

Nonlocal nature of the resistance in classical ballistic transport

E. V. Sukhorukov and I. B. Levinson

*Institute of Problems in Technology of Microelectronics and Ultrapure Materials,
Academy of Sciences of the USSR*

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An investigation is made of the resistance of ballistic microstructures formed in the two-dimensional electron gas of a GaAs/AlGaAs heterojunction representing combinations of long channels. It is shown that the nonlocal nature of the resistance (dependence on the measurement method) is unrelated to the quantum nature of the electron behavior, but is solely due to the ballistic nature of microstructures and does not disappear in the classical limit. An analog of the Landauer equation is obtained for the resistance measured by the four-probe method allowing for the geometry of the measuring probes.

INTRODUCTION

The lateral transport in microstructures formed in a two-dimensional electron gas (2DEG) of GaAs/AlGaAs heterojunctions has been attracting intense interest in the last few years. The lateral transport in microstructures characterized by a high mobility of 2DEG electrons ($\mu \gtrsim 10^6 \text{ cm}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1}$) receiving special attention. In this case the transport may be ballistic (if the “dimensional” size of a structure is less than the momentum mean free path l , which in such structures is of the order of $10 \mu\text{m}$) or even quantum (if there is a “constriction” of size of the order of the Fermi wavelength $\lambda_F \approx 50 \text{ nm}$ in the path of electrons in a structure). The interest in the ballistic transport has increased strongly since the discovery of the conductivity quantization effect.^{1,2}

Various experiments have revealed that the resistance of ballistic microstructures has a number of unusual properties which can be labeled “nonlocal behavior of the resistance.” For example, the resistance of two resistors connected in series is not equal to the sum of the resistances but to the larger of the two resistances.³ Another type of nonlocal behavior is exhibited by structures in which the conducting region of the 2DEG is a network of long channels with the “end” channels spreading into large contact areas (banks).^{4–6} In systems of this kind the resistance between two nodes of a network depends on the path along which the current enters one node and emerges from another node in the network.⁶

The experimental results show that the nonlocal behavior of the resistance in a network of long channels becomes weaker as the mobility decreases and the number of the conducting modes increases (i.e., as the number of filled transverse quantization levels in a channel increases). An impression may therefore arise that the nonlocal behavior is a quantum effect. We shall show that this is not true, i.e., that the nonlocal behavior is associated entirely with the ballistic nature of the transport and does not disappear when the number of the conducting modes (equal to the integral part of the fraction $N = 2d/\lambda_F$, where d is the channel width) rises without limit.

The nonlocal behavior is closely linked to the relationship between the resistances $R_{(2)}$ and $R_{(4)}$ measured by the two- and four-probe methods. It is shown in Ref. 7 that the

measured value of $R_{(4)}$ for a scatterer in a single-mode channel depends strongly on the degree of interference of electron waves reflected from a scatterer and potential probes. We shall investigate the question of determining $R_{(4)}$ in a classical system ($N \gg 1$) where there is no interference and we shall show that in this case the measured value of $R_{(4)}$ still depends on the nature of the probes (even when the probes perturb the current-conducting channel only slightly).

1. CONDUCTANCE MATRIX AND CLASSICAL SCATTERING MATRIX

Multipole systems are described⁸ by a matrix of conductances $G_{s's'}$ governing the currents J_s emerging from contact areas due to the potentials φ_s of these areas:

$$J_s = \sum_{s'} G_{s's'} \varphi_{s'} \quad (1)$$

The conductance matrix has the following properties:

$$G_{s's'} = G_{s's}, \quad \sum_{s'} G_{s's'} = 0 \quad (2)$$

These properties ensure invariance of the currents J_s with respect to the reference point from which the potentials φ_s are measured and also satisfy the obvious requirement that the sum of all the currents should be zero.

In a semiclassical description of the ballistic transport a calculation of the matrix G reduces to a study of the classical trajectories of electrons emerging from one bank and entering the other.^{9,10} The problem is in principle elementary, but extremely messy. It becomes particularly involved when the system contains long channels because then—due to the large number of reflections from the channel walls—a small shift of the point at which an electron enters may strongly distort its path. However, this circumstance can be turned to advantage and a simpler method can be developed for calculating the resistance in the case of long channels.

