# Critical behavior of layered superconductors

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We investigate the properties of layered superconductors with Josephson interactions between the layers as the transition temperature is approached, assuming that the coupling between layers is so weak that the transition from two-dimensional to three-dimensional fluctuations comes about in the strong-fluctuation regime. Precisely this situation is realized in high-temperature superconductors made with Bi, Tl, and Pb. We obtain the temperature dependences of the penetration depths and critical currents in various crystallographic directions.

## 1. INTRODUCTION

Recently, there has been renewed interest in the physical properties of layered superconductors with Josephson interactions between the layers, in connection with the discovery of high-temperature super-conductivity (HTSC). It is well-known that the fluctuation properties of layered superconductors are subject to the phenomenon known as dimensional crossover: fluctuations far from the transition temperature have a two-dimensional character, while near it they are three-dimensional. Layered superconductors therefore have several characteristic values of temperature, each of which can be experimentally estimated: a "mean-field" transition temperature $T_{BCS}$, the true transition temperature $T_c$ (at which the resistivity reduces to zero), the crossover temperature $T_{cr}$, and the boundary of the fluctuation region $T_f$ (i.e., for $T < T_f$ the fluctuations of the order parameter are considerably smaller than its average value). In what follows, we will measure all temperatures in relative units, i.e., $\tau = (T - T_{BCS})/T_{BCS}$.

We can identify two different characteristic situations, depending on the ratio of the relative crossover temperature $\tau_{cr}$ to the fluctuation-induced width of the transition $\tau_f$. For $|\tau_{cr}| \gg |\tau_f|$, the crossover from three-dimensional to two-dimensional behavior takes place in the region where mean-field theory is applicable, and occurs when the temperature decreases to a point where the coherence length perpendicular to the layers $\xi_\perp(\tau) \sim \xi_{0\perp}/|\tau|^{1/2}$ becomes comparable to the interlayer spacing $d$, i.e., for $\tau_{cr} = -(\xi_{0\perp}/d)^2$ This case is discussed in detail in Ref. 1.

In the opposite limit $|\tau_{cr}| \ll |\tau_f|$, the crossover takes place in the fluctuation region; this is the case that will be treated in the following section of this paper. For these superconductors, there is a range of temperatures for which the temperature dependence of the superconducting parameters is determined by strong quasi-two-dimensional fluctuations. We will discuss the behavior of the critical current and the London penetration depth. In order to obtain the temperature dependence we will use the scaling analysis first applied to quasi-two-dimensional systems by Pokrovski and Uimin in their paper[2] (see also Ref. 3).

It is noteworthy that until the discovery of the HTSC, all the known layered superconducting compounds apparently belonged to the first class ($|\tau_{cr}| \gg |\tau_f|$); this is because the fluctuation region in a typical superconductor is unusually narrow. In the HTSC, because of their high transition temperature and small concentration of conduction electrons, the fluctuation region is found to be several orders of magnitude wider ($\tau_f \sim 10^{-2}$).[4] Nevertheless, in compounds with the 123 structure, the crossover takes place in the mean-field region, since the electron anisotropy of these compounds is insufficiently high (according to the estimate of Ref. 5, $\tau_{cr} \sim 0.1$). However, in the new recently-synthesized classes of superconductors $Tl_2Ba_2Ca_nCu_nO_{6+2n}$ (Ref. 6), $Bi_2Sr_2Ca_nCu_nO_{6+2n}$ (Refs. 7–9), and $Pb_2Sr_2ACu_3O_{8+\delta}$ (where $A$ is a lanthanide, a mixture of La and Sr, or Ca; see Ref. 1), the electron anisotropy is much higher; these materials apparently belong to the second class ($|\tau_{cr}| \ll |\tau_f|$). Recent experiments which favor this assumption are studies of the temperature dependence of the resistivity for $T > T_c$ (Ref. 11; these experiments are analyzed in Ref. 12) and the nonlinear behavior of the IV characteristics for $T < T_c$ (Ref. 13) of the compound $Bi_2Sr_2CaCu_2O_8$.

