Coercivity of a Bloch line

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The motion of a Bloch line along a domain wall in a ferromagnet characterized by a high uniaxial anisotropy and the presence of strongly localized microinhomogeneities is investigated using perturbation theory of solitons. An asymptotic dependence of the dynamic coercivity of a Bloch line on its velocity v is found. At high values of v the coercivity is proportional to v^{-1}, whereas at low velocities it is proportional to $v^{-1/2}$. Moreover, the coercivity of a domain wall creates an additional drag force which acts on a Bloch line and reduces its mobility. The dependence of the static coercivity of a Bloch line on the parameters of a material is derived in a model postulating pinning of this line by fluctuations of the number of microinhomogeneities. It is shown that the coercivity of a Bloch line is less sensitive to changes in the density of defects and in the energy of their interaction with the line than the coercivity of a domain wall. This result is used to predict different temperature dependences of the coercivity of a Bloch line and a domain wall on approach to the Curie temperature T_C.

1. Magnetic imperfections of a magnetically ordered crystal associated with fluctuations of the composition, magnetic vacancies, and other factors are known to result in local pinning of topologically stable entities, such as domain walls and the coercivity of these walls.[1–3] This is the reason for the coercivity of Bloch lines in a domain wall. In contrast to the coercivity of a domain wall, which has been the subject of many experimental and theoretical investigations (see, for example, Refs. 4–9), the coercivity of a Bloch line has attracted attention only recently because discrete memory devices have been formed using vertical Bloch lines.[10] A study of the coercivity of a Bloch line is important not only for the determination of the requirements in respect of the homogeneity of materials used in practice. The problem is of interest also from the general physical point of view, because—in contrast to the essentially one-dimensional dynamics of solitons in an inhomogeneous medium (for example, Josephson vortices in a distributed semiconductor junction[11])—we have to allow for the multidimensional nature of fluctuations influencing both a Bloch line and a domain wall in which the line is channeled. However, a detailed analysis of the micromagnetic nature of the coercivity of a Bloch line has not yet been provided, although experimental investigations of the coercivity are proceeding.[10,12]

A Bloch line, which in the simplest case is a two-dimensionally localized object, can move under the influence of a magnetic bias field causing the motion of a domain wall (because of the gyroscopic force), as well as under the influence of an orthogonal (to the bias field) magnetic field which can reverse the magnetization of a domain wall without causing its overall motion. It is therefore possible that the coercivity of a Bloch line will be different in these two cases. Moreover, it is necessary to distinguish the static and dynamic coercivity of a Bloch line. The former is related to the pinning of a Bloch line by an ensemble of magnetic inhomogeneities and is characterized by an unpinning field H_s^{BL}. The latter is associated with the additional dissipation of energy during the motion of a Bloch line in an inhomogeneous medium. The dynamic coercivity of a Bloch line H_d^{BL} depends on the Bloch line velocity v and, in general, because of the inertia we have $H_d^{BL}(v=0)\neq H_s^{BL}$.

We shall now analyze the coercivity of a Bloch line by considering the example of an isolated Bloch line during magnetization reversal in an untwisted domain wall, the periphery of which remains at rest during the reversal process. We shall consider a model of microdefects of size not exceeding the domain wall thickness and characterized by a high density N in the interior of a sample. This model describes satisfactorily the coercivity of a domain wall in "perfect" single-crystal films such as epitaxially grown iron garnet films.[7,8] Moreover, it is assumed that the anisotropy energy exceeds considerably the Winter demagnetization energy, i.e., we shall postulate that $K_u \gg 2\pi M^2$, where K_u is the anisotropy constant and M is the magnetization. These are mainly the materials used currently in studies of Bloch lines.

2. We shall consider a uniaxial ferromagnet in which the z axis coincides with the anisotropy axis and the domain wall is the $y=0$ plane. We shall assume that a magnetic field H_x which reverses the domain wall magnetization is directed along the x axis and a Bloch line separating two subdomains in a Bloch-type domain wall is parallel to the z axis.

The Lagrangian of a ferromagnet corresponding to the Landau–Lifshitz equations is

$$\mathscr{L}=\iiint dx\,dy\,dz\left[\frac{M}{\gamma}\varphi\dot{\theta}\sin\theta-\Phi\right], \qquad (1)$$

where θ and φ are the polar and azimuthal angles measured from the x axis and representing the direction of the magnetization in a crystal; γ is the magnetomechanical ratio; Φ is the thermodynamic potential. In the Winter approximation,[13] we can represent the potential Φ as follows:

$$\begin{aligned}\Phi=&A(1+\varepsilon_A)[\theta_x^2+\theta_y^2+\theta_z^2+\sin^2\theta(\varphi_x^2+\varphi_y^2+\varphi_z^2)]\\&+[K_u(1+\varepsilon_K)+2\pi M^2(1+2\varepsilon_M)\sin^2\varphi]\sin^2\theta\\&-H_zM(1+\varepsilon_M)\cos\theta-H_xM(1+\varepsilon_M)\sin\theta\cos\varphi,\end{aligned} \qquad (2)$$

where A is the exchange interaction constant; $H_z(y)$ is the inhomogeneous bias field which pins a domain wall; $\varepsilon_p=\delta p/p$ is the relative change in the magnetic parameters ($p=A, K_u$, or M).

