Symmetry and dynamics of domain walls in weak ferromagnets

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Dynamic equations for the antiferromagnetic vector **l** are used in a study of the structure of moving domains in antiferromagnets whose symmetry allows for the occurrence of weak ferromagnetism. It is shown that inclusion of terms linear in respect of $d\,\mathbf{l}/dt$ ensures a satisfactory description of lowering of the symmetry of a moving domain wall the structure of which is then described by equations which are not Lorentz-invariant. The absence of the Lorentz invariance of these dynamic equations leads also to the possibility of instability of a moving domain wall. Consequently, phase transitions involving changes in the symmetry and structure of a domain wall may occur at some critical value of its velocity. This critical velocity can be low compared with the limiting velocity of the wall.

The dynamics of domain walls in weak ferromagnets has been investigated experimentally quite thoroughly. A theoretical description of the experiments on steady-state motion of a domain wall has been based on Lorentz-invariant dynamic equations for the antiferromagnetic vector **l** (for a review, see Ref. 1). In the Lorentz-invariant models the only effect of the motion of a domain wall is a reduction of its thickness x_0 in accordance with the law

$$x_0 \to x_0 (1 - v^2/c^2)^{1/2},$$

where v is the domain wall velocity and c is the velocity of spin waves in the linear part of the dispersion law. The symmetry of a moving domain wall is exactly the same as that of a wall at rest. However, the symmetry approach developed in Ref. 2 shows that these representations are of limited validity. It is shown there that the motion of a domain wall usually lowers its symmetry.

A preliminary analysis of the dynamics of a domain wall in a magnetic material of the MnF_2 type demonstrates[3] that allowance for the magnetic anisotropy of a weak ferromagnet in a phenomenological model can lower the symmetry of a domain wall, and the limiting velocity v_c is then considerably less than c. The example used in Ref. 3 demonstrates that inclusion of invariants such as ml^3, ml^5, etc. (where l and m are the components of the antiferromagnetic **l** and weak ferromagnetic **m** vectors) in the expression for the energy of a weak ferromagnet can alter significantly the dynamic characteristics of a domain wall even when the constants in front of these invariants are very small.

Since the symmetry of specific weak ferromagnets admits a large number of different invariants—consisting of the components of the vectors **m**, **l**, and their derivatives—we face the question as to which approximation is capable of providing a satisfactory description of the nonlinear dynamics of a weak ferromagnet. We shall adopt the following criterion: we shall assume that the model is satisfactory if it can predict correctly the lowering of the symmetry of a moving domain wall (compared with the one at rest) predicted in Ref. 2. It is found that a consistent phenomenological description of the symmetry of a moving domain wall can always be provided by inclusion in the energy density of a weak ferromagnet of a term which is of the Dzyaloshinskii interaction type

$$M_0^2 w_d = M_0^2 D_{ij}(\mathbf{l})\, m_i l_j, \qquad (1)$$

where M_0 is the sublattice magnetization and the nature of the tensor D_{ij} is governed by the magnetic structure of a weak ferromagnet.[4]

A phenomenological model developed using Eq. (1) can be used to predict a number of new never before discussed phenomena, such as an instability and dynamic modification of a domain wall (phase transition) at a finite value of the wall velocity, and to identify the actual conditions that must be satisfied in order to observe experimentally these phenomena.

1. MODEL. GENERAL CONSIDERATIONS

The dynamics of a weak ferromagnet can be described by a Lagrangian $\mathscr{L}\{\mathbf{l}\}$

$$\mathscr{L} = M_0^2 \int d\mathbf{r} \left\{ \frac{\alpha}{2} \left[\frac{1}{c^2} \left(\frac{\partial \mathbf{l}}{\partial t} \right)^2 - (\nabla \mathbf{l})^2 \right] - w_a(\mathbf{l}) + \frac{2}{g M_0 \delta} D_{ij} \left[\mathbf{l} \frac{\partial \mathbf{l}}{\partial t} \right]_i l_j \right\}. \qquad (2)$$

Here, $c = gM_0(\alpha\delta/2)^{1/2}$; $g = 2\mu_0/\hbar$; μ_0 is the Bohr magneton; α and δ are the inhomogeneous and homogeneous exchange constants, respectively; $M_0^2 w_a(\mathbf{l})$ is the magnetic anisotropy energy. The first three terms in Eq. (2) represent the usual Lorentz-invariant Lagrangian of a weak ferromagnet[5,6] and the last term is related to the Dzyaloshinskii interaction of Eq. (1) (see Refs. 3 and 7). In this description the magnetization M of a weak ferromagnet is determined by the vector **l** and by the derivative $\partial \mathbf{l}/\partial t$:

$$\mathbf{M} = 2M_0 \mathbf{m} = \frac{4}{g\delta} \left[\mathbf{l} \frac{\partial \mathbf{l}}{\partial t} \right] + \frac{2M_0}{\delta} [(\mathbf{D}, \mathbf{l})\mathbf{l} - \mathbf{D}] \qquad (3)$$

$$D_j = D_{ij} l_j.$$

The structure of the tensor D_{ij} is governed by the symmetry of a weak antiferromagnet[4] and the magnitudes of its components are determined by the physical nature of the Dzyaloshinskii interaction. The main term in w_d is due to the antisymmetric exchange: $w_d^{(\mathrm{ex})} = d^{(\mathrm{ex})}(\mathbf{n}[\mathbf{ml}])$ (**n** is a unit vector along the selected axis), but its contribution to the Lagrangian reduces to the total derivative with respect to time and does not appear at all in the equations of motion. In addition to $w_d^{(\mathrm{ex})}$, the expression for w_d includes a large num-

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ber of relativistic invariants and these invariants determine the effects observed in the dynamics of a domain wall.

We shall use the angular variables $l_1 = \sin\theta\cos\varphi$, $l_2 = \sin\theta\sin\varphi$, and $l_3 = \cos\theta$, for the vector **l** and select the polar axis $\mathbf{e}_3$ along the equilibrium direction of the vector **l**. In terms of these variables, the last term in the Lagrangian $\mathscr{L}$ becomes

$$\mathscr{L}_D=(M_0/g\delta)[(\partial\theta/\partial t)\Delta_1(\theta,\varphi)+(\partial\varphi/\partial t)\Delta_2(\theta,\varphi)],\quad (4)$$

where the functions $\Delta_i(\theta,\varphi)$ and $\Delta_2(\theta,\varphi)$ are determined by the structure of w_d, i.e., by the tensor $D_{ij}(\mathbf{l})$. The dynamic equations for the angular coordinates θ and φ can be expressed conveniently in terms of the following "Lorentz-invariant" variables

$$\zeta=(\zeta_0-vt)(1-v^2/c^2)^{-1/2},\quad \tau=(t-v\zeta_0/c^2)(1-v^2/c^2)^{-1/2},\quad (5)$$

where ζ_0 is a spatial variable along the normal to the domain wall plane and v is the wall velocity. Using Eqs. (1), (3), and (4), we can write down these equations in the form

$$\alpha\left[\theta''-\frac{\ddot\theta}{c^2}-\sin\theta\cos\theta\left[\varphi'^2-\frac{\dot\varphi^2}{c^2}\right]\right]$$

$$-\frac{\partial w_a}{\partial\theta}+\frac{\dot\varphi-v\varphi'}{(1-v^2/c^2)^{1/2}}\frac{D(\theta,\varphi)}{gM_0\delta}=0,\quad (6)$$

$$\alpha\left[(\varphi'\sin^2\theta)'-\frac{1}{c^2}(\dot\varphi\sin^2\theta)^{\cdot}\right]$$

$$-\frac{\partial w_a}{\partial\varphi}-\frac{\dot\theta-v\theta'}{(1-v^2/c^2)^{1/2}}\frac{D(\theta,\varphi)}{gM_0\delta}=0,\quad (7)$$

where the dot and the prime denote the derivatives with respect to τ and ζ, respectively; $M_0^2 w_a(\theta,\varphi)$ is the anisotropy energy. The term with $D(\theta,\varphi)$ appears because of variation of $\mathscr{L}_d$ [see Eq. (4)]:

$$D(\theta,\varphi)=\frac{\partial\Delta_1(\theta,\varphi)}{\partial\varphi}-\frac{\partial\Delta_2(\theta,\varphi)}{\partial\theta}.$$

A uniformly moving domain wall corresponds to the solution of the type $\theta=\theta(\zeta)$ and $\varphi=\zeta$. The structure of a domain wall is governed by a system of second-order equations

$$\alpha\theta^{\cdot\cdot}-\alpha\varphi'^2\sin\theta\cos\theta-\frac{\partial w_a}{\partial\theta}-\frac{v\varphi'}{(1-v^2/c^2)^{1/2}}\frac{D(\theta,\varphi)}{gM_0\delta}=0,\quad (8)$$

$$\alpha(\varphi'\sin^2\theta)'-\frac{\partial w_a}{\partial\varphi}+\frac{v\theta'}{(1-v^2/c^2)^{1/2}}\frac{D(\theta,\varphi)}{gM_0\delta}=0.\quad (9)$$

We shall be concerned mainly with uniaxial crystals and for these crystals we have

$$w_a=(\beta/2)(l_x^2+l_y^2)+\tilde w_a(\mathbf{l}),$$

whereas the expressions for $\tilde w_a$ are given in Table I.

