Novel effects in the scattering of charged particles by a cosmic string

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In this paper, we investigate the quantum analog of Lorentz scattering from the "magnetic" field of a local cosmic string, a phenomenon manifested by scattering asymmetry. For low-momentum incident particles (in the rest frame of the string), Lorentz scattering looks like a soft asymmetry in the cross section with a maximum near the classical value. The effect also occurs in the presence of a Bohm-Aharonov amplitude. A caustic appears near the classical scattering direction for high-momentum incident particles. We close with a discussion of possible physical effects.

1. INTRODUCTION

Cosmic strings, whose existence was first proposed by Zel'dovich[1] and Kibble,[2] have been employed in a variety of cosmological structures (e.g., see Ref. 3). In the present paper, we discuss certain novel effects that occur when charged particles interact with a local (gauge) cosmic string. This sort of interaction has been examined previously[4,5] in papers that treated the scattering of fermions carrying charge e by a string with flux $2\pi/e_0$ for integer values of $\alpha = e/e_0$. A calculation of the scattering amplitude for a charged particle with momentum $k \ll m$, where $1/m$ is the characteristic transverse size of the string, can be found in Ref. 4. For $k \gg m$, partial amplitudes have only been found for values of the orbital momentum $j \ll k/m$, while the dominant terms in the total scattering amplitude are those with j of order k/m. The results reported by de Vega[4] for low-energy scattering were subsequently partially reproduced by Everett,[5] but with certain more general assumptions about the form of the interaction between the fermion and the string. Furthermore, the latter paper treated the dynamics of a string in plasma or ionized gas, taking into account bremsstrahlung due to the interaction of charged particles with the string. In Ref. 6, Alford and Wilczek noted that there exist grand unified theories in which $\alpha = e/e_0$ is not necessarily an integer; it is then necessary to make allowance for Bohm-Aharonov scattering,[7] which can turn out to be the dominant effect.

In the present paper, we wish to study a certain effect which, as far as we are aware, has never been discussed in the literature. We have in mind the quantum analog of Lorentz scattering in the "magnetic" field of a string (note that local cosmic strings are basically analogous to Abrikosov filaments in type II superconductors[8]; in the context of field theory, they first appeared in the paper by Nielsen and Olesen,[9] who generalized them to the non-Abelian case), which is manifested by scattering asymmetry. In a quasineutral plasma, this results in the formation of diametrically opposed currents carried by oppositely-charged particles. Formally, we have examined the scattering of electrically charged particles in a magnetic field, but actual cosmic strings (e.g., see Refs. 3,5,6) carry a current associated with the nonelectromagnetic spontaneously broken gauge group G, which does not pose a problem when the particles considered are nontrivially transformed by the spontaneously broken generators of G. In this paper, we shall not discuss specific string models, and shall henceforth simply refer to the magnetic field.

In Sec. 2, we determine the scattering amplitude at low energies ($k \ll m$) and for arbitrary values of $\alpha = e/e_0$. Besides deriving the asymmetric term ($\sin\vartheta$, where ϑ is the scattering angle) responsible for Lorentz scattering—which thus far has not appeared in the literature—we also take account in our complete equation of Bohm-Aharonov scattering for nonintegral α, which was not done in Refs. 4 and 5.

In Sec. 3, the scattering amplitude is calculated at large energies in the rest frame of the string ($k \gg m$). It is then not legitimate to use the Born approximation—the eikonal approximation must be used instead. Diffraction around the string leads to a swallowtail caustic,[10] and the branch of the caustic with the maximum amplitude coincides with the classical Lorentz-scattering trajectory. For large α (ion scattering?), diffraction peaks show up within the caustic.

Section 4 deals with physical applications of the Lorentz scattering at hand. We consider several examples for which either the slow- or fast-scattering approximations are applicable. We discuss charge-separation effects related to string motion in plasma, and in particular the interesting possibility of matter-antimatter separation in an Alfvén wind as a hypothetical alternative to the standard mechanism for generating the baryon asymmetry of the universe.[11,12] We also discuss the vibration of a loop-like string in the presence of oscillations in plasma of particles having different charge-to-mass ratios. We show that the presence of caustics can lead to interesting collective phenomena with strong density fluctuations. At the conclusion of Sec. 4, we discuss the spontaneous generation of angular momentum perpendicular to the plane of a string as it contracts in the vacuum in a theory with parity violation.

2. SLOW-PARTICLE SCATTERING AMPLITUDE

To obtain the scattering amplitude, we solve the Dirac equation in the field of a string assuming minimal coupling,

$$(i\hat{\partial} - e\hat{A} - M)\psi = 0. \qquad (1)$$

For stationary solutions,

$$\omega\psi = (\boldsymbol{\pi\alpha} + p^3\alpha^3 + M\beta)\psi, \quad \boldsymbol{\pi} = \mathbf{p}_\perp - e\mathbf{A}. \qquad (2)$$

