Acoustic self-induced transparency of a metal in a quantized magnetic field

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A nonlinear self-induced transparency effect of a metal for narrow hypersound pulses under conditions of quantum Doppler-shifted acoustic cyclotron resonance (QDSACR) is predicted. It is demonstrated that the nonlinear evolution of the pulse in the vicinity of the QDSACR absorption peaks attributable to resonance electron transitions between different Landau levels produces an acoustical soliton travelling at an amplitude-dependent velocity and capable of reaching Fermi values. The conditions required for experimental observation of the effect, which are virtually identical to the conditions required to observe DSACR quantum oscillations, are discussed.

Reference 1 has demonstrated the fundamental possibility of a quantum coherent "bleaching" of a pure metal manifested as an optical self-induced transparency effect[2] for narrow (shorter than the electron free path length) hypersound pulses which generate acoustical undamped solitons travelling at a velocity other than the acoustic phase velocity. Underlying this effect is the resonance interaction between the sound and the conduction electrons in the metal, which induces electron transitions between the pairs of resonant states identified by the laws of conservation in the continuous electron spectrum of the metal and in linear theory yielding acoustic damping (Landau damping).

In nonlinear theory the total resonant transition probability from electron scattering by an acoustic pulse may vanish under certain conditions so that there will be no energy exchange between the wave packet and the electrons, and interaction will become purely dispersive. Here the pulse envelope will correspond to a nonreflecting potential in the wave equation for the electrons. It is essential that this envelope represent an exact solution of the elasticity equation, while the existence of this solution is due to the integrability, within the framework of the resonance approximation, of the complete system of equations of the problem, which includes, in addition to the elasticity equation, a kinetic equation for the electrons and Maxwell's equations.

One specific characteristic of the acoustic self-induced transparency (ASIT) of a metal compared to the optical effect is the participation of delocalized conduction electrons in the soliton acoustic energy transfer as well as the nonzero momentum transfer to the electron in the resonance transition. Mathematically these features yield a new integrable system of equations representing a generalization of the familiar three-wave system[3] which accounts for an infinite number of electron degrees of freedom.

As noted in Ref. 1 the nonreflecting property of the RF wave packet will occur only with a certain type of acoustic-induced electron scattering when the scattered wave in the acoustic reference system propagates in the same direction as the transmitted wave. The resonant transition probability for the "forward" scattered electrons is an oscillating function of position which periodically vanishes (the analog of Rabi oscillations in optics), which makes it fundamentally possible to produce a nonreflecting potential. On the other hand under "backward" reflection conditions the resonance transition probability varies exponentially and while never vanishing, approaches unity with increasing wave packet length. In this case the reflectionless potential cannot exist in principle.

Another necessary condition for the reflectionless state is that the characteristics of all resonant transitions involved in the interaction be identical. In a zero magnetic field the "forward" scattering process can occur only with a very special Fermi surface type[1] which, moreover, must have a high degree of isotropy of the electron velocity within the resonance "belt," which makes acoustic self-induced tr·.nsparency difficult to achieve in optics. It is demonstratec in the present paper that these limitations can be eliminated by placing the metal in a quantized magnetic field. As we know in the RF range $qR > 1$ (q is the acoustic wave vector, $R = v_F/\Omega$ is the Larmor radius) the law of conservation

$$\varepsilon_n(p_z) - \varepsilon_{n'}(p_z - q) = \omega \quad (\hbar = 1) \tag{1}$$

allows resonant transitions between different Landau levels $n' \neq n$ satisfying the required "forward" scattering (see figure). At low temperatures $T \ll \Omega$ the motion of these pairs of resonance states in the ε, p_z plane resulting from the changing magnetic field will produce quantum oscillations in the sound absorption[4,5] due to the resonance states passing through the Fermi level [quantum Doppler-shifted acoustic cyclotron resonance (QDSACR)]. Only a single resonant transition in the vicinity of each isolated absorption peak is significant, while the conditions for ASIT are most favorable in this area. We note that in this case we should anticipate a large difference between the soliton velocity and the acoustic phase velocity, since the resonant electrons carrying the soli-

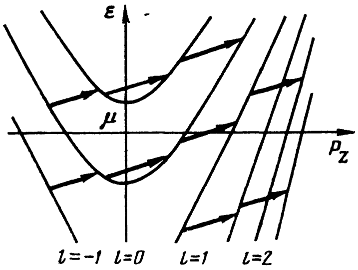

FIG. 1.