The matrix $G_{s's'}$ can be calculated for ballistic devices with long end channels if we know the scattering matrix $S_{s'n, s'n'}$ for a waveguide junction with cutoff banks and the coefficients of reflection $R_{s'n}$ of waves in the end waveguides from the point of junction with the contact area.⁷ Here, n and n' are the numbers of the waveguide modes in the end chan-

nels s and s' . We shall be interested in the situation when the end channels are semiclassical. This means that the number of modes in such channels is large: $N_s = 2d_s/\lambda_F \gg 1$ (here, d_s is the width of the s th channel). In the semiclassical situation we have $R_{sn} = 0$, and we can use the expression obtained in Ref. 11:

$$G_{ss'} = \frac{2e^2}{h} \sum_{nn'} |S_{sn,s'n'}|^2, \quad s \neq s'. \quad (3)$$

This expression applies when the electron temperature is $T = 0$; the scattering matrix S is calculated for an electron energy ε equal to the Fermi energy ε_F .

The matrix S governs the amplitudes of the outgoing waves b_{sn} via the amplitudes of the incoming waves $a_{s'n'}$:

$$b_{sn} = \sum_{s'n'} S_{sn,s'n'} a_{s'n'}. \quad (4)$$

The normalization of the amplitudes in Eq. (3) is selected so that the incoming and outgoing fluxes due to a mode n in a channel s are $|a_{sn}|$ and $|b_{ns}|^2$. In this case the matrix S is symmetric and unitary.

In the semiclassical situation it is convenient to replace the mode number n with an angle θ defined by the condition

$$\sin \theta = k_{\perp}/k_F, \quad k_{\perp} = n\pi/d \quad (0 < \theta < \pi/2). \quad (5)$$

Here, $\hbar k_F$ is the Fermi momentum in a 2DEG. Two plane waves propagate at an angle θ to the waveguide axis and interference between them creates a guided wave. The number of modes within the angular interval $\Delta\theta$ is

$$\Delta n = N \cos \theta \Delta\theta, \quad N = k_F d/\pi, \quad (6)$$

where N is the total number of modes.

Transforming from summation over n to integration with respect to θ in Eq. (3), we obtain

$$G_{ss'} = \frac{2e^2}{h} N_s N_{s'} \int d\theta \int d\theta' \cos \theta \cos \theta' |S_{sn,s'n'}|^2, \quad (7)$$

where $n = N_s \sin \theta$, $n' = N_{s'} \sin \theta'$, and N_s and $N_{s'}$ are the numbers of modes in the channels s and s' .

The next step involves attribution of a classical meaning to the quantity $|S_{sn,s'n'}|^2$ considered as a function of the angles θ and θ' . We do this by considering a point source of classical particles in an infinitely long channel of width d emitting particles at an angle θ to the channel axis within a certain interval $\Delta\theta$. These particles are reflected specularly from the channel walls and conserve the angle which their velocity makes with the channel axis θ as well as the angular interval $\Delta\theta$. However, away from the source the distribution of particles over the channel cross section becomes equalized. At distances from a source $z \gg d/\Delta\theta$ the distribution over the channel cross section can be regarded as uniform. Hence it is clear that in long channels far from inhomogeneities (scatterers, points where these channels join other channels and contact areas) the particle distribution is described by two functions $u(\theta)$ and $v(\theta)$ which represent the angular distribution of the particles moving to the right and to the left.

If we know the distributions $u_s(\theta)$ of the particles entering the end channels, then the distribution of the outgoing particles $v_s(\theta)$ can be expressed linearly in terms of it:

$$v_s(\theta) = \sum_{s'} \int_0^{\pi/2} d\theta' u_{s'}(\theta') T_{s's}(\theta', \theta). \quad (8)$$

The kernel T can be called a classical scattering matrix.

We shall normalize the distributions $u(\theta)$ and $v(\theta)$ so that the flux along the channel axis due to the particles moving at an angle θ to this axis within the interval $\Delta\theta$ is $u(\theta)\Delta\theta$ and $v(\theta)\Delta\theta$. It is then obvious that

$$u(\theta) = |a_n|^2 N \cos \theta, \quad v(\theta) = |b_n|^2 N \cos \theta. \quad (9)$$