The momentum component along the string (directed along the z axis) can be eliminated by a boost, whereupon the single equation for the bispinor

$$\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$$

$$\omega\psi = (\boldsymbol{\pi\alpha} + M\beta)\psi$$

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leads to two independent equations in the spinors

$$\varphi=\begin{pmatrix}\varphi_\uparrow\\ \varphi_\downarrow\end{pmatrix},\quad \chi=\begin{pmatrix}\chi_\uparrow\\ \chi_\downarrow\end{pmatrix}:$$

$$\begin{aligned}\omega\varphi_\uparrow&=(\pi_1-i\pi_2)\chi_\downarrow+M\varphi_\uparrow,\\ \omega\chi_\downarrow&=(\pi_1+i\pi_2)\varphi_\uparrow-M\chi_\downarrow,\\ \omega\varphi_\downarrow&=(\pi_1+i\pi_2)\chi_\uparrow+M\varphi_\downarrow,\\ \omega\chi_\uparrow&=(\pi_1-i\pi_2)\varphi_\downarrow-M\chi_\uparrow.\end{aligned}\tag{3}$$

Thus, the three-dimensional problem can be reduced to two dimensions. The first pair of equations differs from the second only in the sign of the spin-magnetic coupling, and for the minimal coupling case, states of opposite spin differ solely by a shift in the numbering of the levels arising from the super-symmetry of the problem.[13] We ignore effects of any anomalous magnetic moment, which breaks the super-symmetry and gives rise to small changes in that part of the scattering amplitude symmetric under the replacement $\theta\to-\theta$ for particles with spin oriented either parallel or antiparallel to the z axis (which is directed along the string). From here on, we study only the first pair of equations (3).

Solutions are classified according to eigenvalues of the angular momentum operator j,

$$j=-i\partial/\partial\theta+\sigma_3/2,$$

where σ_3 is a Pauli spin matrix. The angular momentum eigenfunctions take the form

$$\frac{e^{ij\theta}}{r^{1/2}}\begin{pmatrix}f_je^{-i\theta/2}\\ g_je^{i\theta/2}\end{pmatrix}.\tag{4}$$

Some straightforward calculations yield the equations for f_j and g_j:

$$\begin{aligned}i(\omega-M)f_j&=g_j'+jg_j/r-\alpha(1-\mathscr{H})g_j/r,\\ i(\omega+M)g_j&=f_j'-\frac{j}{r}f_j+\alpha(1-\mathscr{H})f_j/r\end{aligned}\tag{5}$$

In Eq. (5), $A_\theta(r)=(1-\mathscr{H}(r))/e_0r=a(r)/e_0r$. Expressing g in terms of f, we obtain

$$f_j''+\left[k^2-\frac{(j-\alpha a)(j-\alpha a-1)}{r^2}+e\mathscr{H}\right]f_j=0.\tag{6}$$

The general solution of Eq. (6) contains both incoming and outgoing waves in some ratio fixed by the requirement of regular behavior at $r\to0$. For slow-particle scattering, there is also a range of distances r for which $1/m\ll r\ll1/k$. Within this range of r, one can ignore both the magnetic field term and the r-dependence of $as(r)$ in Eq. (6). The only thing that remains is the long-range Bohm-Aharonov potential, which shifts the angular momentum, $j\to j-\alpha$, so that solutions may be expressed in terms of the Bessel functions J of order $j-\alpha-1/2$. At the same time, for $r\ll1/k$, one can construct a solution by iterating with respect to k^2, starting with the exact solution for $k=0$. Over the indicated range $(1/m\ll r\ll1/k)$, the asymptotic behavior of these two solutions can be matched. This was the approach taken by de Vega[4] for integer α. Following his method, we obtain solutions in the region of interest in the form

$$f_j=r^{1/2}A_j\left[J_{j-\alpha-1/2}(kr)-2\alpha L_j\left(\frac{k}{2}\right)^{2(j-\alpha)+1}\frac{\Gamma(\alpha-j-1/2)}{\Gamma(j-\alpha+1/2)}J_{-(j-\alpha-1/2)}(kr)\right],\quad j>0,\tag{7}$$

$$f_j=r^{1/2}B_j\left(J_{-(j-\alpha-1/2)}(kr)+2\alpha M_j\left(\frac{k}{2}\right)^{2(\alpha-j)+1}\times\frac{\Gamma(j-\alpha+1/2)}{\Gamma(\alpha-j+3/2)}J_{j-\alpha-1/2}(kr)\right),\quad j<0,\tag{8}$$

where L_j and M_j are defined by the integrals

$$L_j=\int_0^\infty dr(r^{j-\alpha}g)^2\mathscr{H},\qquad M_j=\int_0^\infty dr\frac{\mathscr{H}}{(r^{j-\alpha}g)^2}.\tag{9}$$

Here

$$g=\exp\left\{-\alpha\int_r^\infty\frac{\mathscr{H}(t)}{t}dt\right\}.$$

Making use of Eqs. (4), (7), and (8), we obtain the total wave function for the upper component of the spinor φ for $r\gg1/m$ (the lower component is uniquely related to the upper, and we omit it here):

$$\varphi=\sum_{j>0}\exp(ij\theta-i\theta/2)A_j\times\left[J_{j-\alpha-1/2}(kr)-2\alpha L_j\left(\frac{k}{2}\right)^{2(j-\alpha)+1}\frac{\Gamma(\alpha-j-1/2)}{\Gamma(j-\alpha+1/2)}\times J_{-(j-\alpha-1/2)}(kr)\right]+\sum_{j<0}\exp(ij\theta-i\theta/2)B_j\left[J_{-(j-\alpha-1/2)}(kr)+2\alpha M_j\left(\frac{k}{2}\right)^{2(\alpha-j)+1}\frac{\Gamma(j-\alpha+1/2)}{\Gamma(\alpha-j+3/2)}J_{j-\alpha-1/2}(kr)\right].\tag{10}$$