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ton energy have high translational velocities $\sim v_F$. These absorption oscillations together with their associated acoustic velocity oscillations are grouped into series with different values of $l = n' - n$ and differ from the quantum oscillations attributable to transitions within each individual level ($l = 0$) (Ref. 6) in the dependence of their parameters on the acoustic frequency and wave vector, which causes a dispersion of the acoustic wave velocity.

Below we formulate a general scheme together with the primary results of linear QDSACR theory[4] with additional results required for a subsequent analysis of the nonlinear evolution of an acoustic pulse of finite duration and amplitude; we then derive nonlinear equations for the acoustic self-induced transparency in a quantized magnetic field; as we would expect these equations are, in their principal features, identical to the generalized three-wave system found in Ref. 1 and have similar soliton solutions.

1. LINEAR THEORY OF QDSACR

We consider a planar longitudinal sound wave $u(z,t)$ running parallel to the magnetic field $[u(z,t) \| H \| z]$ and satisfying the elasticity equation[7]

$$\rho\partial_z^2 u - \lambda_0 \partial_t^2 u = F, \tag{2}$$

where ρ is the metal density, λ_0 is the longitudinal component of the elasticity modulus of the lattice. The force F on the lattice from the electrons, neglecting the insignificant inertial component, takes the form

$$\mathbf{F}(\mathbf{r},t) = -Ne\mathbf{E}(\mathbf{r},t) - \frac{\delta H(t)}{\delta \mathbf{u}(\mathbf{r},t)},$$
$$H(t) = \mathrm{Sp}(\hat{H}\hat{\gamma}(t)), \tag{3}$$

where $\hat{\gamma}(t)$ is the electron density matrix satisfying the kinetic equation

$$i\partial_t\hat{\gamma} = [\hat{H}, \hat{\gamma}], \tag{4}$$

while $\hat{H}$ is the electron Hamiltonian which, in the quasiclassical electron wave function representation $|n,p_z,p_x\rangle$ is an underformed lattice

$$|n, p_z, p_x\rangle = (2\pi)^{-1}\exp(ip_z z + ip_x x)\sum_\alpha \frac{C_\alpha(p_z)}{|v_y^\alpha|^{1/2}}$$
$$\times\exp\left[-\frac{i}{\sigma}\int dp_x' \, p_y^\alpha(p_x', p_z, \varepsilon_n(p_z))\right], \tag{5}$$
$$\sigma = |e|H/c, \quad |C_\alpha|^2 = \Omega/4\pi$$

takes the form of the matrix

$$\hat{H} = \hat{H}_0 + \delta\hat{H}, \; H_0(nn', p_z p_z') = \varepsilon_n(p_z)\delta(p_z - p_z')\delta_{nn'}\delta(p_x - p_x'),$$
$$\delta H(nn', p_z p_z') = \{[\lambda_{zz}(nn', p_z) + v_z(nn', p_z)p_z]\partial_z u(p_z, p_z') + e\varphi(nn', p_z, p_z')\}\delta(p_x - p_x'). \tag{6}$$

In expression (5) α is the branch number of the multivalued function $p_y(p_x)$ describing the intersection of the isoenergetic surface by the plane $p_z = \text{const}$, and λ_{zz} is the longitudinal component of the deformation potential. Equations (2)–(4) are closed by the electrical neutrality condition for the electrostatic potential φ.

