_0(p,\mathbf{q}). \tag{5.2}$$

Bearing in mind the calculation of the asymptote for large M and S, we expand F_0 in powers of small p and $|\mathbf{q}|$:

$$F_0(p,\mathbf{q})\approx n_0(1-p\overline{M}-i\mathbf{q}\overline{\mathbf{S}}-{}^1/_2 q_iq_k\overline{S_iS_k}). \tag{5.3}$$

The form of the expansion (5.3) suggests that the initial distribution is localized enough to make finite its total mass, the angular momentum and its square, and the product of the mass by the angular momentum. Initial conditions of another type are considered in Sec. 7.

Introducing the dimensionless time $\tau=n_0Ut$ ("number of mergings") and

$$p(\mathbf{q})=-M^{-1}(1/\tau+i\mathbf{q}\mathbf{S}+{}^1/_2q_iq_k\overline{S_iS_k}), \tag{5.4}$$

we separate in $F(p,\mathbf{q};t)$ the pole singularity that determines the form of the solution for long times:

$$F(p,\mathbf{q};t)\approx n(t)-1/Ut+1/(Ut)^2n_0\overline{M}[p-p(\mathbf{q})]. \tag{5.5}$$

In the pole approximation we obtain for the distribution function

$$f(M,\mathbf{S};t)\approx\int\frac{d\mathbf{q}}{(2\pi)^3}\frac{n_0}{\overline{M}\tau^2}\exp[i\mathbf{q}\mathbf{S}+p(\mathbf{q})M]$$
$$=\frac{n_0}{\overline{M}\tau^2}\exp\left(-\frac{M}{\overline{M}\tau}\right)\int\frac{d\mathbf{q}}{(2\pi)^3}\exp\left[i\mathbf{q}\left(\mathbf{S}-\frac{M}{\overline{M}}\overline{\mathbf{S}}\right)\right.$$
$$\left.-\frac{1}{2}q_iq_k\overline{S_iS_k}\frac{M}{\overline{M}}\right]. \tag{5.6}$$

Referring $\overline{S_iS_k}$ to the principal axes with principal values $\overline{S_j^2}$ $(j=1,2,3)$, we factorize the integral (5.6) and obtain upon integration

$$f(M,\mathbf{S};t)=\frac{n_0}{\overline{M}\tau^2}\exp\left(-\frac{M}{\overline{M}\tau}\right)\left(\frac{\overline{M}}{2\pi M}\right)^{3/2}(\mathrm{Det}\,\overline{S_iS_j})^{-1/2}$$
$$\times\exp\left[-\frac{1}{2}\sum_j\frac{M\overline{M}}{\overline{S_j^2}}\left(\frac{S_j}{M}-\frac{\overline{S}_j}{\overline{M}}\right)^2\right]. \tag{5.7}$$

In the simpler case of an isotropic quadratic form $\overline{S_iS_j}=\overline{S^2}\delta_{ik}$, which is perfectly sufficient for our purposes, we obtain

$$f(M,\mathbf{S};t)=\frac{n_0}{\overline{M}\tau^2}\left(\frac{1}{2\pi\overline{S^2}}\right)^{3/2}\left(\frac{\overline{M}}{M}\right)^{3/2}$$
$$\times\exp\left[-\frac{M}{\overline{M}\tau}-\frac{(S-\overline{\mathbf{S}}M/\overline{M})^2}{2\overline{S^2}}\frac{\overline{M}}{M}\right]. \tag{5.8}$$

In particular, in the isotropic case $\overline{\mathbf{S}}=0$ at fixed $S\neq 0$ the mass distribution has with respect to M a maximum that decreases in magnitude and shifts towards larger masses when τ is increased.

Integrating over the angular momenta, we obtain the known nonstationary mass distribution[11] that describes the evolution of the initial perturbation in the absence of a source:

$$f(M;t)\equiv\int d\mathbf{S}\,f(M,\mathbf{S};t)=\frac{n_0}{\overline{M}\tau^2}\exp\left(-\frac{M}{\overline{M}\tau}\right). \tag{5.9}$$

We obtain now the distribution in the angular momenta, integrating over the mass:

$$f(\mathbf{S};t)\equiv\int dMf(M,\mathbf{S};t)$$
$$=\frac{n_0}{2\pi\overline{S^2}\tau^2}\frac{1}{S}\exp\left\{\frac{\mathbf{S}\overline{\mathbf{S}}}{\overline{S^2}}-\left(\frac{2}{\tau}\right)^{1/2}\frac{S}{(\overline{2S^2})^{1/2}}\right.$$
$$\left.\times\left[1+\frac{(\overline{S})^2\tau}{2\overline{S^2}}\right]^{1/2}\right\}. \tag{5.10}$$

We used in the calculation the integral[23]

$$\int_0^\infty dx\,x^{-3/2}\exp\left(-\gamma x-\frac{\beta}{x}\right)=\left(\frac{\pi}{\beta}\right)^{1/2}\exp[-2(\beta\gamma)^{1/2}].$$

