Electron-current modulation in the field of a standing electromagnetic wave

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(Submitted 3 August 1989)
Zh. Eksp. Teor. Fiz. **97**, 162–168 (January 1990)

Interaction of nonrelativistic electrons with a standing electromagnetic wave is considered. The modulation amplitude of an electron current in the field of a standing or traveling electromagnetic wave is calculated in the quantum approach. An expression is obtained for the intensity of the spontaneous coherent radiation.

INTRODUCTION

Elastic scattering of nonrelativistic electrons in the field of a standing electromagnetic field was first considered by Kapitza and Dirac.[1] Their effect in the field of a strong standing electromagnetic wave was investigated by Fedorov[2] and Avetissyan.[3] Gaponov and Miller[4,5] studied the modulation and acceleration of nonrelativistic electrons in the field of counterpropagating waves that are shifted in frequency, and it was noted in Ref. 4 that the electron interaction with the standing wave is described by an effective potential that is quadratic in the electromagnetic-field strength.

The emission and absorption in an effective potential, however, were not considered in the cited studies. They were first investigated in Ref. 6, where the probability of this emission was estimated in first-order perturbation theory. It was shown that in this case the emission probability is proportional to $(V_0/E)^2$, where V_0 is the effective potential produced by the standing wave, and E is the electron kinetic energy ($E > V_0$).

Varshalovich and D'yakonov[7] first noted the possibility of obtaining quantum modulation of a current of nonrelativistic electrons as they pass through a thin dielectric plate in the field of an electromagnetic wave. They calculated the emission intensity at the modulation frequency of such a current near the surface of a metal. This intensity was shown to be proportional to the square of the current density. We shall refer to it as transient coherent emission (TCE).

In Refs. 8 and 9 was considered quantum modulation of a current of slow electrons reflected from a vacuum–dielectric interface, and also by elastic reflection of electrons from the surface of a transparent single crystal (Bragg reflection). The TCE spectra and intensity were calculated.

One can expect the current-density modulation depth and the emission intensity to increase if the electrons are simultaneously acted upon by a spatially periodic field of a diffraction grating and by a traveling electromagnetic wave. The energy and momentum conservation laws are then simultaneously satisfied on account of the grating quasimomentum. The role of this grating can be assumed, for example, by a standing electromagnetic wave.

We consider in the present paper the modulation (classical and quantum) of an electron current in the field of a standing electromagnetic wave. We call this emission by the modulated current diffractive coherent emission (DCE). We shall calculate the DCE intensity.

BASIC RELATIONS

Consider a nonrelativistic electron in the field of a standing linearly polarized wave. We define the wave field by a vector potential

$$A_1 = A_{01}^z \sin \omega_1 t \cos k_1 y, \tag{1}$$

where A_{01}^z is the amplitude of the vector potential, the superscript z indicates the polarization direction, while ω_1 and k_1 are respectively the frequency and field vector of waves counterpropagating along the y axis, and their superposition produces the standing wave.

Let the electron momentum be directed along the y axis. In this geometry, the interaction of the electron with the wave is due to the term $(eA_1)^z$, and the Schrödinger equation for the particle in the field of the wave (1) is

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi + V_0 \cos^2 k_1 y \Psi. \tag{2}$$

Here $V_0 (eA_{01}^z)^2/2mc^2$ is the effective potential and $A_{01}^z = -\mathscr{E}\lambda_1$, where $\mathscr{E}_1$ is the electromagnetic field and λ_1 is the wavelength.

Note that the high-frequency $(2\omega_1)$ term in the interaction is determined by a phase transformation of a Psi function and is disregarded hereafter.

We represent Eq. (2), accurate to an inessential phase shift, in the form

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial y^2} + \frac{V_0}{2} \cos qy \Psi, \tag{3}$$

where $q = 2k_1$. We add to the standing electromagnetic wave a traveling wave with a vector potential

$$A_2 = A_{02}^y \sin(\omega_2 t - k_2 z), \tag{4}$$

where A_{02}^y is the amplitude of the vector potential, the superscript y indicates the polarization direction, while ω_2 and k_2 are respectively the frequency and the wave vector of the traveling wave $A_{02}^y = -\mathscr{E}_0 c/\omega_2$.