The electron force (3) consists of an adiabatic part and a nonadiabatic part proportional to ω. The adiabatic part is generated by all electrons of the Fermi surface and, neglecting the weak de Haas-van Alphen oscillations of the chemical potential μ and the elastic moduli of the metal, gives rise to classical renormalization of the deformation potential $\lambda_{zz}(p) \to \Lambda(p)$ which is responsible for the electrical neutrality, and will also result in a constant renormalization of the acoustic velocity, which we shall henceforth assume is included in the definition of the elastic modulus λ_0. The nonadiabatic part, which is of primary interest and which is determined in s/v_F to lowest order by the resonant contribution of the small denominator, consists of the sum of the partial contributions of the resonant transitions between the Landau zones:

$$F_{res} = \sum_l F_l, \quad l = n' - n. \tag{7}$$

The term with $l = 0$ contains the effect of the giant oscillations[6] attributable to the contribution of the resonant points near the central (in the general case, the extremal) cross-section of the Fermi surface $[v_z(p_z \approx p_0) \sim q/m \ll v_F]$ given by the condition $\varepsilon_n(p_0) = \mu$. The QDSACR oscillations of interest to us correspond to transitions between levels near the resonant cross-sections with characteristic longitudinal velocities $v_z(p_z = p_l) \sim v_F$:

$$\Delta_l(E, p_l) \equiv qv_z(E, p_l) - l\Omega(E, p_l) = \omega, \quad \varepsilon_n(p_l) = E, \tag{8}$$

and determined by the condition

$$\varepsilon_n(p_l(\mu, H, \omega, q)) = \mu. \tag{9}$$

The term with $l = 0$ in the expression for the deformation force as well as the resonant part of the longitudinal electrical field can be dropped near the QDSACR peaks, since the electrical neutrality condition is satisfied only by transitions within the levels. The expression for the partial force F_l corresponding to the lth series of QDSACR oscillations in a frequency range $\omega \ll T$ typical of the experiment takes the form

$$F_l = -\frac{q^2\omega\sigma u}{2\pi^2}\sum_n \int dp_z \frac{|\Lambda_l|^2 n_F'(\varepsilon_n(p_z))}{\Delta_l(\varepsilon_n(p_z), p_z) - i\eta}, \quad \eta \to +0. \tag{10}$$

For simplicity we set $v_z = \text{const}$ on the classical electron trajectory, and then

$$\Lambda_l = \int_0^{T_H} \frac{d\tau}{T_H}\Lambda(\tau)\exp(il\Omega\tau). \tag{11}$$

The imaginary part of expression (10) describes the acoustic wave damping found in Ref. 4:

$$\mathrm{Im}\, F_l = \frac{q^2\omega\sigma u}{8\pi T}|\Lambda_l|^2\left|\frac{\partial\Delta}{\partial p_z}\right|_l^{-1}$$
$$\times\sum_n \mathrm{ch}^{-2}\left[\frac{\varepsilon_n(p_l(\mu, H, \omega, q)) - \mu}{2T}\right], \tag{12}$$

$$\left(\frac{\partial\Delta}{\partial p_z}\right)_l = \left(\frac{q}{m_z}\right)_l,$$
$$m_z^{-1} = v_z\frac{\partial \ln d}{\partial p_z}, \tag{13}$$

where $d(p_z) = v_z T_H$ is the electron shift along the magnetic field over the orbital motion period T_H.

We use the Poisson summation formula[8] to calculate the real part of the force F_l which determines the acoustic velocity renormalization

$$\sum_n \ldots = \sum_k \int \frac{dE}{\Omega(E,p_z)} \exp\left[ik\frac{S(E,p_z)}{\sigma}\right], \tag{14}$$

where $S(E,p_z)$ is the area of intersection of the isoenergetic surface $E=$ const determined by the plane $p_z=$ const. Estimates reveal that the continuously varying term corresponding to $k=0$ in (14) is small, while the oscillating terms are determined by the contribution of the narrow range near the resonance $p_z - p_l \sim \delta p_{zs} \sim q$. Expanding the integrands in the neighborhood of p_l in Re F_l, we find

$$\operatorname{Re} F_l = -\frac{q^2\omega u m^*}{2\pi} \times |\Lambda_l|^2 \left(\frac{\partial\Delta}{\partial p_z}\right)_l^{-1} \operatorname{sign} l \oint dE\, n_F'(E-\mu) \operatorname{ctg}\frac{S_l(E)}{2\sigma}, \tag{15}$$

$$S_l(E,H,\omega,q) = S(E, p_l(E,H,\omega,q)), \tag{16}$$

where $m^* = \sigma/\Omega$ is the cyclotron mass.