Equations (6), (7) or (8), (9) differ from the standard equations[4,5] by the presence of terms with the first derivatives with respect to ζ and τ, proportional to the function $D(\theta,\varphi)$. It is these terms that depend on the domain wall velocity v and govern the characteristics of the domain-wall dynamics. If in the case of some magnetic materials the value of $D(\theta,\varphi)$ is identically equal to zero, the dynamics of a domain wall of any type can be "Lorentz-invariant" and at any velocity $v\neq 0$ the wall structure should be described by expressions that can be deduced from the equations describing a domain wall at rest by applying the Lorentz transformation of Eq. (5). The solution for $v = 0$ is found readily if it corresponds to $\theta=\theta(\zeta)$ and $\varphi=\varphi_0=$ const, where φ_0 is governed by the relationship $\partial w_a(\theta,\varphi)/\partial\varphi_0=0$. We then have to solve only one equation for $\theta(\zeta)$ and the first integral of this equation is known:

$$\alpha\theta'^2/2+w_a(\theta,\varphi_0)=\text{const}.$$

In the case when $D(\theta,\varphi)=0$ we can readily solve also the problem of the stability of a domain wall: a wall at rest is stable if $\partial w_a^2/\partial\varphi_0^2>0$; if it is stable (or unstable) for $v=0$, it remains so at any velocity $v<c$ (Ref. 8). However, if $D(\theta,\varphi)$ is not identically equal to zero [and we shall show that $D(\theta,\varphi)\neq 0$ applies to all magnetic materials in which the Dzyaloshinskii interaction is possible] the simple solution represented by $\theta=\theta(\zeta)$ and $\varphi_0=$ const may not exist for $v\neq 0$. In this case both an analysis of the structure of a domain wall when $v\neq 0$ and an investigation of its stability are not trivial tasks. In particular, there are no general methods for solving the system of equations (8)–(9). The latest progress has been made by finding exactly integrable examples.[9] but Eqs. (8) and (9) do not reduce to these examples for any one of known weak ferromagnets.

It is found that a fairly complete analysis can be carried out for any weak antiferromagnet either exactly or allowing for the natural smallness of the parameters of β/δ, $d/(\beta\delta)^{1/2}$, etc. where d is the Dzyaloshinskii interaction constant. Moreover, it is found that the great variety of the behavior of domain walls in different weak ferromagnets can be divided into several universal classes. It has been found that the variants of lowering of the symmetry of a domain wall compared, with the one at rest, pointed out in Ref. 2 (loss of a geometric center of a domain wall, symmetric or antisymmetric with respect to ζ emergence or tilt of the vector **l** from the $\varphi_0=$ const plane, typical of the $v=0$ case and denoted by SE and AE, respectively) are largely responsible also for the effects at a finite velocity: stabilization of a domain wall on increase in v or a loss of its stability, nature of the dynamic modification of a domain wall of one type into a wall of a different type, etc. We shall now provide a concrete analysis of these relationships.

2. DYNAMICS OF A DOMAIN WALL WITH $\varphi\neq$const, $\theta=\theta(\zeta)$

In an analysis of the motion of a domain wall we face first of all the question of whether when $v\neq 0$ we are still left with the simple solution with $\varphi=\varphi_0=$ const and $\theta=\theta(\zeta)$ for the system of equations (8)–(9) typical of the case when $v=0$. Obviously, Eqs. (8) and (9) have such a solution if $D(\theta,\varphi)$ and $\partial w/\partial\varphi$ vanish for the same value of $\varphi=\varphi_0$. Table I allows us to list readily the corresponding cases; it is sufficient to write down $w(\mathbf{l})$ in terms of the angular variables representing a given ground state of a weak ferromagnet.

In the case of a weak ferromagnet with **l** along the Z axis (easy-axis weak ferromagnets) the dependence of W_a on φ is governed by the anisotropy in the basal plane: $\tilde w(\theta,\varphi)$. In this case a weak antiferromagnet with an n-fold axis is char-

TABLE I.

Crystal symmetry; w_a	Structure of weak ferromagnets; w_d	Direction of **l** in ground state ($\mathbf{e}_3$ axis); $\mathbf{e}_1$ axis; $D(\theta,\varphi)$	Plane of rotation of **l**	Change in symmetry on domain wall, type of phase transition
Orthorhombic; $-{}^1/_2\beta(l_z^2+\varepsilon l_x^2)$	$2_x^{(-)},\ 2_y^{(+)},\ 2_z^{(-)}$; $d(m_x l_z + m_z l_x)$	$\hat{Z};\ \hat{X}$; $-12d\sin^2\theta\cos\theta\cos 2\varphi$	(ZY)	LC
			(ZX)	AE
		$\hat{Y};\ \hat{X}$; $6d\sin^3\theta\sin 2\varphi$	(YX)	K
			(YZ)	K
Tetragonal; ${}^1/_2\beta(l_x^2+l_y^2)+\beta_4 l_x^2 l_y^2$	$4_z^{(+)},\ 2_x^{(-)},\ 2_{xy}^{(+)}$; $d(m_x l_y + m_y l_x)(l_x^2 - l_y^2)$	$\hat{Z};\ \hat{X}$; $5d\sin^5\theta\sin 4\varphi$	(ZX)	K
		$\hat{X};\ \hat{Y}$; $20d\sin^2\theta\cos\theta\cos\varphi\,(\cos^2\theta-\sin^2\theta\cos^2\varphi)$	(XZ)	LC
			(XY)	AE
	$4_z^{(-)},\ 2_x^{(-)},\ 2_{xy}^{(+)}$; $d\,(m_x l_y + m_y l_x)$	$\hat{Z};\ \hat{X}$; $6d\sin^3\theta\sin 2\varphi$	(ZX)	K
			(ZY)	SE
		$\hat{X};\ \hat{Y}$; $12d\sin^2\theta\cos\theta\cos\varphi$	(XY)	AE
			(XZ)	LC
		$\hat{X}Y;\ \hat{Y}X$; $6d\sin\theta\,(\cos^2\theta-\sin^2\theta\cos^2\varphi)$	$(XY\,Z)$	SE
			$(XY\,YX)$	SE
Hexagonal; ${}^1/_2\beta(l_x^2+l_y^2)+{}^1/_{12}\beta_6(l_+^6+l_-^6)$	$6_z^{(+)},\ 2_x^{(-)},\ 2_y^{(-)}$; $id(m_+l_+^5 - m_-l_-^5)$	$\hat{Z};\ \hat{X}$; $28d\sin^7\theta\sin 6\varphi$	(ZX)	K
		$\hat{X};\ \hat{Y}$; $56d\sin^2\theta\cos\theta\cos\varphi\,(3\cos^4\theta-10\cos^2\theta\sin^2\theta\cos^2\varphi+3\sin^4\theta\cos^4\varphi)$	(XY)	AE
			(XZ)	LC
	$6_z^{(-)},\ 2_x^{(+)},\ 2_y^{(-)}$; $idm_z(l_+^3 - l_-^3)$	$\hat{Z};\ \hat{Y}$; $-20d\sin^4\theta\cos\theta\cos 3\varphi$	(ZY)	AE
			(ZX)	LC
		$\hat{X};\ \hat{Y}$; $20d\sin^3\theta\sin\varphi\cos\varphi\,(3\cos^2\theta-\sin^2\theta\cos^2\varphi)$	(XZ)	K
			(XY)	K
		$\hat{Y};\ \hat{Z}$; $20d\sin^3\theta\cos\theta\sin\varphi\,(3\sin^2\theta\sin^2\varphi-\cos^2\theta)$	(YZ)	AE
			(YX)	LC