In the case of an unperturbed medium ($\varepsilon_p=0$) we can use variational equations

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$$\delta\Phi/\delta\theta=0, \quad (3)$$

$$\delta\Phi/\delta\varphi=0 \quad (4)$$

subject to the boundary conditions

$$\begin{aligned} &\theta(y=-\infty)=0,\ \theta(y=+\infty)=\pi,\ \theta_y(y=\pm\infty)=0,\\ &\theta_z(-\infty<z<+\infty)=0,\\ &\varphi(x=-\infty)=0,\ \theta(x=+\infty)=\pi,\ \varphi_x(x=\pm\infty)=0,\\ &\varphi_z(-\infty<z<+\infty)=0, \end{aligned} \quad (5)$$

to formulate the problem of search for a reference solution describing an untwisted domain wall at rest with an isolated Bloch line, parallel to the z axis and also at rest. In the zeroth approximation in terms of a small parameter $Q^{-1}=2\pi M^2/K_u \ll 1$ the solution of Eq. (3) is of the form

$$\theta_0=2\,\mathrm{arctg}\exp\left(\frac{y-q}{\Delta}\right), \quad (6)$$

where $\Delta\approx\Delta_0[1-(\Delta_L{}^2\ \varphi_x^2+\sin^2\ \varphi)/2Q.]$; $\Delta_0=(A/K_u)^{1/2}$ is the domain wall thickness; $\Delta_L=\Delta_0 Q^{1/2}$ is the Bloch line thickness; q is the coordinate of the domain wall center (it is assumed that $q_x, q_z \ll 1$). In this case the angle φ represents the tilt of the magnetization relative to the domain wall plane. The spatial variation of this tilt is described by the second variational equation (4) which is of the first order of smallness in terms of Q^{-1}. The solution of Eq. (4) determines the structure of an unperturbed Bloch line:

$$\varphi^0=2\,\mathrm{arctg}\exp\left(\frac{x-x_L}{\Delta_L}\right). \quad (7)$$

Using the solution (6), we can find from Eq. (1) the variational Lagrangian of a domain wall considered as an infinitely thin elastic membrane with additional degrees of freedom created by the tilt of the magnetization out of the domain wall described by $\varphi(x,z,t)$. This approximation is valid provided the domain wall velocity is not too high, i.e., provided $\partial_t q \ll 4\pi M\gamma\Delta_0$ (adiabatic approximation), and it has been discussed in detail earlier (see, for example, Refs. 14 and 15). We shall use it here to obtain the initial Lagrangian for q and φ allowing for the spatial fluctuations $\varepsilon_p \neq 0$ of the magnetic parameters.

We shall assume that the field of point defects $\{x_i, y_j, z_k\}$ creates fluctuations of the magnetic parameters, which can be represented by

$$\varepsilon_p=\sum_{ijk} a_{ijk}^3\varepsilon_{ijk}^p\delta(x-x_i)\delta(y-y_j)\delta(z-z_k), \quad (8)$$

where $\varepsilon_{ijk}^p=\delta p_{ijk}/p$; $\delta p_{ijk}=\delta A_{ijk}, \delta K_{ijk}$, and δM_{ijk} are the absolute values of fluctuations of the three magnetic parameters in the vicinity of a center of a defect (x_i, y_j, z_k) with the dimensions $a_{ijk}^3=a_i\times a_j\times a_k$.

Subject to these comments, we can transform the initial Lagrangian to

$$\frac{\mathscr{L}}{8\pi M^2\Delta_0\Delta_L{}^2}=L+\delta L, \quad (9)$$

where

$$L=\iint dx\,dz\left[-q\dot\varphi-\frac{1}{2}(\varphi_x{}^2+\varphi_z{}^2+\sin^2\varphi+q_x{}^2+q_z{}^2+b^2q)+h_x\cos\varphi\right], \quad (10)$$

$$\begin{aligned}\delta L=&-\iint dx\,dz\sum_{i,j,k}a_{ijk}^3\Big\{\frac{Q}{4}(\varepsilon_{ijk}^A+\varepsilon_{ijk}^K)+\frac{\varphi_x{}^2}{4}\\ &\times[2\varepsilon_{ijk}^A-(\varepsilon_{ijk}^A+\varepsilon_{ijk}^K)(y_j-q)\,\mathrm{th}(y_j-q)]+\frac{\sin^2\varphi}{4}[\varepsilon_{ijk}^A+2\varepsilon_{ijk}^M\\ &-(\varepsilon_{ijk}^A+\varepsilon_{ijk}^K)(y_j-q)\,\mathrm{th}(y_j-q)]\Big\}\mathrm{ch}^{-2}(y_j-q)\delta(x-x_i)\delta(z-z_k).\end{aligned} \quad (11)$$