## 2. SETTING UP THE PROBLEM: CHARACTERISTIC VALUES OF THE TEMPERATURE IN A QUASI-TWO-DIMENSIONAL SUPERCONDUCTOR

Let us discuss a layered superconductor with weak interactions between its layers. It is known that in a two-dimensional system the fluctuations of the phase of the order parameter for $\tau < \tau_c$ are much stronger than fluctuations in its amplitude. Fluctuation effects in a quasi-two-dimensional superconductor also involve fluctuations of the phase in an essential way;[14] therefore we will neglect the amplitude fluctuations of the order parameter from the beginning and write the energy of the superconductor in the following form:

$$E = \sum_n \int d^2\mathbf{r} \left\{ \frac{1}{2} J \left( \nabla\varphi_n - \frac{2e}{c}\mathbf{A}_\parallel \right)^2 + E_J \left[ 1 - \cos\left( \varphi_{n+1} - \varphi_n - \frac{2e}{c}\int_{z_n}^{z_{n+1}} A_z\,dz \right) \right] \right\} + \int d^3\mathbf{r} \frac{\mathbf{B}^2}{8\pi}, \quad (1)$$

where $J = \Phi_0^2 d/\pi\varepsilon(4\pi\lambda_\parallel)^2$ is the "rigidity" which characterizes the fluctuations in the plane ($\lambda_\parallel$ is the London penetration depth and $\varepsilon \gtrsim 1$ is the effective "dielectric constant" which describes the decrease in the rigidity of the fluctuation vortex pairs[15]), and $E_J$ is the energy of the Josephson interaction between layers. The anisotropy parameter $\gamma = J/E_J d^2$ is assumed to be large, i.e., $\gamma \gg 1$. Near the mean-field

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temperature for the superconducting transition $T_{BCS}$ the following relation obtains

$$J = J_0|\tau|, \quad E_J = E_{J0}|\tau|$$

$[J_0/T_{BCS} \sim \varepsilon_F/T_{BCS} \gg 1$; the parameters $J_0$, $E_{J0}$ have a weak dependence on temperature due to $\varepsilon(\tau)$].

If the Josephson coupling between layers were absent, then for $\tau = \tau_{KT} \approx -2T_{BCS}/(\pi J_0)$ a Berezin-Kosterlitz-Thouless transition should occur.[16,17] Because of the Josephson coupling between layers the super-conducting transition temperature $\tau_c$ lies in the interval $\tau_{KT} < \tau_c < 0$.

In order to estimate the shift in $\tau_c$ relative to $\tau_{KT}$ for small values of $E_J$ we will investigate the influence of interlayer Josephson coupling on the interaction between vortex excitations in a single layer. For the case $E_J = 0$ the interaction energy of a "vortex-antivortex" pair equals

$$\varepsilon_{int}^{(0)} = 2\pi J \ln(r/\xi_\parallel)$$

(where $\xi_\parallel = \xi_{0\parallel}/|\tau|^{1/2}$ is the coherence length). When the spacing between vortices and antivortices is less than a certain characteristic value $r_c = (J/E_J)^{1/2}$, we can neglect the perturbation of the phases in the other layers, and the contribution to the interaction energy caused by the Josephson coupling between layers has the form

$$\varepsilon_{int}^{J} = \pi E_J r^2 \ln(r_c/r).$$

In the case $r > r_c$ the uniform distribution of the phase is broken up into many layers consisting of vortices and antivortices. The region over which this new phase distribution varies is equivalent to a line defect with tension $\sim (J/E_J)^{1/2}$; therefore we have $\varepsilon_{\rm int}^{J} \sim (J/E_J)^{1/2}\Gamma$. From the condition $\varepsilon_{\rm int}^{J} \sim \varepsilon_{\rm int}^{(0)}$ we find that the vortex pair "senses" the other layers for $r < r_c \ln(r_c/\xi_\parallel)$.

The fluctuation properties of a purely two-dimensional system are characterized by a correlation radius $\zeta$, whose temperature dependence for $|\tau - \tau_{KT}| \ll |\tau_{KT}|$ has the form

$$\zeta = \xi_\parallel \exp\{[b|\tau_{KT}|/(\tau - \tau_{KT})]^{1/2}\}.$$

The constant $b$ is not universal; however, it has a definite value within the Ginzburg-Landau theory. Experiments on superconducting films[18] give the estimate $b \sim 2-9$. The "two-dimensional" character of the transition is destroyed when the correlation radius $\zeta$ becomes of order $r_c$. From this we obtain the following estimate for $\tau_c$:

$$\tau_c = \tau_{KT}\left[1 - \frac{b}{\ln^2(J_0|\tau_{KT}|/E_{0J}\xi_{0\parallel}^2)}\right]. \tag{2}$$