Let us now find the relationship between $|S|^2$ and T . We assume that a flux $u_{s'}(\theta')\Delta\theta'$ enters a channel s' within an angular interval $\Delta\theta'$. The flux leaving a channel s within an interval $\Delta\theta$ is, according to Eq. (8),

$$u_s(\theta)\Delta\theta T_{s's}(\theta', \theta)\Delta\theta. \quad (10)$$

On the other hand, the same flux is described by

$$\begin{aligned} \sum_n |b_{sn}|^2 &= \sum_n \sum_{n'} |S_{sn,s'n'}|^2 |a_{s'n'}|^2 \\ &= N_s \cos \theta \Delta\theta |S_{sn,s'n'}|^2 \sum_{n'} |a_{s'n'}|^2 \\ &= N_s \cos \theta \Delta\theta |S_{sn,s'n'}|^2 u_{s'}(\theta') \Delta\theta'. \end{aligned} \quad (11)$$

Here, the sums over n and n' cover the modes lying within the intervals $\Delta\theta$ and $\Delta\theta'$. Comparing the two expressions for the flux, we obtain

$$T_{s's}(\theta', \theta) = N_s \cos \theta |S_{sn,s'n'}|^2. \quad (12)$$

This is the required relationship between T and S . The principle of detailed equilibrium follows from the symmetry of the matrix:

$$N_{s'} \cos \theta' T_{s's}(\theta', \theta) = N_s \cos \theta T_{s's}(\theta, \theta'), \quad (13)$$

and the unitarity of S implies conservation of the flux:

$$\sum_s \int d\theta T_{s's}(\theta', \theta) = 1. \quad (14)$$

Using Eq. (12), we obtain (for $s \neq s'$) the expression

$$G_{ss'} = \frac{2e^2}{h} N_{s'} \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\theta' \cos \theta' T_{s's}(\theta', \theta). \quad (15)$$

This expression allows us to find the conductance matrix if we know the classical scattering matrix.

2. CONDUCTANCE MATRIX OF SOME OF THE SIMPLEST SYSTEMS

We now consider several examples in which the matrices T and G can be calculated.

1. The trivial example of a rectilinear channel connecting banks 1 and 2. In this case we obviously have

$$T_{12}(\theta, \theta') = \delta(\theta - \theta'), \quad T_{11} = T_{22} = 0.$$

According to Eq. (15), we obtain

$$G_{12} = \frac{2e^2}{h} N, \quad N = \frac{k_F d}{\pi}. \quad (16)$$

2. The series connection of two channels of equal width (Fig. 1). In this case if we assume that a flux is incident from the left, we obviously have

$$T_{12}(\theta, \theta') = \delta(\theta - \theta'), \quad T_{11} = 0. \quad (17)$$

If we assume that a flux distributed uniformly over the cross section enters from the right, then

$$T_{21}(\theta, \theta') = \frac{d_1}{d_2} \delta(\theta - \theta'), \quad T_{22}(\theta, \theta') = \frac{d_2 - d_1}{d_2} \delta(\theta - \theta'). \quad (18)$$

We can readily see that the conditions (13) and (14) are satisfied. Using Eq. (15), we find that

$$G_{12} = \frac{2e^2}{h} N_1, \quad N_1 = \frac{k_F d_1}{\pi}, \quad (19)$$

i.e., the conductance is governed only by the width of the narrow channel, as is indeed found experimentally.³

3. Four-armed cross (Fig. 2). We can calculate the matrix T by assuming that the flux of particles traveling at an angle θ and distributed uniformly over the cross section is incident from far on the right and it enters a channel 1. This flux reaches the cross section AA' without perturbation. We introduce an angle θ_0 such that $\tan \theta_0 = d_1/d_2$. For $\theta > \theta_0$, then half the flux emitted by the section AA' escapes to a channel 2 and another half to a channel 2'. However, for $\theta < \theta_0$, then the channel 2 receives only half the particles from the area AB . All the particles from the area $A'B$ escape to channel 1'. No particles return to the channel 1 irrespective of the angle of incidence θ . The result is then

$$T_{11'}(\theta, \theta') = \delta(\theta - \theta') t_{11}(\theta), \\ T_{12}(\theta, \theta') = \delta(\theta + \theta' - \pi/2) t_{12}(\theta), \quad T_{11} = 0, \quad (20)$$

where

$$\theta < \theta_0: t_{12}(\theta) = (d_2/2d_1) \operatorname{tg} \theta, \quad t_{11}(\theta) = 1 - (d_2/d_1) \operatorname{tg} \theta, \\ \theta > \theta_0: t_{12}(\theta) = 1/2, \quad t_{11}(\theta) = 0. \quad (21)$$

