Frequency-angle distribution of radiation in collisions between relativistic atomic particles

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We obtain expressions for the total emission spectrum and angular distribution of photons in the collision of two structured atomic particles. The relative importance of various emission mechanisms in the formation of the frequency-angle distribution of the photons is elucidated. It is shown that a pronounced role in the total emission spectrum is played by polarization radiation.

INTRODUCTION

The radiation processes that accompany the collision of charged structureless particles (electrons, protons, mesons, etc) are well known (see, e.g., Ref. 1). The traditional mechanism, proposed a long time ago (see Ref. 1), for the production of photons is bremsstrahlung. According to this mechanism photons are emitted in the slowing down of the incident particle in the static field of the atom-target. Relatively recently a principally new mechanism, involving polarization, was proposed in Refs. 2 and 3 for radiation of photons in the collision of nonrelativistic electrons with atoms. This radiation is due to the deformation of the atomic shells by the field of the incident particle and will be referred to in the following as polarization radiation (PR).

The polarization mechanism for radiation has been studied in many papers (see Refs. 4 and 5). In this paper the role of PR is studied for the first time in the formation of the full radiation spectrum in the collision of two relativistic atomic particles, and its frequency and angular dependences are established. To take into account all possible radiation processes that contribute to the full spectrum and determine the angular distribution of the photons is no simple task. Its complexity is due, in the first place, to the fact that one must take into account the contribution of a significant number of different radiation mechanisms, since in the collision of the atoms there occur along with PR other radiation processes accompanied by atom-projectile as well as atom-target excitation or ionization. In the second place, the problem considered here is made significantly more complicated by the fact that the colliding particles are relativistic.

For the solution of the problem posed here it is important to not only know how to calculate the full emission spectrum, but to also know the relative contribution to it of PR, since if that contribution is large it would be possible to study PR experimentally. In this paper it is established that there exist broad regions of frequency ω and photon emission angles θ, where PR, in essence, determines the full radiation cross section. At the same time there exist quite substantial regions where the situation is the opposite.

Before starting on the exposition of the essence of this work we would like to recall the recent research devoted to the study of PR in the collision of relativistic atomic particles. In Refs. 6 and 7 the PR produced by relativistic electrons in scattering off atomic targets was considered. The problem was also solved of the PR in the collision of two structured particles (atoms or ions) proceeding at relativistic velocities.[8,9] In these papers production processes were studied in which at the end of the collision the atoms were left in their ground states. Radiation processes accompanied by the excitation or ionization of the atoms were also considered in the literature. The ordinary bremsstrahlung of nonrelativistic electrons with excitation or ionization of the atom-target has, apparently, been considered for the first time in Ref. 10, see also Ref. 1. The process of resonant PR of the electron on the atom with simultaneous excitation of the latter to some discrete level has been studied in Ref. 11. The problem of calculating the full spectrum of PR in the collision of a fast, relativistic as well as nonrelativistic, structureless particle (electron, proton) with an atom in the full interval of photon frequencies was solved in Refs. 12 and 13, and somewhat later in Ref. 14. In the full spectrum of PR are included all possible final states of the atom. The problem of the full radiation spectrum in the collision of two structured particles, relativistic as well as nonrelativistic, has remained unconsidered until now.

1. THE AMPLITUDE OF "INELASTIC" POLARIZATION RADIATION

Let us consider the collision of a pair of relativistic atomic particles (for definiteness — atoms), in which a photon of momentum **k** and polarization **e** is emitted while the atoms go over into some excited state or become ionized. In the following we shall refer to radiation processes, in which the atoms are excited or ionized at the end of the collision, as "inelastic" PR processes for short (IPR), while radiation without exciting the atoms will be called "elastic" PR (EPR). We construct the amplitude for the process under consideration in lowest order of perturbation theory in the inter-atomic interaction (which reduces to one-photon exchange for relativistic collision velocities) and in the interaction between the atoms and the radiation field.

The Feynman diagrams for the amplitude for the process look as follows:

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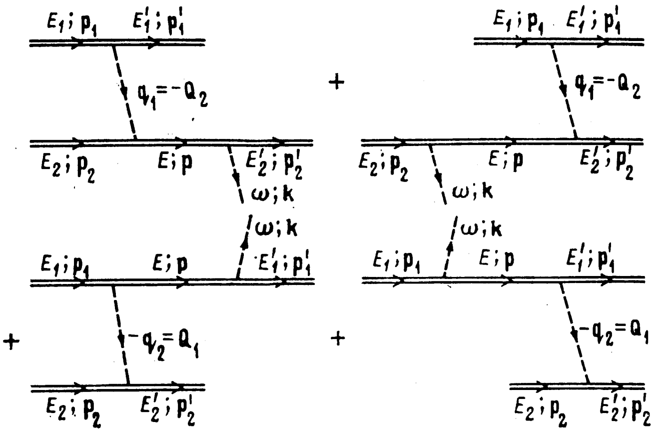

(1)

where $\mathbf{q}_1 = \mathbf{p}_1 - \mathbf{p}_1'$, $\mathbf{q}_2 = \mathbf{p}_2 - \mathbf{p}_2'$, $\mathbf{Q}_1 = \mathbf{p}_1 - \mathbf{p}_1' - \mathbf{k}$, $\mathbf{Q}_2 = \mathbf{p}_2 - \mathbf{p}_2' - \mathbf{k}$.

The double lines in the diagram describe schematically the motion of the atoms. The state of the particles is labeled by two letters, the first of which is the total energy of the particle, and the second — its momentum. The photon exchange processes, as well as the processes of emission of a photon during collision, are denoted by dashed lines. The indices 1 and 2 in the diagrams (1) will in what follows refer to quantities corresponding to the atom-projectile and target respectively.

The diagrams (1) do not take into account exchange effects in the collision of identical atoms. However it is quite natural to ignore these effects when discussing collisions of fast and especially relativistic atoms.

In contrast to Refs. 8 and 9, in which the final states are taken as the ground state only, here we allow at the end of the process for the atoms to be in an excited, as well as the ground, state. The excitation of the atoms as a result of the collision may be connected with a transition of the electron (or electrons) to a discrete level or to the continuum.

