Coherent four-wave mixing of light pulses

A. A. Zobolotskiĭ

Automation and Electrometry Institute, Siberian Division, USSR Academy of Sciences
(Submitted 27 April 1989)
Zh. Eksp. Teor. Fiz. **97**, 127–135 (January 1990)

The dynamics of the polarization of quasimonochromatic electromagnetic waves in a medium with a cubic nonlinearity is studied. It is assumed that two pairs of fields propagate in opposite directions and the sum (difference) of the carrier frequencies of some fields or polarization components is close to the characteristic frequency of the medium. To solve the problem of the evolution of fields with nonvanishing asymptotes the inverse scattering transform method is employed. The relevant technique is developed and soliton and quasi-self-similar solutions are found describing the start of decay of the initially unstable state. The possibility of using the results to describe wave front reflection, spontaneous generation, and related effects is discussed.

1. INTRODUCTION

The four-wave interaction in media with mirror symmetry is basic for values of intense fields below saturation. An important particular case of such interaction is degenerate four-wave mixing (FM), i.e., the nonlinear interaction of waves having the same frequency but otherwise differing in the direction of propagation or polarizations.[1–3] Data exist on the observation of such FM in different media for continuous and pulsed radiation. Degenerate FM leads to different effects such as the amplification of the signal wave running ahead, wave-front reversal (WFR), and the reconstruction of holographic images.[2] Related phenomena arise in the restructuring of polarization of two wave packets having unequal carrier frequencies.[1–6] In a series of studies different partial solutions were obtained for the evolutionary equations describing the above effects.[2,6–8]

The most often used approximation is the inexhaustibility of intensities of reference waves.[6–8] However, in a number of FM systems the coefficient of field transformation (more precisely, of the number of photons) reaches values of order unity, exceeds it and in systems in which an explosive instability occurs.[8] In this connection the necessity arises of constructing a more complete theory describing the evolution of all fields taking part in FM. The excitation of WFR effects and others with FM requires in the majority of cases significant field powers which may be reached often only by use of ultrashort light pulses (USP). This fact also underlines the necessity of considering nonstationary effects.

The most convenient analytic tool for the above purposes is the inverse scattering transform method (ISTM).[9] This method was employed in studies[10–12] of FM of light waves in a nondispersive medium with a cubic nonlinearity. The case of interaction of two counterpropagating waves with a difference of carrier frequencies close to the frequency of the medium was considered in Ref. 10. Topologically stable solutions of the domain-wall type were used for describing the restructuring of the polarization. In the framework of new models of FM, a soliton regime and a self-similar regime describing the decay of the unstable state of the system were studied in Refs. 11 and 12.

The models of FM studied in our work are particular cases of those considered in Refs. 10–12. However, in contrast to preceding studies, nonzero asymptotes are considered here for all fields (components of field polarization) participating in the FM. This fact causes a modification of the ISTM technique and correspondingly of the solutions of the problem. We note that the constructed ISTM technique generalizes the corresponding results obtained by use of this method to include the study of a model of an isotropic ferromagnet.[13–16] The statement of the problem with nonzero asymptotic fields results from specific properties of the problem of FM for which a special choice of initial field polarizations is not required.

The results obtained below can also be used in other areas of physics in which formally equivalent models arise, for example in the theory of a plasma for carrier frequencies much higher than the plasma frequency,[10] and for the theory of elementary particles in the $SU(2)$ model of Vaks-Larkin-Nambu-Iona-Lazino.[17]

The plan of study is the following. In Sec. 2 the studied models of FM are presented. Sec. 3 is devoted to development of the ISTM technique and contains the solutions. Possible physical consequences are discussed in Sec. 4.

