W-bosons and the structure of the interface between the *A* and *B* phases of superfluid ^{3}He at low pressures

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An analog of the electroweak interaction in superfluid ^{3}He, leading to a small W-boson mass and the phenomenon of vanishing charge (Landau ghost, in Western terminology), has a substantial influence on the structure of the boundary between the A and B phases at low temperatures and pressures. Under these conditions the strong-coupling corrections are small and there occurs an additional degeneracy of the order parameter in ^{3}He-A. One of the components of the order parameter in ^{3}He-B, which plays the role of the W-boson in ^{3}He-A, penetrates deeply into the A-phase owing to its small mass, thus enlarging the thickness of the domain wall between the A and B phases. Symmetry breaking in the core of the domain wall, which should occur at low pressures, is also considered.

1. INTRODUCTION

The separation boundaries between ^{3}He-A, and ^{3}He-B (the A–B interface) has been vigorously investigated both theoretically and experimentally. The earliest theoretical papers on the determination of the structure of the order parameter within the interface are due to Osherof and Cross,[1,2] and also to Kaul and Kleinert.[3] The interface was considered at high pressures, when the A phase exists in the absence of a magnetic field, and near T_c, where Ginzburg-Landau theory is applicable. An exact solution of the Ginzburg-Landau equations was afterwards obtained by Schopohl[4] and Salomaa.[5] Experiments to measure the speed of propagation of the A–B-interface into the bulk of the supercooled A-phase were carried out in Refs. 6 and 7. This made it possible to estimate the friction force acting on the moving wall from the side of the normal component of the fluid.

The dynamics of the interface, in which a fundamental role is played by the Andreev reflection of quasiparticles on the boundary, was discussed in Refs. 8–12 at intermediate temperatures, outside the domain of applicability of the Ginzburg-Landau theory, as well as at temperatures which are not too close to zero. In this region the friction force does not depend on the structure of the order parameter within the interface wall, since it is determined by the values of the order parameters on both sides of the interface. For lower temperatures new dissipation mechanisms associated with the wall motion may become more important, including the creation of quasiparticles and solitons, such as the interphasons considered by Salomaa.[5] These processes are already determined by the internal structure of the rigid core of the interface, and therefore it is necessary to investigate this structure at low temperatures, in fact for $T = 0$.

Here we do not calculate the structure of the core of the A–B-interface, but discuss some new features of its structure for low temperatures, features which should manifest themselves at low pressures, when the weak coupling approximation becomes applicable. It will be shown that in this range of temperatures and pressures the size of the rigid core of the domain wall increases for two reasons having an exact analogue in elementary particle physics. One of them is the existence of collective boson fields in ^{3}He-A, corresponding to the W-boson field in the theory of the electroweak interactions.[13,14] These modes are almost Goldstone modes; their mass vanishes in the weak coupling approximation, owing to a hidden symmetry in the BCS model, leading to a degeneracy between the A-phase and the planar phase.

The strong-coupling corrections lift the degeneracy and the mass of the W-boson becomes nonzero. This mechanism for mass generation for the W-boson is thus different from the known Higgs mechanism in the electroweak theory (see, e.g., Huang's book, Ref. 15). The strong-coupling corrections are experimentally small at low pressures. As a result of this one of the components of the order parameter which exists in the B-phase and corresponds to the W boson field in the A phase, penetrates deeply into the bulk of the A-phase, thus increasing the size of the domain wall between the B- and A-phases.

Another mechanism leading to an increase in the size of the domain wall is the vanishing-charge (Landau ghost) phenomenon, well known in quantum electrodynamics (Refs. 15,16). This phenomenon consists in logarithmic shielding of the electric charge by the electron-positron vacuum owing to the dielectric vacuum polarization. Because of this effect the W-component of the order parameter in ^{3}He-A decays more slowly than the usual exponential law. The influence of an effect analogous to the vanishing of the charge on the structure of another soliton-like object in ^{3}He-A—the nonsingular quantum vortex—was discussed in Ref. 17.

The A-phase exists for low pressures and temperatures only in a sufficiently strong magnetic field. Therefore we will consider from the outset domain walls in the presence of such a field. In Sec. 3 we consider maximally symmetric A–B-interfaces, and in Sec. 4 we discuss the symmetry-breaking which must occur at low pressures.