For two-dimensional spin systems analogous estimates were given in Refs. 19,20 (in the case of a layered superconductor an important difference arises as a consequence of the strong temperature dependence of the parameters of the Hamiltonian). The Josephson coupling between layers is characterized by the dimensionless parameter $E_{0J}\xi_{0\parallel}^2/J_0|\tau_{KT}| \ll 1$. The three-dimensional fluctuation region around the transition $\tau_{3D}$ can be estimated from the relation $\tau_{3D}|(\partial\zeta/\partial\tau)(\tau = \tau_c)| = r_c$ (see Ref. 19):

$$\frac{\tau_{3D}}{\tau_c - \tau_{KT}} \approx \frac{2}{\ln(J_0\tau_{KT}/E_{0J}\xi_{0\parallel}^2)} \tag{3}$$

In the temperature region $\tau < \tau_{KT}$ the fluctuations of the order parameter have a two-dimensional character. In purely two-dimensional systems the long-wavelength phase fluctuations lead to the absence of long-range order over the entire temperature range. In the quasi-two-dimensional superconductors the interaction between layers leads to the suppression of the phase fluctuations with wave vectors smaller than $1/r_c$. This leads to the suppression of the finite fluctuation-induced width of the transition

$$|\tau_f| = (|\tau_{KT}|/8)\ln(J_0|\tau_{KT}|/E_{0J}\xi_{0\parallel}^2).$$

In the region of temperatures, $\tau_f < \tau < \tau_{KT}$ the phase fluctuations strongly influence a number of superconducting parameters, while for $\tau < \tau_f$ they lead only to a small correction. From this we see that certain characteristic values of temperature are distinguished in the fluctuation region of a layered superconductor. The temperature intervals corresponding to qualitatively different fluctuation behaviors are illustrated in the figure. Let us emphasize that Fig. 1 refers to the special case we are discussing, i.e., extremely weak interlayer coupling, when the $2D$–$3D$ crossover takes place in the region of strong fluctuations.

## 3. TEMPERATURE DEPENDENCE OF THE PENETRATION DEPTHS $\lambda_\parallel, \lambda_\perp$

The quantity $\lambda_\parallel^{-2}(T)$ is proportional to the density of pure superfluid.[3] In a purely two-dimensional system the superfluid density undergoes a Kosterlitz-Nelson discontinuity, changing at the transition point from zero to a finite value.[21] In a layered superconductor the jump is smeared out into a step with width

$$\tau_{3D} \approx 2|\tau_{KT}|b/\ln^3(J_0|\tau_{KT}|/E_{0J}\xi_{0\parallel}^2).$$

For $\tau < \tau_{KT}$ the vortex excitations decrease the superfluid density by the factor $\varepsilon(\tau)$ compared to its mean-field value. The maximum value of the "dielectric constant" $\varepsilon_c = \varepsilon(\tau_{KT})$ was experimentally estimated in Ref. 18: $\varepsilon_c \sim 1.2-1.8$. Thus, as the temperature decreases the quantity $\lambda_\parallel^{-2}$ changes in the vicinity of $\tau_c$ from zero to the value $\varepsilon_c^{-1}[\lambda_\parallel^{BCS}(\tau_c)]^{-2}$ in an interval of width $\tau_{3D}$.

In the absence of Josephson coupling between the layers a magnetic field applied parallel to the layers is not screened. The Josephson coupling leads to the appearance of a finite value of the transverse penetration depth $\lambda_\perp$, which in mean-field theory is determined by the relation

$$\lambda_\perp^{BCS} = \left[\frac{\Phi_0^2}{2(2\pi)^3 dE_J}\right]^{1/2}. \tag{4}$$

The phase fluctuations cause an increase in the penetration depth $\lambda_\perp$ compared to $\lambda_\perp^{BCS}$. In order to determine the temperature dependence $\lambda_\perp$ we can make use of the scaling rela-

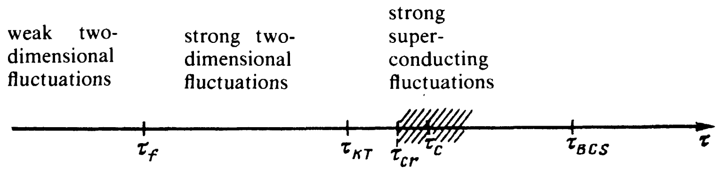


FIG. 1. Hierarchy of characteristic values of the temperature for a layered superconductor with weak interlayer coupling: $\tau_{BCS}$ is the mean-field transition temperature, $\tau_{KT}$ is the transition temperature in an individual layer, and $\tau_c$ is the true transition temperature; the cross-hatched region corresponds to the temperature interval of three-dimensional fluctuations.