The sum of the first two diagrams in (1) is proportional to the product of the 4-current of the atom-projectile and the polarizability 4-tensor of the atom-target, while the rest of (1) is proportional to the 4-current of the target multiplied by the polarizability 4-tensor of the atom-projectile. The calculation of the components of the 4-current and the polarizability 4-tensor of the atom-target is simpler, since it is carried out in the proper reference frame of the atom, where for not too deep atomic shells the nonrelativistic description is applicable. The components of the 4-current and the polarizability 4-tensor of the atom-projectile are found in the laboratory reference frame with the help of Lorentz transformations of these quantities from their proper reference frame, where their calculation presents no difficulties. The introduction of the proper reference frame for the atom-target and the laboratory reference frame for the atom-projectile is possible because the atoms are heavy, and the momentum transfers are small (see Sec. 2), so that the motion of the colliding particles in the processes of elastic as well as inelastic PR may be viewed as rectilinear. To calculate the amplitude for inelastic PR we shall make use, in essence, of the method developed in Refs. 8 and 9. Omitting the intermediate formulas, analogous to those obtained in Refs. 8 and 9 for elastic PR we give right away the expression for the amplitude of inelastic PR, corresponding to the diagrams (1) and expressed in the atomic system of units ($|e| = m_e = \hbar = 1$):

$$F = F_1 + F_2,$$

$$F_1 = \frac{4\pi}{q_1^2 - (\omega + \omega_{m_2 0_2})^2/c^2} S^{(1)}_{m_1 0_1}(\mathbf{q}_1^c)\, \alpha^{(2)}_{m_2 0_2}(\omega, \mathbf{Q}_2) \times \frac{\omega}{c}\left\{\mathbf{e}\mathbf{q}_1 - \frac{\omega}{c}\mathbf{e}\boldsymbol{\beta}\right\}, \tag{2}$$

$$F_2 = \frac{4\pi}{q_2^2 - (\omega_{m_2 0_2})^2/c^2} S^{(2)}_{m_2 0_2}(\mathbf{q}_2)\, \alpha^{(1)}_{m_1 0_1}(\omega^c, \mathbf{Q}_1^c) \times\left\{\frac{\omega^c}{c}\mathbf{e}\mathbf{q}_2^{\perp} + \gamma(\mathbf{e}\boldsymbol{\beta})(\mathbf{k}\mathbf{q}_2^{\perp}) - \gamma^{-2}\frac{\omega^c}{c}\mathbf{e}\mathbf{q}_1^{\parallel}\right\}. \tag{3}$$

The amplitude F_1 describes the radiation of the second atom [the first two diagrams in (1)], while F_2 describes the radiation of the first atom [the third and fourth diagrams in (1)]. Here

$$S^{(1)}_{m_1 0_1}(\mathbf{q}_1^c) = Z_1 \delta_{m_1 0_1} - \left\langle m_1 \left| \sum_{i=1}^{N} \exp(-i\mathbf{q}_1^c \mathbf{r}_i) \right| 0_1 \right\rangle. \tag{4}$$

In the region of large frequencies $\omega^c \gg \omega_{\rm at}$ ($\omega_{\rm at}$ is a characteristic atomic frequency) the expressions for the off-diagonal polarizabilities $\alpha^{(1)}_{m_1 0_1}(\omega^c, \mathbf{Q}_1^c)$ ($\alpha^{(2)}_{m_2 0_2}(\omega, \mathbf{Q}_2)$) are quite simple:

$$\alpha^{(1)}_{m_1 0_1}(\omega^c, \mathbf{Q}_1^c) \cong -\frac{(Z_1 - N_1) S^{(1)}_{m_1 0_1}(\mathbf{Q}_1^c)}{M_1 (\omega^c)^2} - \frac{W^{(1)}_{m_1 0_1}(\mathbf{Q}_1^c)}{(\omega^c)^2}, \tag{5}$$

where

$$W^{(1)}_{m_1 0_1}(\mathbf{Q}_1^c) = \left\langle m_1 \left| \sum_{i=1}^{N_1} \exp(-i\mathbf{Q}_1^c \mathbf{r}_i) \right| 0_1 \right\rangle.$$

In the above formulas $\omega^c = \omega\gamma(1 - \beta\cos\theta)$, θ is the angle between $\mathbf{k}$ and $\mathbf{p}_1$, $\gamma = (1 - \beta^2)^{-1/2}$, $\beta = v_1/c$ (v_1 is the velocity of the first atom, c is the velocity of light), $\omega_{m_1 0_1} = \varepsilon_{m_1} - \varepsilon_{0_1}$ is the frequency of transition between the states $|m_1\rangle$ and $|0_1\rangle$ of the first atom, M_1, Z_1, N_1 are respectively its mass, nuclear charge and number of electrons. The index "c" in Eqs. (2)–(5) and in the following indicates that the corresponding quantity is evaluated in the proper frame of reference of the atom-projectile. Thus, the momenta $\mathbf{q}_1^c$, $\mathbf{Q}_1^c$ are equal to

$$\mathbf{q}_1^c = \gamma^{-1}\mathbf{q}_1^{\parallel} + \frac{\omega_{m_1 0_1}}{c}\boldsymbol{\beta} + \mathbf{q}_1^{\perp} \approx \gamma^{-1}\mathbf{q}_1^{\parallel} + \mathbf{q}_1^{\perp},$$

$$\mathbf{Q}_1^c = \frac{\omega^c}{v_1}\frac{\boldsymbol{\beta}}{\beta} + \frac{\omega_{m_1 0_1}}{c}\boldsymbol{\beta} - \mathbf{k}^{\perp} + \mathbf{q}_1^{\perp} \approx \frac{\omega^c}{v_1}\frac{\boldsymbol{\beta}}{\beta} + \mathbf{q}_1^{\perp} - \mathbf{k}^{\perp}$$

for $\omega^c \gg \omega^c_{\rm at}$. In deriving Eqs. (2) and (3) we made use of the condition $q_1^c \gg \omega_{m_1 0_1}/c$, valid for $\omega^c \gg \omega^c_{\rm at}$.