2. FM EQUATIONS AND STATEMENT OF THE PROBLEM

We represent a field propagating in a homogeneous medium in the following form:

$$\mathscr{E}(z,t)=\sum_{\pm}\sum_{j=1}^{2}E_j^{\pm}(z,t)\exp(i\omega_j^{\pm}\mp k_j^{\pm}x)+\text{c.c.},\tag{1}$$

where $E_j^{\pm}$ are the slowly changing envelopes of wave packets, and $k_j^{\pm}$ and $\omega_j^{\pm}$ are the carrier wave vectors and frequencies, respectively. The frequencies $\omega_j^{\pm}$ are chosen such that their sum or difference is close to the characteristic frequency of the medium:

$$\omega_j^{+}\pm\omega_j^{-}=\omega_0+\nu,\tag{2}$$

where ν is the detuning. In a medium with a cubic nonlinearity the change of the nonlinear part of the permittivity is proportional to the product of the field envelopes. Condition (2) allows one to neglect in the final equations the nonresonant Kerr self-action of the fields. The nature of the nonlinear part of the evolutionary equations can be different.[5–10] In one study[11] these equations were derived by the adiabatic exclusion of a mode of natural oscillations of the medium for times much shorter than relaxation times and for sufficiently large detunings ν. The derivation of the final equations is presented in Ref. 11. Finally, Maxwell's equations in the

 0038-5646/90/010071-05$03.00

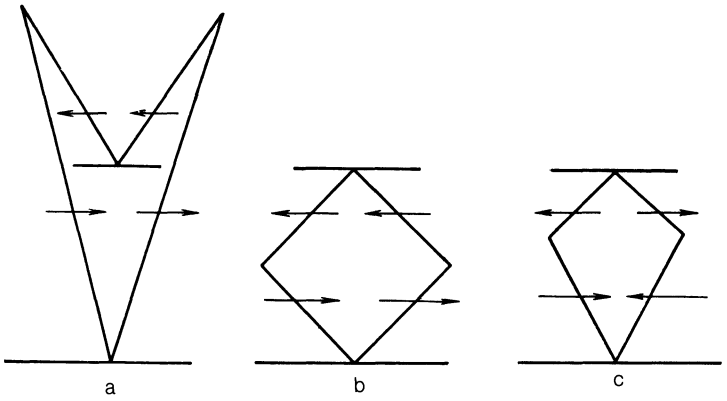

FIG. 1. FM diagrams: a and c are for $\omega_1^+ = \omega_2^+ \neq \omega_1^- = \omega_2^-$; b is for $\omega_1^+ = \omega_2^- = \omega_2^+ = \omega_1^+$. The horizontal lines indicate the energy levels of the medium. The straight lines originating from the lower level correspond to the field with envelope E_i^+, carrier frequency ω_i^+, and wave vector k_i^+, and the lines originating from the upper level correspond to the field with corresponding parameters E_i^-, ω_i^- and k_i^-. The index $i = 1$ distinguishes the left pair of fields from the right pair ($i = 2$). The arrows indicate the direction of field propagation along the z axis.

approximation of a rotating wave reduce to the following form [for definiteness we choose the minus sign in Eq. (2)]:

$$\begin{aligned}
&(\partial_t+V_1^+\partial_z)E_1^+=i\omega_1^+\alpha(E_1^+|E_1^-|^2+E_2^+E_1^-\overline{E_2^-}),\\
&(\partial_t+V_2^+\partial_z)E_2^+=i\omega_2^+\alpha(E_2^+|E_2^-|^2+E_1^+E_2^-\overline{E_1^-}),\\
&(\partial_t+V^-\partial_z)E_1^-=i\omega_1^-\alpha(E_1^-|E_1^+|^2+E_2^-E_1^+\overline{E_2^+}),\\
&(\partial_t+V_2^-\partial_z)E_2^-=i\omega_2^-\alpha(E_2^-|E_2^+|^2+E_1^-E_2^+\overline{E_1^+}),
\end{aligned} \tag{3}$$

where $V_{1,2}^\pm$ are the field group velocities, $V_i^\pm = \omega_i^\pm / k_i^\pm$, $k_1^+ = k_2^+ \neq k_1^- = k_2^-$, $\omega_1^+ = \omega_2^+ \neq \omega_1^- = \omega_2^-$,

$$\alpha = \frac{2\pi N_0}{\hbar\nu}|\varkappa_{ij}(\omega_1^\pm)|^2,$$

and N_0 is the active-particle number density. We used here the assumption of a weak perturbation of the medium and $\varkappa_{ij}$ is the scattering tensor which for the case considered of an isotropic medium is proportional to a δ-function, $\varkappa_{ij} = \delta_i \varkappa$. Diagrams of FM are shown in Fig. 1, where the directions of propagation of the fields along axis z are shown by arrows.