In the isotropic case $\overline{\mathbf{S}}=0$ we obtain from (5.10)

$$f(S;t)=\frac{n_0}{2\pi\overline{S^2}\tau^2}\frac{1}{S}\exp\left[-\left(\frac{2}{\tau}\right)^{1/2}\frac{S}{(\overline{S^2})^{1/2}}\right], \tag{5.11}$$

an expression that can be easily derived independently by considering the isotropic solution. Expression (5.10) is quite complicated, and it is expedient to consider in its place the angle-averaged distribution

$$\overline{f(\mathbf{S};t)}=\int d\Omega f(\mathbf{S};t)$$

$$=\frac{2n_0}{|\mathbf{\overline{S}}|(S\tau)^2}\operatorname{sh}\left(\frac{S|\mathbf{\overline{S}}|}{\overline{S^2}}\right)\exp\left[-\left(\frac{2\mathbf{S}^2}{\tau\overline{S^2}}+\frac{S^2(\overline{S})^2}{(\overline{S^2})^2}\right)\right]^{1/2} \tag{5.12}$$

We have used the fact that

$$\int d\Omega\exp\left(\frac{\mathbf{S}\mathbf{\overline{S}}}{\overline{S^2}}\right)=\frac{4\pi}{\lambda}\operatorname{sh}\lambda,\quad \lambda\equiv\frac{S|\mathbf{\overline{S}}|}{\overline{S^2}}.$$

Various limiting cases can be easily expressed. A maximum with respect to τ is evident.

6. ALLOWANCE FOR COSMOLOGICAL EXPANSION

Following Silk and White,[24] we introduce a symbol for the number of particles in the volume $R^3(t)$ of the proper reference frame

$$\hat{f}=fR^3(t),\quad \hat{U}=UR^{-3}(t). \tag{6.1}$$

Recognizing that the mass and the angular momentum are conserved in the comoving volume, we obtain for the quantities with the carets a Smoluchowski equation of the usual form (2.1). Now U in Eq. (6.1) is likewise a function of the time. Assume that the temporal variable in U differs from the others:

$$U=\tilde{U}\chi(t), \tag{6.2}$$

where $\tilde{U}$ is no longer time-dependent. The KE takes then the form[24]

$$\partial\hat{f}/\partial\tilde{t}=I_{\rm col}\{\hat{f}\}, \tag{6.3}$$

where we must put $U\to\tilde{U}$ in $I_{\rm col}\{\hat{f}\}$ of (2.1). Introducing

$$d\tilde{t}=[R(t_0)/R(t)]^3\chi(t)\,dt,$$

we go over to $\tilde{f}(\tilde{t})\equiv\hat{f}(t(\tilde{t}))$. Equation (6.3) for $\tilde{f}$ leads to the conservation laws

$$\frac{d\mathscr{M}}{d\tilde{t}}=0,\quad \mathscr{M}=\int dM\,d\mathbf{S}\,M\tilde{f},\quad \frac{d\mathbf{\Sigma}}{d\tilde{t}}=0,\quad \mathbf{\Sigma}=\int dM\,d\mathbf{S}\,\mathbf{S}\tilde{f}. \tag{6.4}$$

Using the solution (5.8) of the problem with initial conditions in the absence of a source at $\tilde{U}$ = const, we obtain a solution of the KE (6.3) with the following final substitution in (5.8)

$$n_0\to n_0R^3(t_0)/R^3(t),\quad \tau\to\tilde{\tau}-\tilde{\tau}_0, \tag{6.5}$$

where

$$\tilde{\tau}\equiv\tilde{\tau}(t)=n_0\tilde{U}\tilde{t}(t),\quad \tilde{\tau}_0=\tilde{\tau}(t_0),$$

and return to the initial function f:

$$f(M,\mathbf{S};t)=\frac{n_0R^3(t_0)}{R^3(t)}\frac{1}{\overline{M}}\left(\frac{1}{2\pi\overline{S^2}}\right)^{3/2}\frac{1}{(\tilde{\tau}-\tilde{\tau}_0)^2}$$

$$\times\exp\left[-\frac{M}{\overline{M}(\tilde{\tau}-\tilde{\tau}_0)}\right]\exp\left[-\frac{(\mathbf{S}-\mathbf{\overline{S}}M/\overline{M})^2}{2\overline{S^2}}\frac{\overline{M}}{M}\right]. \tag{6.6}$$

The function $\tau(t)$ is determined both by the expansion of $R(t)$ in accordance with (6.1), and by the time dependence of the probability $U\sim\langle\sigma v\rangle$ determined by the expansion, for example, on account of the change of the average velocity (see the Appendix). The most interesting is the power-law dependence $R^3(t)/R^3(t_0)\propto(t/t_0)^\lambda$, where $\lambda=2$ corresponds to the Einstein–de Sitter solution, $\lambda=3$ to an empty universe, etc. It is also natural to choose $\chi(t)$ in power-law form, $\chi(t)\propto t^\mu$, where the superscript μ describes the time dependence of $\langle\sigma v\rangle$ during expansion. In this case

$$\tilde{\tau}=n_0\tilde{U}t^{\mu+1-\lambda}/(\mu+1-\lambda)$$

and the number of mergings per unit time increases with decrease of t under the realistic condition $\lambda>\mu$.