In the general case it is difficult to solve the dispersion equation (2) subject to (7), (12), (15) due to the dependence of the oscillating terms on the acoustic frequency and wave vector in the resonance conditions (8), (9). The dispersion equation can therefore be solved by iteration only when the small parameter is on the right side. Assuming, in accordance with (13), $\partial\Delta/\partial p_z = q/m_z$ and using the standard estimate $m/M \sim (s/v_F)^2$ (M is ion mass) we obtain the applicability condition of perturbation theory for solving (2) as

$$\frac{m_z}{m}\left[\frac{\Lambda}{\varepsilon_F}\right]^2 \frac{\omega}{TqR} \ll 1. \tag{17}$$

In the general case the acoustic spectrum will contain a large number of singularities[1)], each of which contains an absorption peak

$$\Gamma_{ln} = \frac{qm^*\Omega}{8\pi\rho T} |\Lambda_l m_{zl}|^2 \operatorname{ch}^{-2}\left[\frac{\varepsilon_n(p_l(\mu,H,\omega,q))-\mu}{2T}\right] \tag{18}$$

together with a correction to the acoustic velocity which can have either sign,

$$\delta S_{ln} = \frac{m^*}{4\pi\rho}|\Lambda_l|^2 m_{zl} \operatorname{sign} l J_n\left(\frac{S_l(\mu,H,\omega,q)}{2\sigma}\right),$$

$$J_n(x) = \begin{cases} \operatorname{ctg} x, & |x-\pi n| \gg T/\Omega \\ \dfrac{7\zeta(3)}{8\pi^4}\left[\dfrac{\Omega}{T}\right]^2 (x-\pi n), & |x-\pi n| \ll T/\Omega \end{cases} \tag{19}$$

Due to the dependence of the resonant cross-section area S_l (16) on the magnetic field and wave vector, the QDSACR oscillations in the lth series within the range

$$qR > |l|\left(1-\frac{s \operatorname{sign} l}{v_F}\right)^{-1}, \tag{20}$$

form a quasiperiodic structure with a period varying slowly as a function of the inverse magnetic field:

$$\delta(1/H) = 2\pi e/c\tilde{S}_l(H), \quad \tilde{S}_l(H) = S_l + 2\pi(m^* m_z v_z^2)_l \tag{21}$$

and the wave vector q:

$$\delta q = q^3/|lm_z|\Omega. \tag{22}$$

The oscillating nature of the dependence of the acoustic phase velocity on q (19) and the narrow width of each singularity region $\delta q_T \sim (T/\Omega)\delta q$ causes a sudden jump in the acoustic group velocity, while the maximum value near the singularity

$$\frac{\partial\omega}{\partial q} = v_g \sim \frac{\max(q\delta s)}{\delta q_T} \sim v_F \frac{m^*}{M}|\Lambda_l|^2 \frac{\Omega m_z^2}{T^2 m p_F^2 (qR)^2} \tag{23}$$

for $T \approx 1$ K, $\Omega \sim 10^{12}$ sec^{-1}, $\Lambda \sim \varepsilon_F$ may reach Fermi values. However small values of Λ ($\Lambda \sim 0.1\varepsilon_F$) are typical of the majority of metals, and hence the group velocity of the linear acoustic packet under QDSACR conditions is comparable to s for $qR \geqslant 5$.