The two terms in Eq. (5) for $\alpha^{(1)}_{m_1 0_1}$ ($\alpha^{(2)}_{m_2 0_2}$) are of a different nature. The first arises in taking into account the radiation of the first (second) atom as it accelerates as a whole in the field of the partner. The second term in $\alpha^{(1)}_{m_1 0_1}$ ($\alpha^{(2)}_{m_2 0_2}$) results from taking into account the polarization of the atom in the collision process. For neutral ($Z_{1,2} = N_{1,2}$) or heavy atoms ($M_{1,2}\gg 1$) the first term in $\alpha^{(1)}_{m_10_1}$ ($\alpha^{(2)}_{m_20_2}$) may be ignored, as a result of which these quantities reduce to the generalized dynamical polarizability of the first (second) atom in the region of high frequencies. On the other hand, in the case of a structureless light particle (electron, positron) only the first term remains in $\alpha^{(1)}_{m_10_1}$ ($\alpha^{(2)}_{m_20_2}$).

In the region of nonrelativistic collision velocities, when $\beta\ll 1$, the total amplitude F_1+F_2 goes over into its nonrelativistic limit:

$$F=\frac{4\pi e q_1}{\omega c q_1^2}\left(\frac{Z_1-N_1}{M_1}-\frac{Z_2-N_2}{M_2}\right)S^{(1)}_{m_10_1}(\mathbf{q}_1)S^{(2)}_{m_20_2}(-\mathbf{q}_1)$$
$$-\frac{4\pi e q_1}{\omega c q_1^2}\left(Z_1\delta_{m_10_1}W^{(2)}_{m_20_2}(-\mathbf{q}_1)-Z_2\delta_{m_20_2}W_{m_10_1}(\mathbf{q}_1)\right). \tag{6}$$

The expression (6) serves as the generalization of the corresponding formula for the amplitude of EPR in the collision of two nonrelativistic fast atomic particles[15,16] to the case of an arbitrary final state and goes over into it for $|m_1\rangle=|0_1\rangle$, $|m_2\rangle=|0_2\rangle$. The above formulas permit the determination of the total PR cross sections in the case of atom-atom collisions, as well as in the case when ions participate.

2. TOTAL CROSS SECTION OF POLARIZATION RADIATION

The PR differential cross section is related to its amplitude by

$$\frac{d^5\sigma}{d\omega\, d\Omega\, dq_1\, d\varphi_{q_1}}=\frac{\omega q_1}{(2\pi)^4 c v_1^2}|F|^2. \tag{7}$$

Substituting in Eq. (7) the PR amplitudes found in Sec. 1 [relations (2) and (3)], summing the resultant expression over the photon polarizations and then over the final states of the atoms $\langle m_1|$, $\langle m_2|$ and integrating over φ_{q_1} — the azimuth angle of the vector $\mathbf{q}_1$ in the coordinate system with the z-axis along $\mathbf{v}_1$, we obtain the following expression for the total radiation spectra of the target and the projectile:

$$\frac{d^3\sigma^t}{d\omega\, d\Omega}=\sum_{m_1,m_2}\int_{(q_1^{m_1,m_2})_{min}}^{(q_1^{m_1,m_2})_{max}}\frac{d^4\sigma^{M}_{m_1,m_2}}{d\omega\, dq_1\, d\Omega}dq_1,$$
$$\frac{d^3\sigma^P}{d\omega\, d\Omega}=\sum_{m_1,m_2}\int_{(q_2^{m_1,m_2})_{min}}^{(q_2^{m_1,m_2})_{max}}\frac{d^4\sigma^{CH}_{m_1,m_2}}{d\omega\, d\Omega\, dq_2}dq_2, \tag{8}$$

where

$$\frac{d^4\sigma^t}{d\omega\, dq_1\, d\Omega}=\frac{1}{\pi c^3 v_1^2\omega}q_1\left[q_1^2-\frac{(\omega+\omega_{m_20_2})^2}{c^2}\right]^{-2}$$
$$\times\left\{(q_1^\perp)^2(1+\cos^2\theta)+2\sin^2\theta\left(q_1^\parallel-\beta\frac{\omega}{c}\right)^2\right\}$$
$$\times|(Z_1\delta_{m_10_1}-W^{(1)}_{m_10_1}(\mathbf{q}_1^c))W^{(2)}_{m_20_2}(-\mathbf{q}_1)|^2 \tag{9}$$

$$\frac{d^4\sigma^P_{m_1,m_2}}{d\omega\, dq_2\, d\Omega}=\frac{1}{\pi c^3 v_1^2\omega}\left(\frac{\omega}{\omega^c}\right)^2\{(q_2^\perp)^2(1+\cos^2\theta^c)$$
$$+2(q_2^\parallel)^2\sin^2\theta^c\}$$
$$\times q_2\left[q_2^2-\frac{(\omega_{m_20_2})^2}{c^2}\right]^{-2}|[Z_2\delta_{m_20_2}-W^{(2)}_{m_20_2}(\mathbf{q}_2)]W^{(1)}_{m_10_1}(\mathbf{Q}_1^c)|^2,$$
$$\mathbf{q}_1^c\approx\gamma^{-1}\mathbf{q}_1^\parallel+\mathbf{q}_1^\perp,\quad \mathbf{Q}_1^c\approx\frac{\omega^c}{v_1}\frac{\beta}{\beta}-\mathbf{q}_2^\perp=-\gamma\mathbf{q}_2^\parallel-\mathbf{q}_2^\perp. \tag{10}$$

Here $d^3\sigma^t/d\omega d\Omega$, $d^3\sigma P/d\omega d\Omega$ ($d\Omega$ is the photon emission solid angle) are respectively the total cross sections of the polarization radiation of the target and the projectile. The interference term in the cross section $d^3\sigma^t/d\omega d\Omega$ has been omitted in the intermediate transformations. The upper and lower limits of integration $(q_{1,2}^{m_1,m_2})_{\max}$ and $(q_{1,2}^{m_1,m_2})_{\min}$ in Eq. (8) are found by energy conservation. Thus

$$(q_1^{m_1,m_2})_{min}=(\omega+\omega_{m_10_1}+\omega_{m_20_2})/v_1,$$
$$(q_2^{m_1,m_2})_{min}=(q_1^{m_1,m_2})_{min}-\frac{\omega}{c}\cos\theta,$$

and the upper limits

$$(q_1^{m_1,m_2})_{max}=2p_1-(q_1^{m_1,m_2})_{min},$$
$$(q_2^{m_1,m_2})_{max}=2p_1+k-(q_2^{m_1,m_2})_{min}$$

may be replaced by infinity since as will be shown shortly, the main contribution to the total radiation spectrum comes from the nonrelativistic region of momentum transfers and excitation energies of the atoms.