The problem with a source (constant or with a power-law dependence of the expansion) can be treated in the same manner. We shall draw from this only the (rather obvious) qualitative conclusion that the merging probability, which can be quite high during the galaxy-formation epoch and then decrease rapidly upon expansion, can again increase, albeit not to the same degree, as a result of the galaxy "clustering" that results from gravitational instability and correlates with the activity. It is in fact the activity which reveals the manifestations of these secondary mergings.

7. DISTRIBUTION EVOLVING AFTER TURNING-ON A CONSTANT SOURCE

We assume that a source of constant mass and angular momentum is turned on under the initial conditions $X_0=0$ and $n_0=0$ at $t=0$. The solution for the Fourier transform is [see (3.4)]

$$F(p,\mathbf{q};t)=n(t)-X(p,\mathbf{q};t),$$

$$X(t)=X_\infty\operatorname{th}(UX_\infty t),\quad X_\infty=\{[D(0,0)-D(p,\mathbf{q})]/U\}^{1/2}, \tag{7.1}$$

$$n(t)=n_\infty\operatorname{th}\tau,\quad \tau=Un_\infty t,\quad n_\infty=[D(0,0)/U]^{1/2} \tag{7.2}$$

The arguments p and $\mathbf{q}$ of the quantities X in (7.1) are omitted for brevity.

We consider first the integral with respect to p:

$$\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}dpe^{pM}F(p,\mathbf{q};t)=\frac{n_\infty}{2\pi i}\int_{-i\infty}^{i\infty}dpe^{pM}\Bigg\{\operatorname{th}\tau$$

$$-\left[1-\frac{D(p,\mathbf{q})}{D(0,0)}\right]^{1/2}\operatorname{th}\left(\left[1-\frac{D(p,\mathbf{q})}{D(0,0)}\right]^{1/2}\tau\right)\Bigg\} \tag{7.3}$$

As $p\to\infty$ we have $D(p,\mathbf{q})\to 0$, and the expression in the curly brackets tends to zero, thus ensuring stability. We assume that $D(p,\mathbf{q})$ has no branching points. Equation (7.3)' reduces then to a sum of residues tanh x at the points $x_l=i\pi/2+i\pi l$, $l=0,1,\ldots$. The equation for the coordinates of the poles in the p-plane

$$\tau[1-D(p,\mathbf{q})/D(0,0)]^{1/2}=i\pi/2+i\pi l, \tag{7.4}$$

reduces to

$$1-D(p_l,\mathbf{q})/D(0,0)=-(\pi^2/\tau^2)(l+{}^1/_2)^2 \tag{7.5}$$

and for fixed l it can have in principle several roots. Small p and $\mathbf{q}$ assume a role for large times $\tau\gg 1$ and for $M\to\infty$ and $S\to\infty$:

$$D(p,\mathbf{q})=D(0,0)\{1-pM_1-i\mathbf{q}S_1-{}^1/_2q_iq_k\overline{S_iS_k}\}. \tag{7.6}$$

For the locations of the poles $p_l(\mathbf{q})$ we obtain

$$p_l(\mathbf{q})M_1=-i\mathbf{q}\mathbf{S}_1-{}^1/_2q_iq_k\overline{S_iS_k}-(\pi^2/\tau^2)(l+{}^1/_2)^2. \tag{7.7}$$

It follows that the distribution is

$$f(M,\mathbf{S};t)=\frac{2\pi^2n_\infty}{\tau^3M_1}\sum_{l=0}^{\infty}\left(l+\frac{1}{2}\right)^2\int\frac{d\mathbf{q}}{(2\pi)^3}\exp(i\mathbf{q}\mathbf{S}+p_lM). \tag{7.8}$$

According to (5.6) and (5.7) the integral over $\mathbf{q}$ is equal to

$$\int\frac{d\mathbf{q}}{(2\pi)^3}\exp\left[i\mathbf{q}\left(\mathbf{S}-\mathbf{S}_1\frac{M}{M_1}\right)-\frac{1}{2}\frac{M}{M_1}q_iq_k\overline{S_iS_k}\right]$$
$$=\frac{1}{(2\pi)^{3/2}}\left(\frac{M_1}{M}\right)^{3/2}(\operatorname{Det}\overline{S_iS_k})^{-1/2}$$
$$\times\exp\left[-\frac{1}{2}\frac{M_1}{M}\sum_j\frac{(S_j-S_{1j}M/M_1)^2}{\overline{S_j^2}}\right]. \tag{7.9}$$

We confine ourselves below, however, to isotropic $\overline{S_iS_k}$, $\overline{S_iS_k}=\overline{S^2}\delta_{ik}$. The right-hand side of (7.9) becomes then

$$\frac{1}{(2\pi)^{3/2}}\left(\frac{M_1}{M}\right)^{3/2}\frac{1}{(\overline{S_2^2})^{3/2}}\exp\left[-\frac{M_1}{2M\overline{S_2^2}}\left(\mathbf{S}-\mathbf{S}_1\frac{M}{M_1}\right)^2\right].$$