Electron relaxation processes occurring at a frequency τ^{-1} will suppress quantum oscillations and cause a transition to the classical picture of magnetoacoustic resonance.[10] The estimate of the QDSACR range, accounting for electron scattering by phonons or lattice defects, found in Ref. 4 can be obtained based on the following qualitative considerations which will be used systematically in analyzing the nonlinear situation below. It is clear from the structure of the resonant denominator in (10) that the uncertainty of electron momentum $\delta p_{z\tau}$ related to relaxation processes is of the order of $(\tau\partial\Delta/\partial p_z)^{-1}$ and will result in uncertainty of the resonance transition energy $\delta\varepsilon_\tau \sim v_z \partial p_{z\tau}$. The existence condition for quantum oscillations derives from the requirement that $\delta\varepsilon_\tau$ be small compared to the distance Ω between neighboring resonant transitions in the given series (see figure):

$$\tau^{-1} \gg q^2/|lm_z| \tag{24}$$

and is significantly more stringent than the existence conditions of oscillations corresponding to transitions within the levels.

The finite spectral width of the acoustic signal in the case of pulsed sound excitation, which is required to achieve ASIT, will suppress quantum oscillations in a similar fashion. With a spatial acoustic signal width L less than the free electron path length, the quantity L/v_z plays the role of τ and the corresponding uncertainty of electron momentum $\delta p_{zL} \sim m_z v_z/qL$ imposes the following limit on the existence of QDSACR oscillations

$$Lq^2/m_z v_z \gg 1. \tag{25}$$

This inequality, obviously, coincides with the condition that the spectral width of the acoustic packet be small compared to the oscillation period (22), which takes the form $L\delta q \gg 1$.

2. ACOUSTIC SOLITONS UNDER QDSACR CONDITIONS

The linear theory developed above is applicable with a finite sound amplitude as long as the nonlinear corrections to the resonance linewidth are small compared to its width in the linear approximation; henceforth we will relate the width to the finite spectral width of a short hypersound packet:

$$q^{-1} \ll L \ll \tau v_z. \tag{26}$$

It is convenient to analyze the role of the finite acoustic amplitude near resonance using a wave equation for the Hamiltonian (6) which conserves only the resonant matrix elements for transitions between two levels:

$$i\partial_t\psi_n(z,t)=\varepsilon_n(p_z)\psi_n(z,t)+\Lambda_l\partial_z u(z,t)\psi_{n+l}(z,t),$$
$$i\partial_t\psi_{n+l}(z,t)=\varepsilon_{n+l}(p_z)\psi_{n+l}(z,t)+\Lambda_{-l}\partial_z u(z,t)\psi_n(z,t), \quad (27)$$
$$\psi_n(z)=\int\frac{dp_z}{2\pi}\exp(ip_z z)\psi_n(p_z).$$

Representing the wave field of the pulse as

$$\partial_z u(z,t)={}^1/_2 U(z,t)\exp(iq\chi)+\text{c.c.}, \quad (28)$$
$$\chi=z-(\omega t/q),\quad \partial_z U\ll qU,\quad \partial_t U\ll\omega U,$$

we isolate the coefficients a, b that slowly vary on the scale p_z^{-1} in the wave equations of the resonant approximation of Ref. 11:

$$\psi_n(z,t)=a(z,t)\exp[ip_l z-i\varepsilon_n(p_l)t], \quad (29)$$
$$\psi_{n+l}(z,t)=b(z,t)\exp(i(p_l-q)z-i(\varepsilon_n(p_l)-\omega)t).$$

The equations for coefficients a and b derive from the requirement that no resonant singularities exist in higher orders of perturbation theory in system (27):

$$i\partial_t a=v_+\hat{p}_z a+{}^1/_2\Phi_l b,$$
$$i\partial_t b=v_-\hat{p}_z b+{}^1/_2\Phi_l^* a, \quad (30)$$
$$v_+=v_{z,n}(p_l),\quad v_-=v_{z,n+l}(p_l-q),\quad \Phi_l(z,t)=\Lambda_l U(z,t).$$

It is possible to obtain qualitative information on the nonlinear dynamics of resonant electrons in the acoustical field by examining formal solutions of Eqs. (30) with a fixed field amplitude Φ_l = const when they can be found in explicit form:

$$a,\ b\propto\exp(i\delta p_z z-i\delta\varepsilon t), \quad (31)$$
$$\delta p_z=\frac{v_++v_-}{2v_+v_-}\delta\varepsilon\pm\delta p_0,$$
$$\delta p_0=\frac{[(v_+-v_-)^2\delta\varepsilon^2+|\Phi_l|^2v_+v_-]^{1/2}}{2v_+v_-}.$$