acterized by $\partial w/\partial\varphi \propto \sin(n\varphi)$. Orthorhombic weak ferromagnets of the orthoferrite type and also easy-plane uniaxial weak ferromagnets exhibit a dependence of w on φ governed by the terms which are now quadratic in **l** and we then have $\partial w/\partial\varphi \propto \sin 2\varphi$. In the case of all easy-axis weak ferromagnets with an even [in accordance with the terminology of Turov—see Ref. 1] principal axis we have $D(\theta,\varphi) \propto \sin(n\varphi)$ and $\partial w_a/\partial\varphi \propto \sin(n\varphi)$. Otherwise, if $v\neq 0$, we have all the solutions $\varphi_0 = (\pi/n)k$, where k is an integer, as in the case $v=0$. If the principal axis is odd, then $D(\theta,\varphi) \propto \sin(n\varphi/2)$ [or it is proportional to $\cos(n\varphi/2)$] and if $v\neq 0$ only those domain walls "survive" which are characterized by $k=2m$ ($k=2m+1$), whereas in the case of the remaining walls we have $\varphi\neq$const and **l** emerges from the initial rotation plane (symmetrically or antisymmetrically). The case of a domain wall with a nonplanar rotation of **l** will be considered later and we shall now return to an analysis of the solutions characterized by $\varphi_0=$ const.

The structure of a domain wall for the case when

$\varphi = \varphi_0 = $ const is governed by the function $\theta(\zeta)$ and can be readily found from the first integral of Eq. (8); for example, in the case of an easy-axis weak ferromagnet, we find that

$$\int d\theta[\sin^2\theta+\tilde{w}(\theta,\varphi_0)]^{-1/2}=\zeta/x_0,\quad \cos\theta=\sigma\,\text{th}(\zeta/x_0)[1+R(\theta)], \tag{10}$$

where $x_0 = (\alpha/\beta)^{1/2}$, the function R is small because of the smallness of $\tilde{w}/\beta$, and the number $\sigma = \pm 1$ governs the sign of a domain wall.[1] In writing down Eq. (10) we have allowed for the fact that $w(\theta,\varphi) = 0$ when $\theta = 0$. Although the particular solution of the (10) type with $\varphi = \varphi_0 =$ const is exactly the same as in the Lorentz-invariant case, the absence of the Lorentz invariance in the case of the complete system of equations may be manifested in two ways: a) lowering of the symnetry of a domain wall because of a reduction in the symmetry of the distribution of $\mathbf{M}$; b) loss of the stability of a domain wall at some finite value of the velocity v. We shall see later that these two effects are interrelated. We shall consider how they are manifested.

In some weak ferromagnets the symmetry of a moving domain wall is exactly the same as for a wall at rest and such domain walls can be conveniently called kinematic domain walls (Table I). The following feature should be mentioned: according to Table I, a domain wall is kinematic if $D(\theta,\varphi)$ is an even function of $\cos\theta$, i.e., it is an even function of ζ if we allow for Eq. (10). In the case of some other weak ferromagnets which have the domain wall solution $\varphi = \varphi_0 =$ const, but are characterized by $D(\theta,\varphi) \propto \sin^k\theta\cos\theta$, the symmetry of a moving wall is less than that of a wall at rest because of lowering of the symmetry of the function $\mathbf{M}(\zeta)$. By way of example, we shall consider a domain wall in an orthoferrite characterized by $\theta = \theta(\zeta)$ and by $\varphi = \varphi_0 = 0, \pi$. In this case we readily obtain from Eq. (2) that

$$M_z=M_y=0,\quad M_x=-\frac{2M_0}{\delta}(d_{ex}+d)\cos\theta-\frac{4v}{g\delta M_0}\frac{\partial\theta_0}{\partial\zeta}, \tag{11}$$

where $(d_{ex} + d)$ is the effective Dzyaloshinskii interaction constant. It is clear from the above expression that if $v\neq 0$, the geometric centers of a domain wall found fromthe distributions $\mathbf{l}(\zeta)$ and $\mathbf{m}(\zeta)$ (for the values of ζ characterized by $l_z = 0$ and $\mathbf{m} = 0$) are not identical and we cannot in general introduce the concept of a geometric center of a domain wall. This loss of the center is a manifestation of the lowering of the symmetry of such walls during their motion (Table I).

It therefore follows that all the domain walls characterized by $\varphi = \varphi_0 =$ const and $\theta = \theta(\zeta)$ can be divided into two classes: kinematic walls and those exhibiting a loss of the center. We shall now investigate the stability of these domain walls by writing down

$$\theta=\theta_0(\zeta)+\vartheta(\zeta)e^{i\omega\tau},\quad \varphi=\varphi_0+\mu e^{i\omega\tau}/\sin\theta_0, \tag{12}$$

and finding the spectrum of ω^2 for a problem linearized in terms of ϑ and μ. In view of later applications, we shall consider the problem for an arbitrary domain wall without invoking the condition $\varphi =$ const. The equations for ϑ and μ can be represented in the form

$$(\hat{H}_1+F_{11})\vartheta+\hat{F}_{12}\mu=\Omega^2\vartheta, \tag{13}$$

$$(\hat{H}_2+F_{22})\mu+\hat{F}_{21}\vartheta=\Omega^2\mu, \tag{14}$$

where $\Omega^2 = (\omega x_0/c)^2$, $x_0 = (\alpha/\beta)^{1/2}$,

$$\hat{H}_1=-x_0^2\frac{\partial^2}{\partial\zeta^2}+\frac{1}{\beta}\frac{\partial^2 w_a(\theta_0,\varphi_0)}{\partial\theta_0^2},\qquad \hat{H}_2=-x_0^2\frac{\partial^2}{\partial\zeta^2}+x_0^2(\theta_0''\,\text{ctg}\,\theta-\theta'^2) \tag{15}$$

are the Schrödinger operators, and the other terms are small because of the smallness of the parameters D and $\tilde{w}/\beta$:

$$F_{11}=\varphi_0'^2\cos 2\theta_0+(v\varphi_0'/2)[\beta\delta(c^2-v^2)]^{-1/2}\partial D/\partial\theta_0, \tag{16}$$

$$\hat{F}_{12}\sin\theta_0=(1/\beta)(\partial^2 w/\partial\varphi_0\partial\theta_0)+(v\varphi_0'/2)[\beta\delta(c^2-v^2)]^{-1/2}\partial D/\partial\varphi_0 + [\varphi_0'\sin 2\theta_0+(vD/2)[\beta\delta(c^2-v^2)]^{-1/2}](\partial/\partial x-\theta_0'\text{ctg}\,\theta_0), \tag{17}$$

$$\hat{F}_{21}\sin\theta_0=(1/\beta)(\partial^2 w/\partial\theta_0\partial\varphi_0-\varphi_0'\sin 2\theta_0\partial/\partial x-(\varphi_0'\sin 2\theta_0)' - (v/2)[\beta\delta(c^2-v^2)]^{-1/2}[\theta_0'\partial D/\partial\theta_0+D\partial/\partial x], \tag{18}$$

$$F_{22}\sin^2\theta=(1/\beta)\partial^2 w/\partial\varphi_0^2-(v\theta_0'/2)[\beta\delta(c^2-v^2)^{-1/2}\partial D/\partial\varphi_0. \tag{19}$$

Here, $D = D(\theta_0,\varphi_0)$, $\tilde{w} = \tilde{w}(\theta_0,\varphi_0)$, and the terms linear in Ω are omitted because—according to the calculations—they do not influence the stability of a domain wall.