We obtain ultimately for the solution of interest to us

$$f(M,\mathbf{S};t)=\frac{2\pi^2n_\infty}{M_1(2\pi\overline{S_2^2})^{3/2}}\left(\frac{M_1}{M}\right)^{3/2}\exp\left[-\frac{M_1}{2M\overline{S_2^2}}\left(\mathbf{S}-\mathbf{S}_1\frac{M}{M_1}\right)^2\right]$$
$$\times\sum_{l=0}^{\infty}\left(l+\frac{1}{2}\right)^2\exp\left[-\frac{\pi^2M}{M_1\tau^2}\left(l+\frac{1}{2}\right)^2\right]. \tag{7.10}$$

The sum over l in this integral is the derivative of a Weierstrass elliptic function.[23] Its asymptotes in terms of the parameter $M/M_1\tau^2$ are easily obtained. For $M/M_1\tau^2\gg 1$ it suffices to retain in the sum only the terms with $l=0$. For $M/M_1\tau^2\ll 1$ (for "long" times), on the contrary, contributions are made by many terms, and the sum reduces to an integral and turns out to be proportional to $(M_1\tau^2/M)^{3/2}$. Equation (7.10) becomes then the stationary distribution (4.3).

A transition takes place thus, at $M/M_1\tau^2\sim 1$, from the stationary distribution that managed to set in for smaller masses, to an exponentially small number of large-mass galaxies. It is remarkable that only a transition to a stationary mass distribution takes place, without affecting the angular momentum.

A maximum (with respect to both M and S) is obviously present at $\mathbf{S}=\mathbf{S}_1M/M_1$ at $\mathbf{S}_1\neq 0$. The width of the maximum with respect to S is $(\overline{S}_2^2M/M_1)^{1/2}$. Comparing it with the location of the maximum, we see that the latter is quite peaked at $S_1^2/S_2^2\gg M_1/M$. The distributions in M and in S are then quite rigidly related. In the opposite case the momentum flux is small and the relation between the distributions is weaker.

Let us see now what happens after turning off a source that operated for a time t_* long enough to cause enough collisions ($\tau_*=n_\infty Ut_*\gg 1$) to establish the stationary intermediate asymptote (4.3). We confine ourselves first to the distribution in mass. A relaxation front corresponding to $M\sim M_1\tau^2$ passes then in the direction of the larger masses when the time after shutoff is such that $\tau=n_\infty Ut\ll\tau_*$. In fact, from (2.1) and (4.3) follows an estimate for the time of relaxation due to mergings:

$$1/t_{\rm rel}(M)\sim MUf(M)\sim Un_\infty(M_1/M)^{1/2}. \tag{7.11}$$

After a time t, the part of the spectrum for which $t>t_{\rm rel}(M)$ relaxes, i.e.,

$$\tau=Un_\infty t>(M/M_1)^{1/2}. \tag{7.12}$$

The spectrum remains practically unperturbed if $M\gg M_1\tau^2$.

The asymptotes of the solution for $M\gg M_1\tau^2$ and $M\ll M_1\tau^2$ and at $1\ll\tau\ll\tau_*$ can be easily found by using the solution of the initial problem (5.1):

$$f(M,t)=\frac{1}{2\pi iUt}\int_c\frac{dpe^{pM}}{1+X^{(0)}(p,\mathbf{q}=0)Ut},\qquad \operatorname{Re}p|_c\leqslant 0, \tag{7.13}$$

where $X^{(0)}(p,\mathbf{q}=0)$ is the Laplace transform of the distribution that sets in by the instant the source is turned off. According to the foregoing, if $M_1\ll M$ it suffices to retain in $D(p,0)$ the linear term in the expansion over p, and then, according to (7.1) and (7.6),

$$1+X^{(0)}(p,\mathbf{q}=0)Ut=1+(pM_1)^{1/2}\tau\,\operatorname{th}[(pM_1)^{1/2}\tau_*]. \tag{7.14}$$

The nonlinear, in contrast to the preceding, dependence of the denominator of (7.13) on p is connected with the additional scale $M\tau_*^2$ of the initial distribution that is an example of a delocalized initial condition. This distribution contains an intermediate power-law asymptote corresponding formally to an infinite initial mass (the square root term of the expansion in p). The mass corresponding to the exact condition is of course finite.

It is convenient to express the roots of the denominator of (7.13), which determine the poles, in terms of the quantity x given by

$$x=-i(pM_1)^{1/2}\tau_*,\qquad x\operatorname{tg}x=\tau_*/\tau. \tag{7.15}$$

We see that the roots form a set x_l ($l=0,1,\ldots$), where $\pi l<x_l<\pi l+\pi/2$. For $\tau_*/\tau\ll 1$ we have $x_0\approx(\tau_*/\tau)^{1/2}$, $x_l\approx\pi l+\tau_*/\tau\pi l$; for $\tau_*/\tau\gg 1$ we have $x_l\approx\pi l+\pi/2-\tau/\tau_*(\pi l+\pi/2)$. In final analysis we have (the contribution of the residues):

$$f(M,t)=\frac{2n_\infty}{M_1\tau_*\tau^2}\sum_{l=0}^{\infty}\frac{x_l^2\exp(-x_l^2M/M_1\tau_*^2)}{(\tau_*/\tau)^2+\tau_*/\tau+x_l^2}. \tag{7.16}$$