Here, $b^2=H_z'\Delta/4\pi M$, H_z is the magnetic field gradient which stabilizes a domain wall, $h_x=H_x/8M$, time is measured in units of $(\gamma 4\pi M)^{-1}$, and the dimensionality of the spatial variables in as follows: $[x]=\Delta_L$, $[y,q]=\Delta_0$, $[a_{i,k}]=\Delta_L$, $[a_j]=\Delta_0$. The Lagrangian (11) does not include terms of the $\varepsilon^M hx$ and $\varepsilon^M\dot q\varphi$ type, since we shall consider later the range of weak magnetic fields ($h_x \ll 1$) and low domain wall and Bloch line velocities ($\dot q$, $vq_x \ll 1$). Moreover, in the case of a weakly dissipative medium we can ignore fluctuations of the magnetic parameters in the dissipation function. Therefore, we shall use the conventional form of this function (see, for example, Ref. 16):

$$R=\frac{\alpha}{2}\iint dx\,dz(\dot q^2+\dot\varphi^2), \quad (12)$$

where α is the magnetic relaxation parameter. We shall henceforth assume that $\alpha \ll 1$.

The use of the Lagrangian (9)–(11) and of the dissipation function (12) leads to the following dynamic equations:

$$\dot\varphi-q_{xx}-q_{zz}+b^2q=-\alpha\dot q+f^{(1)}, \quad (13)$$

$$-\dot q-\varphi_{xx}-\varphi_{zz}+{}^1/_2\sin 2\varphi=-\alpha\dot\varphi-h_x\sin\varphi+f^{(2)}, \quad (14)$$

where

$$f^{(1)}=-\frac{Q}{2}\sum_{i,j,k}a_{ijk}^3(\varepsilon_{ijk}^A+\varepsilon_{ijk}^K)\frac{\mathrm{th}(y_j-q)}{\mathrm{ch}^2(y_j-q)}\delta(x-x_i)\delta(z-z_k) \quad (15)$$

is a random force associated with the change in the energy of a domain wall near magnetic inhomogeneities, whereas

$$\begin{aligned}f^{(2)}=&\sum_{i,j,k}a_{ijk}^3\Big\{\frac{1}{2}[2\varepsilon_{ijk}^A-(\varepsilon_{ijk}^A+\varepsilon_{ijk}^K)(y_j-q)\,\mathrm{th}(y_j-q)]\\ &\times\mathrm{ch}^{-2}(y_j-q)[\varphi_{xx}\delta(x-x_i)+\varphi_x\delta_x(x-x_i)]\\ &-\frac{1}{4}\sin 2\varphi[\varepsilon_{ijk}^A+2\varepsilon_{ijk}^M\\ &-(\varepsilon_{ijk}^A+\varepsilon_{ijk}^K)(y_j-q)\,\mathrm{th}(y_j-q)]\mathrm{ch}^{-2}(y_j-q)\delta(x-x_i)\Big\}\delta(z-z_k)\end{aligned} \quad (16)$$

is a random force to an inhomogeneous variation of the energy of a Bloch line near microdefects. In view of the condition $Q\ll 1$, Eq. (13) is simplified by dropping the less important fluctuation terms.

3. In the following calculations we shall use the smallness of the dissipative and fluctuation parameters (α, $|\delta p/p| \ll 1$) and apply the perturbation theory used in Ref. 16 to provide a compact description of the dynamics of an isolated Bloch line. We shall assume that the zeroth-order solution of the system of equations (13) and (14) is the self-similar solution of a dissipation-free system in the absence of fluctuations when the velocity of a Bloch line is low ($v \ll 1$). Then, in view of the smallness of the parameter b^2 (a typical value of this parameter for iron garnet films is 10^{-1}–10^{-3}), this solution can be described approximately by the expressions

$$\varphi^0 = 2 \operatorname{arctg} \exp(x - x_L), \qquad q^0 = \frac{\pi v}{2b} \exp(-|x - x_L|), \tag{17}$$

where x_L is the position of the Bloch line center and $v = \partial_t x_L$.

Obviously, local fluctuations of the magnetic parameters give rise to fluctuations not only of the position of the Bloch line center in the course of its motion, but they also excite fluctuation-induced flexural vibrations of a domain wall as a result of dynamic bending which accompanies a moving Bloch line. Such fluctuation-induced flexural vibrations of a domain wall ($q_{\rm fl}$, $\varphi_{\rm fl}$), created because of its encounter with microdefects and unrelated fluctuations of the Bloch line center, determine the dynamic coercivity of a domain wall in the absence of a Bloch line. The coercivity field of a domain wall is then governed by the average fluctuation force on the right-hand side of Eq. (12), i.e., $\langle f^{(1)} \rangle = h_k^{\rm DW}$. We shall now write down the equations describing separately the fluctuation-induced vibrations of a domain wall bearing in mind that the main source of these vibrations is the force $f^{(1)}$:

$$\partial_t \varphi_{\rm fl} + (b^2 - \partial_x^2 - \partial_z^2 + \alpha \partial_t) q_{\rm fl} = f^{(1)} - \langle f^{(1)} \rangle, \qquad -\partial_t q_{\rm fl} - (1 - \partial_x^2 - \partial_z^2 + \alpha \partial_t) \varphi_{\rm fl} = 0, \tag{18}$$

where $\langle f^{(1)} \rangle$ is the average force of the fluctuations.