As we see from Eqs. (31) these solutions oscillate in accordance with the discussion in the Introduction for all deviations of $\delta\varepsilon$ from exact resonance. The nonlinear resonant broadening is estimated as

$$\delta\varepsilon_\Phi\sim\frac{\Phi v_z}{|v_+-v_-|}\sim\frac{\Phi\Omega ml}{q^3}.$$

A comparison of this quantity to the linear width obtained previously

$$\delta\varepsilon_L\sim v_z\delta p_{zL}\sim m_z v_z^2/qL$$

[see Eq. (25)] establishes the nonlinear region $\delta\varepsilon_\Phi\gg\delta\varepsilon_L$:

$$\Phi_l\gg\Omega/qL. \quad (32)$$

We require $\delta\varepsilon_\Phi\ll\Omega$ in order to assure that the nonlinearity will not cause smearing of the quantum oscillations, and consequently the sound amplitudes with which oscillations are possible must lie in the range

$$\Omega/qL\ll\Phi_l\ll q^2/m_z. \quad (33)$$

The existence of such an interval is guaranteed by the smallness of the oscillation period in q, Eq. (22), compared to q itself: $\delta q\ll q$.

We note that the second equalty in (33) guarantees linearity of the nondissipative part of the resonant force (16) since, in accordance with the estimate obtained above, the width of its generating resonance range $\delta\varepsilon_s\sim v_z\delta p_{zs}\sim l\Omega$ substantially exceeds $\delta\varepsilon_\Phi$.

The following is a simple interpretation of the "Rabi oscillations" of the solutions (31) $L_R=v_z/\Phi$: The nonlinear range is determined by the ratio of L_R to the pulse length and corresponds to the maximum values of this parameter; here in the nonlinear range L_R, rather than L, is the factor responsible for resonance blurring.

An investigation of the limits of applicability of the resonance approximation (27)–(30) determined by the zero resonance overlap conditions demonstrates that these conditions are most stringent for resonances with different n [i.e., in order to distinguish between the pair of equations (27)]: $\delta p_0\ll q^3/\Omega m_z$. This inequality is identical to the second inequality in (33), while the zero overlap condition takes the form $\Phi\ll\Omega$ for resonances with different l and for multiphonon resonances. Therefore the condition responsible for quantization of transverse electron motion simultaneously assures the adequacy of the two-level Hamiltonian (27) and the resulting quantum nature of the longitudinal electron motion. This fact, which plays the decisive role in achieving ASIT conditions, may be surprising at first glance given the large longitudinal momentum of the resonance electrons $p_z\sim p_F$. However it is important to remember that the two-component wave equation (27) contains an additional characteristic scale: the "coherence length" of the resonance pair $v_z/\Omega\sim d$, analogous to the Cooper pair dimension in superconductivity theory, which functions as the new quantum length.

It is necessary to add to Eqs. (30) an equation for the envelope Φ_l by expressing the dissipative part of the resonance force through the functions a, b in order to describe nonlinear evolution of a wave packet of arbitrary shape. We therefore expand the electron density matrix in the neighborhood of the wave packet in one-electron wave functions

$$\gamma(\mathbf{r},\mathbf{r}',t)=\sum_\lambda f(\lambda)\psi_\lambda(\mathbf{r},t)\psi_\lambda^*(\mathbf{r}',t),$$
$$\psi_\lambda(t\to-\infty)=|\lambda\rangle,\quad \lambda=(n,p_z), \quad (34)$$

where f is the electron distribution function prior to scattering on the packet. Assuming that the scattered nonequilibrium electrons thermalize before rescattering, and substituting (34), (29) into Eq. (9) we obtain

$$F_l^{res}=\frac{iq\sigma}{2\pi^2}\sum_\varkappa\int dp_z\,n_F(\varepsilon_\varkappa)[\Lambda_{-l}a_\varkappa b_\varkappa\exp(iq\chi)+\text{c.c.}], \quad (35)$$

where χ takes the value " + " or " − ", with $\varepsilon_+=\varepsilon_n(p_l)$, $\varepsilon_-=\varepsilon_+-\omega$,

$$a_+(t\to-\infty)=\exp[i\delta p_z(z-v_+t)],\quad b_+(t\to-\infty)=0,$$
$$a_-(t\to-\infty)=0,\quad b_-(t\to-\infty)=\exp[i\delta p_z(z-v_-t)].$$