The coefficients A and B may be determined from the continuity conditions (10) at large r using a wave function of the form

$$\exp(-ikx+i\alpha\theta)+\frac{ie^{ikr}}{(2\pi ikr)^{1/2}}\frac{e^{-i\theta/2}}{\cos\theta/2}\sin\pi\alpha+\frac{f}{r^{1/2}}e^{ikr}\tag{11}$$

which consists of the incident plane wave and a scattered wave (the Bohm-Aharonov contribution to the scattering amplitude is given explicitly). Equation (10) contains terms of different order in the small parameter k/m. We can also expand the coefficients A_j, B_j, $A_j^{(0)}$, and $B_j^{(0)}$ in k/m; these are determined by matching the leading terms in Eq. (10) with the plane wave and the Bohm-Aharonov scattered wave in Eq. (11). A recursion relation between $A_j^{(n)}$ and $B_j^{(n)}$ can then be obtained by requiring, at each iteration, that there be no incoming waves. Note that the second term in the sum over positive momenta is reduced by powers of k/m only for $j>\alpha-1/2$, while for $j=[\alpha]+1/2$ the suppression is weaker than k^2. For the latter waves, a finite number of iterations is never sufficient; for terms that die out more rapidly than k^2, on the other hand, just one iteration is enough. The added precision of a second iteration would not be justified, since in deriving Eq. (10) from (4), (7), and (8), we have already neglected terms of order k^2.

We now examine the case $0<\alpha\leqslant1$. The result for arbitrary α will be presented at the end of this section.

Using the relation (see Ref. 7)

$$\sum_{j>0} \exp(-i\pi/2(j-\alpha)+i\pi/4) J_{j-\alpha-1/2}(kr)\exp(ij\theta-i\theta/2)$$
$$+\sum_{j<0} \exp(i\pi/2(j-\alpha)-i\pi/4) J_{-(j-\alpha-1/2)}(kr)\exp(ij\theta-i\theta/2)$$
$$\to \exp(-ikx+i\alpha\theta)+i\frac{e^{ikr}}{(2\pi ikr)^{1/2}}\frac{e^{-i\theta/2}}{\cos\theta/2}\sin\pi\alpha, \tag{12}$$

we obtain from (10)

$$A_j^{(0)}=\exp(-i\pi(j-2)/2+i\pi/4),\quad B_j^{(0)}=\exp(i\pi(j-\alpha)/2-i\pi/4), \tag{13}$$

and for the recursion relations, we obtain

$$A_j^{(n)}=2\alpha L_j\left(\frac{k}{2}\right)^{2(j-\alpha)+1}\frac{\Gamma(\alpha-j-1/2)}{\Gamma(j-\alpha+1/2)}\times\exp[-i\pi(j-\alpha-1/2)]A_j^{(n-1)},$$
$$B_j^{(n)}=-2\alpha M_j\left(\frac{k}{2}\right)^{2(\alpha-j)+1}\frac{\Gamma(j-\alpha+1/2)}{\Gamma(\alpha-j+3/2)}\times\exp(i\pi(j-\alpha-1/2))B_j^{(n-1)}. \tag{14}$$

Some simple manipulations (for $j=1/2$, we sum a geometric series) yield the scattering amplitude

$$f=\frac{i}{(2\pi ik)^{1/2}}\Bigg[\sum_{j\geq 3/2}-4\alpha L_j\left(\frac{k}{2}\right)^{2(j-\alpha)+1}\times\frac{\pi\exp(ij\theta-i\theta/2+i\alpha\pi)}{\Gamma(j-\alpha+3/2)\Gamma(j+1/2-\alpha)}+\sum_{j<0}4\alpha M_j\left(\frac{k}{2}\right)^{2(\alpha-j)+1}\frac{\pi\exp(ij\theta-i\theta/2-i\alpha\pi)}{\Gamma(\alpha-j+3/2)\Gamma(\alpha-j+1/2)}\Bigg]+f_{1/2}. \tag{15}$$

The function $f_{1/2}$ is of the form

$$f_{1/2}=\frac{i}{(2\pi ikr)^{1/2}}4\alpha L_{1/2}\left(\frac{k}{2}\right)^{2(1-\alpha)}\times\frac{\Gamma(\alpha-1)}{\Gamma(1-\alpha)}\frac{e^{i\alpha\pi}\sin\pi\alpha}{1-2\alpha L_{1/2}\left(\frac{k}{2}\right)^{2(1-\alpha)}\frac{\Gamma(\alpha-1)}{\Gamma(1-\alpha)}e^{i\pi\alpha}}, \tag{16}$$

where we have made the substitution $\theta\to 0+\pi$, so that now, in contrast to Eq. (11), θ is the scattering angle. As $\alpha\to 1$, the expression for $f_{1/2}$ becomes

$$f_{1/2}=-\frac{i}{(2\pi ik)^{1/2}}\frac{\pi}{\ln(k/2m)+\gamma+2\int_0^\infty dr\ln(nr)\mathscr{H}g^2/r-i\pi/2} \tag{17}$$

where γ is Euler's constant.