tions. In Ref. 2 it is shown that for a purely two-dimensional degenerate system the scale invariance occurs over the entire region of temperatures below the transition temperature. The scaling dimension[1] of the order parameter $\Psi = \exp(i\varphi)$ in this case equals

$$\Delta(\Psi) = T/4\pi J. \tag{5}$$

Let us estimate the scaling dimension of the other parameters. The correction to the Hamiltonian connected with the Josephson interaction between layers has the form

$$\Delta H = E_J \int d^2\mathbf{r} \sum_j \Psi_j \Psi_{j+1}^*.$$

From scale invariance of the Hamiltonian it follows that the dimension of $E_J$ equals

$$\Delta(E_J) = 2 - 2\Delta(\Psi). \tag{6}$$

From the expression for the transverse supercurrent

$$j_\perp = 2eE_J \,\mathrm{Im}(\Psi_l \Psi_j^*)$$

we obtain the scaling dimension of $j_\perp$:

$$\Delta(j_\perp) = 2. \tag{7}$$

The quantity $\lambda_\perp^{-2}$ is proportional to the ratio of $j_\perp$ to the corresponding component of the superfluid density, whose scaling dimension equals zero. Therefore the dimensions of the quantities $\lambda_\perp^{-2}$ and $j_\perp$ coincide, i.e.,

$$\Delta(\lambda_\perp) = -1. \tag{8}$$

The requirement of scale invariance leads to the following dependence of $\lambda_\perp^{-2}$ on $E_J$:

$$\lambda_\perp^{-2} \propto (E_J)^{-2\Delta(\lambda_\perp)/\Delta(E_J)} \sim (E_J)^{1/[1-\Delta(\Psi)]}, \tag{9}$$

or, in dimensional units,

$$\frac{(\lambda_\perp^{BCS})^2}{\lambda_\perp^2} \approx \left(\frac{E_{0J}\xi_{0\parallel}^2}{J_0|\tau|}\right)^{\Delta(\Psi)/[1-\Delta(\Psi)]}. \tag{10}$$

The result (10) can be carried over to variational methods.[2,3]

In the temperature range $\tau_f < \tau < \tau_{KT}$ the fluctuations lead to a considerable increase in $\lambda_\perp$ compared to its mean-field value $\lambda_\perp^{BCS}$ and to the appearance of a temperature dependence of the anisotropy of the penetration depth.

At low temperatures $\tau \ll \tau_f$ the fluctuation-induced correction to $[\lambda_\perp^{BCS}]^{-2}$ can be obtained by direct calculation:

$$\lambda_\perp^{-2} = [\lambda_\perp^{BCS}(T)]^{-2}\left(1 - \frac{T}{4\pi J}\ln\frac{E_J\xi_\parallel^2}{J}\right). \tag{11}$$

For $\tau < \tau_f$ the result (11) matches with Eq. (10). Let us note that at the point $\tau = \tau_{KT}$ the equation $\Delta(\Psi) = 1/8$ is satisfied; therefore the exponent in Eq. (10) at this point is approximately equal to 1/7.

## 4. CRITICAL CURRENT

Analogous considerations can be used to estimate the temperature dependence of the critical current density in various crystallographic directions. In a purely two-dimensional system the critical current equals zero, since under the action of an arbitrarily small current a dissociation into vortex pairs occurs. This leads to a finite magnetic intensity, which depends on the magnitude of the current as a power law $U \propto j$, where $a = \pi J/T$ (Refs. 22,23). It should be noted that, strictly speaking, a finite intensity also appears even in the three-dimensional case at small currents and finite temperatures due to the generation of vortex ring currents.[24] However, this intensity is exponentially small (i.e., $U \propto \exp(-j/j_0)$, where the quantity $j_0$ considerably exceeds the value of the depairing current density) and therefore is unobservable.

In quasi-two-dimensional superconductors the minimum characteristic current is that for which exponential behavior of the intensity shifts to power-law behavior. In the region $\tau < \tau_f$ an estimate of this critical current can be obtained from simple considerations. The current "drags apart" the vortex-antivortex pairs in the layer with a force $f_j = d\Phi_0 j_\parallel/c$; on the other hand, for $r > r_c$ there is an attractive force between vortex and antivortex equal to $f_{\rm int} \approx (JE_J)^{1/2}$. For this reason, the current is capable of breaking the pairs for $j_\parallel > j_{c\parallel}$, with

$$j_{c\parallel} = j_\parallel^{BCS}(T)\,(E_J\xi_\parallel^2/J)^{1/2}, \tag{12}$$