In this case the solution of the initial system of equations (13) and (14) should be sought in the form

$$q = q^0 + q_{\rm fl} + \tilde{q}, \qquad \varphi = \varphi^0 + \varphi_{\rm fl} + \tilde{\varphi}, \tag{19}$$

where $\tilde{q}$ and $\tilde{\varphi}$ are the corrections to the main solution. If we assume slow changes in the coordinate of the Bloch line center with time, i.e., if v, $\partial_t v \ll 1$, and also bear in mind the smallness of $|\varphi_{\rm fl}|$, $|\tilde{\varphi}| \ll \pi$, we find that Eqs. (13) and (14) yield the following equations for the corrections to the zeroth-order solution described by the system (17):

$$\partial_t \tilde{\varphi} + (b^2 - \partial_x^2 - \partial_z^2) \tilde{q} = \alpha v q_x^0 + \langle f^{(1)} \rangle,$$
$$-\partial_t \tilde{q} + (\cos 2\varphi^0 - \partial_x^2 - \partial_z^2) \tilde{\varphi} = \partial_v q \partial_t^2 x_L + \alpha \varphi_x^0 \partial_t x_L - \varphi_x^0 \partial_z^2 x_L - h_x \sin \varphi^0 + 2\varphi_{\rm fl} \sin^2 \varphi + f^{(2)}. \tag{20}$$

Applying the procedure of elimination of the secular terms from the solution of the system of equations (20) by orthogonalization of the right-hand side relative to a vector (q_x^0, φ_x^0), which is the solution of the conjugate homogeneous system, we obtain the following equations for the compact description of a Bloch line in an inhomogeneous medium:

$$\frac{\pi^2}{2b} \partial_t^2 x_L + 2\alpha \partial_t x_L \left(1 + \frac{\pi^2 (\partial_t x_L)^2}{8b}\right) - 2\partial_z^2 x_L - 2h_x = -\pi \varphi_{\rm fl} - F^{(1)} - F^{(2)}, \tag{21}$$

where

$$F^{(1)} = \int_{-\infty}^{+\infty} q_x^0 \langle f^{(1)} \rangle \, dx, \qquad F^{(2)} = \int_{-\infty}^{+\infty} f^{(2)} \varphi_x^0 \, dx.$$

In the case of a film of thickness d $(0 < z < d)$ this equation must generally be supplemented by boundary conditions of the $\partial_z x_L |_{z=0,d} = 0$ type. However, bearing in mind that the Q factor of the vibrations of a Bloch line across a magnetic film is usually small and that the flexural vibrations are usually damped out before they travel from one surface of a film to the other, i.e., assuming that $v_L \tau_L d^{-1} = \Delta_L (\alpha d)^{-1} \ll 1$, where v_L is the velocity of flexural vibrations of a Bloch line and τ_L is the relaxation time of these vibrations, we can ignore the influence of the boundary conditions on the Bloch line dynamics and assume that the motion of a Bloch line is unrestricted.

The first term on the right-hand side of the resultant equation (21) describes the gyroscopic effect of the fluctuation-induced vibrations of a domain wall on a Bloch line. When domain wall is bent only slightly, so that $q_x \ll 1$, this effect is weak and we can ignore it in subsequent analysis. The second term also originates from the fluctuation-induced vibrations of a domain wall, but it results in an accumulation of changes in the average position of the moving center of a Bloch line. It can be regarded as a contribution of the dynamic coercivity of a domain wall to the drag of a Bloch line because of a moving dynamic deflection of a domain wall which accompanies a moving Bloch line. The expression for this additional drag force can be found explicitly if we assume that the domain wall coercivity is independent of the velocity, i.e., if we assume that $\langle f^{(1)} \rangle = h_k^{\rm DW} = \text{const}$. We then have

$$F^{(1)} = \int_{-\infty}^{+\infty} q_x^0 \langle f^{(1)} \rangle \, dx = \frac{\pi v}{b} h_k^{\rm DW}. \tag{22}$$

In general, the domain wall coercivity is a function of the velocity of its motion: $h_k^{\rm DW} = h_k^{\rm DW}(\dot{q})$. At sufficiently high domain wall velocities, the dependence can be found by using the condition of smallness of the amplitude of fluctuations of the domain wall $q_{\rm fl} \ll 1$, and the relationship

$$h_k^{\rm DW} = \left\langle \frac{\partial f^{(1)}}{\partial q} q_{\rm fl} \right\rangle,$$

where $q_{\rm fl}$ is the solution of a linearized system of equations (18) with $q = \dot{q}t$. The velocity dependence of the dynamic domain wall coercivity is discussed, for example, in Ref. 8, so that we shall confine ourselves to a general comment without a detailed discussion of this topic. In general, the additional drag force exerted on a Bloch line and associated with the domain wall coercivity is a nonlinear function of the velocity, which applies particularly to the dynamics of Bloch line clusters creating a large deflection of a domain wall.[17] In the simplest case of a small domain wall deflection it results in renormalization of the linear Bloch line mobility by reducing it.