Let l_* correspond to the root x_{l*}, for which the argument of the exponential in (7.16) is closest to unity: $x_{l*}^2M/M_1\tau_*^2\sim 1$. Then

$$x_{l*}^2 \sim M_1\tau_*^2/M \ll (\tau_*/\tau)^2, \quad M_1\tau^2 \ll M. \tag{7.17}$$

The sum over l in (7.16), accurate to exponentially small terms, can be terminated with l_*, leaving only $(\tau_*/\tau)^2$ in the denominators of (7.16), by virtue of (7.17), after which the summation can be extended to infinity. Ultimately τ drops out from this part of the spectrum,

$$f(M,t) \approx \frac{2n_\infty}{M_1\tau_*^3}\sum_{l=0}^{\infty} x_l^2 \exp\left(-x_l^2\frac{M}{M_1\tau_*^2}\right), \quad M_1\tau^2 \ll M. \tag{7.18}$$

(the sum here is the derivative of a Weierstrass function), corresponding to the unperturbed spectrum part due to the source.

We consider now smaller masses — the region that had relaxed prior to turning off the source:

$$M_1 \ll M \ll M_1\tau^2, \quad 1 \ll \tau^2 \ll \tau_*^2. \tag{7.19}$$

We now define $l_0 \ll l_*$ such that $x_{l0}^2 \approx (\tau_*/\tau)^2$. The sum over l can then be taken first from 0 to $l_0 + k$, where k is chosen such that $x_{l_0+k}^2 \gg (\tau_*/\tau)^2$ but $x_{l_0+k}^2 M/M_1\tau^2 \ll 1$, and from $l_0 + k + 1$ to l_*. In the first sum the exponential can be replaced by unity, and in the second we need retain in the denominator only x_l^2, after which the summation can be extended to infinity. This enables us to express the sum in terms of an analytic Weierstrass function, and the principal term of the asymptote takes, if (7.19) is satisfied, the form

$$f(M,t) \approx \frac{n_\infty}{M_1\tau^2}\left(\frac{M_1}{M}\right)^{1/2}. \tag{7.20}$$

We see that after the relaxation the slope becomes less steep because the small-mass contribution is decreased. The motion of the front is particularly clearly seen on the plot of $Mf(M,t)$, which has at $M \sim M_1\tau^2$ a maximum at which the square-root increase $Mf(M,t) \propto M^{1/2}$ changes to a square-root decrease $Mf(M,t) \propto M^{-1/2}$. The relaxation front moves along the stationary part of the spectrum towards larger M. The relaxed part decreases with increase of τ at fixed M (Fig. 3).

Returning to the distributions in mass and in angular momentum, we note that under our assumptions the dependence on S, just as above [see (7.10) and others], is manifested by an additional factor that is independent of time. When the source is turned off, the distributions in mass and angular momentum are obtained from (7.16), (7.18), and (7.20) by multiplying by

$$\left(\frac{M_1}{2\pi M\overline{S_2^2}}\right)^{3/2} \exp\left[-\frac{M_1}{2M\overline{S_2^2}}\left(\mathrm{S}-\mathrm{S}_1\frac{M}{M_1}\right)^2\right]. \tag{7.21}$$

If the distribution part that is unperturbed by relaxation (at $M \gg M_1\tau^2$) is to remain unperturbed after the source is shut off, it is therefore necessary that the number of collisions that have formed this distribution (τ_*) be considerably larger than the number of collisions produced after the shutoff ($\tau_* \gg \tau$).

This condition can be met relatively easily in an expanding universe, since the collision probability decreases rapidly with the expansion.

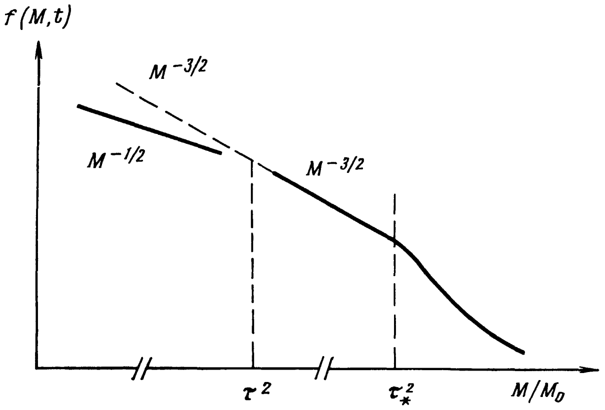

FIG. 3.

8. SELF-SIMILAR VARIABLES FOR A MASS-DEPENDENT COAGULATION COEFFICIENT

In the general case when the merging coefficient depends on M (and S) it is impossible[2)] to obtain exact solutions of the nonlinear KE for the coagulation. Important information can be obtained, however, by separating the self-similar variables, particularly those describing the motion of a nonstationary front.