In the subsequent analysis it is important to note that the operators $\hat{H}_1$ and $\hat{H}_2$ have zero eigenvalues $\hat{H}_1\sin\theta_0 = 0$ for any domain wall and $\hat{H}_2(\partial\theta_0/\partial\zeta) = 0$ for a domain wall characterized by $\varphi_0 =$ const. In the case of an easy axis or an orthorhombic weak ferromagnet characterized by $\tilde{w}_a \ll \beta$ the operators $\hat{H}_1$ and $\hat{H}_2$ are close to $\hat{H}_0$, where $\hat{H}_0$ is the Schrödinger operator with a nonreflection potential and the familiar complete set of the eigenfunctions $\{\psi_0,\psi_k\}$ is described by

$$\hat{H}_0=-\partial^2/\partial x^2+1-2/\text{ch}^2 x,\ x=\zeta/x_0;\ \hat{H}_0\psi_0=0,\Psi_0=1/2^{1/2}\,\text{ch}\,x;$$

$$\hat{H}_0\psi_k=(1+k^2)\psi_0,\quad \psi_k=\{(\text{th}\,x-ik)/[L(1+k^2)]^{1/2}\}\exp(ikx), \tag{20}$$

where L is the size of a magnetic sample alcng the normal to the domain wall.

The general system of equations (13)–(14) will be used to study the stability of kinematic domain walls and those exhibiting loss of their center. In this case we have $\varphi = \varphi_0 =$ const, $D(\theta_0,\varphi_0) = 0$, and $\partial D/\partial\theta_0 = 0$, so that $F_{11} = 0$, $\hat{F}_{12} = 0$, $\hat{F}_{21} = 0$, and it is sufficient to consider the spectrum of the problem described by $(\hat{H}_2 + F_{22})\mu = \Omega^2\mu$ in the case of these domain walls.

Kinematic domain walls

In the case of all kinematic domain walls both terms in F_{22} are even functions of ζ (Table I). In view of this and because of the smallness of $\tilde{w}/\beta D(\beta\sigma)^{1/2}$, the minimum value of the eigenvalue of the problem Ω^2 is governed by a first-order perturbation theory correction to the zeroth level of $\hat{H}_2$:

$$\Omega^2=\langle\sin^2\theta_0\rangle^{-1}\left\langle\frac{1}{\beta}\frac{\partial^2\tilde{w}}{\partial\varphi_0^2}-\frac{v\theta_0'}{2[(c^2-v^2)\beta\delta]^{1/2}}\frac{\partial D(\theta_0,\varphi_0)}{\partial\varphi_0}\right\rangle, \tag{21}$$

where

$$\langle \ldots \rangle = \int_{-\infty}^{+\infty} (\ldots)\, d\zeta.$$

The following natural result is obtained from Eq. (21): if $v=0$, a domain wall is stable if $\partial^2 \tilde{w}/\partial\varphi_0^2 > 0$, i.e., the stability depends on the energy conditions. This answer is valid for any domain wall at rest, including a domain wall exhibiting loss of its center or nonplanar rotation of **l** (specific expressions for Ω^2 will be given later).

However, if $v \neq 0$, contributions of the second term to Eq. (21) begin to manifest themselves and at some value of the velocity we find that Ω^2 may reverse sign, meaning domain-wall instability. It is interesting that the sign of the second term in the case of these kinematic domain walls depends on the sign of the velocity, so that the stability of a domain wall depends on the direction of its motion. This result can be made clearer by noting that in the case of a kinematic domain wall we have $D(\theta,\varphi) \propto d(\sin\theta)^{2m+1}$, where $m = 1, 2, \ldots$ (Table I). We therefore have $\theta' D(\theta,\varphi) \propto d x_0 \sigma (\sin\theta)^{2(m+1)}$, where $\sigma = \pm 1$ determines the sign of a domain wall, i.e., the sign of the vector **l** in the ground state of a weak ferromagnet when $\zeta = +\infty$ [see Eq. (10)]. Therefore, the velocity occurs in the combination $v\sigma d$ in the condition of stability of a kinematic domain wall is given by Eq. (21), which once again demonstrates that if $\mathbf{M}(\pm\infty) = 0$, the sign of the quantity $d\mathbf{l}(\pm\infty)$ can be found by dynamic experiments (the question of determination of the sign of $d\mathbf{l}$ is discussed in Ref. 10). Therefore, an experimental study of the dynamics of a kinematic domain wall can be used to find the sign of $d\mathbf{l}$ (forced motion of a domain wall in antiferromagnetic phases of weak ferromagnets was achieved in the experiments described in Ref. 11 and other methods for an experimental study of such dynamics are proposed in Ref. 12).

By way of example we shall give the value of Ω^2 for the easy-axis phase of tetragonal weak ferromagnets with the even principal axis for which (see Table I) we have $\partial^2 w/\partial\varphi_0^2 = 2\beta_4 \sin^4\theta \cos 4\varphi$ and $\partial D/\partial\varphi_0 = 4d\sin^6\theta\cos 4\varphi$:

$$\Omega^2 = \cos 4\varphi_0 \left[\frac{2\beta_4}{3\beta} + \frac{32 v\sigma d}{15[\beta\delta(c^2-v^2)]^{1/2}}\right]. \tag{22}$$

A similar result [presence of identical angular factors in front of both terms in Eq. (21)] applies to all those magnetic phases of weak ferromagnets which have only kinematic domain walls. For example, in the case of kinematic domain walls in orthorhombic weak ferromagnets or in the easy-axis phase of hexagonal weak ferromagnets with an even axis we have, respectively,

$$\Omega^2 = \cos 2\varphi_0 \left[\varepsilon + \frac{6 v\sigma d}{[\beta\delta(c^2-v^2)]^{1/2}}\right]$$

or

$$\Omega^2 = -\frac{16}{5}\cos 6\varphi_0 \left[-\frac{\beta_6}{\beta} + \frac{24 v\sigma d}{[\beta\delta(c^2-v^2)]^{1/2}}\right].$$

Such "symmetry-governed" kinematic domain walls behave as follows (Fig. 1). If $v=0$, one possible kinematic domain wall is stable [for example, in the case of a weak ferromagnet with the $4_z^{(+)}$ axis and with $\beta_4 > 0$ this is a domain wall with $\cos 4\varphi_0 > 0$, i.e., with $\varphi_0 = (\pi/2)k$]. This domain wall loses its stability at a specific value of the wall velocity which is $v = +v_c$ if $\sigma d < 0$ or $v = -v_c$ if $\sigma d > 0$; at these values of v another domain wall of the same type becomes stable [$c\varphi_0 = (\pi/4)(2k+1)$]. It should be noted that according to Eq. (22) the quantity

$$v_c = c\,\frac{5|\beta_4|}{16d}\left(\frac{\delta}{\beta}\right)^{1/2}\left[1+\left(\frac{5\beta_4}{16d}\right)^2\frac{\delta}{\beta}\right]^{-1} \tag{23}$$

contains a ratio of small parameters $\beta_4/d \propto \beta_4/\beta$ and $(\beta/\delta)^{1/2}$, and can be small compared with the value of c. Expressions of the same type can readily be obtained for other kinematic domain walls. The value of v_c is particularly small in the case of hexagonal weak ferromagnets characterized by $\beta_6/\beta \leqslant 10^{-3}$.

The stability diagram in Fig. 1 is typical of all the symmetry-governed kinematic domain walls. However, in the case of some weak ferromagnets (those with the $4_z^{(-)}$ axis, or cubic materials—see Table I) kinematic domain walls may coexist with domain walls exhibiting a symmetric emergence of **l** out of a plane. Their behavior is different. We shall consider this behavior by discussing the example of an easy-phase axis of a weak ferromagnet of the MnF^2 type with the $4_z^{(-)}$ axis. In this phase a domain wall is kinematic only if $\sin 2\varphi_0 = 0$, i.e., if $\cos 4\varphi_0 = 1$. Allowing for possible values of φ_0 and the explicit form of $D(\theta,\varphi)$, we find from the general expression for Ω^2 that

$$\Omega^2 = \frac{4}{3}\left[\frac{\beta_4}{\beta} - \frac{3 v\sigma d\cos 2\varphi_0}{[\beta\delta(c^2-v^2)]^{1/2}}\right] \tag{24}$$

and this means that the dependence of Ω^2 on φ_0 is different from that given by Eq. (22). It follows from Eq. (24) that if for $v=0$ there are two stable kinematic domain walls with rotation of **l** in a plane containing the z axis and one of the odd twofold x ($\varphi_0 = 0,\pi$) or y ($\varphi_0 = \pi/2, 3\pi/2$) axes, then both these kinematic walls remain stable only if $|v| < v_c$, where

$$v_c = c[1+9d^2\beta/\beta_4^2\delta]^{-1/2}. \tag{25}$$

A further increase in the absolute value of the velocity causes each of these domain walls to lose their stability: one when $v > 0$ and the other when $v < 0$ (Fig. 2). However, if $\beta_4 < 0$, then at low velocities all kinematic domain walls are unstable, but they may become stable in the range $|v| > v_c$ (one when $v > v_c$ and the other when $v < -v_c$), as demonstrated below in Fig. 4.