2. THE ASYMPTOTIC BEHAVIOR OF *W*-BOSON COLLECTIVE MODES IN THE *A*-PHASE

The bosonic fields in superfluid ^{3}He-A are the deviations $\delta A_{\alpha i} = A_{\alpha i} - A^{(0)}_{\alpha i}$ of the order parameter, defined by a 3×3 matrix $A_{\alpha i}$, from its equilibrium vacuum value

$$A^{(0)}_{\alpha i} = \Delta_A d_\alpha (e_{1i} + ie_{2i}), \qquad (1)$$

where $\mathbf{d}$ is a unit vector along the axis of the magnetic anisot-

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ropy; the mutually orthogonal unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ represent the orbital part of the order parameter with the orbital anisotropy vector $\mathbf{l}=\mathbf{e}_1\times\mathbf{e}_2$, and Δ_A is the gap amplitude in the spectrum of Bogolyubov quasiparticles. We use Greek subscripts for the components of a vector in spin space, and latin subscripts for the components in orbital space. The deviations corresponding to W bosons can be expressed in terms of a complex spin vector $\mathbf{W}=\mathbf{W}_1+i\mathbf{W}_2$ which is perpendicular to $\mathbf{d}$ and thus contains four real components:

$$\delta A_{\alpha i}=\Delta_A e_{\alpha\beta\gamma}W^\beta d_\gamma l_i. \tag{2}$$

The real components $\mathbf{W}_1$ and $\mathbf{W}_2$ of the complex vector $\mathbf{W}$ form a vector with subscript i in the plane perpendicular to $\mathbf{l}$.

The analogy with W-bosons stems form the fact that the fields W_i^α interact with the Bogolyubov fermions near the zero of the gap in a manner similar to the interaction of the $SU(2)$ gauge field with the Weyl fermions, i.e., via a covariant derivative $D_i=\partial_i-\boldsymbol{\tau}\cdot\mathbf{W}_i$, where $\boldsymbol{\tau}$ are the Pauli matrices.

The Lagrangian for these four collective modes has two important components. One of them is the gradient term, which is nonanalytic in the gradients of the order parameter. The nonanalytic behavior follows from the massless character of the Bogolyubov fermions in the A-phase: there are gap zeroes in the fermion spectrum at $\mathbf{k}=\pm k_F\mathbf{l}$, near which the Bogolyubov equation reduces to the Weyl equation for chiral fermions in the presence of an electromagnetic, weak, and gravitational fields. These chiral fermions play the role of massless left-handed electrons and right-handed positrons in quantum electrodynamics (QED). In the same manner as in QED, the vacuum of the massless fermions exerts a logarithmic screening action both on the "electric" and the "weak" charges in ^{3}He-A. This also differs from the vacuum polarization effects of the electroweak interaction, where in place of screening, there occurs antiscreening of the weak charge, i.e., asymptotic freedom (see Ref. 15). The gradient term, which is obtained as a result of integration over the fermion vacuum, near the gap zeroes, can be expressed in terms of the field strength of the magnetic $SU(2)$ field, which in the linear approximation has the form

$$F_{ij}{}^\alpha=\partial_i W_j{}^\alpha-\partial_j W_i{}^\alpha. \tag{3}$$

Since the Bogolyubov equations near a zero of the gap exhibit a local gauge symmetry as well as general covariance, the result of integration over the fermions also has these symmetry properties and has the following covariant form

$$L_{\text{grad}}=\frac{(-g)^{1/2}}{24\pi^2}F_{ij}{}^\alpha F^{\alpha ij}\ln\frac{\Delta_A^2}{(F_{ij}{}^\alpha F^{\alpha ij})^{1/2}}, \tag{4}$$

where g is the determinant of the metric tensor in ^{3}He-A (see Ref. 14). In terms of the usual variables we obtain the following expression for the nonanalytic gradient term:

$$L_{\text{grad}}^{(1)}=\frac{k_F{}^2v_F}{24\pi^2}|(\mathbf{l}\nabla)\mathbf{W}|^2\ln\frac{\Delta_A}{v_F|(\mathbf{l}\nabla)\mathbf{W}|}, \tag{5}$$

where k_f and v_F are the Fermi momentum and velocity.