The field dynamics may qualitatively depend on resonance conditions and the direction of field propagation.[11] Diagrams a and b in Fig. 1 are described by systems of equations which are isomorphic to each other and do not reduce to equations corresponding to the FM diagram in Fig. 1(c). One can represent the system of equations (3) in the form

$$\begin{aligned}
&\partial_T R_+=-\varepsilon\partial_X F_+=i(\varepsilon R_3F_+-R_+F_3),\\
&\partial_T R_3=-\partial_X F_3=i(R_+F_--R_-F_+)/2,
\end{aligned} \tag{4}$$

where $\overline{F}_+ = F_-$, $\overline{R}_+ = R_-$, $\partial_\eta = \partial_t + V\partial_z$, $\partial_\xi = \partial_t - V\partial_z$; for diagrams a and b, $\varepsilon = 1$, $V = V_1^+ = V_2^+ = -V_1^- = -V_2^-$,

$$T=\omega_1^+\alpha\int_{-\infty}^{\eta} I_1(\eta)\,d\eta,$$

$$X=\omega_1^-\alpha\int_{-\infty}^{\xi} I_2(\xi)\,d\xi,$$

$$I_1(\eta)=|E_1^-|^2+|E_2^-|^2/\beta^2,$$

$$I_2(\xi)=|E_1^+|^2+\beta^2|E_2^+|^2,$$

$$R_3=(\beta^2|E_2^+|^2-|E_1^+|^2)/I_2,\quad \beta^2=\omega_1^+/\omega_1^-,$$

$$F_3=(|E_2^-|^2/\beta^2-|E_1^-|^2)/I_1,$$

$$R_+=2E_1^+\overline{E_2^+}\beta/I_2,\quad F_+=2E_1^-\overline{E_2^-}/\beta I_1;$$

for diagram c, ξ, T, V_1^+, β, and α are the same as above, $\varepsilon = -1$, $V_2^- = V = -V_2^+ = -V_1^-$,

$$X=\omega_2^+\alpha\int_{-\infty}^{\xi} I_2(\xi)\,d\xi,\qquad I_1(\eta)=|E_1^+|^2/\beta^2-|E_2^-|^2,$$

$$I_2(\xi)=|E_2^+|^2\beta^2-|E_1^-|^2,\quad F_3=-(|E_2^-|^2+|E_1^+|^2/\beta^2)/I_1,$$

$$R_3=-(|E_1^-|^2+\beta^2|E_2^+|^2)/I_2,$$

$$R_+=2E_1^+E_2^-/\beta I_2,\ F_+=2\beta E_1^-E_2^+/I_1^+.$$

The integrals of motion of Eqs. (4) are

$$F_3^2+\varepsilon|F_+|^2=1,\quad R_3^2+\varepsilon|R_+|^2=1. \tag{5}$$

We consider the following stationary solutions of Eqs. (4):

$$F_3^0R_+^0=\varepsilon R_3^0F_+^0,\quad R_+^0F_-^0=R_-^0F_+^0, \tag{6}$$

$$R_+^0=F_+^0=0,\quad R_3^0F_3^0=\pm1. \tag{7}$$

We find from Eqs. (5) and (6) the possible solutions for $\varepsilon = 1$:

$$r_1^+r_2^-=r_2^+r_1^-\beta^2,\quad r_1^+r_1^-=-r_2^-r_2^+,\quad \varphi_1^+-\varphi_2^+-\varphi_1^-+\varphi_2^-=0,$$

and for $\varepsilon = -1$:

$$r_1^-r_2^+=-r_1^-r_2^-,\quad r_1^+r_1^-=-r_2^+r_2^-\beta^2,\quad \varphi_1^++\varphi_2^--\varphi_1^--\varphi_2^+=0,$$

where $E_{1,2}^\pm = r_{1,2}^\pm \exp(i\varphi_{1,2}^\pm)$. In the linear approximation the stable states are

$$r_1^+r_2^-=r_2^+r_1^-\beta^2,\quad \varepsilon=1,\quad r_1^+r_2^+=-r_1^-r_2^-\beta^2,\quad \varepsilon=-1. \tag{8}$$