We thus find that in spite of some differences an analysis demonstrates a number of common properties of domain walls such as the asymmetry of the ranges of their stability relative to the sign of the velocity v and a linear dependence of v_c on the anisotropy constant $\tilde{\beta}$ in the basal plane: $v_c \approx (\tilde{\beta}/d)(\delta/\beta)^{1/2}$ when $\tilde{\beta} \to 0$.

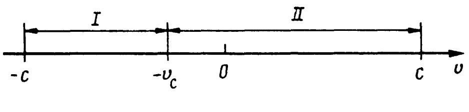

FIG. 1. Ranges of stability of kinematic domain walls in a weak ferromagnet of the $4_z^{(+)}$ type plotted for the parameters $\beta_4 > 0$ and $\sigma d > 0$. Region I: $\varphi_0 = (\pi/4)(2k+1)$; region II: $\varphi_0 = (\pi/2)k$.

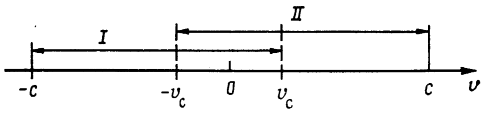

FIG. 2. Ranges of stability of kinematic domain walls in a weak ferromagnet of the $4_z^{(-)}$ type when $\beta_4 > 0$ and $\sigma d > 0$. Region I: $\varphi_0 = \pi/2, 3\pi/2$; region II: $\varphi_0 = 0, \pi$.

Domain walls with loss of the center

In the case of domain walls exhibiting a loss of their center the dependence $\Omega^2(v)$ is different. As pointed out above (see also Table I), they are characterized by $D(\theta_0(\zeta),\varphi_0) = -D(\theta_0(-\zeta)\varphi_0)$, i.e., $F_{22}(\zeta) = -F_{22}(-\zeta)$. Therefore, the dynamic term in F_{22} contributes to Ω^2 only in the second order of perturbation theory. This contribution is automatically negative. The value of Ω^2 for domain walls with loss of their center is obtained quite readily:

$$\Omega^2=\langle\tilde{\beta}\rangle-D^2v^2/(c^2-v^2), \tag{26}$$

where

$$\langle\tilde{\beta}\rangle\langle\sin^2\theta_0\rangle=\langle\partial^2\tilde{w}_a(\theta_0,\varphi_0)/\partial\varphi_0^2\rangle, \tag{27}$$

$$D^2\approx\int_{-\infty}^{+\infty}dk\frac{J_kJ_k^*}{1+k^2},\qquad J_k=\int_{-\infty}^{+\infty}dx\frac{L^{1/2}D(\theta_0,\varphi_0)\psi_0\psi_k}{2(\beta\delta)^{1/2}\sin\theta_0},$$

and ψ_0 and ψ_k are the eigenfunctions of $\hat{H}_0$ [see Eq. (16)]. It follows from Eq. (21) that the stability conditions of a domain wall are not dependent on the sign of the velocity or on the value of σd: if $\langle\tilde{\beta}\rangle > 0$, it is found that a domain wall is stable if $v = 0$, but when the velocity is increased the value of Ω^2 decreases and such a wall becomes unstable at velocities $|v| > v_c$, where

$$v_c=c\left(\frac{\langle\tilde{\beta}\rangle}{\beta}\right)^{1/2}\frac{1}{D}\left[1+\frac{\langle\tilde{\beta}\rangle}{\beta D^2}\right]^{-1/2}, \tag{28}$$

However, if $\langle\tilde{\beta}\rangle < 0$, then a domain wall with loss of the center is unstable for any value of the velocity. We shall show below that in this case domain walls with an antisymmetric emergence of **l** from a plane are stable. The values of D^2, $\langle\tilde{\beta}\rangle$, and v_c can easily be found for any weak ferromagnets in which domain walls with loss of the center can exist. For example, in the case of a weak ferromagnet of the $2_z^{(-)}2_y^{(+)}$ type (orthoferrites), we have

$$\langle\tilde{\beta}\rangle=\beta\varepsilon,\quad D^2=8d^2/3\beta\delta,\quad v_c=c(1+8d^2/3\beta\delta\varepsilon)^{1/2}. \tag{29}$$

It should be noted that domain walls with loss of the center are of special interest because they have been observed experimentally (for example, in dysprosium orthoferrite at temperatures $T < 150$ K—see Ref. 13) and they separate domains with different values of the magnetization **M**, so that it is easy to set such domain walls in forced motion by application of an external magnetic field.

Having considered the dynamics and stability of all domain walls characterized by $\varphi = \text{const}$ and by the Lorentz-invariant dependence of $x - vt$, we shall turn back to Table I. We can easily see from this table that in the case of some easy-axis weak ferromagnetic phases we can expect only kinematic domain walls. They exhibit the behavior shown in Fig. 1: at any velocity one of the domain walls with the rotation of **l** in one of two crystallographically inequivalent planes, for example (YZ) [or (XY) for orthoferrites], is stable. When the velocity is altered, one domain wall may lose its stability and then the second becomes stable. A full description of such kinematic domain walls is given above.

In the case of other magnetic phases of weak ferromagnets (of the $4_z^{(-)}$ type, $\mathbf{l}\|\mathbf{e}_z$; cubic) we can expect both kinematic domain walls as well as domain walls with symmetric emergence (SE) of **l** from a plane. In this case we shall show that there are values of v when none of the kinematic domain walls is stable [see Eq. (24) at $\beta_4 < 0$].

It follows from Eq. (26) that a domain wall with loss of the center becomes unstable on increase in v. In accordance with Table I, such domain walls exist only in those weak ferromagnetic phases for which domain walls with an antisymmetric emergence (AE) from a plane are possible. From these two observations we can easily deduce that in the case of such weak ferromagnets a change in the velocity should modify a domain wall with R.p. 315 E into a domain wall with emergence of l from a plane: a kinematic domain wall changes to one with SE and a domain wall with loss of the center changes to one with AE. We shall now analyze the dynamics and stability of these walls.

3. MOTION OF A DOMAIN WALL WITH ANTISYMMETRIC EMERGENCE

It is clear from Table I that domain walls with AE are typical of many weak ferromagnets. The majority of the experiments on dynamics of high-velocity walls have been carried out in orthorhombic weak ferromagnets such as yttrium and thulium orthoferrites[1] in which these walls are of the AE type. We shall therefore consider the example of an orthoferrite in a weak ferromagnetic phase $\mathbf{l}\|\mathbf{a},\mathbf{m}\|\mathbf{c}$,**a**,**c** are the orthoferrite axes).[1,14] The directions of x, y, and z will be selected along the **c**, **b**, and **a** axes. If we allow for the specific form of $\tilde{w}$ and of $D(\theta,\varphi)$, we can write down the equation for φ of an orthoferrite in the form

$$x_0^2(\varphi'\sin^2\theta)'+\varepsilon\sin^2\theta\sin\varphi\cos\varphi+x_0D(v)\theta'\sin^2\theta\cos\theta\cos\varphi=0, \tag{30}$$

where

$$x_0=(\alpha/\beta)^{1/2},\quad D(v)=6dv[\beta\delta(c^2-v^2)]^{-1/2}.$$

Equation (30) has a solution $\varphi = \pm\pi/2$ corresponding to a domain wall with loss of the center and rotation of **l** in the ab plane (domain wall of the ab type.) If $v = 0$, there is also a solution $\varphi = 0$ describing a static domain wall of the ac type (such domain walls occur in practically all orthoferrites at room temperature—see Ref. 1). However, if $v\neq 0$, there is clearly no solution of Eq. (30) of the $\varphi = 0$ type.