A second important contribution to the Lagrangian is the mass term. A Lagrangian which is invariant with respect to local $SU(2)$ gauge transformations is not allowed to contain a mass term, which would violate this symmetry. Therefore in the weak-coupling approximation the W-bosons in the A-phase are Goldstone bosons, i.e., the corresponding fields can penetrate into the A-phase. However, a mass term may arise if one takes into account the strong-coupling corrections in the BCS model, corrections which in principle do not have a gauge invariant form. We write this term in a form convenient for comparison with the gradient term:

$$L_{\text{bulk}}=\eta^2\frac{k_F{}^2v_F}{24\pi^2}\frac{|\mathbf{W}|^2}{\xi^2}. \tag{6}$$

Here $\xi=\hbar v_F/\Delta_A$ is the coherence length and η is a dimensionless parameter which vanishes in the weak coupling approximation, and is therefore small for low pressures, where the strong coupling corrections are experimentally small. In a literal expression this parameter is of the order $\Delta_A/E_F\ll 1$ (E_F is the Fermi energy); therefore, although the W field is expelled from the bulk of the A-phase (an analog of the Meissner effect), the penetration depth of this field may be quite large compared to the coherence length.

The gradient term in Eq. (5) contains the gradient along the vector $\mathbf{l}$, since the transverse gradients can be neglected in Eq. (4) on account of the strong anisotropy of the A-phase. The transverse gradients are, however, present among the noninvariant terms in the Lagrangian; these terms are obtained by integrating with respect to the fermions far from the zero gap, where the Bogolyubov equation already coincides with the Weyl equation. The logarithmic divergence is absent from these terms and they are analytic in the gradients. We write these terms in the following form

$$L_{\text{grad}}^{(2)}=\frac{k_F{}^2v_F}{24\pi^2}\{a^2(\partial_i\mathbf{W}_i)^2+b^2(e_{ijk}l_k\mathbf{W}_j)^2\}, \tag{7}$$

where a and b are dimensionless parameters of the order of unity. In the weak-coupling approximation we have

$$a^2=\frac{1}{4},\quad b^2=\frac{1}{6}\left(1+\frac{3}{2}\frac{m^*}{m_3}\right),$$

where m^* is the mass of the excitation in the normal Fermi liquid and m_3 is the mass of the ^{3}He atom. These expressions for a and b can be obtained from the expression for gradient energy of the vector field $\mathbf{l}$, since the hidden symmetry of the BCS model in the weak coupling approximation unifies the "photons"—the oscillations of the field $\mathbf{l}$ and the W-bosons into one multiplet (see Ref. 18).

From Eqs. (5)–(7) one can find the asymptotic behavior of the W-mode in the A-phase. If the propagation direction is perpendicular to $\mathbf{l}$ then the W-mode decays according to the usual exponential law, and depending on the polarization of $\mathbf{W}_i$ relative to the orbital vectors e_{1i} and e_{2i}, either as

$$\mathbf{W}(x)\sim\mathbf{W}(0)\exp\left(-\frac{x}{\xi}\frac{\eta}{a}\right), \tag{8a}$$

or as

$$\mathbf{W}(x)\sim\mathbf{W}(0)\exp\left(-\frac{x}{\xi}\frac{\eta}{b}\right). \tag{8b}$$

Since we have $\eta\ll 1$, the penetration depth of this collective mode is large compared to the coherence length ξ. If, however, the mode propagates along the vector $\mathbf{l}$, then on account of the logarithmic divergence of the gradient energy the asymptotic behavior is changed

$$\mathbf{W}(x)\sim\mathbf{W}(0)\exp\left[-\left(\frac{\eta x}{\xi}\right)^{4/3}\right]. \tag{9}$$

3. THE W-MODES IN MAXIMALLY SYMMETRIC A–B INTERFACES

Since for low pressures the A-phase exists only in the presence of a magnetic field, one must investigate to what extent the magnetic field modifies the structure of the domain wall. In a vanishing magnetic field an analysis based on symmetry of the structure of the domain wall was carried out in Ref. 5. According to that paper the combinations of symmetry elements possible in a wall separating two phases with different symmetries of the vacuum state belong to the two groups $D_2^{(1)}$ and $D_2^{(2)}$:

$$D_2^{(1)}=\{1,\ C_{2x}T,\ C_{2y}P,\ C_{2z}PT\}, \tag{10}$$

$$D_2^{(2)}=\{1,\ C_{2x},\ C_{2y}PT,\ C_{2z}PT\}. \tag{11}$$

Here C_{2x}, C_{2y}, and C_{2z} denote rotations by an angle π around the x,y,z axes, respectively, with the x axis normal to the wall, and the y and z axes arbitrarily oriented in the plane of the domain wall; T is the time reversal operation and P denotes space inversion; thus the operations $C_{2y}P$ and $C_{2z}P$ denote reflections in planes perpendicular to the wall. The action of the group elements P and T on the components of the order parameter are the following:

$$PA_{\alpha i}=A_{\alpha i}, \tag{12}$$

$$T\,\mathrm{Re}\,A_{\alpha i}=\mathrm{Re}\,A_{\alpha i},\qquad T\,\mathrm{Im}\,A_{\alpha i}=-\mathrm{Im}\,A_{\alpha i}, \tag{13}$$

and the action of the rotations yields

$$\begin{gathered} C_{2x}A_{xx}=A_{xx},\quad C_{2x}A_{yy}=A_{yy},\quad C_{2z}A_{zz}=A_{zz},\\ C_{2x}A_{yz}=A_{yz},\quad C_{2x}A_{zy}=A_{zy},\\ C_{2x}A_{xy}=-A_{xy},\quad C_{2x}A_{yx}=-A_{yx},\\ C_{2x}A_{xz}=-A_{xz},\quad C_{2x}A_{zx}=-A_{zx} \end{gathered} \tag{14}$$

and similarly for the operations C_{2y} and C_{2z}.

A nonzero magnetic field which is either parallel or perpendicular to the boundary separating the phases does not violate these symmetry elements, if one neglects a small term in the energy which is linear in the magnetic field. Under these conditions the fundamental term is quadratic in the field:

$$F_H=g_2|A_{\alpha i}H_\alpha|^2. \tag{15}$$

It does not violate the time-reversal symmetry and consequently conserves all the symmetries in Eqs. (10) and (11). Thus, in the presence of a longitudinal or transverse magnetic field the classification of the domain walls according to their symmetry does not differ from the classification in the absence of the field.

According to this classification there are three different solutions having the maximally possible symmetry, i.e., either $D_2^{(1)}$ or $D_2^{(2)}$. We write down the asymptotic behavior of these solutions on both sides of the boundary in a field $\mathbf{H}\|\hat{\mathbf{x}}$ (here $\hat{\mathbf{x}},\hat{\mathbf{y}},\hat{\mathbf{z}}$ are the unit vectors along the coordinate axes). For the first solution the asymptotic behavior in the regions of the A-phase ($x=-\infty$) and the B-phase ($x=+\infty$) have the form

$$\begin{aligned} 1:\quad & A_{\alpha i}(x=-\infty)=\Delta_A\hat{x}_\alpha(\hat{x}_i-i\hat{z}_i),\\ & A_{\alpha i}(x=+\infty)=\Delta_\|\hat{x}_\alpha\hat{x}_i+\Delta_\perp(\hat{y}_\alpha\hat{y}_i+\hat{z}_\alpha\hat{z}_i). \end{aligned} \tag{16}$$

Here it was taken into account that the gap in the B-phase is anisotropic in the presence of a magnetic field. The longitudinal gap $\Delta_\|$ is smaller than the transverse gap $\Delta_\perp$ for $H\neq 0$. The $D_2^{(1)}$ symmetry of these asymptotic forms requires that in the maximally symmetric solution with a given asymptotic behavior the following components of the order parameter should be different from zero:

$$\mathrm{Re}\,A_{xx},\quad \mathrm{Re}\,A_{yy},\quad \mathrm{Re}\,A_{zz},\quad \mathrm{Im}\,A_{xz},\quad \mathrm{Im}\,A_{zx}. \tag{17}$$

The first three components are the order parameter of the B-phase to the left of the wall. The components $\mathrm{Re}A_{xx}$ and $\mathrm{Im}A_{xx}$ represent the A-phase to the left of the wall. The component $\mathrm{Im}A_{zx}$ vanishes on both sides of the boundary and are slightly different from zero inside the wall only for symmetry reasons, since it obeys the symmetry $D_2^{(1)}$. The component corresponding to the W-boson in the region of the A phase is $\mathrm{Re}A_{yy}$. It is one of the components of the B phase to the right of the wall and decays in the A phase.