For the stationary states (7) the condition of stability has the form

$$R_3^0F_3^0=1\ . \tag{8'}$$

In the present study we will seek soliton solutions describing deviations of the system from the stable asymptotic form (6) or (7). As will be soon from the following, production of a soliton by FM requires synchronization of two USP propagating in the same direction or the introduction of pulsed fields with nonzero polarization components. A stable soliton regime occurs only for $\varepsilon = 1$ (for arbitrary asymptotic forms). In this case if the initial state of the system is unstable, a weak perturbation, for example quantum fluctuations of the medium, initiates the decay of such a state and intense transfer of energy between components of field polarization. The final state of the system depends on the sign of ε and on the values of asymptotic forms (6) and (7). In the latter case, for a sufficiently small perturbation, one succeeds in finding in explicit form the shape of the first spike of the quasi-self-similar solution which, as computer calculations show, consists of an infinite train of damped oscillations.[11]

3. CONSTRUCTION OF ISTM

The system of equations (4) can be represented in the form of the compatibility condition ($\partial_z L - \partial_x A + [L,A] = 0$) of two systems of linear equations:

$$\partial_z\varphi = A\varphi = i\begin{pmatrix} \lambda n & \lambda\mu \\ \varepsilon\lambda\bar{\mu} & -\lambda n \end{pmatrix}\varphi(z,x,\lambda), \tag{9}$$

$$\partial_x\varphi = L\varphi = \begin{pmatrix} -\dfrac{i}{2}\left(1+\dfrac{1}{2\lambda\varepsilon-1}\right)R_3 & \dfrac{\lambda\varepsilon R_+}{2\lambda\varepsilon-1} \\ -\dfrac{\lambda R_-}{2\lambda\varepsilon-1} & \dfrac{i}{2}\left(1+\dfrac{1}{2\lambda\varepsilon-1}\right)R_3 \end{pmatrix}\varphi, \tag{10}$$

where $n = F_3$, $\mu = i\varepsilon F_+$, $\bar{\mu} = -i\varepsilon F_-$, $z = -\varepsilon T$, and λ is a spectral parameter. An ISTM technique for the spectral problem (9) with $\varepsilon = 1$ was constructed in Refs. 11–14, 16 for the asymptotic conditions $\mu n = 0$. In the present study these results are generalized to the case of arbitrary constant values $E_i^{\pm}(z,x)$ as $z\to\pm\infty$.

Let

$$n\to n_0^{\pm},\quad \mu\to\mu_0^{\pm},\quad z\to\pm\infty. \tag{11}$$

The spectral problem (9) has the involute

$$\psi' = M\bar{\psi}(\lambda)M^{-1}, \tag{12}$$

where

$$M = \begin{pmatrix} 0 & 1 \\ -\varepsilon & 0 \end{pmatrix}.$$

The unperturbed Eq. (9) has fundamental solution matrices for $z\to\pm\infty$:

$$\varphi_0^{\pm}(z) = \begin{pmatrix} \mu_0^{\pm} & \varepsilon(n^{\pm}-1) \\ 1-n_0^{\pm} & \bar{\mu}_0^{\pm} \end{pmatrix}\begin{pmatrix} e^{i\lambda z} & 0 \\ 0 & e^{-i\lambda z} \end{pmatrix}. \tag{13}$$