The Schrödinger equation takes now the form

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial y^2} + i \frac{e\hbar}{mc} \sin(\omega_2 t - k_2 z) A_{02}^y \frac{\partial \Psi}{\partial y} + \frac{V_0}{2} \cos qy\, \Psi \tag{5}$$

[We have neglected in (5) the high-frequency $(2\omega_2)$ term determined by a phase transformation of a Psi function and disregarded hereafter, since it contains the small parameter eA_{02}^y/mc^2 compared with the retained terms].

We seek the solution of (5) in the form

$$\Psi(y,t) = \sum_n a_n(y) \Psi_n(y) \exp[ink_2 z - iE_n t/\hbar], \tag{6}$$

where $E_n = E_0 + n\hbar\omega_2$.

 0038-5646/90/010090-04$03.00

We seek the wave function $\Psi_n(y)$ in the semiclassical approximation

$$\Psi_n(y)=\exp\left[\frac{i}{\hbar}\int^{y}(p_n^2-2meV_0\cos qy')^{1/2}dy'\right]. \quad (7)$$

Here $p_n=(p_0^2+2mn\hbar\omega)^{1/2}$ and $p_0(2mE_0)^{1/2}$. We assume hereafter $\omega_2=\omega$, $k_2\equiv k$.

The function $a_n(y)$ must be determined. We find it for $E\gg V_0$ by using an approximation from Refs. 7–9. In this approximation, first, $(p_n,p)\gg p_n-p\pm q$, a condition met in the case of resonance $p-p_n\pm q\approx 0$, and second, $p_n\approx p$, which introduces in $a_n(y)$ an error order $n\hbar\omega/E$. Taking this into account, we get

$$\frac{da_n(y)}{dy}\frac{\partial\Psi_n}{\partial y}=-\frac{e\mathscr{E}_0}{2\hbar\omega}\left(a_{n+1}\frac{\partial\Psi_{n+1}}{\partial y}-a_{n-1}\frac{\partial\Psi_{n-1}}{\partial y}\right). \quad (8)$$

Expanding p_n in terms of the small parameter V_0/E_n ($p_0\equiv p, E_0\equiv E$) and using expression (7) for ψ_n, we get

$$\frac{da_n(y)}{dy}=-\frac{e\mathscr{E}_0}{2\hbar\omega}\left\{a_{n+1}\exp\left[\frac{i}{\hbar}(p_{n+1}-p_n)y+i\frac{V_0}{4E}\sin qy\right]\right.$$
$$\left.-a_{n-1}\exp\left[\frac{i}{\hbar}(p_{n-1}-p_n)y-i\frac{V_0}{4E}\sin qy\right]\right\}. \quad (9)$$

For $V_0/E\ll 1$, Eq. (9) takes the simpler form

$$\frac{da_n(y)}{dy}=-\frac{e\mathscr{E}_0}{8\hbar\omega}\frac{V_0}{E}\left\{a_{n+1}(y)\exp\left[\frac{i}{\hbar}(p_{n+1}-p_n-\hbar q)y\right]\right.$$
$$\left.-a_{n-1}(y)\exp\left(\frac{i}{\hbar}(p_{n-1}-p_n+\hbar q)y\right]\right\}. \quad (10)$$

We have thus obtained for $a_n(y)$ the finite-difference differential equation (10).

MODULATION OF THE ELECTRON-CURRENT DENSITY

We shall solve (10) for the case $n=\pm 1$. Assuming that $|a_{\pm 1}|\ll 1$ and $a_0\approx 1$, we obtain, if the interaction is turned-on instantaneously ($Lq\gg 1$)

$$a_{\pm1}(y)=\mp\frac{e\mathscr{E}_0}{8\hbar\omega}\frac{V_0}{E}\int_0^y\exp\left[\frac{i}{\hbar}(p-p_{\pm1}\pm\hbar q)y'\right]dy'$$
$$=\mp\frac{e\mathscr{E}_0}{4\hbar\omega}\frac{V_0}{E}\exp\left[\frac{i}{\hbar}(p-p_{\pm1}\pm\hbar q)\frac{y}{2}\right]$$
$$\times\frac{\sin\left[(p-p_{\pm1}\pm\hbar q)\frac{y}{2}\right]}{p-p_{\pm1}\pm\hbar q}. \quad (11)$$