The second solution with the symmetry $D_2^{(2)}$ has the asymptotic behavior

$$\begin{aligned} 2:\quad & A_{\alpha i}(x=-\infty)=\Delta_A\hat{y}_\alpha(\hat{y}_i+i\hat{z}_i),\\ & A_{\alpha i}(x=+\infty)=\Delta_\|\hat{x}_\alpha\hat{x}_i+\Delta_\perp(\hat{y}_\alpha\hat{y}_i+\hat{z}_\alpha\hat{z}_i). \end{aligned} \tag{18}$$

and the nonvanishing components of the order parameter in the wall are

$$\mathrm{Re}\,A_{xx},\quad \mathrm{Re}\,A_{yy},\quad \mathrm{Re}\,A_{zz},\quad \mathrm{Im}\,A_{yz},\quad \mathrm{Im}\,A_{zy}. \tag{19}$$

A third solution has the same symmetry and the same components of the order parameters as the first, but differs from it in asymptotic behavior:

$$\begin{aligned} 3:\quad & A_{\alpha i}(x=-\infty)=\Delta_A\hat{z}_\alpha(\hat{z}_i+i\hat{x}_i),\\ & A_{\alpha i}(x=+\infty)=\Delta_\|\hat{x}_\alpha\hat{x}_i+\Delta_\perp(\hat{y}_\alpha\hat{y}_i+\hat{z}_\alpha\hat{z}_i). \end{aligned} \tag{20}$$

Correspondingly, for a field aligned with the wall, say $\mathbf{H}\|\hat{\mathbf{y}}$, we have

$$\begin{aligned} 1:\quad & A_{\alpha i}(x=-\infty)=\Delta_A\hat{x}_\alpha(\hat{x}_i-i\hat{z}_i),\\ & A_{\alpha i}(x=+\infty)=\Delta_\|\hat{y}_\alpha\hat{y}_i+\Delta_\perp(\hat{x}_\alpha\hat{x}_i+\hat{z}_\alpha\hat{z}_i). \end{aligned} \tag{21}$$

with the same components and symmetry as the solution for the field $\mathbf{H}\|\mathbf{x}$. Further,

$$\begin{aligned} 2:\quad & A_{\alpha i}(x=-\infty)=\Delta_A\hat{z}_\alpha(\hat{x}_i-i\hat{y}_i),\\ & A_{\alpha i}(x=+\infty)=\Delta_\|\hat{y}_\alpha\hat{y}_i+\Delta_\perp(\hat{x}_\alpha\hat{x}_i+\hat{z}_\alpha\hat{z}_i) \end{aligned} \tag{22}$$

with the components listed in Eq. (19). The qualitative behavior of these components inside the wall is shown in Fig. 1. Finally,

$$\begin{aligned} 3:\quad & A_{\alpha i}(x=-\infty)=\Delta_A\hat{z}_\alpha(\hat{z}_i+i\hat{x}_i),\\ & A_{\alpha i}(x=+\infty)=\Delta_\|\hat{y}_\alpha\hat{y}_i+\Delta_\perp(\hat{x}_\alpha\hat{x}_i+\hat{z}_\alpha\hat{z}_i) \end{aligned} \tag{23}$$

with the components listed in Eq. (17). Each maximally symmetric Ansatz contains five components. Owing to the maximal symmetry there always exists a solution for the order parameter within the domain wall for the given Ansatz. However, not all the solutions are local minima of the energy. Some of them correspond to saddle points of the energy functional. Thus, according to Refs. 4 and 5, in a vanishing field and in the Ginzburg-Landau temperature range, the solution 1 corresponds to minimal energy, such that

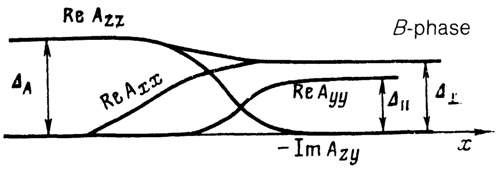

FIG. 1. The qualitative behavior of the four most important of the five components of the order parameter, corresponding to the maximally symmetric structure of the interface between the A and B phases of superfluid ^{3}He in a magnetic field situated in the plane of the interface (the field is $\mathbf{H}\|\hat{\mathbf{y}}$ and the normal to the surface is along the unit vector $\hat{\mathbf{x}}$). The A_{xx} component of the order parameter of the B phase to the right of the wall corresponds to the gauge field of a W boson with small mass, in the A phase to the right of the wall. This field penetrates deeply into the bulk of the A phase on account of its small mass, and decays asymptotically according to the law (9), which is nonstandard on account of the vanishing charge phenomenon, which is valid both in QED and in ^{3}He-A.