We begin with the case $U = \text{const}$ and confine ourselves to the distribution in mass. Joining together the asymptotes of the solution (7.10) obtained above, we get

$$f(M,\ t) \sim M^{-3/2}\exp(-\pi^2 M/4M_1\tau^2). \tag{8.1}$$

This equation can be regarded as an interpolation formula corresponding to correct asymptotes of the exact solution of the KE. It coincides with the Schechter function[28] used to describe the distributions, in the galaxy luminosity, of the fields and of the clusters. This luminosity is usually written in the form (we convert from luminosity to masses)

$$f(M) \approx \varphi^*(M/M_*)^\alpha \exp(-M/M_*). \tag{8.2}$$

Comparing, we obtain the parameters of the Schechter function

$$\alpha = -3/2, \quad M_* = 4M_1\tau^2/\pi^2. \tag{8.3}$$

It can thus be assumed that the power law in (8.2) corresponds to a stationary spectrum, while the exponential decrease corresponds to a nonstationary coagulation front. This attempt to interpret the Schechter function presupposes that during the "action" time of the source (i.e., during the period of separation of the Jeans masses $\delta\rho/\rho \sim 1$ and of their fragmentation) the number of collisions with merging was substantially larger than after the source is "turned off."

We note also that the physical meaning of the solution used in (8.1) differs substantially from the variant of Ref. 24, where the Schechter function corresponds to a self-similar asymptote of the initial coagulation problem at $U \sim M_1 + M_2$ (Ref. 26).

The superscript $\alpha = -3/2$ is in quite fair agreement with the observed values,[29,30] but a detailed evaluation seems premature.

We confine ourselves hereafter to the case when a constant source is turned on, and regard the merging probability as a homogeneous function, of degree u, of the mass:

$$U_{\lambda M|\lambda M_1\,\lambda M_2}=\lambda^u U_{M|M_1M_2}, \quad (8.4)$$

or in abbreviated form $U = VM^u$ (this must not be taken literally). We shall neglect the dependence on the angular momenta, which can be substantial for mergings. The values of u depend on the interaction mechanism and on the distribution in velocity, and are given in Ref. 31 and in the Appendix.

The stationary distribution corresponding to a constant mass flux J over the spectrum can be easily constructed from dimensionality considerations and from the definition of the flux:

$$f_{\rm col}(M)=(J/V)^{1/2}M^{-(u+3)/2}, \quad (8.5)$$

thereby determining the exponent α (corresponding to the Lushnikov–Smirnov spectrum[11]) in the Schechter function:

$$\alpha=-(u+3)/2. \quad (8.6)$$

The dimensionless combination containing the time

$$\eta=M/JUt^2=M^{1-u}/JVt^2, \quad (8.7)$$

defines the self-similar variable η (Ref. 22), with the aid of which we obtain in the self-similar asymptotic region an isotropic solution that describes the nonstationary front:

$$f(M,t)=f_{\rm col}(M)g(\eta),$$
$$g(\eta)\to 1,\quad \eta\to 0;\qquad g(\eta)\to 0,\quad \eta\to\infty. \quad (8.8)$$

This makes it possible to set the quantity $\eta^{1/(1-u)}$ in correspondence with the argument M/M_* of the Schechter function, whence

$$M_*=(JVt^2)^{1/(1-u)}. \quad (8.9)$$

The quantity φ^* in (8.2) makes it also possible to determine J/V.

Note the different character of the solutions for $u<1$ and $u>1$ (cf. Ref. 32). For $u<1$ a self-similar distribution with a mass flux can be interpreted as motion of the front (8.9) towards large masses, while behind the front a stationary distribution (8.5) is established. For the case $u=0$, which admits of an exact solution, this can be seen directly from the asymptote (7.10) (see also Ref. 22). The integral that determines the total mass diverges in the region of large masses like $M^{(1-u)/2}$ (on a stationary distribution), i.e., the analog of the "energy containing" region for the turbulent spectrum, meaning the "mass-containing" region is located at $u<1$ on the large-mass side ($M\to\infty$).

For $u>1$, on the contrary, the total mass diverges as $M\to 0$. The self-similar substitution (8.7) can in this case not be interpreted as the motion of the front of a stationary distribution. As noted in Ref. 32, in the corresponding weak-turbulence situation the stationary distribution is formed "explosively" for all M, as is confirmed by the computations of Zakharov and Musher for a model problem.

9. ACTIVITY INDUCED BY MERGING

The extinction of the angular momentum on merging may be the most important reason why part of the matter of the disk-subsystem colliding galaxies drops out to the center of the combined system. A few preliminary remarks are in order.

To estimate the disk-mass "defect" Δm produced upon merging, we shall assume that the disk has a smaller mass m than the spheroidal subsystem, that their radii R are equal, and the angular momentum $S=m\Omega R^2$ is contained in the disk system rotating with average angular velocity Ω. Using the condition that the disk is in equilibrium

$$m\Omega^2R^2/2=GmM/R, \quad (9.1)$$

which leads to $\Omega=(G\rho)^{1/2}$, where $\rho=M/R^3$ (we omit coefficients of order unity), we obtain for the angular momentum

$$S=m(G\rho)^{1/2}(M/\rho)^{2/3}. \quad (9.2)$$

The power 2/3 reflects the ratio of the dimensionalities of the disk and spheroidal subsystems.