The last term on the right-hand side of Eq. (21) determines the intrinsic Bloch line coercivity, which is the main contribution if $(7/2\alpha b)\, h_k^{DW} \ll 1$. The field representing the dynamic coercivity of a Bloch line is equal to the average value of the force F^2, i.e.,

$$h_d^{BL} = \left\langle \int_{-\infty}^{+\infty} f^{(2)} \varphi_x^0 \, dx \right\rangle. \tag{23}$$

We can find it explicitly by deriving from Eq. (21) the expression for $x_L\ (t, x_i, y_j, z_k)$ and substituting it in Eq. (23). This is not possible to do in general. We shall therefore consider a simplified situation when the fluctuation-induced vibrations of a Bloch line are weak, i.e., $|\delta x_L| \ll 1$. This is possible if the Bloch line velocity is sufficiently high, so that $|\dot{x}_L| > a^3 (n\langle \varepsilon^2 \rangle)^{1/2} \alpha^{-1}|$ (see below), where $n = N\Delta_0 \Delta_L^2$ is the normalized density of defects and $\langle \varepsilon^2 \rangle$ is the variance of the relative fluctuations of the magnetic parameters. The last requirement is not in conflict with the adopted adiabaticity condition $|\dot{x}_L| \ll 1$, because $\langle \varepsilon^2 \rangle \ll 1$. Then, after linearization of Eq. (21), we obtain the following equation for stochastic vibrations of a Bloch line moving at an average velocity v:

$$\frac{\pi^2 v^2}{2b} \partial_\xi^2 \partial x_L + 2\tilde{\alpha} v \partial_\xi \partial x_L - 2\partial^2_z \delta x_L = -F^{(2)}_{(\xi)}, \tag{24}$$

where

$$\tilde{\alpha} = \alpha\left(1 + \frac{3\pi v^2}{8b} + \frac{\pi h_k^{DW}}{2b\alpha}\right), \quad \xi = vt,$$

and the Bloch line velocity is now given by an equation describing the averages

$$\tilde{\alpha} v = h_x - h_d^{BL}. \tag{25}$$

The dynamic coercivity of a Bloch line is obtained in the second order in respect of the fluctuation interaction from Eq. (23) as follows:

$$h_d^{BL} = {}^1/_2 \langle (G * F^{(2)}) \partial_\xi F^{(2)} \rangle, \tag{26}$$

where $G(\xi, z)$ is the Green function of Eq. (24) and (...*...) denotes a convolution of the functions. Averaging is carried out over the whole field of defects on the assumption of an equiprobable spatial distribution characterized by the Poisson statistics of the number of defects in a bounded volume. Similar statistical properties are exhibited by a pulsed steady-state process with a random number of pulses in a limited time interval (see, for example, Ref. 18). We then find from Eqs. (24) and (26) that

$$h_d^{BL} = \frac{1}{2} \iint G_\mathbf{k} \langle |F_\mathbf{k}^{(2)}|^2 \rangle\, ik\, dk\, dk_z = \frac{b^{1/2} n \langle \varepsilon^2 \rangle \langle \overset{6}{a}_{ijk} \rangle}{2^{15/2} v} \times \int_{-\infty}^{+\infty} \frac{k^{9/2} [(k^2 + k_0^2)^{1/2} + k]^{1/2}}{(k^2 + k_0^2)^{1/2} \,\mathrm{sh}^2(\pi k/2)} dk, \tag{27}$$

where

$$F_\mathbf{k}^{(2)} = (2\pi)^{-2} \iint F^{(2)} \exp[-i(k\xi + k_z z)]\, d\xi\, dz$$

is the Fourier transform of the random force, $G_k = \frac{1}{2}\varkappa\,(k_z^2 \varkappa - k^2 + ik_0 k)^{-1}$ is the Fourier transform of the Green function, $\varkappa = 4b/\pi^2 v^2$, $k_0 = 4\tilde{\alpha} b/\pi^2 v$, and

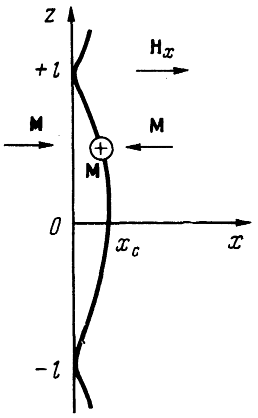

FIG. 1. Pinned segment of a Bloch line $x_L\ (z)$, where x_c is the maximum (critical) deflection of the line when $h_x = h_c^{BL}$.

$$\langle \varepsilon^2 \rangle = \frac{1}{3} \langle (4\overset{M}{\varepsilon}_{ijk} + 5\overset{A}{\varepsilon}_{ijk} - \overset{K}{\varepsilon}_{ijk})^2 \rangle + \langle (\overset{A}{\varepsilon}_{ijk} + \overset{K}{\varepsilon}_{ijk})^2 \rangle \left(\frac{4\pi^2}{45} - \frac{1}{3}\right).$$

The asymptote of the dependence of the Bloch line coercivity on the velocity is as follows:

$$h_d^{BL} = \begin{cases} \dfrac{b^{1/2} n \langle \varepsilon^2 \rangle \langle \overset{6}{a}_{ijk} \rangle}{60\pi^2 v} & \text{if} \quad v \gg 4\tilde{\alpha} b/\pi^2 \\ \dfrac{9!!\, \zeta(^9/_2)\, n \langle \varepsilon^2 \rangle \langle \overset{6}{a}_{ijk} \rangle}{2^{21/2} \pi^4 (\tilde{\alpha} v)^{1/2}} & \text{if} \quad v \ll 4\tilde{\alpha} b/\pi^2, \end{cases} \tag{28}$$

where $\zeta(s)$ is the Riemann zeta function.