As pointed out already, a rigorous analysis of the system of equations (8)–(9) is not possible in the general case, but because of the inequality $D(v)\ll 1$, we can find an approximate solution of Eq. (30). This solution corresponds to $\varphi \propto [D(v)/\max(1,\varepsilon)]\ll 1$. We recall that we always have $D(v)\ll 1$ with the exception of a narrow range of values of v

close to c where the condition $|c-v| \lesssim c(d^2/\beta\delta) \ll c$ is satisfied. Therefore, we can find φ using the linearized variant of Eq. (30):

$$x_0^2(\varphi' \sin^2\theta)' + \varepsilon\varphi \sin^2\theta = -x_0 D(v)\theta' \sin^2\theta \cos\theta. \quad (30')$$

The differential operator on the left-hand side of Eq. (30) can be modified by the substitution $\mu = \varphi \sin\theta_0$ which reduces it to the form $\hat{H}_0 + \varepsilon U(x)$, where $\hat{H}_0$ is the Schrödinger operator of Eq. (16) derived above. Therefore, the explicit solution of this equation can easily be found by expanding $\mu(\zeta)$ in terms of known eigenfunctions of $\hat{H}_0$. In view of the antisymmetry of the right-hand side of Eq. (30), the term with ψ_0 is missing from the expansion and we have

$$\varphi \sin\theta_0 = -D(v) \int_{-\infty}^{+\infty} \frac{dk}{2\pi} \psi_k \frac{\langle \psi_k{}^* \theta' \sin\theta_0 \cos\theta_0 \rangle}{1+k^2+\varepsilon}. \quad (31)$$

Hence, we obtain the above inequality $\varphi \propto D(v) \ll 1$ (the exception to this rule is represented by the case $\varepsilon \cong -1$, i.e., by a region near the temperature of lability of the phase with weak ferromagnetism). Next, assuming that $\theta = \theta_0 + \Delta\theta$, where θ_0 is the solution of Eq. (8) when $\varphi = 0$ and linearizing Eq. (8) with respect to $\Delta\theta$, we can easily find the structure of a domain wall of the ac type with precision sufficient for investigating its stability [$\varphi \propto D(v)$, $\Delta\theta \propto D^2(v)$].

For the sake of brevity, we shall consider only the limiting cases. If $\varepsilon \ll 1$, and $\varepsilon \gg 1$, Eq. (31) simplifies greatly and the value of φ can be written explicitly. For example, if $\varepsilon \gg 1$, we then have

$$\varphi \approx -[x_0 D(v)/\varepsilon]\theta' \cos\theta, \quad \varepsilon \gg 1. \quad (31')$$

The solution of Eq. (31) is obtained for the case of a domain wall with AE easy plane wea antiferromagnet and an orthoferrite in the limit $\beta \to 0$ (near a transition of the Morin type). At low values of ε the integral in Eq. (31) has no singularities so that we can assume that $\varepsilon = 0$. We then have

$$x_0\varphi' = -{}^1\!/_3 D(v) \sin\theta, \quad \varepsilon \ll 1. \quad (31'')$$

In both cases the solution exists right up to $v = v_c$. The value of v_c can be estimated from the condition $D = 1$ and it is close to the value of c (Ref. 15). According to the expressions obtained earlier [Eqs. (26)–(29)] it is the $\varepsilon \ll 1$ case which is most interesting from the point of view of an instability of a domain wall with loss of the center, because then $v_c \to 0$ when $\varepsilon \to 0$. We shall confine our analysis to this case. It follows from Eqs. (31') and (8) that if $\varepsilon \to 0$, we can readily obtain the equation for $\theta(\zeta)$:

$$x_0^2\theta'' - \sin\theta \cos\theta + {}^1\!/_3 D^2 \sin^3\theta \cos\theta = 0.$$

Its first integral is

$$x_0^2\theta'^2 = \sin^2\theta - {}^1\!/_6 D^2 \sin^4\theta,$$

which makes it possible to find the solution of $\theta(\zeta)$ explicitly, i.e., to determine the structure of a domain wall with AE.

We shall now study the stability of this wall. It should be pointed out that because of Eq. (31) and structural equations for $\theta(\zeta)$, we have F_{12}, $F_{21} \propto D$, F_{22}, $F_{11} \propto \tilde{w}$, D^2 in the eigenvalue problem represented by Eqs. (13) and (14). We shall seek $\vartheta(\zeta)$ and $\mu(\zeta)$ in the form of an expansion in terms of eigenfunctions of $\hat{H}_0$:

$$\vartheta = A\psi_0 + \sum_k a_k\psi_k, \quad \mu = B\psi_0 + \sum_k b_k\psi_k. \quad (32)$$

The coefficients A, B, a_k, and b_k are described by an infinite series of algebraic equations. We can easily show that in the first approximation with respect to $D(v)$, we have

$$a_k = -[B/(1+k^2)]\langle \psi_k{}^* F_{12}\psi_0 \rangle,$$
$$b_k = -[A/(1+k^2)]\langle \psi_k{}^* F_{21}\psi_0 \rangle,$$

i.e., $a_k \propto DB$, $b_k \propto DA$. This makes it possible to reduce the problem to two algebraic equations for A and B. These equations can be solved giving two values of Ω^2, one of which vanishes and represents translation of a domain wall as a whole, whereas in the case of the other we obtain an expression

$$\Omega^2 = \langle F_{22}\psi_0{}^2 \rangle - \int_{-\infty}^{+\infty} \frac{dk}{1+k^2} \langle \psi_0 F_{21}\psi_k \rangle \langle \psi_k{}^* F_{12}\psi_0 \rangle,$$

which contains cumbersome integrals. Direct calculation of these integrals yields an expression similar in structure to Eq. (26), but with the opposite signs of the coefficients in front of $D^2(v)$ (i.e., in front of v^2):

$$\Omega^2 = -\varepsilon + 0.192 D^2(v) = -\varepsilon + 6.9 \frac{d^2}{\beta\delta} \frac{v^2}{(c^2-v^2)}. \quad (33)$$

It follows from Eq. (33) that Ω^2 rises on increase in v. This means that if a static domain wall with AE is stable in orthoferrite ($\varepsilon < 0$), it remains stable at all velocities right up to $v = v_c \approx c$. Therefore, although the symmetry of a domain wall with AE increases under dynamic conditions and its structure becomes more complex, its limiting velocity is practically the same as in the Lorentz-invariant theory.

Even if a static domain wall with AE is unstable [$\Omega^2(0) < 0$, i.e., when $\varepsilon > 0$], it becomes stable for $v > v_*$, where

$$v_* = c[1 + 6.9 d^2/|\varepsilon|\beta\delta]^{-1/2}. \quad (33')$$

This means that an interesting dynamic stabilization effect appears in the case of a domain wall which is unstable in the static case.

These effects—the onset of instability of a domain wall with loss of the center which occurs at velocities $v > v_c$ and stabilization of a domain wall with AE when $v > v_*$—may result in a dynamic modification of the structure of a domain wall. This modification can be regarded as a characteristic phase transition. A comparison of the values of v_c and v_* defined respectively by Eqs. (28) and (33), leads to the conclusion that $v_c > v_*$, i.e., that the regions of stability of domain walls with loss of the center and those with AE overlap.

We shall now calculate the energy of both domain walls and express it as a function of the momentum P of a domain wall, which is defined as the total momentum of the field of a vector $\mathbf{l}$ based on the Lagrangian of Eq. (2) (for details see Ref. 16). The values of the energy, velocity, and momentum are related by

$$E^2=E_0^2(\varphi_0)+c^2P^2\left(1-\frac{4d^2}{\beta\delta}\cos^2\varphi_0\right),$$

$$P=\frac{Ev}{c^2}\left(1+\frac{8d^2}{3\beta\delta}\cos^2\varphi_0\right), \qquad (34)$$

$$P_{LC}=\frac{E_{LC}v}{c^2}, \quad P_{AE}=\frac{E_{AE}v}{c^2}\left[1+\frac{8d^2}{3\beta\delta}\right].$$