Let us perform the summation over $\langle m_1|$, $\langle m_2|$ in Eq. (8). Here we note that in the region of momentum transfers ($\omega_{m_10_1}\sim q_1^2/2$, $\omega_{m_20_2}\sim q_2^2/2$) of the most significance for the sum one may ignore the quantities $(\omega_{m_10_1}/c)^2$, $(\omega_{m_20_2}/c)^2\sim q_{1,2}^4/c^2\ll q_{1,2}^2$ and $(\omega_{m_10_1}/c)(\omega/c)$, $(\omega_{m_20_2}/c)(\omega/c)\sim q_{1,2}^2\omega/c^2\ll q_{1,2}^2$ in the integrand. As a result we obtain for $d^3\sigma^t/d\omega d\Omega$ the following expression:

$$\frac{d^3\sigma^t}{d\omega\, d\Omega}=\frac{1}{\pi c^3 v_1^2\omega}\left\{\int_{\omega/v_1}^{\infty}\frac{dq_1 q_1}{q_1^2-\omega^2/c^2}|Z_1-W_1(q_1)|^2|W_2(q_1)|^2\right.$$
$$\times\left[1+\cos^2\theta+\frac{(\omega/v_1)^2\gamma^{-2}}{q_1^2-\omega^2/c^2}\left(1-3\cos^2\theta-2\frac{v_1^2}{c^2}\sin^2\theta\right)\right]$$
$$+\int_{\omega/v_1}^{2v_1}\frac{dq_1 q_1}{q_1^2-\omega^2/c^2}[N_1+F_1(q_1)-|W_1(q_1)|^2]|W_2(q_1)|^2$$
$$\times\left[1+\cos^2\theta+\frac{(\omega/v_1)^2\gamma^{-2}}{q_1^2-\omega^2/c^2}\left(1-3\cos^2\theta-2\frac{v_1^2}{c^2}\sin^2\theta\right)\right]$$
$$+\int_{\omega/v_1}^{2v_1}\frac{dq_1 q_1}{q_1^2-\omega^2/c^2}|Z_1-W_1(q_1)|^2[N_2+F_2(q_1)-|W_2(q_1)|^2]$$
$$\times\left[1+\cos^2\theta+\frac{(\omega/v_1)^2\gamma^{-2}}{q_1^2-\omega^2/c^2}\left(1-3\cos^2\theta-2\frac{v_1^2}{c^2}\sin^2\theta\right)\right]$$
$$+\int_{\omega/v_1}^{v_1}\frac{dq_1 q_1}{q_1^2-\omega^2/c^2}[N_1+F_1(q_1)-|W_1(q_1)|^2]$$
$$\times[N_2+F_2(q_1)-|W_2(q_1)|^2]$$
$$\left.\times\left[1+\cos^2\theta+\frac{(\omega/v_1)^2\gamma^{-2}}{q_1^2-\omega^2/c^2}\left(1-3\cos^2\theta-2\frac{v_1^2}{c^2}\sin^2\theta\right)\right]\right\}, \tag{11}$$

where

$$F_{1,2}(q_1)=\left\langle 0_{1,2}\left|\sum_{i\neq j}\exp[i\mathbf{q}_1(\mathbf{r}_i-\mathbf{r}_j)]\right|0_{1,2}\right\rangle,$$

$$W_{1,2}(q_1)=W^{(1,2)}_{0_{1,2},0_{1,2}}(q_1).$$

In writing Eq. (11) we have replaced the argument of the functions $F_1(q_1^c)$ and $W_1(q_1^c)$ by q_1, which is correct in the region $(\omega/v_1)\gamma^{-1}R_{at}\sim(\omega/c)\gamma^{-1}R_{at}\ll 1$ (R_{at} is the size of the atom) or if the condition $q_1^\perp \gg q_1^\parallel$ is satisfied, when $\exp(i\mathbf{q}_1^\parallel\gamma^{-1}\mathbf{r}_j)\approx 1$.

The expression that we have obtained consists of a sum of four terms (integrals) of different nature. The first integral takes into account the process of EPR of the target without exciting the projectile. The second term in Eq. (11) describes the contribution of processes of EPR of the target with simultaneous excitation or ionization of the atom-projectile. The upper limit of integration in this term may be replaced by infinity, since the main contribution to the integral comes from the region $q_1\sim R_{at}^{-1}$. Outside this region either $|W_2(q_1)|^2\approx 0$ ($q_1R_{at}\gg 1$), or $N_1+F_1(q_1)-|W_2(q_1)|^2\approx 0$ ($q_1R_{at}\ll 1$). In the third term in Eq. (11) we take into account processes of IPR of the target with the projectile unexcited. And, finally, in the last term are summed processes of IPR of the target with simultaneous excitation of the atom-projectile. The appearance of different limits of integration over q_1 in the third and fourth terms in Eq. (11) is connected with the fact that in the first case we are taking into account radiation processes by target electrons in collision with the heavy particle-projectile, while in the second case radiation processes by target electrons in collision with the projectile electrons only. The difference in the reduced mass of the colliding particles in these two cases is the reason for the difference in the values $q_{1_{max}}=2v_1$ and $q_{1_{max}}=v_1$.