At $\alpha=1$, amplitudes with j not equal to 1/2 agree with the amplitudes in Ref. 4 (up to an overall sign on the S matrix); for $j=1/2$, our result (17) differs from de Vega's Eq. (3.20) by the term $i\pi/2$ in the denominator. The importance of this term lies not in the fact that it renormalizes the logarithmic constant, but that by virtue of its being imaginary, it gives rise to an asymmetry in the cross section under the transformation $\theta\to-\theta$. This same asymmetry is responsible for Lorentz scattering.

Taking terms of order k^2 into account, the cross section is

$$\frac{d\sigma}{d\theta}=\frac{\pi}{2k(\ln^2(k/\Lambda)+\pi^2/4)}\times[1-2L_{1/2}k^2\ln(k/\Lambda)\cos\theta-\pi L_{3/2}k^2\sin\theta], \tag{18}$$

$$\Lambda=2m\exp\left\{-\gamma-2\int_0^\infty dr\frac{\ln mr}{r}\mathscr{H}g^2\right\}.$$

The angular dependence arises from interference between the S and P waves ($j=1/2, j=3/2$), where the term in $\sin\theta$ results solely from the fact that the amplitude (17) has an imaginary part (due to the term $i\pi/2$ in the denominator). The sign of the contribution to (18) that is odd in θ obviously agrees with the sign of the Lorentz force.

For $\alpha<1$, the Bohm-Aharonov terms must also be taken into consideration; in return, one can ignore the P wave, which falls off faster than k^2. The corresponding cross section takes the form

$$\frac{d\sigma}{d\theta}=\frac{\sin^2\pi\alpha}{2\pi k\sin^2\theta/2}\Bigg(1+8\alpha L_{1/2}\left(\frac{k}{2}\right)^{2(1-\alpha)}\times\frac{\Gamma(\alpha-1)}{\Gamma(1-\alpha)}\cos\pi\alpha\sin^2\theta/2+4\alpha L_{1/2}\left(\frac{k}{2}\right)^{2(1-\alpha)}\frac{\Gamma(\alpha-1)}{\Gamma(1-\alpha)}\sin\pi\alpha\sin\theta+O(k^{4(1-\alpha)})\Bigg). \tag{19}$$

We see that the asymmetry has the same sign, but due to the Bohm-Aharonov terms, forward scattering makes the dominant contribution to the cross section.

Finally, let us consider essentially arbitrary positive values of α. Let α lie in the range $(n,n+1)$. The present case does not reduce to the previous shift of momentum j by n. We see from the discussion preceding Eq. (11) that partial waves with momenta in the range from 1/2 to $n-1/2$ must be iterated in the same was as the partial wave with momentum $n+1/2$ (or 1/2 in the case $n=0$). Formally summing the series over negative powers of k/m, we obtain for these waves an additional contribution to the amplitude

$$\frac{i}{(2\pi ik)^{1/2}}\frac{2c_j}{1-c_j}\sin\pi\alpha\exp(ij\theta-i\theta/2), \tag{20}$$

where

$$c_j=\exp[-i\pi(j-\alpha-1/2)]2\alpha L_j\left(\frac{k}{2}\right)^{2(j-\alpha)+1}\frac{\Gamma(\alpha-j-1/2)}{\Gamma(j-\alpha+1/2)}$$

with $c_j\gg 1$ for $j<n$. Summing the dominant terms in (20) for j from 1/2 to $n-1/2$, we obtain the correction to the Bohm-Aharonov amplitude, so that the resulting amplitude differs from the usual one by a phase factor; it takes the form

$$-\frac{1}{(2\pi ik)^{1/2}}\frac{\exp(-i\theta/2+in\theta)}{\sin\theta/2}\sin\pi\alpha \tag{21}$$

[compared with (12) there has been a shift $\theta\to\theta+\pi$].

Note that when it undergoes interference with the amplitudes (20), this phase factor is cancelled by the corresponding angular dependence of (20). The same can also be said about interference with the unscattered wave. To leading order in k, and taking into account the interference of the amplitude (21) with the wave (20) having $j = n + 1/2$, the cross section is

$$\frac{d\sigma}{d\theta}=\frac{\sin^2\pi\alpha}{2\pi k\sin^2\theta/2}\left(1+8\alpha L_{n+1/2}\left(\frac{k}{2}\right)^{2(\alpha-n)}\right.$$
$$\times\frac{\Gamma(\alpha-n-1)}{\Gamma(n+1-\alpha)}\cos\pi(\alpha-n)\sin^2\theta/2+$$
$$\left.+4\alpha L_{n+1/2}\left(\frac{k}{2}\right)^{2(\alpha-n)}\frac{\Gamma(\alpha-n-1)}{\Gamma(n+1-\alpha)}\sin\pi(\alpha-n)\sin\theta\right). \quad (22)$$

As α tends to $n + 1$, the Bohm-Aharonov amplitude vanishes, and it then becomes necessary to take account of waves with momentum $n - 1/2$ or $n + 3/2$, which are of order k^2. In that event, the cross section will have the form

$$\frac{d\sigma}{d\theta}=\frac{\pi}{2k(\ln^2(k/\Lambda)+\pi^2/4)}$$
$$\times\left[1-\left(2\alpha L_{n+3/2}+\frac{1}{2\alpha L_{n-1/2}}\right)k^3\ln\frac{k}{\Lambda}\cos\theta\right.$$
$$\left.-\frac{\pi}{2}\left(2\alpha L_{n+3/2}-\frac{1}{2\alpha L_{n-1/2}}\right)k^2\sin\theta\right]. \quad (23)$$