Characteristic values of the velocity v_c and v_* correspond to the values of the momentum $P_c=P_{LC}(v_c)$, $P_*=P_{AE}(v_c)$, and $P_*<P_c$, where LC denotes the loss of the center and AE denotes asymmetric emergence from a plane. The dependences $E_{AE}(P)$ and $E_{LC}(P)$ can be represented in the form

$$E_{LC}^2(P)=E_1^2+c^2P^2, \quad E_{AE}^2(P)=E_2^2+c^2P^2(1-4d^2/\beta\delta), \qquad (35)$$

where

$$E_1=E_{LC}(0)=2M_0^2(\alpha\beta)^{1/2}, \quad E_2=E_{AE}(0)=2M_0^2[\alpha\beta(1+\varepsilon)]^{1/2}$$

are the known energies of static domain walls. In the case of interest to us the occurrence of a phase transition ($\varepsilon<0$) at $v=0$ is more likely from the energy point of view in the case of a domain wall with loss the center than one with AE. An increase in the momentum causes the energy of a domain wall with AE to rise more slowly than that with the loss of the center and they become equal when $P=P_t\approx(P_c+P_*)/2$ (see Fig. 3). Consequently, the (domain wall with the loss of the center) ⇄ (domain wall with AE modification) is typically first-order phase transition (E is the "thermodynamic potential" dependent on the external parameter P). The conclusion that they are first-order phase transition agrees with the circumstance that the symmetry groups of domain walls with the loss of the center and with AE are not related by a subgroup.[2)]

The "growth" phase transition (domain wall with AE) ⇄ (domain wall with loss of the center) may be manifested experimentally as an anomaly (kink) in the dependence of the velocity of forced motion of a domain wall on the applied field H or it may give rise to a hysteresis of this dependence in the range of velocities $v_*<v<v_c$. The values of v_* and v_c can be small in the limit $\varepsilon\to 0$, i.e., near the point of a phase transition one static domain to another. Such a transition has been observed experimentally in dysprosium orthoferrite $DyFeO_3$ at $T=155$ K (Ref. 13). A crystal of $DyFeO_3$ is a good candidate for an object in which forced motion of a

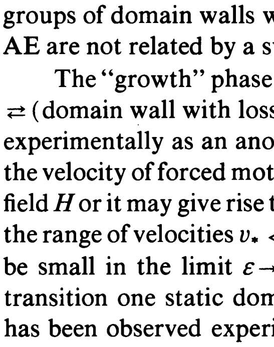

FIG. 3. $E(P)$ dependences for a domain wall with asymmetric emergence (curve 1) and for a domain wall with loss of the center (curve 2) in the case when the energies of these domain walls obey $E_{AE}(0)>E_{LC}(0)$ (schematic representation). In the region where the domain wall is unstable the dependence is shown by a dashed line.

domain wall may give rise to a (domain wall with loss of the center) ⇄ (domain wall with AE) phase transition.

The above results of an analysis of the structure of a domain wall with AE in orthoferrites and the concept of a (domain wall with loss of the center → domain wall with AE) phase transition as a result of a change in the velocity may be applied also to other weak ferromagnets in which domain walls with AE and those with loss of the center can exist (Table I). The main relationships are still conserved: destabilization of a domain wall with loss of the center and stabilization of a domain wall with AE on increase in the velocity, the order of the phase transition, etc. Only the values of v_* and v_c may be different, for example, in the case of easy-plane weak ferromagnets v_* and v_c are close to c. We can effectively speak of a certain unified class of behavior of domain walls in all weak ferromagnets in which domain walls with loss of the center and those with AE can exist.

4. MOTION OF A DOMAIN WALL WITH SYMMETRIC EMERGENCE

An analysis of the behavior of a domain wall with symmetric emergence (SE) of l from a plane will be made by considering a tetragonal weak ferromagnet with an odd easy axis (easy axis of the $4_z^{(-)}$ type). Such a structure is encountered, for example, in MnF_2 which is a thoroughly investigated magnetic material.

Equation (9) for the angle φ derived allowing for the explicit form of $\tilde{w}(\theta,\varphi)$ and $D(\theta,\varphi)$ for a weak ferromagnet of the MnF_2 type becomes

$$\alpha[\varphi'\sin^2\theta]'-\beta_4\sin^4\theta\sin 4\varphi$$

$$+\frac{v\,dx_0\theta'}{g\delta M_0[1-v^2/c^2]^{1/2}}\sin^3\theta\sin 2\varphi=0. \qquad (36)$$

It follows from Eq. (11) that

$$x_0\theta'\sin\theta\approx\sigma\sin^2\theta[1+(\beta_4/2\beta)\sin^2\theta].$$

If we drop the small term with β_4/β we find that Eq. (36) has the exact solution $\varphi'=0$, $\varphi=\varphi_0(v)$, where

$$2\beta_4\cos 2\varphi_0=[v/(c^2-v^2)^{1/2}][\sigma d/(\beta\delta)^{1/2}],$$

or

$$\cos 2\varphi_0=-\text{sign}(\sigma d)(v/v_c)(c^2-v_c^2)^{1/2}(c^2-v^2)^{-1/2},$$
$$v_c=c[1+9\beta d^2/\delta\beta_4^2]^{-1/2}. \qquad (37)$$

If β_4/β is small but finite, there is no $\varphi=$ const solution but we can show that the angle φ varies over distances of the order of $(\alpha/\beta_4)^{1/2}\gg(\alpha/\beta)^{1/2}$ and in the region of a domain wall ($\xi\lesssim x_0$) the approximate solution $\varphi=\varphi_0(v)$ describes the situation satisfactorily. It follows from Eq. (37) that in the limit $v\to 0$ we have $\varphi_0=(\pi/4)(2k+1)$ which corresponds to a static domain wall with SE. An increase in the velocity causes $\varphi_0(v)$ to change and in the limit $v\to\pm v_c$ we have $\cos 2\varphi_0\to\mp$ sign (σd), i.e., $\varphi_0=(\pi/2)k$. Therefore, if $|v|=v_c$ a domain wall with SE reduces to one of the kinematic domain walls discussed in Sec. 2. If, for example, we have $\sigma d>0$ and $\varphi_0=\pi/4$ when $v=0$, then for $v\to+v_c$ or $v\to-v_c$ we have $\varphi_0\to\pi/2$ or $\varphi_0\to 0$, respectively. Hence it is clear that the value of v_c represents the limiting velocity of motion of a domain wall with SE. It should be noted that v_c is identical with the characteristic value of the velocity of a kinematic domain wall introduced above [see Eq. (25)] for a magnetic material of the $4_z^{(-)}$ type.

We shall estimate v_c for MnF_2 which is characterized by $c = c_\parallel = 2.3\times 10^3$ m/s and $c = c_\perp = 1.7\times 10^3$ m/s in the case of an inhomogeneity directed along and at right-angles to the z axis, respectively. If we assume in our estimates that $c \approx 2\times 10^3$ m/s, $H_e = \delta M_0/2 = 560$ kOe, $H_{sf}(\beta\delta)^{1/2} M_0/2 \approx 93$ kOe, as well as $dM_0 \approx 2$ kOe and $bM_0 \approx 30$ Oe (Ref. 18), we find that $v_c \approx 60$ m/s and this value is much less than c.

In an analysis of the stability of a domain wall SE of the type described by Eq. (37) we note that, in contrast to a domain wall with AE, in this case the value of F_{22} for the former all contains a term which is linear in Dv and proportional to the symmetric function ζ:

$$F_{22}=\{\beta_4/\beta+vd\sigma[\beta\delta(c^2-v^2)]^{-1/2}\cos 2\varphi_0\}\sin^2\theta_0. \tag{38}$$

The remaining F_{ik} are either small ($\propto D^2$) or contain antisymmetric functions ζ. Therefore, the instability effects appear even in the first order of perturbation theory and the following expression is readily found for Ω^2:

$$\Omega^2=\tfrac{4}{3}\beta_4\cos 4\varphi_0-4v\sigma d[\beta\delta(c^2-v^2)]^{-1/2}\cos 2\varphi_0. \tag{39}$$