The calculation of the sum over $|m_1\rangle$, $|m_2\rangle$ in the cross section $d^3\sigma P/d\omega d\Omega$ [see Eqs. (8), (10)] is performed analogously to what was done above in the derivation of $d^3\sigma^t/d\omega d\Omega$ [see Eq. (11)] and gives rise to the following result:

$$\frac{d^3\sigma^P}{d\omega\, d\Omega}=\frac{1}{\pi c^3v_1^2\omega}\left(\frac{\omega}{\omega^c}\right)^2\left\{\int_{\omega^c/\gamma v_1}^{\infty}\frac{dq_2}{q_2}|Z_2-W_2(q_2)|^2|W_1(q_2)|^2\right.$$
$$\times\left[1+\cos^2\theta^c+(1-3\cos^2\theta^c)\frac{(q_2^{\parallel})^2}{q_2^2}\right]+\int_{\omega^c/\gamma v_1}^{\infty}\frac{dq_2}{q_2}[N_2+F_2(q_2)$$
$$-|W_2(q_2)|^2]|W_1(q_2)|^2\left[1+\cos^2\theta^c+\left(\frac{q_2^{\parallel}}{q_2}\right)^2(1-3\cos^2\theta^c)^2\right]$$
$$+\int_{\omega^c/\gamma v_1}^{2v_1}\frac{dq_2}{q_2}|Z_2-W_2(q_2)|^2[N_1+F_1(q_2)-|W_1(q_2)|^2]$$
$$\times\left[1+\cos^2\theta^c+\left(\frac{q_2^{\parallel}}{q_2}\right)^2(1-3\cos^2\theta^c)\right]$$
$$+\int_{\omega^c/\gamma v_1}^{v_1}\frac{dq_2}{q_2}[N_2+F_2(q_2)-|W_2(q_2)|^2]$$
$$\times[N_1+F_1(q_2)-|W_1(q_2)|^2]\left[1+\cos^2\theta^c\right.$$
$$\left.\left.+\frac{(q_2^{\parallel})^2}{q_2^2}(1-3\cos^2\theta^c)\right]\right\}. \qquad (12)$$

Here, as in Eq. (11), we have ignored the difference between the vectors $\mathbf{Q}_1^c$ and $\mathbf{q}_2$ in the argument of the functions $W_1(Q_1^c)$, $F_1(Q_1^c)$, which is valid provided $(\omega^c/c)R_{at}\ll 1$ or $q_2^\perp\gg q_2^\parallel$.

The first term (integral) in $d^3\sigma P/d\omega d\Omega$ describes EPR of the projectile without exciting the target, the second — EPR of the projectile with simultaneous excitation or ionization of the atom-target. In the third term in Eq. (12) we take into account processes of IPR of the projectile with the target left unexcited, while in the fourth term are summed all possible processes of IPR of the projectile with simultaneous excitation of the atom-target. The choice of the limits of integration over q_2 in Eq. (12) is carried out analogously to what was done in Eq. (11).

The resultant expressions for $d^3\sigma^t/d\omega d\Omega$ and $d^3\sigma P/d\omega d\Omega$ may be simplified noting that the terms proportional to $(\omega^2/v_1^2)\gamma^{-2}(q_1^2-\omega^2/c^2)^{-1}$ and $(q_2^\parallel)^2/q_2^2$ in the integrands of Eqs. (11) and (12) respectively are of the same order as the remaining expressions in the integrand only in the region $q_1\sim q_{1_{min}}$, $q_2\sim q_{2_{min}}$. In the remaining region of integration, i.e., for $q_{1_{max}}>q_1\gg q_{1_{min}}$ in Eq. (11) and for $q_{2_{max}}>q_2\gg q_{2_{min}}$ in Eq. (12), the indicated terms are small and therefore do not contribute significantly to $d^3\sigma^t/d\omega d\Omega$, $d^3\sigma P/d\omega d\Omega$. If we ignore these terms in Eqs. (11) and (12) we obtain the following expressions for the total cross sections of the polarization radiation of the projectile and the target:

$$\frac{d^3\sigma^t}{d\omega\, d\Omega}=\frac{1+\cos^2\theta}{\pi c^3v_1^2\omega}\left\{\int_{\omega/v_1}^{\infty}\frac{dq_1q_1}{q_1^2-\omega^2/c^2}|Z_1-W_1(q_1)|^2|W_2(q_1)|^2\right.$$
$$+\int_{\omega/v_1}^{\infty}\frac{dq_1q_1}{q_1^2-\omega^2/c^2}[N_1+F_1(q_1)-|W_1(q_1)|^2]|W_2(q_1)|^2+$$
$$+\int_{\omega/v_1}^{2v_1}\frac{dq_1q_1}{q_1^2-\omega^2/c^2}|Z_1-W_1(q_1)|^2[N_2+F_2(q_1)-|W_2(q_1)|^2]$$
$$+\int_{\omega/v_1}^{v_1}\frac{dq_1q_1}{q_1^2-\omega^2/c^2}[N_1+F_1(q_1)-|W_1(q_1)|^2]$$
$$\left.\times[N_2+F_2(q_1)-|W_2(q_1)|^2]\right\}, \qquad (13)$$

$$\frac{d^3\sigma^P}{d\omega\, d\Omega}$$
$$=\frac{(1+\cos^2\theta^c)}{\pi c^3v_1^2\omega}\left(\frac{\omega}{\omega^c}\right)^2\left\{\int_{\omega^c/\gamma v_1}^{\infty}\frac{dq_2}{q_2}|Z_2-W_2(q_2)|^2|W_1(q_2)|^2\right.$$
$$+\int_{\omega^c/\gamma v_1}^{\infty}\frac{dq_2}{q_2}[N_2+F_2(q_2)-|W_2(q_2)|^2]|W_1(q_2)|^2$$
$$+\int_{\omega^c/\gamma v_1}^{2v_1}\frac{dq_2}{q_2}|Z_2-W_2(q_2)|^2[N_1+F_1(q_2)-|W_1(q_2)|^2]$$
$$+\int_{\omega^c/\gamma v_1}^{v_1}\frac{dq_2}{q_2}[N_2+F_2(q_2)-|W_2(q_2)|^2]$$
$$\left.\times[N_1+F_1(q_2)-|W_1(q_2)|^2]\right\}. \qquad (14)$$