The scattering matrix T is defined by the equation

$$\varphi^- = \varphi^+ T, \tag{14}$$

where $\varphi^{\pm}$ are fundamental Jost-solution matrices, $\varphi^{\pm}\to\varphi_0^{\pm}$ as $z\to\pm\infty$. We have from Eqs. (12) and (14)

$$T = \begin{pmatrix} \bar{a} & b \\ -\varepsilon\bar{b} & a \end{pmatrix}. \tag{15}$$

The coefficient $a(\lambda)$, the first column of the Jost matrix φ^+, and the second column φ^- have analytic expressions in the upper half plane. The zeros of a (λ_k) determine the discrete spectrum of the problem. The matrix of Jost solutions has the following triangular representation:

$$\varphi^+(z) = \varphi_0^+(z) + \int_z^{\infty}\begin{pmatrix} \lambda K_1(z,s) & \lambda K_2(z,s) \\ -\lambda\varepsilon\bar{K}_2(z,s) & \lambda\bar{K}_1(z,s) \end{pmatrix}\varphi_0^+(s)\,ds. \tag{16}$$

Because in all the following expressions contain values of constants only for $z\to+\infty$, we omit the "plus" superscript from n_0^+ and μ_0^+. Substituting Eq. (16) in Eq. (9) we find that $K_{1,2}$ are solutions of the problem

$$\mu_0 K'_{1,z}(z,s) - n_0 K'_{2,z}(z,s) + nK'_{2,s}(z,s) + \mu\bar{K}'_{1,s}(z,s) = 0, \tag{17}$$

$$K''_{2,zs}(z,s) + \mu_0 nK''_{1,ss}(z,s) - inn_0K_{2,zs}(z,s)$$
$$= i\mu\mu_0\varepsilon\bar{K}_{2,ss}(z,s) + in_0\mu\bar{K}_{1,ss}(z,s), \tag{18}$$

with $K_{1,2}(z,s)\to 0$ as $s\to\pm\infty$ and the following conditions on the diagonal

$$\mu = (1-n)\frac{i\mu_0 + K_2(z,z)(n_0-1) - \mu_0K_1(z,z)}{i(1-n_0) + \bar{K}_1(z,z)(n_0-1) + \varepsilon\mu_0\bar{K}_2(z,z)}, \tag{19}$$

$$K'_{2,z}(z,z) = -\mu_0 nK'_{1,s}(z,s=z) + K'_{2,s}(z,s=z)nn_0$$
$$+ \mu\mu_0\varepsilon\bar{K}'_{2,s}(z,s=z) + n_0\mu\bar{K}_{1,s}(z,s=z). \tag{20}$$

Integrating Eq. (14) along the real axis with multiplier $\exp(i\lambda y)/(2\pi\lambda)$, we finally obtain the Marchenko equations:

$$V(z,y) + \mu_0F(z+y) + \int_z^{\infty}\bar{U}(z,s)F(y+s)\,ds = 0, \tag{21}$$

$$\bar{U}(z,y) + (1-n_0)F(z+y) + \int_z^{\infty}(-\varepsilon)\bar{V}(z,y)F(y+s)\,ds = 0, \tag{22}$$

where

$$U(z,y) = (1-n_0)\bar{K}_2(z,y) + \bar{\mu}_0\bar{K}_1(z,y),$$
$$V(z,y) = \bar{\mu}_0K_2(z,y) - \varepsilon(1-n_0)K_1(z,y).$$

The continuous and discrete spectra λ_k of problem (9) ($\operatorname{Im}\lambda_k > 0$) contribute to the kernel F:

$$F(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{b(\lambda)\exp(i\lambda x)\,d\lambda}{a(\lambda)\lambda} - i\sum_k\frac{b(\lambda_k)\exp(i\lambda_k x)}{a_\lambda'(\lambda_k)\lambda_k}. \tag{23}$$