$E_1 < E_2 < E_3$. However, in the field $\mathbf{H}\|\hat{\mathbf{x}}$ this solution is obviously a saddle point, since the direction of the vector $\mathbf{d}$ in the bulk of the A phase, $\mathbf{d}\|\mathbf{H}$, contradicts the requirement derived from Eq. (15) that at equilibrium the vector $\mathbf{d}$ should be perpendicular to the field (the coefficient satisfies $g_2 > 0$). This means that the solution 1 with the vector $\mathbf{l}$ lying in the plane of the wall becomes unstable either with respect to breaking of the symmetry, or, according to Ref. 5, with respect to the solution 2 with a vector $\mathbf{l}$ perpendicular to the wall. If the field is weak, the first instability mechanism becomes operative. Near the wall, on the side of the A phase there appears a region where the vector $\mathbf{d}$ revolves from the position $\mathbf{d} = \hat{\mathbf{x}}$ to directions either along $\mathbf{d} = \hat{\mathbf{y}}$ or $\mathbf{d} = \hat{\mathbf{z}}$. In the first case the symmetry of the order parameter in the wall reduces to the two-element group $\{1, C_{2z}PT\}$, and in the second case it reduces to the other two-element group $\{1, C_{2y}P\}$. As a result, the total solution for the domain wall contains in the first case nine components, four in addition to the components (17):

$$\mathrm{Re}\,A_{yx},\ \mathrm{Im}\,A_{yz},\ \mathrm{Re}\,A_{xy},\ \mathrm{Im}\,A_{zy}, \tag{24}$$

and in the second case the solution contains ten components, the components (17) plus another five:

$$\mathrm{Re}\,A_{zx},\ \mathrm{Im}\,A_{zz},\ \mathrm{Re}\,A_{xz},\ \mathrm{Im}\,A_{xx},\ \mathrm{Im}\,A_{yy}. \tag{25}$$

As the magnetic field increases the region where a reorientation of the vector $\mathbf{d}$ occurs decreases and thus the energy of the wall increases. Thus, one may expect a phase transition from the solution 1 with broken symmetry to the solution 2, where the vector $\mathbf{d}$ is already correctly oriented to begin with, and therefore there is no need to break the maximal symmetry $D_2^{(2)}$.

Each of the maximally symmetric solutions contains one of the three components of the B-phase, which transforms into the W-boson mode of the A-phase. This is the component ReA_{yy} in the solutions 1 and 3, and the component ReA_{xx} in the solution 2. In order to determine the asymptotic behavior of this mode in the A-phase one must take into account the fact that owing to the energy of the magnetic anisotropy the magnetic field itself leads to a mass of the W boson for that polarization of the latter which is parallel to the magnetic field. This can be seen if one rewrites Eq. (15) in terms of the W field:

$$F_A = g_2\Delta_A^2|\mathbf{WH}|^2.$$

Therefore, for $\mathbf{H}\|\hat{\mathbf{x}}$ the field of the W-boson penetrates deeply into the A phase only for the domain walls 1 and 3, and for $\mathbf{H}\perp\hat{\mathbf{x}}$, only for the solution 2. In the cases 1 and 3 the W field decays according to the usual exponential law (8), whereas for the solution 2 the decay occurs according to the law (9). Hence in a phase transition which must take place from solution 1 with the additional components (24) to solution 2 as the magnetic field perpendicular to the wall increases, the penetration depth of the W-field, and accordingly, the size of the core of the domain wall must decrease. We note that the deep penetration of the W component into the A phase, as compared to the other components, can already be seen from the numerical solutions in Refs. 4 and 5, in spite of the fact that the solutions were obtained for high pressures where the strong coupling parameter η is not so very small.

4. A DOMAIN WALL WITH BROKEN SYMMETRY

For low pressures, when the parameter η is small, the approximate degeneracy between the A-phase and the planar phase may also lead to symmetry-breaking in the A–B interface. This follows from the scenario of the transition from the B phase to the A phase via a planar phase, first discussed by Cross[2] in the weak-coupling approximation. If one exactly follows his line of reasoning one is led to an A–B-interface with two cores. In the first core which has a size of the order of ξ, a transition occurs from the B phase into the planar phase, which means that, in fact, the longitudinal gap $\Delta_{\|}$ vanishes. The subsequent transition from the planar phase into the A phase occurs through intermediate degenerate states, and so the size of the corresponding region is determined by the strong coupling parameter η which removes this degeneracy. This region forms the second core of size $\sim\xi/\eta$.