Notice that the modified amplitude (21) can be derived from a generalization of Eq. (12),

$$\sum_{n<0}\exp(i\pi(n+\alpha)/2)J_{-(n+\alpha)}(kr)e^{in\theta}$$
$$+\sum_{n=0}^{m-1}e^{i\pi(n+\alpha)/2}J_{-(n+\alpha)}(kr)e^{in\theta}$$
$$+\sum_{n>m}e^{-i\pi(n+\alpha)/2}J_{n+\alpha}(kr)e^{in\theta}$$
$$\to e^{-ikx-i\alpha\theta}-i\frac{e^{ikr}}{(2\pi ikr)^{1/2}}\frac{e^{-i\theta/2-im(\theta-\pi)}}{\cos\theta/2}\sin\pi\alpha. \quad (24)$$

3. FAST-PARTICLE SCATTERING AMPLITUDE ($k/m \gg 1$)

As we did for slow-particle scattering, we shall write the squared equation for the upper component of the spinor:

$$k^2\varphi=[\pi^2-e\mathscr{H}]\varphi. \quad (25)$$

It makes no sense to use the Born approximation to find the fast-particle scattering amplitude, since it is appropriate only for partial waves whose momentum is much less than k/m, while the important momenta here are of order k/m. Instead, we make use of the eikonal approximation by assuming a wave function of the form $\varphi = \exp\{iS\}$. We then obtain

$$k^2=[(\nabla S-e\mathbf{A})^2-i\nabla^2S-e\mathscr{H}]. \quad (26)$$

Representing S as a series in negative powers of k,

$$S = S^{(0)} + S^{(1)},$$

we obtain the equations for $S^{(0)}$ and $S^{(1)}$:

$$k^2-(\nabla S^{(0)}-e\mathbf{A})^2=0,$$
$$2(\nabla S^{(0)}-e\mathbf{A})\nabla S^{(0)}-i\nabla^2S^{(0)}-e\mathscr{H}=0. \quad (27)$$

The solution of the first of these must be of the form

$$\nabla S^{(0)}-e\mathbf{A}=k\mathbf{n}, \quad (28)$$

which then yields the equation for $S^{(1)}$:

$$(\mathbf{n}\nabla)S^{(1)}={}^1/_2(\operatorname{div}\mathbf{n}+e\mathscr{H}/k), \quad (29)$$

where $\mathbf{n}$ is the vector field of unit normals to the wavefront. From (28), we obviously have

$$k\operatorname{rot}\mathbf{n}=-e\mathbf{H},\ |\mathbf{H}|=\mathscr{H}.$$

Taking advantage of the vector identity $[\mathbf{n},\operatorname{rot}\mathbf{n}] + (\mathbf{n}\nabla)\mathbf{n} = \frac{1}{2}\operatorname{grad}\mathbf{n}^2$, we obtain

$$(\mathbf{n}\nabla)\mathbf{n}=[\mathbf{n},\mathbf{H}]e/k. \quad (30)$$

We next introduce new variables (l,t) such that the tangent vectors to the curves of constant t (rays) coincide with $\mathbf{n}$:

$$\partial x(l,t)/\partial l=n_1,\ \partial y(l,t)/\partial l=n_2,$$
$$x(l=-\infty,t)=-\infty,\ y(l=-\infty,t)=t. \quad (31)$$

In these new coordinates, Eq. (30) takes the form

$$\frac{\partial n_1(l,t)}{\partial l}=\frac{e}{k}n_2(l,t)\mathscr{H}(x(l,t);\ y(l,t)),$$
$$\frac{\partial n_2(l,t)}{\partial l}=-\frac{e}{k}n_1(l,t)\mathscr{H}(x(l,t);\ y(l,t)). \quad (32)$$

We proceed to solve this system to leading order in $1/k$. Then $n_1 = 1 + O(1/k^2)$, $n_2 = 1 + O(1/k)$. Accordingly,

$$x(l/t)\approx l,\quad y(l,t)\approx t+\int_{-\infty}^{l}dl'\,n_2(l',t).$$

Since any rotation of $\mathbf{n}$ takes place within a narrow zone near the center of the string (where the magnetic field is located), while $y(l,t)$ begins to deviate from t only after leaving that region, to leading order in Eq. (32) it is legitimate to replace $\mathscr{H}(x(l,t);\ y(l,t))$ by $\mathscr{H}(x(l,t),t)$. Keeping in mind that $x(l,t)\approx l$, we find

$$n_2(l,t)\approx-\frac{e}{k}\int_{-\infty}^{l}\mathscr{H}(x,t)\,dx+O(1/k^3), \quad (33)$$

$$y(l,t)=t-l\frac{e}{k}\int_{-\infty}^{l}\mathscr{H}(x,t)\,dx+\frac{e}{k}\int_{-\infty}^{l}x\mathscr{H}(x,t)\,dx. \quad (34)$$