The remaining elements of the matrix T can be obtained from the principle of detailed equilibrium (13) by transposing the indices 1 and 2. For example, we find that

$$t_{21}(\theta) = (d_1/d_2) \operatorname{tg} \theta t_{12}(\pi/2 - \theta). \quad (22)$$

The conductance matrix of the cross (in units of $2e^2/h$) is

$$G_{11} = -\frac{k_F}{\pi} d_1, \quad G_{11'} = \frac{k_F}{\pi} [(d_1^2 + d_2^2)^{1/2} - d_2], \\ G_{12} = \frac{k_F}{2\pi} [d_1 + d_2 - (d_1^2 + d_2^2)^{1/2}]. \quad (23)$$

The other elements of the matrix are obtained by transposition of the indices 1 and 2. For $d_1 = d_2 = d$, then in units of $(2e^2/h)k_F d/\pi$, we have

$$G_{11} = -1, \quad G_{11'} = 2^{1/2} - 1 = 0.414, \quad G_{12} = 1 - 2^{-1/2} = 0.293, \quad (24)$$



FIG. 1.

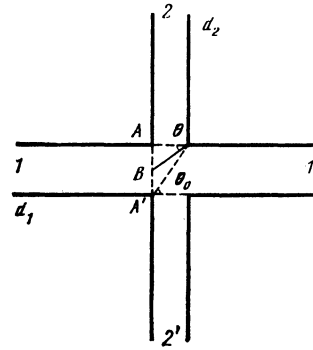


FIG. 2.

4. Six-armed cross (Fig. 3). The simplest system in which one can study the nonlocal behavior of the type found experimentally and reported in Ref. 6 is the six-pole system in Fig. 3. If not only are the end channels $A1, A3, A4, B2, B5$, and $B6$ long, but this also applies to an intermediate channel AB , the matrix T of the whole six-pole can be expressed in terms of the matrices T of two quadripoles which are crosses obtained by cutting the channel AB . The problem is greatly simplified because there are no reflections in the cross. We shall assume that a flux $u(\theta)$ enters a channel 1. When this flux branches at a node A , then channels 3 and 4 receive the fluxes

$$v_3(\theta) = v_4(\theta) = u(\bar{\theta}) t_{12}(\bar{\theta}),$$

whereas a channel AB receives

$$a(\theta) = u(\theta) t_{11}(\theta)$$

(here, $\bar{\theta} = \pi/2 - \theta$). The flux $a(\theta)$ branches at a node B , so that channels 5 and 6 receive fluxes

$$v_5(\theta) = v_6(\theta) = a(\bar{\theta}) t_{12}(\bar{\theta}),$$

whereas channel 2 is reached by the flux

$$v_2(\theta) = a(\theta) t_{11}(\theta).$$

The result is then

$$T_{12}(\theta, \theta') = \delta(\theta - \theta') t_{11}^2(\theta), \\ T_{13}(\theta, \theta') = T_{14}(\theta, \theta') = \delta(\theta + \theta' - \pi/2) t_{12}(\theta), \\ T_{15}(\theta, \theta') = T_{16}(\theta, \theta') = \delta(\theta + \theta' - \pi/2) t_{12}(\theta) t_{11}(\theta). \quad (25)$$

The other matrix elements are found similarly:

$$T_{34}(\theta, \theta') = \delta(\theta - \theta') t_{22}(\theta), \\ T_{35}(\theta, \theta') = T_{36}(\theta, \theta') = \delta(\theta - \theta') t_{21}(\theta) t_{12}(\pi/2 - \theta). \quad (26)$$

The conductance matrix of such a six-armed cross will

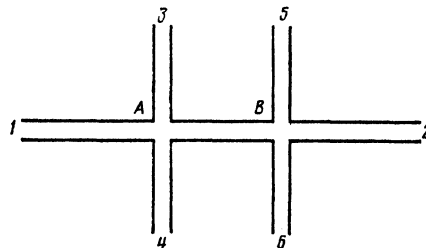


FIG. 3.