Before finding the solution of Eqs. (21) and (22) using Eqs. (19) and (5), we represent the connection between the "potentials" $V(z,z)$ and $U(z,z)$ in the form

$$\mu = 2Q/(1+\varepsilon|Q|^2), \tag{24}$$
$$n = 1-2/(1+\varepsilon|Q|^2), \tag{25}$$

where

$$Q = [i\mu_0 - \bar{U}(z,z)]/[i(1-n_0) - \varepsilon\bar{V}].$$

The soliton single-pole solution satisfies the following kernel form:

$$F_1(z) = c\exp(i\lambda_1 z),\quad c = -ib(\lambda_1)/a_\lambda'(\lambda_1)\lambda_1, \tag{26}$$

where c and λ_1 are some complex numbers and $\operatorname{Im}\lambda_1>0$. In this case the solution of Eqs. (21) and (22) has the form

$$V(z,y)=[-\mu_0F_1(y+z)-(1-n_0)F_1(y+z)\bar{F}_1(2z)/i(\lambda_1-\bar{\lambda}_1)]D^{-1}, \tag{27}$$

$$U(z,y)=[-(1-n_0)F_1(y+z)-\varepsilon\bar{\mu}_0F_1(y+z)\bar{F}_1(2z)/i(\lambda_1-\bar{\lambda}_1)]D^{-1}, \tag{28}$$

where

$$D=1-\varepsilon F_1(2z)\bar{F}_1(2z)/(\lambda_1-\bar{\lambda}_1)^2.$$

Finally we obtain from Eqs. (24) and (25)

$$\mu=\mu_0\frac{D^2-A^2|f|^2-\varepsilon|f|^2(1+iDA)-\varepsilon(iD+A|f|^2)(\bar{\mu}_0\bar{f}-\mu_0f)/(1-n_0)}{1+|f|^4(A^2+A^4)+\varepsilon|f|^2(1+2A^2)+A|f|^2(\mu_0f+\bar{\mu}_0\bar{f})}, \tag{29}$$

$$n=n_0\frac{1+(A^2+A^4)|f|^4+(2A^2-1)|f|^2\varepsilon-2\varepsilon Di(\bar{\mu}_0\bar{f}-\mu_0f)/n_0}{1+(A^2+A^4)|f|^4+(2A^2-1)|f|^2\varepsilon+A|f|^2(f\mu_0+\bar{\mu}_0\bar{f})}, \tag{30}$$

where $A=1/[i(\lambda_1-\bar{\lambda}_1)]$ and $f=F_1(2z)$.

For $\mu_0\to0$ and $n_0\to1$ Eq. (30) reduces to the form

$$n=\frac{2\operatorname{ch}\psi+\varepsilon(2A^2-1)/(A^2+A^4)^{1/2}}{2\operatorname{ch}\psi+\varepsilon(2A^2+1)/(A^2+A^4)^{1/2}}; \tag{31}$$

here $\Psi=2iz(\lambda_1-\bar{\lambda}_1)+\ln(A^2+A^4)^{1/2}$. The nonsingular (for some μ_0 and λ) solution (29), (30) describes the "divergence" of initial wave polarizations and a return to the initial stable state (11). The obtained soliton solution, in contrast to the domain-wall solution described in Ref. 10, does not have a topological charge, but is nevertheless stable. The dependence on the variable x is found from the system (10). For asymptotic states satisfying Eq. (6), we have

$$\rho(\lambda,x)=\rho(\lambda,0)\exp\frac{2i\lambda x}{2\lambda-\varepsilon},\qquad \rho(\lambda,x)=\frac{b(\lambda,x)}{a(\lambda)}. \tag{32}$$

Correspondingly for the asymptotes (7) we obtain

$$\rho(\lambda,x)=\rho(\lambda,0)\exp\left[i\left(1+\frac{1}{2\lambda\varepsilon-1}\right)\int_{-\infty}^{x}R_3(0,x')dx'\right]. \tag{33}$$

We consider the case of small F and $K_{1,2}$, i.e., we neglect in the Marchenko equations (21) and (22) the product $FK_{1,2}$. We find from Eqs. (24) and (25) that

$$n\approx n_0+i(\mu_0F(2z)-\bar{\mu}_0\bar{F}(2z))+O(|F(2z)|^2), \tag{34}$$

$$(\mu-\mu_0)\bar{\mu}_0=-i(1+n_0)\mu_0F(2z)-i(1-n_0)\bar{\mu}_0\bar{F}(2z)+O(|F(2z)|^2). \tag{35}$$