where $j_\parallel^{BCS} \sim c\Phi_0/\pi(4\pi\lambda_\parallel)^2\xi_\parallel$ is the depairing current density. In order to obtain the temperature dependence of the critical current density $j_{c\parallel}$ which is valid over the entire temperature range $\tau < \tau_{KT}$, we will determine the scaling dimension of the current. From the expression $j_\parallel \sim J\,\mathrm{Im}(\Psi^*\nabla\Psi)$ we obtain $\Delta(j_\parallel) = \Delta(J) + 2\Delta(\Psi) + 1$. The relation $\Delta(J) + 2\Delta(\Psi) = 0$ follows from the invariance of the Hamiltonian relative to scale transformations. From this we see

$$\Delta(j_\parallel) = 1. \tag{13}$$

Using the value of the scaling dimension $\Delta(E_J)$, see (5) and (6), we obtain the following estimate of the renormalized fluctuations of the critical current:

$$j_{c\parallel} = j_\parallel^{BCS}(T)\left(\frac{E_J\xi_\parallel^2}{J}\right)^{1/2[1-\Delta(\Psi)]}. \tag{14}$$

It should be noted that for $j_\parallel > j_{c\parallel}$ the resistivity $R$ does not immediately rise to its normal value. Over a wide interval of values of the current density $j_{c\parallel} \ll j_\parallel \ll j_\parallel^{BCS}$ a characteristic "two-dimensional" behavior of the resistivity is observed: $R \propto (j_\parallel)$.$^a$ Only for $j_\parallel \sim j_\parallel^{BCS}$ does the resistivity reach its normal value.

Now let us estimate the critical current density $j_{c\perp}$ in a direction perpendicular to the layers. In the region $\tau_f < \tau < \tau_{KT}$ the phase fluctuations strongly decrease the critical current density compared to the Josephson critical current density $j_J(T) = 2eE_J(T)$. In this region scaling analysis leads to the following estimate:

$$j_{c\perp} \approx j_J(T)\left(\frac{E_{0J}\xi_{0\parallel}^2}{J_0|\tau|}\right)^{\Delta(\Psi)/[1-\Delta(\Psi)]}. \tag{15}$$

For $\tau \lesssim \tau_f$ the difference between the critical current and $j_J$ becomes small. In this region we should include the fact that a transverse current decreases the effective energy of the Josephson coupling:

$$E_J(j_\perp) = [E_J^2 - (j_\perp/2e)^2]^{1/2}, \tag{16}$$

thereby increasing the intensity of the phase fluctuations. Repeating the analysis given previously with the replacement $E_J \to E_J(j_\perp)$, we obtain in place of Eq. (15) a transcendental equation for determining $j_{c\perp}$:

$$j_{c\perp} \approx j_J\left\{\frac{\xi_\parallel^2}{J}[E_J^2 - (j_{c\perp}/2e)^2]^{1/2}\right\}^{\Delta(\Psi)/[1-\Delta(\Psi)]}. \tag{17}$$

In particular, for low temperatures $\tau \ll \tau_f$ we can obtain from

Eq. (17) the fluctuation correction to the transverse critical current:

$$j_{c\perp}-j_J \approx -j_J \frac{T}{4\pi J} \ln\left[\frac{J}{E_J \xi_{\parallel}^2}\left(\frac{4\pi J}{T}\right)^{1/2}\right]. \quad (18)$$

## 5. SUPPRESSION OF THREE-DIMENSIONAL FLUCTUATIONS BY A MAGNETIC FIELD PARALLEL TO THE LAYERS

It is well-known that in "ordinary" layered superconductors (in which $\tau_{cr} \gg \tau_f$) there exists a crossover in the temperature dependence of the upper critical field $H_{c2}^{\parallel}$ at $T = T_{cr}$. It reveals itself as a shift from the dependence $H_{c2}^{\parallel} \propto |\tau|$, which is observed for $\tau > \tau_{cr}$ (and coincides with the dependence characteristic of bulk material) to a different dependence $H_{c2}^{\parallel}(\tau)$ which corresponds to an individual layer and is observed for $\tau < \tau_{cr}$. The field corresponding to this crossover is

$$H_{cr} = \frac{\Phi_0}{\pi d}\left(\frac{E_J}{J}\right)^{1/2}. \quad (19)$$

Physically, this crossover is due to suppression of the Josephson interaction between layers by fields $H > H_{cr}$.