be given here only for the case when $d_1 = d_2 = d$. In units of $(2e^2/h)k_F d/\pi$, we have

$$\begin{aligned} G_{ss} &= -1, \quad s=1, \dots, 6, \\ G_{12} &= 2^{1/2} - 2 + \Lambda = 0.296, \quad \Lambda = \frac{1}{2} \ln \frac{2^{1/2} + 1}{2^{1/2} - 1} = \ln \operatorname{ctg} \frac{\pi}{8}, \\ G_{13} &= G_{14} = G_{23} = G_{26} = 1 - 2^{-1/2} = 0.293, \\ G_{15} &= G_{16} = G_{25} = G_{24} = \frac{1}{2}(1 - \Lambda) = 0.059, \\ G_{34} &= G_{36} = 2^{1/2} - 1 = 0.414, \\ G_{35} &= G_{36} = G_{45} = G_{46} = \frac{1}{4}(1 - 2^{1/2} + \Lambda) = 0.117. \end{aligned} \quad (27)$$

3. NONLOCAL BEHAVIOR OF THE RESISTANCE

The dependence of the resistance on the current path can be demonstrated using just the example of the four-armed cross. If the current flows along the "straight line" (i.e., for $J_2 = J_2' = 0$), the resistance is

$$R_{11',11'} = \frac{\Phi_1 - \Phi_{1'}}{J_{1'}} = 2^{1/2} = 1.414. \quad (28)$$

However, if the current turns (i.e., if $J_{1'} = J_2' = 0$), the resistance is higher:

$$R_{12,12} = \frac{\Phi_1 - \Phi_2}{J_2} = \frac{2 + 3 \cdot 2^{1/2}}{4} = 1.561. \quad (29)$$

To compare with the experimental results of Ref. 6, let us calculate the "resistance" of the central channel AB of the six-armed cross for different current paths, i.e., let us find quantities of the type

$$R_{ss',ss'} = \frac{\Phi_s - \Phi_{s'}}{J_{ss'}}, \quad (30)$$

where $J_{ss'}$ is the current entering an end channel s and leaving to reach an end channel s' ; all the other currents are assumed to vanish. Using the matrix G of Eq. (27), we find that

$$R_{12,12} = 0.49, \quad R_{15,16} = 0.57; \quad R_{35,36} = 0.65.$$

The ratio of these quantities are

$$R_{12,12} : R_{15,16} : R_{35,36} = 1 : 1.36 : 1.55.$$

The first quantity corresponds to a straight current path, the second corresponds to turning at one of the potential probes, and the third represents the effect of turning at both potential probes. The larger the number of turns of the current, the greater the resistance. This is in full agreement with the experimental results of Ref. 6, although the ratios reported there are larger, amounting approximately to 1:2:3 (for channels with the mode number $N \approx 5$). It is therefore possible that as the number of modes N increases the nonlocal behavior becomes weaker but does not disappear in the limit $N \rightarrow \infty$.

The resistance of a six-pole $R_{12,12}$ (in the quantum case $N < 3$) was calculated earlier¹² from the resistance of the quadrupole $R_{11',11'}$ using the relationship

$$R_{12,12} = R_{11',11'} - \frac{\hbar}{2e^2} \frac{1}{N}, \quad (31)$$

which is valid (as shown in Ref. 12) if during the propagation in the channel AB : 1) the mutual coherence of the modes is lost; 2) the populations of all the modes become equal. The condition 1 is the criterion for a semiclassical

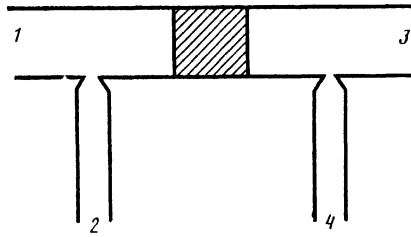


FIG. 4.

description, whereas the condition 2 means that $u(\theta)$ and $v(\theta)$ in the channel AB are independent of θ . Obviously, the condition 2 is more stringent than the condition 1. Our calculations do not postulate leveling of the populations, so that the resistances [in units of $(\hbar/2e^2)N^{-1}$] will have found

$$R_{12,12} = \frac{1 - 2^{1/2} + \Lambda}{5 \cdot 2^{1/2} - 7 + \Lambda} = 0.491, \quad R_{11',11'} = 2^{1/2} = 1.414$$

do not satisfy the relationship (31).