Taking the inverse Fourier transform, we reconstruct the scattering coefficient ρ from Eqs. (34) and (35) from small deviations from asymptotic states:

$$2\mu_0\rho(\lambda,0)=i\lambda\int_{-\infty}^{\infty}e^{i\lambda z}[(\mu(z)-\mu_0)\bar{\mu}_0-(1-n_0)(n(z)-n_0)]dz. \tag{36}$$

In the case of an unstable initial system state a small perturbation is sufficient to initiate the process of nonlinear FM and can be a weak signal pulse or fluctuations of the medium. In this case the coefficient ρ contains all the necessary information because the discrete spectrum of problem (5) is absent. As shown by a computer calculation, the general solution for $|\mu(z,x)|$ in this case has the form of damped pulsations.[11] It is often sufficient for practical purposes to find only the form of the leading edge of the pulse formed in FM. The leading edge and apex of the first "spike" can be found in the framework of the above stated theory. Linear analysis shows that for $\mu_0=0$, $\varepsilon=1$ and

$$\ln\left(\int_{-\infty}^{\infty}|\mu(0,z)|dz\right)^{-1}\gg1 \tag{37}$$

the solution describing $\mu(x,z)$ is concentrated in the region $\eta\gtrsim\eta_0\gg1$, where

$$\eta=\left|z\int_0^x R_3(x,0)dx\right|^{1/2}$$

is the self-similar variable.

The solution describing the leading edge and apex of the first spike can be found from the Marchenko equations (21) and (22) for $\eta\gg1$ in which case the coefficient $\rho(\lambda,0)$ is determined by expression (36) (in this case for $\mu_0=0$) because this solution joins with the linear one for small η. To find the solution we exclude $U(z,s)$ from Eqs. (21) and (22) and then, integrating over s with weight $\exp(i\lambda s)$, we find the function

$$\int_{-\infty}^{\infty}V(z,s)\exp(i\lambda s)ds.$$

The integrals over λ and $\bar{\lambda}$ contained in the obtained expression are calculated by the saddle-point method,[17] by deforming the contour so that it passes through the saddle points

$$(2\lambda_{\pm}-1)\left|z\Big/\int_0^x R_3(x,0)dx\right|^{1/2}=\pm i.$$

With an accuracy to an unimportant constant phase multiplier, which is determined by the behavior of $\rho(\lambda,0)$ at zero,[17] we find

$$n=n_0\left[1-\frac{y^2}{\eta(y^2+\eta^2)^{1/2}\operatorname{ch}\psi_a+\eta^2+y^2/2}\right]\left[1+O\left(\frac{1}{\eta^{1/2}}\right)\right], \tag{38}$$

where

$$y=\int_0^x R_3(x,0)dx,\quad \psi_a=2\eta+\ln\frac{|\rho(\lambda_+,0)|^2}{(4\pi\eta)^{1/2}},$$

$$\lambda_+=\frac{1}{2}+\frac{i}{2}\left|\frac{y}{z}\right|^{1/2}.$$

4. CONCLUSION

The solutions obtained in this study show that the dynamics of USP and the nonlinear stage of FM are to a considerable degree determined by the choice of direction of propagation of reference waves (opposite or parallel) and the field asymptotes. For $\varepsilon = 1$ the maximum coefficient of transformation of photon number $\Delta = n - n_0$ does not exceed unity. Often the experimental value of Δ is small: $\Delta \ll 1$. It can be seen from expression (30) that in these conditions it is more advantageous to use an elliptical field polarization. Actually, the photon-number change characterized by the value Δ for $\mu_0 \sim 1$ has the order of magnitude

$$\Delta_l \approx i(\mu_0 F - \bar{\mu}_0 \bar{F}),$$

while for $\mu_0 = 0$ (circular polarization)

$$\Delta_c \approx O(|F|^2) \ll \Delta_l.$$

For the use of FM as a mechanism of reversing a wave front one needs to determine the degree of distortion of the signal wave front in the course of amplification. The self-similar solution (38) describes amplification of a weak bare field $\sim \mu(0,z)$ for which the information on its leading edge is contained in the coefficient $\rho(\lambda)$. It can be seen from Eq. (38) that significant distortion of the signal wave front (amplified in the FM process) does not occur up to values $\Delta \sim 0.5$.