Given the amplitudes for PR of the projectile and the target [see Eqs. (2) and (3)] one may find the interference term in the total cross section $d^3\sigma^i/d\omega d\Omega$. In the same approximation as was used in the derivation of $d^3\sigma^t/d\omega d\Omega$ and $d^3\sigma P/d\omega d\Omega$ the calculations result in the following:

$$\frac{d^3\sigma^i}{d\omega\, d\Omega} = -\frac{2(1+\cos\theta\cos\theta^c)}{\pi c^3 v_1^2\omega}$$

$$\times\frac{\omega}{\omega^c}\Big\{\int\limits_{\omega^c/\gamma v_1}^{\infty}\frac{dq_2}{q_2}[Z_1 - W_1(q_2)]W_1(q_2)$$

$$\times[Z_2 - W_2(q_2)]W_2(q_2)$$

$$-\int\limits_{\omega^c/\gamma v_1}^{\infty}\frac{dq_2}{q_2}[Z_1 - W_1(q_2)]W_1(q_2)[N_2 + F_2(q_2)$$

$$-|W_2(q_2)|^2] - \int\limits_{\omega^c/\gamma v_1}^{\infty}\frac{dq_2}{q_2}[N_1 + F_1(q_2) - |W_1(q_2)|^2]$$

$$\times[Z_2 - W_2(q_2)]$$

$$\times W_2(q_2) + \int\limits_{\omega^c/\gamma v_1}^{v}\frac{dq_2}{q_2}[N_1 + F_1(q_2) - |W_1(q_2)|^2]$$

$$\times[N_2 + F_2(q_2) - |W_2(q_2)|^2]\Big\}. \qquad (15)$$

The above expressions for the total cross sections show that the angular distributions of the radiation of the projectile and the target are principally different in the region of sufficiently large velocities. For the target it has the form $d^3\sigma^t/d\omega d\Omega \propto (1+\cos^2\theta)$, i.e., close to the isotropic distribution. On the contrary, the radiation of the projectile is concentrated in the narrow interval of angles $\theta \lesssim \gamma^{-1}$ near the direction of motion, since the angular dependence of the cross section contains a singularity for small θ:

$$\frac{d^3\sigma^{cn}}{d\omega\, d\Omega} \propto \frac{1+\cos^2\theta}{\gamma^2(1-\beta\cos^2\theta)^2}$$

Upon comparison of the cross sections (13) and (14) it is not hard to see that in the region $\theta \gg \gamma^{-1}$ ($\omega^c \gg \omega_{at}$) the radiation intensity of the target exceeds by a factor γ^2 the radiation intensity of the projectile. On the other hand, for small angles of emission of the photon $\theta \ll \gamma^{-1}$ the emission of the projectile is more intense by a factor of γ^2. We see thus that the emission of high frequencies of each of the atoms (ions) of the pair is concentrated in its own angular region. The interference between the radiations from the projectile and the target is important only in the intermediate region $\theta \sim \gamma^{-1}$.

Let us illustrate with ion-ion collision as the example the role of the coherent radiation of the electrons in the process of EPR. To this end we replace $W_{1,2}(q)$ by $N_{1,2}$ and $(N_{1,2} + F_{1,2}(q) - |W_{1,2}(q)|^2)$ by zero everywhere in the cross sections (13)–(15) in the region of small momentum transfers $q_{1,2} \lesssim R_{at}^{-1}$, while in the integrals over $q_{1,2} \gg R_{at}^{-1}$ we use the approximate relations $W_{1,2}(q) \approx 0$, $N_{1,2} + F_{1,2}(q) - |W_{1,2}(q)|^2 \approx N_{1,2}$. Further, integrating (13)–(15) and keeping only logarithmically large terms we obtain under the conditions $q_{1_{min}} \ll R_{at}^{-1}$, $q_{2_{min}} \ll R_{at}^{-1}$ the following expressions for the differential cross sections:

$$\frac{d^3\sigma^t}{d\omega\, d\Omega} = \frac{1+\cos^2\theta}{\pi c^3 v_1^2\omega}\Big\{(Z_1-N_1)^2N_2^2\ln\frac{v_1\gamma}{\omega R_{at}}$$

$$+ Z_1^2N_2\ln(2v_1R_{at}) + N_1N_2\ln(v_1R_{at})\Big\}, \qquad (16)$$

$$\frac{d^3\sigma^p}{d\omega\, d\Omega} = \frac{1+\cos^2\theta^c}{\pi c^3 v_1^2\omega}\left(\frac{\omega}{\omega^c}\right)^2\Big\{(Z_2-N_2)^2N_1^2\ln\frac{v_1\gamma}{\omega^c R_{at}}$$

$$+ Z_2^2N_1\ln(2v_1R_{at}) + N_1N_2\ln(v_1R_{at})\Big\}, \qquad (17)$$

$$\frac{d^3\sigma^i}{d\omega\, d\Omega} = -\frac{2(1+\cos\theta\cos\theta^c)}{\pi c^3 v_1^2\omega}\frac{\omega}{\omega^c}\Big\{(Z_1-N_1)(Z_2-N_2)$$

$$\times N_1N_2\ln\frac{v_1\gamma}{\omega^c R_{at}} + N_1N_2\ln v_1R_{at}\Big\}. \qquad (18)$$

In the case that $q_{1_{min}} \gg R_{at}^{-1}$, $q_{2_{min}} \gg R_{at}^{-1}$ the expressions (13)–(15) for the cross sections simplify:

$$\frac{d^3\sigma^t}{d\omega\, d\Omega} = \frac{1+\cos^2\theta}{\pi c^3 v_1^2\omega}\Big\{Z_1^2N_2\ln\frac{2v_1^2\gamma}{\omega} + N_1N_2\ln\frac{v_1^2\gamma}{\omega}\Big\}, \qquad (19)$$

$$\frac{d^3\sigma^p}{d\omega\, d\Omega} = \frac{1+\cos^2\theta^c}{\pi c^3 v_1^2\omega}\left(\frac{\omega}{\omega^c}\right)^2$$

$$\times\Big\{Z_2^2N_1\ln\frac{2v_1^2\gamma}{\omega^c} + N_1N_2\ln\frac{v_1^2\gamma}{\omega^c}\Big\}, \qquad (20)$$

$$\frac{d\sigma^i}{d\omega\, d\Omega} = -\frac{2(1+\cos\theta\cos\theta^c)}{\pi c^3 v_1^2\omega}\frac{\omega}{\omega^c}N_1N_2\ln\frac{v_1^2\gamma}{\omega^c}$$

$$v_1R_{at}^{-1} \ll \omega \ll c^2,\quad \gamma v_1R_{at}^{-1} \ll \omega^c \ll c^2. \qquad (21)$$

The first term in the braces in expression (16) describes the contribution of EPR of the target without excitation of the projectile. The second and third terms in that cross section represent respectively the contribution of processes of IPR of the target without and with excitation of the projectile. The meaning of the terms in the braces in Eq. (17) is analogous to that in Eq. (16) after exchanging the words target and projectile.