4. DETERMINATION OF THE RESISTANCE BY THE FOUR-PROBE METHOD

We consider a channel (of width d_1) carrying a current (Fig. 4). This channel contains a scatterer, which for the sake of simplicity is assumed to be symmetric, described by a transmission coefficient $T(\theta, \theta')$ and a reflection coefficient $R(\theta, \theta')$. The resistance of this scatterer is determined using symmetrically located potential probes in the form of channels of width d_2 coupled weakly to the current channel. This coupling is described by the coefficient $t(\theta, \theta')$ of transmission from the channel to the probe, where $t \ll 1$. The measured resistance is

$$R_{12,12} = \frac{\Phi_2 - \Phi_1}{J_1} \Big|_{J_2 = J_2' = 0}, \quad (32)$$

i.e., it is equal to the difference between the potentials on the contact areas adjoining the probe channels, divided by the current in the current channel when the currents in the probe channels vanish.

In the case of a weak coupling with the probes in a symmetric system,⁷ we have

$$R_{12,12} = \frac{1}{G_{13}} \frac{G_{23} - G_{21}}{G_{23} + G_{21}}. \quad (33)$$

Then, G_{13} can be calculated in the absence of both probes, whereas G_{23} and G_{21} can be found in the absence of the probe 4. Obviously, G_{13}^{-1} is the resistance of the scatterer measured by the two-probe method. It follows from Eq. (15) that

$$G_{13} = \frac{2e^2}{\hbar} \frac{k_F d_1}{\pi} \int d\theta \cos \theta \int d\theta' T(\theta \rightarrow \theta'). \quad (34)$$

If we use the law of conservation (14) and introduce the total probability of reflection through an angle θ , i.e.,

$$R(\theta) = \int d\theta' R(\theta, \theta'), \quad (35)$$

we have

$$G_{13} = \frac{2e^2}{\hbar} N_1 [1 - \langle R \rangle], \quad \langle R \rangle = \int d\theta \cos \theta f(\theta). \quad (36)$$

We now find T_{12} and T_{32} required for the calculation of G_{21} and G_{23} . Since the probes perturb weakly the current channel, the scattering of the fluxes by the probes can be ignored in this channel. We then readily find that

$$\begin{aligned} T_{12}(\theta, \theta') &= t(\theta, \theta') + \int d\theta'' R(\theta, \theta'') t(\theta'', \theta), \\ T_{32}(\theta, \theta') &= \int d\theta'' T(\theta, \theta'') t(\theta'', \theta). \end{aligned} \quad (37)$$

Then, using the conservation law (14) we can calculate

$$\frac{G_{23} - G_{21}}{G_{23} + G_{21}} = \frac{\int d\theta \cos \theta R(\theta) \int d\theta' t(\theta, \theta')}{\int d\theta \cos \theta \int d\theta' t(\theta, \theta')}. \quad (38)$$

The quantity

$$\int d\theta' t(\theta, \theta') = t(\theta) \quad (39)$$

occurring here is the total probability of escape from a channel to a probe at an angle θ . Using this quantity, we can rewrite Eq. (38) in the form

$$\langle Rt \rangle / \langle t \rangle = \langle R \rangle_t. \quad (40)$$

The notation $\langle R \rangle_t$, stresses that the ratio given by Eq. (40) is the average of R with the weight $t(\theta) \cos \theta$, in contrast to the average $\langle R \rangle$ occurring in Eq. (36) where the averaging is carried out using the weight $\cos \theta$. We finally obtain

$$R_{(4)} = R_{13,24} = \frac{\hbar}{2e^2} \frac{1}{N_1} \frac{\langle R \rangle_t}{1 - \langle R \rangle}. \quad (41)$$

It is clear from the above expression that the measured value $R_{(4)}$ is independent of the absolute coupling of the current channel to the potential probes. However, $R_{(4)}$ depends on which modes of the channel are coupled more strongly to the probe and which less strongly. In the case of $\langle R \rangle_t$, a major contribution is made by those channel modes which are coupled more strongly to the probe. If all the modes reflected by the scatterer are not coupled to the probe (i.e., if $Rt = 0$),

then we naturally have $R_{(4)} = 0$, since in this case the probes are not affected by the scatterer.

The result given by Eq. (41) resembles the Landauer expression. The only differences are the presence of a factor N_1 and the different nature of the averaging in the numerator and denominator, and the nature of the averaging in the numerator depends on the probe geometry. This is the basic difference between the result given by Eq. (41) and the expressions of the Landauer type with the denominator representing multimode channels.^{13,14} If summation is replaced with integration in Eq. (4.11) of Ref. 14, we can show that the resistance is described by an expression similar to Eq. (41), but instead of an average $\langle R \rangle_t$, with the weight $t(\theta) \cos \theta$ we have an average with the weight 1.

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