It is natural to define the mass defect Δm as

$$\Delta m=m_1+m_2-m, \quad (9.3)$$

where m is the disk mass after merging, from which we obtain in our model (at constant density, $\boldsymbol{\sigma}=\mathbf{S}/S$)

$$\Delta m=m_1+m_2-(M_1+M_2)^{-2/3}[m_1^2M_1^{4/3}+m_2^2M_2^{4/3}+2m_1m_2(M_1M_2)^{2/3}\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2]^{1/2}. \quad (9.4)$$

It appears that a small fraction ε of this disk-mass defect falls to the center within a time $t_{\rm act}$, thereby determining the accretion rate $\varepsilon\Delta\dot m=\varepsilon\Delta m/t_{\rm act}$.

The analysis of the preceding section allows us to find the probability (per unit time) of obtaining after the collision an active galaxy with a defect Δm:

$$\frac{\partial f(M,\mathbf{S};\Delta m)}{\partial t}=\int dM_1\,dM_2\,d\mathbf{S}_1\,d\mathbf{S}_2 f(M_1\mathbf{S}_1)f(M_2\mathbf{S}_2)U\delta_M\delta_{\mathbf{S}}\delta_{\Delta m}\equiv I, \quad (9.5)$$

where

$$\delta_{\Delta m}\equiv\delta[\Delta m-(m_1+m_2-m)],\qquad m_i=m(M_i,\mathbf{S}_i).$$

Writing out only the arrival term under the assumption that the activity time is significantly shorter than the interval between collisions (a generalization is obvious), and in (9.5) one has in mind a short time scale connected with the activity scale. To describe this, we must introduce into the equation a term that describes the activity damping:

$$\frac{\partial f(M,\mathbf{S};\Delta m)}{\partial t}=I-\frac{1}{t_{act}}f(M,\mathbf{S};\Delta m). \quad (9.6)$$

The activity time is apparently longer than the free-fall time $(G\rho)^{-1/2}$ (the galaxy-collision time) and is independent of Δm. (The time responsible for the fine structure of the activity is considerably shorter and is determined by the effective density in the vicinity of the central compact object; in the case of formation of an accretion disk it is determined by its lifetime.) The temporal argument in f, which corresponds to evolution due to collisions, is in the present analysis a parameter and has been left out of (9.5) and (9.6).

If $f(M,\mathbf{S})$ are known, we can estimate the number of active galaxies of a given activity level determined by Δm, by considering the "stationary" solution of (9.6):

$$f(M, S; \Delta m) = t_{act} I\{f(M, S)\}. \tag{9.7}$$

According to (9.5) and (9.2) we have

$$f(M,S;\Delta m) = t_{act} \int dM'\, dS' f(M', S') f(M-M', S-S')\, U\delta_{\Delta m}, \tag{9.8}$$

where

$$\delta_{\Delta m} \equiv \delta\left\{ \Delta m - \frac{\rho^{2/3}}{(G\rho)^{1/2}} \left[\frac{S'}{(M')^{5/3}} + \frac{|S-S'|}{(M-M')^{5/3}} - \frac{S}{M^{5/3}} \right]\right\}. \tag{9.9}$$

The integrals of (9.8) contain the mass and angular-momentum distributions obtained in the preceding sections. Account must be taken here, of course, of the "clustering" effect, which increases the merging probability in view of the increase of the concentrations n (thus, quasars are encountered predominantly in galaxy groups, while radiogalaxies are encountered in centers of rich clusters[26]).

As to the quantity ε indicative of the mass-defect fraction falling to the center, it includes in all probability an exceedingly small parameter — the ratio of the scales of the central compact object and of the entire galaxy.[3)]

APPENDIX

The merging probability $U\delta_M\delta_S$ is proportional to $U = \langle \sigma v \rangle$, where the angle brackets denote averaging over the momenta p, σ is the cross section, and v is the relative velocity:

$$\sigma v = \pi r^2 v [1 + GM/rv^2]\varphi, \quad \varphi = (1 + rv^2/GM)^{-\xi}, \quad \xi > 0, \tag{A1}$$

Here $r = R_1 + R_2$, and φ is a factor that takes into account the dependence of the merging probability on the relative velocity in frontal collision.[4)]

We assume here the "elastic" variant of this quite unreliably determined quantity. It follows for a homogeneity degree $U \propto VM^u$ that

$$u = \begin{cases} \beta + 2 - \alpha, & GM/rv^2 \gg 1, \\ 2\beta - 1 + \xi(3-\beta) + \alpha(1-2\xi), & GM/rv^2 \ll 1. \end{cases}$$

The exponent α (not to be confused with the Schechter exponent) is connected here with the velocity distribution used in the averaging,[31] viz., $\alpha = 1$ for a collisionless distribution and $\alpha = 1/2$ for a Maxwellian one. The exponent β describes the change of the radius as a function of the mass: $R \propto M^\beta$. At $\beta = 1/3$ the value $\xi = 0$ corresponds to $u > 1$ for gravitational collisions and $u < 1$ for contact collisions. At $\xi = 1/2$ (i.e., the decrease of the merging probability in frontal collision is inversely proportional to the velocity) we have $u = 1$ for contact collisions for all α. For $\xi = 0$ and $\alpha = 1/2$ even a small compression with increase of mass ($\beta = 1/4$) leads to $u = 0$ for contact collisions. On the other hand if $\xi \gg 1$ for $\alpha = 1$ the exponent u vanishes at $\beta = 1$, i.e., in the situation $R \propto M$ which is realized in large scales of mass and density changes.[21] In this case, however, the value of u is quite sensitive to the difference between β and unity.