Let us elucidate the relative roles of the processes of EPR and IPR in the total cross section of the polarization radiation. To this end we form the ratio of the contributions of EPR and IPR for the target and projectile respectively. Making use of Eqs. (16) and (17) we obtain

$$\xi_M = \frac{(Z_1-N_1)^2N_2\ln(v_1\gamma/\omega R_{at})}{Z_1^2\ln(2v_1R_{at}) + N_1\ln(v_1R_{at})}$$

$$\lessapprox \frac{(Z_1-N_1)^2N_2\ln(v_1\gamma/\omega R_{at})}{Z_1^2\ln(v_1R_{at})}, \qquad (22)$$

$$\xi_{CH} = \frac{(Z_2-N_2)^2N_1\ln(v_1\gamma/\omega^c R_{at})}{Z_2^2\ln(2v_1R_{at}) + N_2\ln(v_1R_{at})}$$

$$\lessapprox \frac{N_1(Z_2-N_2)^2\ln(v_1\gamma/\omega^c R_{at})}{Z_2^2\ln(v_1R_{at})}. \qquad (23)$$

The quantity ξ_t characterizes the relative role of elastic and inelastic processes in the full radiation spectrum in the angular region $\theta \gg \gamma^{-1}$, where the radiation by the target dominates. In the region $\theta \ll \gamma^{-1}$ the radiation by the projectile is more intense and the relative role of EPR and IPR in the full radiation spectrum is described by Eq. (23).

The ratios (22) and (23) demonstrate the increased role of the coherent radiation of the electrons in the process of PR at relativistic velocities. For example, in the collision of a pair of identical or nearly identical ions ($N_1 \approx N_2$) with charges $(Z_2 - N_1) \approx Z_1 \gg N_1$ and $(Z_2 - N_2) \approx Z_2 \gg N_2$ it follows from (22) and (23) that EPR dominates in fact in the entire region of angles θ, since $\xi_t \gtrsim N_2 \gg 1$, $\xi_p \gtrsim N_1 \gg 1$.

For comparison we recall that at nonrelativistic velocities the EPR of identical ions is strongly suppressed due to the destructive interference of the radiation of the projectile and the target, while processes of IPR proceed with quite significant probability.

Sometimes the domination by EPR takes place only in one of the ratios ξ_t and ξ_p, i.e., either only in the region of large angles $\theta \gg \gamma^{-1}$ or only small angles $\theta \ll \gamma^{-1}$. Indeed, in the case that $Z_1 - N_1 \sim 1$, $Z_2 - N_2 \sim 1$, $Z_1 \gg Z_2$ we have $\xi_t \sim N_2/Z_1^2 \ll 1$ while $\xi_p \sim N_1/Z_2^2 \gg 1$, i.e., in the radiation of the projectile in the region of small angles $\theta \ll \gamma^{-1}$ the EPR dominates while in the radiation of the target for $\theta \gg \gamma^{-1}$ IPR processes are prevalent.

One must not use values of ξ_t and ξ_p given by Eqs. (22) and (23) to estimate the relative role of EPR and IPR in the frequency region $\omega \lesssim \omega_{at}$. In that case the contribution of EPR to the total radiation cross section is described by the formulas obtained in Refs. 8 and 9. The contribution of IPR, on the other hand, remains to logarithmic accuracy the same as before, since these processes proceed mainly at large momentum transfers, when the atomic electrons radiate as if they were free. In the region $\omega \sim \omega_{at}$ the role of EPR may be enhanced by the fact that at these frequencies atomic polarizabilities are especially large, and the EPR cross sections are expressed in terms of them.

Beginning with Eqs. (13) and (14) it is easy to verify that the role of EPR in $d^3\sigma/d\omega d\Omega$ increases quite noticeably also in the case of collisions of relativistic neutral atoms as a result of coherence of the electron radiation for EPR and the change in the angular distribution of the atom-projectile's radiation as compared to the nonrelativistic case.

3. THE TOTAL RADIATION SPECTRUM

By integrating the cross sections (13)–(21) one may obtain the corresponding expressions for the total radiation spectra. For ion-ion collisions the spectra were found in the logarithmic approximation:

$$\frac{d\sigma^t}{d\omega} = \frac{16}{3c^3 v_1^2 \omega}\Big\{ (Z_1 - N_1)^2 N_2^2 \ln\frac{v_1\gamma}{\omega R_{at}} + Z_1^2 N_2 \ln(2v_1 R_{aT}) + N_1 N_2 \ln(v_1 R_{at})\Big\}, \tag{24}$$

$$\frac{d\sigma^p}{d\omega} = \frac{16}{3c^3 v_1^2 \omega}\Big\{ (Z_2 - N_2)^2 N_1^2 \ln\frac{v_1\gamma^2}{\omega R_{at}}(1+\beta) + Z_2^2 N_1 \ln(2v_1 R_{aT}) + N_1 N_2 \ln(v_1 R_{at})\Big\}, \tag{25}$$

$$\frac{d\sigma^i}{d\omega} = -\frac{16}{c^3 v_1^2 \omega}\frac{1}{\beta^2\gamma}\Big(1 - \frac{\ln\gamma(1+\beta)}{\beta\gamma^2}\Big) \times\Big\{ (Z_1 - N_1)(Z_2 - N_2) N_1 N_2 \ln\frac{(1+\beta)v_1\gamma^2}{\omega R_{at}} + N_1 N_2 \ln v_1 R_{at}\Big\}, \tag{26}$$

$w_{at}\gamma \ll \omega \ll v_1 R_{at}^{-1}$.