Turning to Eq. (29), the right-hand side of which contains the divergence of $\mathbf{n}$, we use (33) and (34) to find that

$$\operatorname{div}\mathbf{n}=\frac{\partial n_2}{\partial t}\left(\frac{\partial y}{\partial t}\right)^{-1}$$

where

$$\frac{\partial y}{\partial t}=1-l\frac{e}{k}\int_{-\infty}^{l}\frac{\partial\mathscr{H}(x,t)}{\partial t}dx+\frac{e}{k}\int_{-\infty}^{l}x\frac{\partial\mathscr{H}(x,t)}{\partial t}dx,$$

whereupon it is clear that on the curves $x(l(t),t)$ and $y(l(t),t)$, with $l(t)$ given by

$$1-l\frac{e}{k}\int_{-\infty}^{l}\frac{\partial\mathscr{H}(x,t)}{\partial t}dx+\frac{e}{k}\int_{-\infty}^{l}x\frac{\partial\mathscr{H}(x,t)}{\partial t}dx=0, \quad (35)$$

the right-hand side of Eq. (29) diverges, which indicates the presence of a caustic, near which the eikonal (geometrical optics) approximation ceases to hold.[10,14] It can also be seen from Eq. (35) that we have $l\sim k/m^2$. Then since the magnetic field $\mathscr{H}(x,y)$ is confined to distances of order $1/m$, and furthermore since $\mathscr{H}(x,y)$ is an even function of x, the equation for $l(t)$ simplifies considerably:

$$l(t)=k/q'(t),\qquad q(t)=e\int_{-\infty}^{+\infty}\mathscr{H}(x,t)\,dx. \tag{36}$$

Note that $q(t)$ is the classical momentum transfer (for fast particles). The parametric equation of the caustic in variables x and y is

$$x(t)=k/q'(t),\qquad y(t)=t-q(t)/q'(t). \tag{37}$$

As $t\to 0$, $q'(t)\to 0$, $q(t)\to q_{\max}$, and $y(t)/x(t)\to -q_{\max}/k$. As $t\to-\infty$ ($l(t)$ exists only for $t<0$), $q(t)\sim\exp\{-mt\}$. Accordingly, $x(t)\sim\exp\{mt\}$ and $y\sim t$. The caustic that appears is thus a swallowtail catastrophe (see Ref. 10 and Fig. 1). There should be a luminous region within this type of caustic; it attains its maximum intensity, as we shall show below, at the lower limit of the caustic.

The scattering amplitudes are calculated in two stages. First, we find the wave function in the region $1/m\ll x\ll k/m^2$, solving the eikonal equation. In order to find the wave function in the region of interest ($x\gg k/m^2$), we make use of Huygen's principle,[14] having first carried out a gauge transformation to eliminate the Bohm-Aharonov potential from the region of positive x. We then have for the wave function

$$\varphi(x,y)=\left[\frac{k}{2\pi i}\right]^{1/2}\int\frac{df}{[R(f)]^{1/2}}e^{ikR(f)}\varphi(f),$$

where df is the integration measure along the wave front, $\varphi(f)$, is the wave function on the front, regarded as the source of secondary waves in the Huygens principle, and $R(f)$ is the distance from a point on the front to the point of observation. Substituting the gauge-transformed function

$$\varphi(f)=\exp\{iS^{(0)}(f)-i\alpha\theta(f)\},$$

and expressing $R(f)$ in terms of the ratio x/y (forward scattering is significant), we obtain

$$\varphi(x,y)=\left[\frac{k}{2\pi ix}\right]^{1/2}\exp(ikx+iky^2/2x)\times\int_{-\infty}^{+\infty}ds\exp\{-ik\theta s+iks^2/2x+iS^{(0)}(f)-i\alpha\theta(f)-ikx_f\}. \tag{38}$$

From Eq. (28) for $S^{(0)}$ and the asymptotic behavior of S as $x\to-\infty$,

$$S(x)\approx kx+\alpha\theta,$$

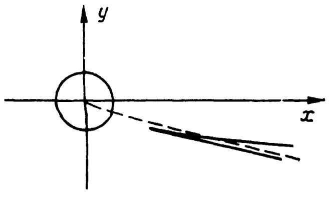

FIG. 1. The solid curve shows the location of the caustic; the dashed curve indicates the direction of classical scattering with maximum momentum transfer.

we find for $x\gg 1/m$, neglecting exponentially small corrections like $\exp\{-mx\}$,

$$S^{(0)}(x,y)=kx+\alpha\theta(x,y)-\int_0^y q(t)dt.$$

Substituting this expression into (38), we obtain the wave function in the scattering region:

$$\varphi(x,y)=\left[\frac{k}{2\pi ix}\right]^{1/2}\exp\{ikx+iky^2/2x\}\times\int_{-\infty}^{+\infty}ds\exp\left\{ik\theta s+iks^2/2x-i\int_0^s q(t)\,dt\right\}, \tag{39}$$

where y/x is the scattering angle. Calculating the value of (39) by the method of steepest descent, we obtain the condition for an extremum in s,

$$k\theta+q(s)=ks/x.$$

If the scattering angle is positive, then s has but one extremum, of order $y=\theta x$, and then $q(s)$ is of order $\exp\{-my\}\ll 1$. This extremum corresponds to the unscattered wave in the upper half-plane. In general, for a negative scattering angle, there are three extrema, with one also corresponding to the unscattered wave (s will then be of order y, $y<0$), and the other two being solutions of the equation

$$k\theta+q(s)=0,$$

[there are two solutions because $q(s)$ is even], with $s\ll y$. These two solutions give rise to the scattered wave, and exist only in the range of scattering angles from 0 to $-q_{\max}/k$. In other words, in the fast-particle approximation, the maximum scattering angle is of order $m/k\ll 1$. The latter is the same as the angle defined by the asymptote of the caustic equation, and it is also the maximum angle in classical scattering. Outside this range, solutions of the equation for the extremum lie in the complex plane, leading to exponential damping of the scattered wave (shadow region). Below we will discuss only the scattered wave.