This scenario due to Cross was not realized in his calculations, since he considered a vanishing field, where an A–B-interface exists only for high pressures and η is not particularly small. Therefore a more convenient solution had maximal symmetry, i.e., dependent on five parameters (in Cross' solution the number of components was four, since the component which vanished on both sides of the wall was not taken into account). For low pressures this scenario should be realized. We consider it for the case of the state with the asymptotic behavior 2 in a field perpendicular to the wall, the solution defined by Eq. (18). This is exactly the state to which the solution 1 must go over as the magnetic field is increased. We now determine into what the solution 2 transforms as the pressure is lowered.

The qualitative structure of the order parameter for a domain wall with two cores is represented in Fig. 2. In this structure only the components of the order parameter which are perpendicular to the field are essential. As one moves from the side of the B phase by a distance of the order of the coherence length, the only parallel component of the B phase, ReA_{xx}, vanishes, as a result of which a planar phase is formed with the order parameter

$$\Delta_\perp(\hat{y}_\alpha\hat{y}_i + \hat{z}_\alpha\hat{z}_i). \tag{26}$$

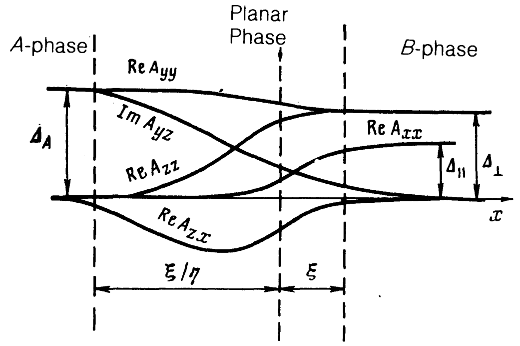

FIG. 2. The qualitative behavior of the five most important of the nine components of the order parameter which exist in the core of the A–B-interface with broken symmetry. Such an interface must exist in a field perpendicular to the interface at low pressures, when the strong-coupling parameter η is small. It consists of two cores: in one core, having a size of order the coherence length ξ, there occurs a transition from the B phase into a planar phase; in the second core, which has a larger size $\sim\xi/\eta$, the transition into the A phase occurs. This wall exhibits a spontaneous superfluid flow along the surface.

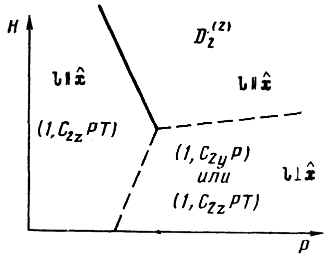

FIG. 3. A qualitative phase diagram of the states of the A–B-interface in the H–P (magnetic field, pressure) plane, with fields perpendicular to the wall. The states of the wall differ both in symmetry and in the orientation of the vector $\mathbf{l}$; the appropriate symmetry group is indicated in the picture. The maximally symmetric state 2 with symmetry $D_2^{(2)}$ is expected for large pressures and strong fields. As the pressure or the field is decreased one should expect spontaneous breaking of the symmetry $D_2^{(2)}$, as a result of which at low pressures the state with two cores is formed whose structure is depicted in Fig. 2. The phase diagram does not depend essentially on the symmetry of the wall at large pressures and weak fields, discussed in Sec. 3. Even if this symmetry does not differ from the symmetry $(1, C_{2z}PT)$ of the wall with two cores, there still must exist a first-order phase transition between the two broken-symmetry states (the dashed line), since the orientation of the vector $\mathbf{l}$ is different in these two states, and the transition between these states requires a further symmetry reduction in the intermediate regions. For a transition from the phase $D_2^{(2)}$ to the phase with two cores which exhibits a lower symmetry and the same orientation of the vector $\mathbf{l}$, the phase transition may be of second-order (full line).

Further, the deformation of the planar state into the A phase occurs through degenerate states characterized by a coordinate-dependent order parameter Φ:

$$A_{\alpha i}(x) = {}^1\!/_2\Delta_A(\hat{y}_\alpha + i\hat{z}_\alpha)(e_{1i\uparrow} + ie_{2i\uparrow}) + {}^1\!/_2\Delta_A(\hat{y}_\alpha - i\hat{z}_\alpha)(e_{1i\downarrow} + e_{2i\downarrow}) = {}^1\!/_2\Delta_A(\hat{y}_\alpha + i\hat{z}_\alpha)\{\hat{y}_i - i[\hat{z}_i\cos\Phi(x) + \hat{x}_i\sin\Phi(x)]\} + {}^1\!/_2\Delta_A(\hat{y}_\alpha - i\hat{z}_\alpha)(\hat{y}_i + i\hat{z}_i). \quad (27)$$