It follows from (11) that a_1 increases with y if $p_1-p=q$ or if $\hbar\omega\ll E$, $\omega/v=q+\pi/L$, where $v=p/m$ is the electron velocity and $L=(8\pi\hbar/p)(E/\hbar\omega)^2$ is the modulation length.[7] The analogous condition for the increase of the amplitude a_{-1} is $\omega/v=q-\pi/L$. The conditions for a_1 and a_{-1} to increase are thus incompatible. That is to say, if the electron beam were ideally monochromatic it would be possible to "tune" it only to absorption ($a_1\neq 0$, $a_{-1}=0$) or only to stimulated emission ($a_1=0$, $a_{-1}\neq 0$), depending on the relations between the parameters ω, v, and q. This asymmetry of the coefficients is a quantum effect manifested only when the length l of the intersection of the standing wave with the electron beam is much larger than L, since L contains the Planck constant $\hbar$. Let us calculate the electron-current density, using expressions (6) and (7) with $n=\pm 1$ and taking (11) into account:

$$j=j_0\left(1-\frac{e\mathscr{E}_0}{2\hbar\omega}\frac{V_0}{E}\left(\frac{\sin D_{+1}y/2}{D_{+1}}\cos(\varphi-D_{+1}y/2)\right.\right.$$
$$\left.\left.-\frac{\sin D_{-1}y/2}{D_{-1}}\cos(\varphi-D_{-1}y/2)\right)\right), \quad (12)$$

where j_0 is the incident-beam current density, defined by

$$j_0=\frac{I_0}{\pi ab}\exp\left(-\frac{x^2}{a^2}-\frac{z^2}{b^2}\right).$$

Here I_0 is the total beam current, and a and b are the effective beam widths in the x and z directions,

$$D_{\pm1}=\frac{\omega}{v}-q\mp\frac{\pi}{L},\quad \varphi=\omega t-qy-kz-\frac{V_0}{2E}\sin qy.$$

A real electron beam, however, is not monochromatic, so that expression (12) for the current density must be averaged over the initial-electron-beam distribution function $F(v)$ in the velocities. We put

$$F(v)=\frac{1}{\pi^{1/2}\Delta v}\exp\left(-\frac{(v-v_0)^2}{(\Delta v)^2}\right),$$

where Δv is the electron-velocity scatter and v_0 is the average electron velocity in the beam ($\Delta v\ll v_0$). The averaged current can be written in the form

$$j=j_0\left[1-\frac{e\mathscr{E}_0}{4\hbar\omega}\frac{V_0}{E}\operatorname{Re}\{\exp(i\varphi)(f_{+1}(y)-f_{-1}(y))\}\right], \quad (13)$$

where

$$f_{\pm1}=2\int_{-\infty}^{+\infty}F(v)\frac{\sin D_{\pm1}x/2}{D_{\pm1}}\exp(-iD_{\pm1}x/2)\,dv$$
$$=\int_0^y\int_{-\infty}^{+\infty}F(v)\exp(-iD_{\pm1}y')\,dy'\,dx.$$

Let us analyze expression (13) for various limiting cases.

1. Let the energy width of the distribution function be $\Delta E=mv_0\Delta v\gg\hbar\omega$ and let $y>(\pi/q)(4E/h\omega)=L/2$. The second condition can be written in a different form by recognizing that $yq/2\pi=N$ is the number of the standing-wave periods spanned by the length y. We have then $N>2E/\hbar\omega$, i.e., the standing wave must have a large enough number of periods.

Calculation of the current leads in this case to the expression

$$j=j_0\left[1-\frac{\pi V_0}{2E}\frac{e\mathscr{E}_0}{mq}\frac{\partial F}{\partial v}\bigg|_{v=\omega/q}(i\sin\varphi+\cos\varphi)\right]. \quad (14)$$

This result is obviously purely classical. In the case $y<L/2$ and under the condition $(\omega/v_0-q)y\ll 1$ we have

$$j=j_0\left[1+\frac{e\mathscr{E}_0}{16E_0}qy^2\frac{V_0}{E_0}\sin\varphi\right]. \quad (15)$$

Thus, the electron-current-density modulation amplitude first increases like y^2, and then assumes the constant value (14).

2. The most interesting case is $\Delta E<\hbar\omega$. Note that for $y<L/2$ and $(\omega/v_0-q)y<1$ the averaging leads to relation

(15) and does not differ at all from the case $\Delta E > \hbar\omega$. For a standing wave satisfying the condition $y > L/2$, i.e. $N > 2E/\hbar\omega$, as already mentioned, the conditions for stimulated emission and absorption are not compatible. One of these effects will therefore predominate, depending on which velocity, $\omega/(q-\pi/L)$ or $\omega/(q+\pi/L)$, turns out to be closer to v_0. We assume hereafter that the "exact resonance" condition $\omega/v_0 = q - \pi/L$ or $\omega/v_0 = q + \pi/L$ is met and that averaging leaves only the function f_{+1} or f_{-1}, respectively.