The total radiation spectra corresponding to Eqs. (19)–(21) have the following form (for $\gamma^2 v_1 R_{at}^{-1} \ll \omega \ll c^2$):

$$\frac{d\sigma^t}{d\omega} = \frac{16}{3c^3 v_1^2 \omega}\Big\{ Z_1^2 N_2 \ln\frac{2v_1^2\gamma}{\omega} + N_1 N_2 \ln\frac{v_1^2\gamma}{\omega}\Big\}, \tag{27}$$

$$\frac{d\sigma^p}{d\omega} = \frac{16}{3c^3 v_1^2 \omega}\Big\{ Z_2^2 N_1 \ln\frac{2(1+\beta)v_1^2\gamma^2}{\omega} + N_1 N_2 \ln\frac{(1+\beta)v_1^2\gamma^2}{\omega}\Big\}, \tag{28}$$

$$\frac{d\sigma^i}{d\omega} = -\frac{16}{c^3 v_1^2 \omega}\frac{1}{\beta^2\gamma}\Big[1 - \frac{1}{\beta\gamma^2}\ln(\gamma(1+\beta))\Big] \times N_1 N_2 \ln\frac{(1+\beta)v_1^2\gamma^2}{\omega}. \tag{29}$$

The structure of the spectra, Eqs. (24)–(29), is similar to the structure of the differential cross sections, Eqs. (16)–(21). The interference term in the total spectrum is smaller by a factor γ than the contributions to it of the projectile and target radiations, which appears natural in view of what has been said above about the angular distribution of the radiation. The argument of one of the logarithms in $d\sigma P/d\omega$ [see Eq. (25)] has been changed as compared to the cross section (17)— in it appears now the large quantity $\gamma(1+\beta)$. This change favors enhancement EPR of the projectile in the background of IPR. The relative role of the EPR and IPR processes in the total radiation spectrum is determined by relations analogous to Eqs. (22) and (23). By making use of Eqs. (16)–(21) in the case of $\gamma \gg 1$ it is not hard to write down formulas for the total radiation spectrum also in the intermediate region $v_1 R_{at}^{-1} \ll \omega \ll \gamma^2 v_1 R_{at}^{-1}$.

The quantity R_{at}, characterizing the atom, has completely disappeared from the spectra, Eqs. (27)–(29). This is not an accident. In the region $\omega \gg \gamma^2 v_1 R_{at}$ the minimal momentum transfers q_1 and q_2 are much larger than the corresponding characteristic atomic quantities, proportional to R_{at}^{-1}. The frequencies of the photons under study are also much larger than ω_{at}. Under these conditions the total spectrum of PR is due to inelastic processes of radiation by effectively free electrons of each of the ions on the charge of the nucleus and electrons of its partner. The radiation spectrum of an ultrarelativistic electron (recoil electron) in collision with a heavy particle at rest (ultrarelativistic), as well as the radiation spectra of an ultrarelativistic electron and recoil electron in the collision of two electrons, are well known.[1] In ion-ion collisions the total radiation spectra of projectile and target in the region $\gamma^2 v_1 R_{at} \ll \omega$ should represent the sum of the above-mentioned radiation spectra with coefficients determined by the number N_1, N_2 of the electrons in the ions and the nuclear charges Z_1, Z_2, as is the case in Eqs. (27)–(29). Thus, e.g., $d\sigma^t/d\omega$ represents the sum of N_2 radiation cross sections of a recoil electron in collision with an ultrarelativistic heavy particle of charge Z_1 and $N_1 N_2$ radiation cross sections of a recoil electron in collision with an ultrarelativistic electron. The cross section $d\sigma P/d\omega$ may be interpreted analogously. The correct asymptotics of the resultant total spectrum, which permits the passage to the problem of bremsstrahlung of an ultrarelativistic electron on an electron and a heavy particle, confirms that the above-assumed nonrelativistic description of the ionized electron in IPR processes is correct, at least in the logarithmic approximation.

The structure of the expressions (27) and (28) suggests a simple way in which they could be made more precise and extended into the region of relativistic frequencies $\omega \gtrsim c^2$. To this end it is necessary to replace in Eqs. (27) and (28) the radiation spectra of the ultrarelativistic electron and the recoil electron, calculated in the logarithmic approximation, by the corresponding exact expressions for the spectra which include, naturally, also the relativistic frequency region $\omega \gtrsim c^2$. The properties of the atoms in the total radiation spectra will, as before, be determined only by the numbers N_1, N_2, Z_1, Z_2.

The simple relation established between the obtained PR cross sections and the cross sections for ordinary bremsstrahlung of an ultrarelativistic electron on an electron in the region of high frequencies of the photon demonstrates that the radiation intensity in the collision of two heavy relativistic atomic particles may exceed by many factors the radiation intensity of an ultrarelativistic electron, or even a heavy particle of charge Z, in collision with an electron. For example, if the radiation cross section of an ultrarelativistic electron on an electron for $\gamma = 10^2$, $\omega = 10^3$ $(27.21\cdot 10^3$ eV) equals $\omega d\sigma/d\omega \approx 4.2\cdot 10^{-26}$ cm^2, and the cross section of the recoil electron is $\omega d\sigma/d\omega \approx 2.5\cdot 10^{-26}$ cm^2, then the radiation cross section of a helium atom on krypton for the same values of γ and ω equals $\omega d\sigma^{\rm He}/d\omega \approx 1.1\cdot 10^{-22}$ cm^2, $\omega d\sigma^{\rm Kr}/d\omega \approx 5.3\cdot 10^{-24}$ cm^2. The interference term is $\omega d\sigma^i/d\omega \approx 2.9\cdot 10^{-26}$ cm^2.

The expressions for the total spectrum and cross section of polarization radiation obtained in this work generalize the corresponding formula of the nonrelativistic theory[12,15,16] and go over into them in the limit of nonrelativistic velocities, when $\beta \ll 1$.

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