At very low velocities the fluctuation-induced vibrations of a Bloch line become very large, since in the limit $v \to 0$ the mean-square fluctuations

$$\langle \delta x_L^2 \rangle = \iint |G_\mathbf{k}|^2 \langle |F_\mathbf{k}^{(2)}|^2 \rangle\, dk\, k_z = \frac{15\zeta(^5/_2)\, n}{2^{17/2} \pi^3 (\tilde{\alpha} v)^{3/2}} \langle \varepsilon^2 \rangle \langle \overset{6}{a}_{ijk} \rangle$$

grow without limit. Therefore, if the velocity obeys

$$v \gtrsim \frac{15^{2/3} \zeta^{2/3}(^5/_2)}{2^{17/3} \pi^2} \frac{(n \langle \varepsilon^2 \rangle \langle \overset{6}{a}_{ijk} \rangle)^{2/3}}{\tilde{\alpha}},$$

the theory ceases to be valid. In this case a more suitable characteristic of the coercivity is the start or unpinning field of a Bloch line.

4. The start field of a Bloch line is governed by the mean-square value of the force of the fluctuation interaction acting on a characteristic Bloch-line pinning length l. This length can be related to the maximum quasistatic reversible displacement of a Bloch line in a segment of length $2l$, pinned at its ends as shown in Fig. 1. It follows from Eq. (21) that a static deflection of a Bloch line can be described by the equation $\partial_z^2 x_L = h_x$, the solution of which subject to the boundary conditions $x_L(l) = 0$ and $x_L(-l) = 0$ makes it possible to find the required relationship

$$l = (2x_c / h_s^{BL})^{1/2},$$

where x_c denotes the maximum reversible deflection of a Bloch line $x_L(z)$. This critical deflection x_c generally depends on the force of the interaction of a Bloch line with point defects, as well as on the density of defects and its variance, and on the "rigidity" of a Bloch line under bending conditions. Numerical and statistical methods were used in Refs. 4–6 to calculate the quasistatic displacement of a filamentary dislocation and a one-dimensionally bent domain wall in the field of strongly localized inhomogeneities occurring under the action of a constant force. Use has been made of the equation

$$\partial_z^2 x_L + h_x = {}^1/_2 F^{(2)}, \qquad (29)$$

which describes also a quasistatic displacement of a Bloch line [see Eq. (21)]. An analysis of the results of the calculations reported in Refs. 4 and 5 shows that the critical deflection x_c of such objects (dislocation and domain wall) considered in the case of pinning by fluctuations of the number of inhomogeneities inside an object is proportional to the thickness of the object, so that $x_c \approx 0.28$. In this case the characteristic pinning length normalized to the length of a Bloch line depends on the coercivity field: $l \approx 0.75\,(h_x^{\rm BL})^{-1/2}$.

The static coercivity of a Bloch line (start field) represents the root of the variance of a random force acting as a segment of length $2l$, i.e.,

$$h_s^{\rm BL} = \frac{1}{2}\left[\left\langle\left(\frac{1}{2l}\int_{-l}^{l} F^{(2)}\,dx\right)^2\right\rangle\right]^{1/2} = \frac{(n\langle\varepsilon^2\rangle\langle \overset{6}{a}_{ijk}\rangle)^{1/2}}{2^{3/2}\cdot 15^{1/2} l^{1/2}}. \qquad (30)$$

The above expression is valid if the characteristic length l does not exceed the thickness of a magnetic film d. In the opposite case, we have to replace l in Eq. (30) with the thickness d. Bearing in mind that l is a function of the coercivity, we can find the Bloch line coercivity for the case when $d \gg l$ from Eq. (30) using a self-consistent approach, such as that employed in Ref. 6 to find the domain wall coercivity.

It therefore follows from Eq. (30) that in the case of weak pinning of a Bloch line by defect clusters, when $d \ll l$ and a Bloch line behaves as a "rigid" entity, we have

$$h_s^{\rm BL} = \frac{n\langle\varepsilon^2\rangle\langle \overset{6}{a}_{ijk}\rangle^{1/2}}{2^{3/2}\cdot 15^{1/2} d}. \qquad (31)$$

In the case of films characterized by $l \ll d$, when a Bloch line bends between microinhomogeneity clusters pinning them, we find that

$$h_s^{\rm BL} \approx 0{,}05\,(n\langle\varepsilon^2\rangle\langle \overset{6}{a}_{ijk}\rangle)^{2/3}. \qquad (32)$$