The calculations lead to the expression

$$j=j_0\left[1\mp\frac{\pi^{1/2}e\mathscr{E}_0}{4\hbar\omega q}\frac{V_0}{E_0}\frac{v_0}{\Delta v}\operatorname{erf}\left(\frac{q\Delta v}{2v_0}y\right)\cos\varphi\right], \qquad (16)$$

where

$$\operatorname{erf}(\alpha)=\frac{2}{\pi^{1/2}}\int_0^\alpha \exp(-\beta^2)\,d\beta$$

is the error integral. The plus and minus signs correspond to emission and absorption, respectively. The modulation is in this case a quantum effect, since the amplitude contains the Planck constant $\hbar$. For $y < v_0/q\Delta v$ or $N < E_0/\Delta E$ (more accurately, $2E_0/\hbar\omega < N < 2E_0/\pi\Delta E$), expression (16) simplifies to

$$j=j_0\left(1\mp\frac{e\mathscr{E}_0 y}{4\hbar\omega}\frac{V_0}{E}\cos\varphi\right). \qquad (17)$$

The modulation amplitude, as follows from (17), increases linearly with the length of the standing wave (or with the number of periods). For $y > 2v_0/q\Delta v$ or $N > 2E_0/\pi\Delta E$ the modulation amplitude assumes an asymptotic value and is independent of the coordinate y. In this case we have

$$j=j_0\left(1\mp\frac{\pi^{1/2}}{4}\frac{e\mathscr{E}_0}{\hbar\omega q}\frac{V_0}{E_0}\frac{v_0}{\Delta v}\cos\varphi\right). \qquad (18)$$

When the beam is modulated by the periodic structure of the standing wave the dependence of the modulation amplitude on the coordinate y is in principle different from that in all other cases of modulation, for example Bragg reflection from a crystal surface.[9] The reason is that the expression of Ref. 9 for the modulation amplitude contains the factor

$$\exp\left\{-\frac{y^2(\Delta v)^2q^2}{4v^2}\right\},$$

which leads to damping of the modulation with increasing distance from the crystal boundary. In our case, however, the modulation amplitude is independent of y and is large enough even if $\Delta v/v \sim 1$. The physical cause of this difference is the resonant character of the interaction of the electron beam with the standing wave. A standing electromagnetic wave selects a narrow part of the electron beam from the entire velocity spectrum ($\omega/v = q \pm \pi/L$), and it is this part which is modulated in the radiation field.

EMISSION OF ELECTROMAGNETIC WAVES BY ELECTRONS IN THE FIELD OF A STANDING ELECTROMAGNETIC WAVE

As noted in Refs. 7–9, interaction between electrons of a modulated beam with a "third body" produces spontaneous coherent emission at a frequency ω or at frequencies that are multiples of ω. In the case considered here, the body in question is the same standing wave on which the modulation takes place.

The energy of coherent emission of frequency ω' is determined from the equation (see Ref. 9)

$$dE=\frac{e^2}{4\pi^2}\left|\int[\mathbf{n}'\mathbf{j}]\exp(i\omega't-i\mathbf{k}'\mathbf{r})\,d\mathbf{r}\,dt\right|^2 d\mathbf{k}', \qquad (19)$$

where $\mathbf{k}'$ is the spontaneous-emission wave vector, $\mathbf{n}' = \mathbf{k}'/k'$ is a unit vector, and $\omega = k'c$.

Equation (19) is valid when $LN^{1/3} > 1$ (quantum case) or $N^{1/3} > q\Delta v/v$ (classical case), where N is the electron density.

Let us calculate the coherent-emission power $d\Pi = dE/\tau$, where τ is the interaction time. For a current density j given, for example, by Eq. (14) we get

$$d\Pi=d\mathbf{k}'\frac{\pi}{32}e^2I_0^2\left(\frac{V_0}{E}\right)^4\left(\frac{e\mathscr{E}_0}{mq}\right)^2\left(\frac{\partial F}{\partial v}\right)^2\delta(\omega'-\omega)$$
$$\times\exp\left(-\frac{k_x'^2a^2}{2}\right)\exp\left(-\frac{(k-k_z')^2b^2}{2}\right)\frac{(\sin k_y'l/2)^2}{k_y'^2}. \qquad (20)$$

The angular distribution of the emission is obtained by making in (20) the substitutions $k_z' = k\cos\theta$, $k_y' = k\sin\theta\sin\varphi$, $k_x' = k\sin\theta\cos\varphi$, $d\Omega = \sin\theta d\theta d\varphi$, $d\mathbf{k}' = k'^2dk'd\Omega$ and by integrating over $dk' = d\omega'/c$.