It should be noted that the above expression applies to any domain wall characterized by $\varphi = \varphi_0 =$ const in a weak ferromagnet of the MnF_2 type. In particular, if $\varphi_0 = \pi k/2$, it reduces to Eq. (24) given above and describing Ω^2 for a kinematic domain wall in similar weak ferromagnets. Employing the explicit form of $\varphi_0(v)$ given by Eq. (37) for a domain wall with SE and using the value of v_c, we can rewrite the above expression in the form

$$\Omega^2=\begin{cases}\beta_4+\beta_4(v/v_c)[(c^2-v_c^2)/(c^2-v^2)]^{1/2}, & \text{KDW}, \quad \varphi_0=0,\\ -\beta_4+\beta_4(v^2/v_c^2)[(c^2-v_c^2)/(c^2-v^2)], & \text{DWSE}, \quad 0<\varphi=\varphi_0(v)<\pi/2,\\ \beta_4-\beta_4(v/v_c)[(c^2-v_c^2)/(c^2-v^2)]^{1/2}, & \text{KDW}, \quad \varphi_0=\pi/2,\end{cases} \tag{40}$$

where KDW denotes a kinematic domain wall and DWSE denotes a domain wall with symmetric emergence, a form that can be used to analyze a domain wall of any type and transitions between domain walls [it is assumed specifically in Eq. (40) that $\sigma d > 0$]. It readily follows from the summarizing expression that the form of the stability diagram of a domain wall depends strongly on the sign of β_4. If $\beta_4 < 0$, then a domain wall with SE characterized by $\varphi = \varphi_0(v) =$ const is never unstable and there are only solutions of the kinematic domain wall type with $\varphi_0 = \pi k/2$, discussed in Sec. 2 (see Fig. 2). However, if $\beta_4 < 0$, then for $v = 0$ we find that a domain wall with SE and with $\varphi_0 = \pi/4$ is stable. An increase in its velocity changes the value of φ_0 (in the case when $\sigma d > 0$ it rises for $v < 0$ and falls for $v > 0$ and becomes equal to $\pi/2$ or 0 respectively for $v = -v_c$ and $v = +v_c$). It follows from Eq. (40) that a domain wall with SE is stable throughout the range of its existence. The stability regions of domain walls with SE do not overlap the stability regions of kinematic domain walls, but they are in contact at points $v = v_c$ (domain walls with SE and those with $\varphi = \pi/2$) and $v = -v_c$ (domain walls with SE and those with $\varphi = 0$). This resembles a pattern of two second-order (kinematic domain wall ⇄ domain wall with SE) phase transitions accompanied by lowering of the symmetry of a domain wall when the angles $\varphi_0(v)$ in the range $\pi/2 - \varphi_0(v)$ represent the order parameters.

This interpretation is confirmed if we consider energies of domain walls (kinematic E for $\varphi = 0$, $\pi/2$ and domain walls with AE) as a function of the momentum of a domain wall. In the case of all these domain walls if we use the condition $\varphi_0 \approx$ const, we obtain

$$E=\frac{E(\varphi_0)}{(1-v^2/c^2)^{1/2}}, \qquad P=\frac{vE(\varphi_0)}{c(c^2-v^2)^{1/2}},$$
$$E(\varphi_0)=2M_0^2(\alpha\beta)^{1/2}\{1+(\beta_4/6\beta)\sin^2 2\varphi_0\}. \tag{41}$$

The relationship (41) for kinematic domain walls gives rise to the usual Lorentz-invariant relationship between E and P:

$$E=E(P)=[E_0^2+c^2P^2]^{1/2},$$

where $E_0 = 2M_0^2(\alpha\beta)^{1/2}$ is the energy of a kinematic domain wall at rest. In the case of a domain wall with SE the relationship between E and P is more complex because of the dependence $\varphi = \varphi_0(v)$ or $\varphi = \varphi_0(P)$. The results of an analysis are plotted in Fig. 4. We can see that the function $E(P)$ demonstrates a behavior typical of the dependence of the thermodynamic potential E on an external parameter P in the case of a second-order phase transition. At the transition point, where

$$P=P_c=(E_0v_c/c)/[c^2-v_c^2]^{1/2},$$

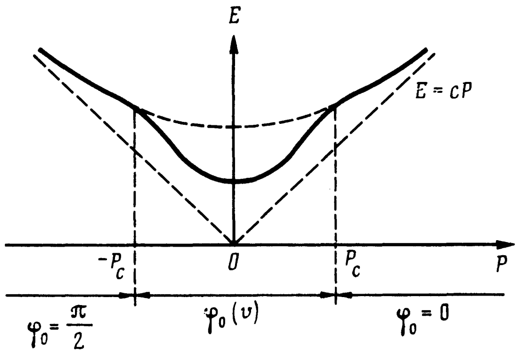

FIG. 4. $E(P)$ dependence for a domain wall in a weak ferromagnet of the $4_z^{(-)}$ type when $\beta_4 < 0$; the specific case when $\sigma d > 0$ is plotted. The dashed line is the dependence for a kinematic domain wall in the region of its instability.

the derivative $\partial E/\partial P$ is continuous, whereas $\partial^2 E/\partial P^2$ has a discontinuity. It should be pointed out that the $E(v)$ dependence at $v = v_c$ exhibits a discontinuity of the derivative $\partial E/\partial v$, indicating that it is the momentum which is the natural parameter for the description of motion-induced phase transitions in domain walls.

5. DISCUSSION OF RESULTS

The above phenomenological analysis makes it possible, firstly, to consider quantitatively the predicted[2,3] lowering of the symmetry of a moving domain wall, compared with a static wall, which appear at a velocity no matter how low and, secondly, to predict the loss of stability by domain walls and phase transitions between different walls at a finite value of the velocity. We shall now consider the general relationships governing the manifestation of these effects.

Our analysis shows that at a low velocity the distortions of the structure of a domain wall are usually small. An indication of the occurrence or otherwise of symmetry lowering in the spirit of Refs. 2 and 15 is insufficient to judge the limiting velocity of a domain wall. The experimentally determined limiting velocity is governed in the case of all walls by the dynamic loss of the wall stability. The only exception is a domain wall with a symmetric emergence on a plane in a tetragonal weak ferromagnet with the $4_z^{(-)}$ axis; such a domain wall is stable throughout the existence of the relevant solution.

Our analysis has revealed a number of general relationships on dynamic destabilization of domain walls. These relationships can be formulated by identifying three types of behavior.

1. Kinematic domain walls become unstable in all weak ferromagnets only for one sign of the velocity, more accurately the sign of the quantity $dv\mathbf{l}(\pm\infty)$. If the symmetry admits only the existence of kinematic domain walls, we then have the behavior illustrated in Fig. 1.

2. Domain walls with loss of the center can exist only in those phases of weak ferromagnets which contain domain walls with asymmetric emergence from a plane. An increase in the velocity always destabilizes the former and stabilizes the latter. If at $v = 0$ a domain wall with loss of the center is stable, then at a finite value of the velocity a first-order phase transition takes place from such a domain wall to one with asymmetric emergence. The value of the velocity at this transition may be low for orthoferrites and easy-axis weak ferromagnets, but it may be close to c for easy-plane weak ferromagnets.

3. A domain wall with symmetric emergence from a plane becomes stabilized on increase of velocity in the case of weak ferromagnetic phases listed in Table I, becomes unstable at $|v| = v_c$ and transforms at $v = +v_c$ and $v = -v_c$ into two different but equivalent kinematic domain walls (Figs. 2 and 4), where the value of v_c is low. In all other weak ferromagnets, both those listed in Table I and also orthorhombic and low-symmetry phases not listed, the behavior of a domain wall can be described by assigning it to one of the above classes.

The dynamic-phase-transition effects have been observed experimentally in forced motion of domain walls involving translation and vibrations of these walls.[1] They are manifested, for example, by a dependence of the domain-wall velocity on the applied field. The typical velocities v_c and v_* can be low in the case of many weak ferromagnets. One other possible manifestation follows from an analysis of quasi-one-dimensional weak ferromagnets in which domain walls have a finite energy, play the role of elementary perturbations, and determine—for example—dynamic neutron scattering.[19] The thermodynamic and response functions of such weak ferromagnets should have singularities in the case when the thermal velocity of a domain wall reaches the value v_c.

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[1)]In writing down Eq. (10) we must generally speaking include invariants of the type l_z^4, l_z^6, etc. and the contribution of these to $\theta(\zeta)$ may be comparable with the contribution of $\tilde{w}$. However, their inclusion does not affect the symmetry of a domain wall or, which is most important, the stability criterion of a domain wall, so that for the sake of brevity they are omitted.

[2)]If we allow for the fourth-order anisotropy, we find that a (domain wall with loss of the center ⇄ domain wall with asymmetric emergence) phase transition may occur in the form of two second-order phase transitions via the least symmetric domain wall, in accordance with the scheme (domain wall with loss of the center ⇄ domain wall with least symmetry ⇄ domain wall with asymmetric emergence). A discussion of details of this transition is outside the scope of the present paper, but it is considered in Ref. 17 in the case of static domain walls.

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