Calculating the Gaussian approximation to Eq. (39) in the scattering region, we obtain diffraction peaks for the scattered wave at

$$\varphi_{\rm scat}=2\left[\frac{k}{i|q'(s(\theta))|x}\right]^{1/2}\exp(ikx+iky^2/2x)\cos(k\theta s(\theta)+\alpha\mathfrak{M}(s(\theta))-\pi/4) \tag{40}$$

where $s(\theta)$ is the positive root of the equation for the extremum,

$$\alpha\mathfrak{M}(s)=\int_0^s q(t)\,dt,$$

$\mathfrak{M}=\pi$ for $s\gg 1/m$. This is a valid equation when

$$|q''|/|q'|^{3/2}\ll 1,$$

a condition satisfied for large α and solutions not too close to the caustic. The caustics in this approach are the curves which (40) approaches at infinity ($q'\to 0$), the Gaussian approximation then being inapplicable. Near the caustic, the Airy approximation holds.[10,14] The amplitude at the (lower) caustic is given by

$$f(\theta_{max})=2\left(\frac{2k}{i}\right)^{1/2}\left(\frac{2}{|q''(0)|}\right)^{1/3}\phi(0)\left(1+O\left[|q^{\rm IV}_{(0)}|/|q''_{(0)}|^{5/3}\right]\right), \tag{41}$$

where $\phi(x)$ is the Airy function, $\phi(0) = 0.629,...$, and for $\alpha \gg 1$ the correction goes as $\alpha^{-2/3}$.

If it is assumed that the mass of the Higgs field forming the cladding of the string is much greater than that of the vector field m, then the region over which the magnetic field falls off is much larger than that cladding. In that region, the magnetic field is

$$\mathscr{H} = m^2 K_0(mr)/e_0$$

and

$$q(s) = \pi\alpha m e^{-m|s|}.$$

Expanding (39) in powers of q, one can obtain the scattering amplitude explicitly in series form. For $\pi = 1$, in which case there is no Bohm-Aharonov scattering,

$$f(\theta) = \frac{1}{m}\left(\frac{k}{2\pi i}\right)^{1/2}\left[\sum_{l=0}^{\infty}\frac{(-1)^l(\pi\alpha)^{2l+1}}{(2l+1)!}\frac{2k\theta/m}{(2l+1)^2+(k\theta/m)^2} + \sum_{l=1}^{\infty}\frac{(-1)^l(\pi\alpha)^l}{(2l)!}\frac{4l}{(2l)^2+(k\theta/m)^2}\right]. \qquad (42)$$

The term linear in the angle is responsible for the asymmetry of interest.

From (42), the optical theorem yields the total cross section, which in the present case takes the field

$$\sigma = (8\pi/k)^{1/2}\mathrm{Im}\, f e^{-i\pi/4}.$$

Straightforward calculation with (42) gives the forward-scattering amplitude:

$$\sigma \approx 4\pi/m. \qquad (43)$$

This cross section falls off with increasing k, like the Lorentz force, leading to particle scattering which itself is proportional to the velocity of the incoming particle. Note that for scattering from the usual short-range potential, the cross section would be down by a factor $1/k^n$.

Equation (43) is consistent with the estimated amplitude of the cross section at the caustic (41), assuming that all scattering takes place at angles between 0 and $\theta_{\max}$.

4. PHYSICAL EFFECTS

We now discuss several effects for which angular asymmetry can turn out to be significant in the scattering of charged particles by a string moving through a plasma or ionized gas. In the rest frame of the string we have some incident particle flux scattered by the string.

The fast-scattering case is applicable to particles that are heavy enough—those whose mass M satisfies the condition

$$\frac{MV\gamma}{m} \gg 1.$$

As we have shown in Sec. 3, this type of scattering in the rest frame of the string engenders a caustic whose focus is at a distance $R = k/m^2$ from the center of the string, at an angle of order m/k from the direction of motion of the incident particle. Interference between the scattered and unscattered waves can be neglected at distances greater than R. The flux density at distance r' from the string then takes the form

$$\mathbf{j} = n'V\hat{x}' + n'V\frac{|f|^2}{r'}\hat{\theta}', \qquad (44a)$$

where $\hat{x}', \hat{\theta}'$ are the unit vectors in the rest frame of the string. The density is determined by the ratio of the absolute value of the flux density to the velocity

$$n' + \delta n' = n'\left(1 + \frac{|f|^2}{r'}\cos\theta'\right), \qquad (44b)$$

where n' is the unperturbed gas density and θ' is the scattering angle, both taken in the rest frame of the string. A Lorentz transformation is required in order to go over into the laboratory coordinate system. The density and flux then transform as a four-vector, and from (44) we have

$$n + \delta n = n\left(1 + \frac{|f|^2}{r'}\cos\theta'\right), \qquad (45)$$

where n is the unperturbed density in the laboratory frame, while r' and θ', which are defined in the rest frame of the string, are related to the lab coordinates r and θ by

$$\mathrm{tg}\,\theta = \gamma\,\mathrm{tg}\,\theta', \quad r\sin\theta = r'\sin\theta'.$$