The angular distribution of the spontaneous-emission flux is given by

$$\frac{d\Pi}{d\Omega}=\frac{\pi}{32}e^2I_0^2\left(\frac{V_0}{E}\right)^4\frac{\omega^2}{c^3}\left(\frac{e\mathscr{E}_0}{mq}\right)^2\left(\frac{\partial F}{\partial v}\right)^2$$
$$\times\exp\left(-\frac{k^2b^2(1-\cos\theta)^2}{2}\right)$$
$$\times\exp\left(-\frac{k^2a^2\sin^2\theta\cos^2\varphi}{2}\right)\frac{\sin^2\left(k\frac{l}{2}\sin\theta\sin\varphi\right)}{k^2\sin^2\theta\sin^2\varphi}. \qquad (21)$$

The most important in (21) are the exponential factors. They show that the emission propagates within a small solid angle in the z direction. It is interesting to note that in the (y,z) plane, i.e., at $\varphi = \pi/2$, the characteristic radiation propagation angle is $\theta_0 \approx (kb)^{-1/2}$. In the (x,z) plane ($\varphi = 0$) the radiation propagates within an angle $\chi_0 \approx (ka)^{-1}$. If $a \approx b$, then $\theta_0 \gg \chi_0$ for $kb \gg 1$ and $ka \gg 1$.

CONCLUSION

Let us assess the current-density modulation amplitudes in the classical and quantum cases, using estimates based on Eqs. (14) and (18). Let the standing-wave electromagnetic field intensity be $\mathscr{E}_1 = 3.10^7$ V/cm and the wavelength $\lambda_1 = 10^{-5}$ cm, so that the effective potential is $V_0 = (e\mathscr{E}_1\lambda_1)^2/2mc^2 = 0.4$ eV and $q = 10^5$ cm^{-1}. Assume an electron velocity $v = 10^9$ cm/s ($E \simeq 1$ keV) and $\Delta v/v = 10^{-3}$. Then $\omega = qv = 10^{14}$ s^{-1}, $L \approx 1$ cm, and the modulation amplitude is $(e\mathscr{E}_0/qE)(V_0/E)(v/\Delta v)^2 \approx 10^{-6}e\mathscr{E}_0$. Assuming a traveling-wave electromagnetic-field intensity $\mathscr{E}\mathscr{E}_0 = 10^5$ V/cm we find that the modulation amplitude reaches 10%. It must be borne in mind that this estimate has been made for the classical case $\Delta E \gg \hbar\omega$. In the opposite limiting case $\hbar\omega \gg \Delta E$ the quantum modulation amplitude takes, according to (18), the form

$$\frac{e\mathscr{E}_0}{\hbar\omega q}\frac{V_0}{E}\frac{v}{\Delta v}.$$

At $\mathscr{E}_1 = 5\cdot 10^7$ V/cm, $\hbar\omega = 1$ eV, $V_0 = 0.04$ eV, $q = 5\cdot 10^5$ cm^{-1}, $v = 3\cdot 10^9$ cm/s ($E = 2.5\cdot 10^3$ eV), $v/\Delta v = 3\cdot 10^3$ and $L = 5\cdot 10^{-2}$ cm, the modulation amplitude reaches 10% at $\mathscr{E}_0 = 10^6$ V/cm. Let us compare the current-density modulation amplitude produced in Ref. 7 with expression (16) for the case

$$\operatorname{erf}\left(\frac{q\Delta v}{2v}l\right) = 1.$$

For $\omega = qv$ this relation is determined by the quantity $(E/V_0)(\Delta v/v) \ll 1$ if E does not exceed V_0 greatly and $\Delta v/v \ll 1$. The interaction of the electron current with the traveling electromagnetic wave is obtained, e.g., using (16), by measuring the change of the current when the laser is turned on. Another possibility is observation of the DCE. It must be borne in mind here that observation of the DCE at the frequency ω is difficult if the angle θ_0 is close to the divergence angle of the modulating wave. It is therefore best to observe the DCE at $kb \sim 1$, $ka \sim 1$, and an angle $\theta_0 \sim 1$ rad.

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Translated by J. G. Adashko