Here Φ varies from zero, corresponding to the planar phase with the order parameter (26), to π, which corresponds to the A phase to the left of the wall, where $A_{\alpha i} = \Delta_A \hat{y}_\alpha(\hat{y}_i + i\hat{z}_i)$. The intermediate states correspond to a rotation of the orbital vector of the Cooper pairs with spin $\hbar\hat{\mathbf{x}}$:

$$\mathbf{l}_\uparrow = [\mathbf{e}_{1\uparrow}\times\mathbf{e}_{2\uparrow}] = -\hat{\mathbf{x}}\cos\Phi(x) + \hat{\mathbf{z}}\sin\Phi(x).$$

The wave function of these pairs is described by the first terms in Eq. (27). This rotation does not change the energy in the weak coupling approximation, in which the Cooper pairs with spins up and spins down are independent. In the planar state ($\Phi = 0$) we have $\mathbf{l}_\uparrow = -\mathbf{l}_\downarrow$, where $\mathbf{l}_\downarrow = \mathbf{e}_{1\downarrow}\times\mathbf{e}_{2\downarrow}$ is the orbital angular momentum vector of Cooper pairs with opposite spin $-\hbar\hat{\mathbf{x}}$, which are described by the second term in Eq. (27). In the A phase, where $\Phi = \pi$ holds, we have $\mathbf{l}_\uparrow = \mathbf{l}_\downarrow = \mathbf{l}$.

The expression (27) contains the components ReA_{zx} and ImA_{yx}, which are absent from (19). This shows that the symmetry $D_2^{(2)}$ of the maximally symmetric solution 2 is spontaneously broken. The symmetry that is broken is C_{2x}, since $C_{2x}A_{zx} = -A_{zx}$, and only the symmetry element $C_{2y}PT$ is conserved. Thus, the group $D_2^{(2)}$ reduces either to the two-element subgroup $(1, C_{2y}PT)$, or to $(1, C_{2z}PT)$, if a transformation of the axes is carried out. This symmetry also admits the existence of the components ReA_{xz} and ImA_{xy}, which thus must be present in the exact solution. Among the four additional components only ReA_{zx} corresponds to the W-field which penetrates into the A phase according to the law (9).

The phase diagram of the states of the A–B interface in the p–H (pressure-magnetic field) plane, is shown in Fig. 3. It does not depend on the state which is realized for weak fields at high pressures, i.e., whether it is the state (24) or (25). Even if this state does not differ in symmetry from the state (27) at low pressures, a first-order phase transition nevertheless occurs between them, owing to the different orientations of the vector $\mathbf{l}$ in these states.

The broken symmetry of the rigid core of the domain wall between the A and B phases leads to the existence of a spontaneous mass flow along the y axis, since the residual symmetry does not change this component of the flow: $C_{2y}PTj_y = j_y$. The spontaneous flow is forbidden in a maximally symmetric a-b interface, on account of the C_{2x} symmetry which transforms this component into its opposite $C_{2x}PTj_y = -j_y$, but it is allowed in the walls 1 and 3. The symmetry breaking also leads to a two-fold degeneracy of the states of the core, as a result of which the plane of the interface may contain singular lines which separate portions with different directions of the superfluid flow, having identical asymptotic behavior far from the wall.

5. CONCLUSION

The low-temperatures low-pressure region in ^{3}He-A remains one of the most interesting areas in the study of superfluid ^{3}He, on account of the far-reaching analogy with quantum field theory in elementary particle physics, as well as because of the hidden symmetry leading to an additional degeneracy of the order parameter. In addition to the chiral anomaly and the vanishing-charge phenomenon, these properties modify significantly the structure of extended objects, such as quantized vortices in ^{3}He-A (Ref. 17) and the interface between the A and B phases (the latter exhibited in the present paper). For a magnetic field perpendicular to the A–B interface one should expect a series of phase transitions in which the structure of the order parameter inside the rigid core is changed, similar to what happens in vortices in superfluid ^{3}He. If the symmetry in the core of the wall is broken, a spontaneous superfluid flow appears. In some walls one of the components of the order parameter, which plays the role of the W-boson, penetrates deeply into the bulk of the A phase, thus increasing the size of the core of the wall. An interesting possibility would be to use a moving wall as a source of W-boson collective modes of the order parameter.

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