Note that if in the rest frame of the string the caustic coincided with the direction of motion of the particles, the latter should now have a velocity component in the direction of motion of the string and should cross the caustic (see Fig. 2). If the string is moving, this density nonuniformity extends behind it. The motion of this density inhomogeneity is a typical collective effect (propagation of the density inhomogeneity is not accompanied by motion of the scattered particles in the same direction). For a string moving in plasma, two such nonuniformities can arise, symmetrically disposed about the string's plane of motion. This effect occurs both for motion of a planar segment of a string and for an oscillating loop. In the latter case, (45) may be generalized in a natural way:

$$\delta n/n \sim |f|^2 L/r^2, \qquad (46)$$

where L is the length of the string.

Another interesting effect we wish to discuss here is the charge separation that takes place when a string moves through a plasma. The effect shows up for both fast and slow scattering, i.e., the mass of the plasma particles is immaterial. For the slow-scattering case with $\alpha = 1$, the cross section for particles of one sign is given by Eq. (18), and the charge excess is determined by twice the asymmetric term

$$\frac{d\Delta\sigma}{d\theta} = -\frac{\pi^2 k L_{1/2}}{\ln^2(k/\Lambda) + \pi^2/4}\sin\theta. \qquad (47)$$

Multiplying (47) by the flux nV, where n is the plasma density and V is the string velocity in the laboratory frame we obtain the charge excess accumulated on the upper half-plane per unit string length per unit time:

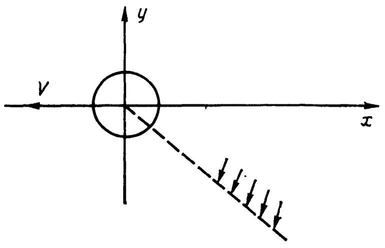

FIG. 2. The dashed line indicates the location of the lower branch of the caustic in the lab frame; arrows indicate the direction of particle motion at the caustic.

$$\frac{d(N_+-N_-)}{dt}=-2\frac{\pi^2 nVkL_{\nu_2}}{\ln^2(k/\Lambda)+\pi^2/4}. \quad (48)$$

The cross section has been calculated in the rest frame of the string, but for slow scattering, relativistic corrections are negligible, and the total excess charge does not depend on the reference frame. As for the angular distribution and velocities of the particles, we have

$$\theta=\theta'/2,\ V=2V'\cos\theta'/2,$$

where the primed variables are measured in the rest frame, and the unprimed variables are measured in the laboratory frame.

For Bohm-Aharonov scattering it is necessary to make use of Eq. (19); instead of (48), we then obtain

$$\frac{d(N_+-N_-)}{dt}=\frac{4\alpha\sin^3\pi\alpha}{\pi k\sin^2\theta/2}\left(\frac{k}{2}\right)^{2(1-\alpha)}L_{\nu_2}\frac{\Gamma(\alpha-1)}{\Gamma(1-\alpha)}nV. \quad (49)$$

Here, in passing to the laboratory frame, small-angle scattering becomes scattering at 90°.

In the fast-scattering case, the total charge excess per unit string length per unit time in the laboratory frame of reference [see (43)]

$$\frac{d(N_+-N_-)}{dt}\sim\frac{\alpha}{m}nV. \quad (50)$$

This charge distribution can lead to the generation of baryon asymmetry, for example when a string passes through an electron-quark plasma. The magnitude of the baryon charge excess generated by the string over its entire history is inversely proportional to the distance from the horizon to the plane of motion of the string.

For a loop oscillating in plasma, there will be oscillations of the sign of the charge in the scattered particle fluxes; in addition, if the different kinds of charge-carriers are also of different mass (as for example in an electron-proton plasma), the loop as a whole will vibrate in a direction perpendicular to the plane of confinement. This results from the fact that the momenta carried by positively- and negatively-charged particles are not equal.

Finally, we note that in odd-P theories, a confined loop in vacuum will acquire an angular momentum perpendicular to the plane of confinement. This effect is associated with the creation of pairs accelerated by a moving string. In a theory with parity violation (left-handed currents), newly created pairs carry unit angular momentum; pairs can only have zero momentum when the chirality of a massive fermion changes (as in the decay[15] $\pi\to\mu\nu$), which provides additional suppression M/E, where E is the energy of the pair. A massless particle (neutrino?) always conveys unit angular momentum. Because of the charge separation induced by a magnetic field, strings are preferentially created with a certain projection of their angular momentum on the axis perpendicular to the plane of the confined loop. Among the possible manifestations of this phenomenon might be the information of a Kerr black hole when a loop disappears beyond the horizon. Note that in that event we are dealing not with a point singularity but a loop singularity,[16] a more natural one for a string, as opposed to a Schwarzschild point singularity, which would be quite natural for particles.

As the present paper went to press, the authors learned of a preprint[17] in which the modified Bohm-Aharonov amplitude (21) was derived for $\alpha > 1$.

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