According to (9.2), the morphologic type should apparently correspond to S/M^k, where $k = (3+\beta)/2$, so that at $\rho = \text{const}$ we have $k = 5/3$ (see Refs. 20 and 34).

[1)] Of course, difficulties are encountered in this approach too, see the corresponding discussion and citations in Ref. 4.

[2)] Solutions of the initial problem have been obtained for particular cases.[25-27,11]

[3)] The complicated hierarchy of the processes that lead to falling of a fraction of the masses to a center, and a numerical analysis of this problem, are treated in a brief review by Hernquist[33] and in the literature cited therein.

[4)] Equation (A1) contains, generally speaking, also dimensionless factors made up of the ratios of the masses, radii, and velocities, which do not influence the degree of homogeneity but are of importance for the asymptotes.

[1] S. F. Shandarin, A. G. Doroshkevich, and Ya. B. Zel'dovich, Usp. Fiz. Nauk **139**, 83 (1983) [Sov. Phys. Usp. **26**, 46 (1983)].
[2] V. G. Gorbatskii, *Introduction to the Physics of Galaxies and of Galaxy Clusters* [in Russian], Nauka, 1986.
[3] A. Toomre and J. Toomre, Astrophys. J. **178**, 623 (1972).
[4] S. Veeraghavan and S. White, *ibid.* **296**, 336 (1985).
[5] N. Roos, *ibid.* **294**, 479 (1984).
[6] B. V. Komberg, Astrofizika **20**, 73 (1984).
[7] J. B. Hutchings, Publ. Astr. Soc. Pac. **95**, 799 (1983).
[8] D. Sanders, B. Soifer, J. Elias, *et al.*, Astrophys. J. **325**, 74 (1988).
[9] A. Dressler, Ann. Rev. Astr. Astrophys. **22**, 185 (1984).
[10] *Large Scale of the Universe*, M. Longaire and J. Einasto, eds., Russ. transl., Mir, 1981.
[11] V. M. Voloshchuk, *Kinetic Theory of Coagulation* [in Russian], Gidrometeoizdat, 1984.
[12] G. B. Field and W. C. Saslaw, Astrophys. J. **142**, 568 (1965).
[13] J. Silk, *ibid.* **220**, 390 (1978).
[14] T. Nakano, Progr. Theor. Fiz. **36**, 515 (1966).
[15] S. White, Mon. Not. Roy. Astr. Soc. **185**, 185 (1978); **189**, 731 (1979).
[16] A. G. Doroshkevich, Astrofizika **3**, 175 (1967).
[17] L. M. Genkina, Tr. Astrofiz. In-ta AN KazSSR **12**, 106 (1969).
[18] G. Tammann, A. Yahil, and A. Sandage, Astrophys. J. **234**, 775 (1979).
[19] S. Van den Berg and McClure, Astrophys. J. **231**, 671 (1979).
[20] S. M. Fall, in *Internal Kinematics and Dynamics of Galaxies*, E. Athanosoula, ed., Reidel, 1983, p. 391.
[21] W. C. Saslaw, *Gravitational Physics of Stellar and Galactic Systems*, Cambridge U. Press, 1987.
[22] A. V. Kats, Dokl. Akad. Nauk. Ukr. SSR A, No. 5, 53 (1986).
[23] I. S. Gradshtein and I. M. Ryzhik, *Tables of Integrals, Sums, Series, and Products*, Academic, 1965.
[24] J. Silk and S. White, Astrophys. J. **223**, L59 (1978).
[25] V. S. Sofronov, *Evolution of Preplanetary Cloud and Formation of the Earth and Planets* [in Russian], Nauka, 1966.
[26] B. A. Trubnikov, Dokl. Akad. Nauk SSSR **196**, 1316 (1971) [Sov. Phys. Doklady **16**, 124 (1971)].
[27] A. A. Lushnikov and V. N. Piskunov, Kolloid. Zh. No. 3, 1166 (1978).
[28] P. Schechter, Astrophys. J. **203**, 297 (1976).
[29] R. E. de Souza, G. Chincarini, and G. Vettolani, Preprint, Eur. South. Obs. No. 396, 1985.
[30] J. Choloniewski, Fermilab Preprint No. 179-A, 1985.
[31] L. I. Vinokuorv, A. V. Kats, and V. M. Konotorovich, J. Stat. Phys. **38**, 217 (1985).
[32] V. E. Zakharov, in *Basic Plasma Physics*, M. N. Rosenbluth and R. Z. Sagdeev, eds., North-Holland, 1983, Vol. 2, p. 3.
[33] L. Hernquist, Preprint, Inst. Adv. Study, Princeton, IASSNS-AST 89/2, 1989
[34] I. L. Genkin and L. M. Genkina, Tr. Astrofiz. In-ta KazSSSR **20**, 36 (1973).

Translated by J. G. Adashko