The interaction scheme for $\varepsilon = -1$ is especially interesting for practical applications because the corresponding solutions may be singular, i.e., an explosive instability may occur. As a result, the coefficient of field transformation may be (theoretically) much larger than unity. For $\mu_0 = 0$ the singularity occurs for any value of the soliton parameters (31). A difference of μ_0 from zero leads to vanishing of the singularity at some values of λ and μ_0, i.e., to a qualitative change of the character of the FM.

We present parameters values for which observation of coherent FM and the above-described effects are possible. We choose the interaction diagram depicted in Fig. 1c. For the 4*S*–5*S* transition in KI vapor we have $\omega_{1,2}^{+} = 1.8 \cdot 10^{15}$ s^{-1} and $\varkappa(\omega_1^{+}) = 1.2 \cdot 10^{-22}$ cm^2 (Ref. 18). For a pressure of 10 Torr we have $\alpha\omega_1^{+2}/k_1^{+}) = 2.0 \cdot 10^{-5}$ CGSE. The field intensity is $|E_{1,2}^{+}|^2 \cong 10^2$ V/cm^2. The front length of USP is in this case 1–2 cm.

[1]S. A. Akhmanov and N. I. Koroteev, *Nonlinear Optics Methods in the Spectroscopy of Light Scattering* [in Russian], Nauka, Moscow, 1981.
[2]F. Reintjes, *Nonlinear Optics Parametric Processes in Liquids and Gases*, Academic, 1984.
[3]A. E. Kaplan, Opt. Lett. **8**, 560 (1983).
[4]M. V. Tratnik and J. E. Sipe, Phys. Rev. A **35**, 2965 (1987).
[5]B. Ya. Zel'dovich, N. F. Pilipetskii, and V. V. Shkunov, *Phase Conjugation* [in Russian] Nauka, Moscow, 1985.
[6]M. Cronin-Golomb, J. O. White, B. Fisher, and A. Yariv, IEEE J. Quant. Electr. **QE-20**, 12 (1984).
[7]P. M. Peterson, Opt. Comm. **68**, 449 (1988).
[8]B. Ya. Zel'dovich, M. A. Orlova, and V. V. Shkunov, Dok. Akad. Nauk SSSR **252**, 592 (1980) [Sov. Phys. Dokl. **25**, 390 (1980)].
[9]V. E. Zakharov, S. P. Novikov, S. V. Manakov, and L. P. Pitaevskiĭ, *Soliton Theory, Inverse Scattering Method*, Plenum, 1984.
[10]V. E. Zakharov and A. V. Mikhaĭlov, Pis'ma Zh. Eksp. Teor. Fiz. **45**, 279 (1987) [JETP Lett. **45**, 349 (1987)].
[11]A. A. Zabolotskii, Physica D., in press, 1989.
[12]A. Zabolotskii, Phys. Lett. A **127**, 83 (1988).
[13]M. Lakshmanan, Phys. Lett. A **61**, 53 (1977).
[14]L. A. Takhtajan, Phys. Lett. A **64**, 235 (1977).
[15]V. E. Zakharov and L. A. Takhtadzhyan, Teor. Mat. Fiz. **38**, 4 (1979) [in Russian].
[16]A. E. Borovik and S. I. Kulinich, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 320 (1984) [JETP Lett. **39**, 384 (1980)].
[17]M. V. Fedoryuk, *Saddle Point Method* [in Russian], Nauka, Moscow, 1977.
[18]A. G. Ford and J. C. Browne, Atom. Data **5**, 305 (1973).

Translated by D. F. Smith