Substituting (35), (29) into the dispersion equation (2) and accounting for the difference between the group acoustic velocity v_g and the phase velocity caused by quantum velocity oscillations (19), after averaging over the fast variables we find

$$i(\partial_t+v_g\partial_z)\Phi_l=\frac{q^2\sigma}{2\pi^2\rho}|\Lambda_l|^2\int dp_z\sum_{\varkappa}n_F(\varepsilon_\varkappa)a_\varkappa b_\varkappa^*. \quad (36)$$

The resulting complete system of equations (30), (36) is nearly identical to the system found in Ref. 1, which is an integrable generalized three-wave system. When the condition

$$(v_+-v_g)(v_--v_g)>0 \quad (37)$$

holds, it has soliton solutions which in the simplest one-soliton case take the form

$$\Phi=\frac{[(v_+-v)(v_--v)]^{1/2}}{T_0v}\mathrm{ch}^{-1}\left[\frac{t-z/v}{T_0}\right],$$
$$v=\frac{v_g}{1-{}^1/_2\Gamma T_0\,\mathrm{sign}\,l}. \quad (38)$$

where T_0 is the soliton lifetime and Γ is the linear acoustic damping (22). The soliton velocity as a function of the sign of l may be either less than or greater than v_g. Here in the first case ($l<0$) it drops to zero as T_0 goes to infinity, while in the second case ($l>0$) the soliton lifetime is limited to a maximum value of

$$T_{0\,max}=\frac{2}{\Gamma}\left[1-\frac{qv_g}{b\Omega}\right], \quad (39)$$

where the soliton velocity reaches the maximum value $v=v_F|l|qR$ while the amplitude vanishes.

Evidently the primary problem for achieving an ASIT mode under QDSACR conditions is the problem of combining high resolution of the acoustical pulse (with respect to resonant transitions with different l) with the limit imposed by relaxation processes, i.e., a combination of inequalities (25) and (26):

$$\Omega m_z/q^2\ll qL\ll\tau\Omega. \quad (40)$$

In reality in very pure metals with $\tau\sim10^{-8}$ sec and magnetic fields of $H\sim100$ kGauss ($\Omega\sim10^{12}$ sec^{-1}) the parameter $\tau\Omega$ is 10^4, while the parameter $\Omega m_z/q^2$ in the hypersound range $\omega\sim10^{10}$ sec^{-1} and for $ms^2\sim1$ K is on the order of 10^3, which is in agreement with (40). The acoustic amplitudes given here must be less than $\Phi_0\sim10^{-2}$ K in accordance with (37), which corresponds to an acoustic power $\leqslant1$ W/cm^2. It is, however, important to remember that according to (20) the QDSACR oscillation system consists of several overlapping series and the distance between the singularities is in fact less than that determined by relation (22). With the parameters noted above $qR\sim5$ and hence it is desirable to reduce the number of series by, for example, using a metal with a highly symmetrical deformation potential [having few harmonics (11)] and to increase the oscillation period by means of small m_z. We note that it is generally desirable to have small m_z since this makes it easier to satisfy inequality (17), reduces v_g given by (23) and the damping Γ given by (18), thereby increasing the maximum possible soliton lifetime $T_{0\,max}$, Eq. (39).

[1)]When inequality (17) is not satisfied the acoustical spectrum $\omega(q)$ becomes a multivalued function whose branches describe the quantum acoustic modes analogous to the quantum electromagnetic waves examined in Ref. 9.

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