For the sake of comparison, we shall give the expression for start field of a domain wall, which is obtained from Eqs. (13)–(15) on the basis of similar considerations:

$$h_s^{\rm DW} = \left[\left\langle\left(\mathscr{A}^{-1}\iint_{\mathscr{A}} f^{(1)}\,dx\,dz\right)^2\right\rangle\right]^{1/2}$$

$$\approx (15\pi)^{-1} n\langle\varepsilon_0^2\rangle\langle \overset{6}{a}_{ijk}\rangle Q^2, \qquad (33)$$

where $\mathscr{A} \approx \pi/h_c^{\rm DW}$ is the region of pinning of a domain wall and $\langle\varepsilon_0^2\rangle = \langle(\varepsilon_{ijk}^A + \varepsilon_{ijk}^K)^2\rangle$. A comparison of Eqs. (31) and (32) shows that the nature of the dependences of the coercivity on the density of defects and on the variance of the magnetic parameters is different for a domain wall and a Bloch line and this is due to the different dimensionalities of these two physical objects. Moreover, it follows from a comparison of Eqs. (15) and (16) that in the case of a strongly anisotropic ferromagnet considered in the present paper the difference becomes greater because the local forces representing the interaction of a domain wall and a Bloch line with inhomogeneities are quite different. The coercivity of a domain wall is governed primarily by the anisotropy field and its fluctuations, whereas the coercivity of a Bloch line is governed by the magnetization. This is particularly clear from the dimensional expressions for the coercivity given below.

5. It follows from the above discussion that under dynamic conditions the coercivity of a Bloch line is characterized by additional drag forces related to the coercivity of a domain wall and the intrinsic coercivity of a Bloch line. A compact dimensional description of a Bloch line subject to these forces is:

$$m_L\ddot{x}_L + \frac{m_L\dot{x}_L}{\tau_L}\left(1 + \frac{H_k^{\rm DW}}{8M\alpha b} + \frac{\pi^2\dot{x}_L^2}{8bs^2}\right) - \sigma_L\partial_z^2 x_L$$

$$= 2\pi M\Delta_0(H_x - H_d^{\rm BL}), \qquad (34)$$

where

$$m_L = \pi(4b\gamma^2 Q^{1/2})^{-1}, \quad \tau_L^{-1} = 16\pi^{-1}\alpha b\gamma M,$$
$$\sigma_L = 8AQ^{-1/2}, \quad s^2 = 8\pi A\gamma^2.$$

The dynamic coercivity of a Bloch line then depends on the velocity and creates an additional nonlinearity of its dynamics in accordance with Eq. (28). Moreover, the mobility of a Bloch line decreases considerably under these conditions. We shall demonstrate this by considering a specific example.

Theile and Engemann[10] measured the mobility of Bloch lines in $(\rm YSmBi)_3\cdot(FeGa)_5O_{12}$ films of thickness $2\,\mu$m with a magnetization $4\pi M = 100$ G, a quality factor $Q = 4.5$, $\alpha = 0.086$, and $\gamma = 1.78\times 10^7\ {\rm Oe^{-1}\cdot s^{-1}}$; the value obtained by them was 15 ${\rm m\cdot s^{-1}\cdot Oe^{-1}}$. In the investigated range of the Bloch line velocities, $v \sim 20$ m/s, the Bloch line mobility was a linear function of the velocity and could be described by the usual expression $\mu_0^{\rm BL} = \pi\gamma\Delta_0/2\alpha$, which yielded 70 ${\rm m\cdot s^{-1}\cdot Oe^{-1}}$. However, a calculation carried out using the expression $\mu^{\rm BL} = \mu_0^{\rm BL}(1 + H_h^{\rm DW}/8M\alpha b)^{-1}$, which follows from Eq. (34), gave the value $\mu^{\rm BL} = 25\ {\rm m\cdot s^{-1}\cdot Oe^{-1}}$ on substitution of the domain wall coercivity $H_k^{\rm DW} = 0.7$ Oe given in Ref. 10 and of a domain wall rigidity estimated from the approximate expression $b^2 = 2d\Delta_0/\pi w^2$, where d is the film thickness and $w = 4.8\ \mu$m is the width of a stripe domain. The latter value of the mobility was closer to the experimental results, particularly if we bear in mind that mobility of the Bloch line was measured under transient conditions and the rigidity of a domain wall was estimated only very roughly.

Since essentially the coercivity of a Bloch line and a domain wall have the same origin, there must be a relationship between them which can be found from Eqs. (32) and (33). Before doing this, we shall adopt more usual dimensional forms of these expression:

$$\frac{H_s^{\rm BL}}{8M} \approx 0.05\,\frac{(N\langle\varepsilon^2\rangle\langle \overset{6}{a}_{ijk}\rangle)^{2/3}}{(\Delta_0\Delta_L^2)^{2/3}} \approx 0{,}077\,\frac{(\langle f_{\rm BL}^2\rangle N)^{2/3} Q^{1/6}}{M^2\sigma_L^{1/3}}, \qquad (35)$$

$$\frac{H_s^{DW}}{4\pi M} \approx 0.021 \frac{N\langle\varepsilon_0^2\rangle\langle a_{ijk}^6\rangle Q}{\Delta_0^3} \approx \frac{\langle f_{DW}^2\rangle N}{2\pi^2 M^2 \sigma_0}, \quad (36)$$

where

$$\langle f_{BL}^2\rangle = \frac{4}{15}\left(\frac{2\pi M^2}{\Delta_L}\right)^2 \langle a_{ijk}^6\rangle \langle\varepsilon^2\rangle,$$

$$\langle f_{DW}^2\rangle = \frac{4}{15}\left(\frac{K}{\Delta_0}\right)^2 \langle a_{ijk}^6\rangle \langle\varepsilon_0^2\rangle -$$

are the mean-square forces of the interaction of an isolated defect with a Bloch line and a domain wall, respectively, and $\sigma_0 = 4(AK)^{1/2}$ is the surface energy density in a domain wall. An expression for the domain wall coercivity resembling Eq. (36) was obtained in Ref. 6 and used in Refs. 7 and 9 in an analysis of the experimental results.