In the case we are discussing, i.e., a strongly layered superconductor ($|\tau_f|) \gg |\tau_{cr}|$), a parallel magnetic field significantly affects the fluctuation-related properties only in a small portion of the fluctuation region

$$|\tau - \tau_c| \lesssim \tau_c - \tau_{KT}. \quad (20)$$

For $H > H_{cr}$, the decrease in the Josephson interaction energy mentioned above leads to a compression of the region of three-dimensional fluctuations $\tau_{3D}(H)$ and the approximation of the transition temperature $\tau_c(H)$ by the temperature $\tau_{KT}$ which does not depend on $H$. In order to calculate the function $\tau_c(H)$ we must determine the $H$-dependence of the effective interaction energy of the "vortex-antivortex" pairs $\varepsilon_{\rm int}^J$ (see Section 2). In the presence of a field $H \gg H_{cr}$ the equilibrium value of the phase difference between adjacent layers

$$\varphi_{n+1} - \varphi_n \sim \frac{E_J}{J}\left(\frac{\Phi_0}{dH}\right)^2 \sin\left(2\pi \frac{Hdx}{\Phi_0}\right) \quad (21)$$

oscillates rapidly in space and is small in magnitude. Therefore, in order to determine the effective interaction energy $E_{\rm int}^J$ of a vortex-antivortex pair of size $r \gg \Phi_0/Hd$ we must replace $E_J$ by its effective value $E_{\rm int}^J$ determined from Eq. (1) by using Eq. (21):

$$E_J^{\rm eff} \approx E_J\left(\frac{H_{cr}}{H}\right)^2. \quad (22)$$

Repeating the discussion of Section 2, we find

$$\tau_c(H) - \tau_{KT} = |\tau_{KT}|\, b \Big/ \ln^2\left(\frac{J_0|\tau_{KT}|H^2}{E_{0J}\xi_{0\parallel}H_{cr}^2}\right). \quad (23)$$

Equation (23) describes the shift of the transition temperature under the action of a magnetic field $H \gg H_{cr}$. In fields $H \ll H_{cr}$ this shift is negligibly small. Note that we have included the effect of $H$ on the orbital motion of the electrons and neglected the spin effects.

## CONCLUSION

An analysis of the temperature dependences of the resistivity[11,12] and the IV characteristics[13] of the compound $Bi_2Sr_2CaCu_2O_8$ reveals a similarity between the fluctuation behavior of these compounds and that of thin superconducting films. In particular, in Ref. 13 there is observed a Kosterlitz-Nelson discontinuity characteristic of a two-dimensional system near the transition temperature.[21] The parameters of this compound are estimated to be as follows: $T_{BCS} \approx 82$ K, $T_{BCS} - T_c \approx 3.2$ K, $d = 12$ Å; the anisotropy parameter $\gamma$ introduced in Section 2 is found to be $\approx 10^4$ (obtained from measuring the conductivity anisotropy at $T = 300$ K), and $\xi_{0\parallel} \sim 20$ Å (from resistance measurements in a magnetic field[25,26]).

Using these values of the parameters, we can estimate the values of the characteristic temperatures. The shift in the transition temperature $T_c$ relative to the Kosterlitz-Thouless temperature $T_{KT}$ comes to $T_c - T_{KT} \sim 1$ K, and the three-dimensional width of the transition equals $T_c - T_{cr} \sim 0.4$ K. The fluctuation in the phase of the order parameter is found to be significant in the temperature interval $T_c - T_f$, amounting to several degrees. The characteristic values of the critical currents come to

$$j_{c\parallel}(0) \approx 10^6 \text{ A/cm}^2,$$
$$j_{c\perp}(0) \approx 10^4 \text{ A/cm}^2.$$

We emphasize that the current density $j_{c\parallel}$ corresponds to a transition from the exponential (three-dimensional) portion of the IV characteristic to a power-law (two-dimensional) portion (see Section 4). In fact this crossover is observable only near the transition point. The value of the critical current density $j_{c\parallel}$ for $T \sim T_{KT}$ comes to $10^4$ A/cm$^2$, which agrees with the value obtained in experiments.[27] However, it is noteworthy that power-law IV characteristics were observed at these current densities in Ref. 13 although they should appear only in the region $j \gg j_{c\parallel}$. The reason for this discrepancy is unclear.

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[1)] The quantity $\Delta(A)$ is called the scaling dimension of the parameter $A$ if under the scaling transformation $r \to \lambda r$ this parameter transforms according to the law $A \to \lambda^{-\Delta(A)} A$.


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