The final forms of Eqs. (35) and (36) for the Bloch line and domain wall coercivities are valid in a wider range of conditions, because they apply also to materials with $Q<1$, where Bloch lines were also investigated (see, for example, Refs. 19 and 20).[1)] In this case it is however essential to specify the micromagnetic dependence of the forces of the interaction of a Bloch line or a domain wall with defects. In the $Q \gg 1$ case considered in the present paper the relationship between the pinning forces

$$\langle f_{BL}^2\rangle = \langle f_{DW}^2\rangle Q^{-3}\langle\varepsilon^2\rangle / \langle\varepsilon_0^2\rangle$$

leads to the following relationship between the coercivity fields of a Bloch line and a domain wall:

$$H_s^{BL} \approx BM^{1/3}(H_s^{DW})^{2/3}Q^{-2/3}, \quad (37)$$

where $B = (\langle\varepsilon^2\rangle/\langle\varepsilon_0^2\rangle)^{2/3}$.

We shall now obtain numerical estimates using the results reported in Refs. 10 and 11. We shall assume that the relative magnitude of the fluctuations of the magnetic parameters A, K, and M is approximately the same and we shall postulate that $B = 3$. Then, if $H_s^{DW} = 1$ Oe, $Q = 4.5$, and $4\pi M = 103$ G (Ref. 10), we find from Eq. (37) that $H_s^{BL} = 2$ Oe and if $H_x^{DW} = 2.1$ Oe, $Q = 7.2$, and $4\pi M = 185$ G (Ref. 11), we have $H_s^{BL} \approx 3$ Oe. The experimentally determined values of the start fields of a Bloch line were $H_s^{BL} \lesssim 3$ Oe (Ref. 10) and $H_s^{BL} \lesssim 4$ Oe (Ref. 11), respectively, in agreement with the theoretical estimates. However, it should be pointed out that the assumptions made about the relative magnitudes of the fluctuations of the selected three magnetic parameters are arbitrary and a satisfactory comparison of the theory and experiment requires additional studies, for example, a study of the temperature dependence of the coercivity of a Bloch line and a domain wall.

Our expressions thus establish the dependence of the coercivity of a Bloch line on the magnetic parameters and their fluctuations, and allow us to relate it to the coercivity of a domain wall. It is clear from Eqs. (35) and (36) that the coercivity of a Bloch line is less sensitive to changes in the magnetic parameters and in the pinning forces than is the coercivity of a domain wall. Consequently, the temperature dependences are also different. For example, on approach to the Curie point T_C, when the main contribution to $\langle\varepsilon^2\rangle$ is associated with fluctuations of the demagnetization energy and of the anisotropy, so that

$$\varepsilon \propto \frac{\delta M}{M} \propto \frac{T\delta T_c}{T_c}\left(1-\frac{T}{T_c}\right)^{2\beta-1},$$

where β is the critical index (in the molecular-field approximation this index is $\beta = 0.5$) and δT_C is the amplitude of local fluctuations of the Curie point T_C, and if $K \propto M^2$ (for example, when the uniaxial anisotropy is dominated by the single-ion contribution), the change in the coercivity of a Bloch line occurs in accordance with the law $H_s^{BL} \propto (1 - T/T_C)^{13\beta/3-4/3}$, whereas $H_s^{DW} \propto (1 - T/T_C)^{6\beta-2}$.

In the case of films with an open domain structure we have to allow for the twisted shape of the domain walls because $\langle f_{BL}^2\rangle$ and σ_L vary across the film thickness. However, in the case of clusters with an even number of lines N_L such variation is slight because of the symmetry of the magnetic structure and Eq. (35) is still applicable. We must also bear in mind that if the pinning forces acting on the individual Bloch lines are weaker than the forces of the interaction in a cluster, it follows that $\langle f^2(N_L)\rangle = N_L\langle f^2(1)\rangle$ and $\sigma_L(N_L) = N_L\sigma_L(1)$, so that $H_s^{BL} \propto N_L^{1/3}$.

Undoubtedly the Bloch line coercivity may also be affected by the appearance of Bloch points,[21] which create additional pinning. However, this is outside the scope of the present paper.

[1)] Equation (36) applies to materials with a moderately strong magnetization (see Ref. 6), when $l_{DW}M^2/\sigma_0 \ll 1$, where $l_{DW} = \Delta_L \mathscr{A}^{1/2}$ is the characteristic size of the pinned part of a domain wall. In the opposite case the Bloch line coercivity must be compared with the domain wall coercivity obtained using the model of its one-dimensional deflection.[4,5]

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