Contributions of bosons of various types to the anomalous magnetic moment of charged leptons

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The contributions of various types of bosons to the anomalous magnetic moment of a charged lepton are calculated. The dependence of the contributions to the anomalous magnetic moment on the energy of the relativistic lepton and the strength of the external electromagnetic field is investigated. The restrictions on the mass of the neutral scalar Higgs boson which appear in a comparison of the theoretical and experimental values for the anomalous magnetic moment of the muon are discussed.

1. INTRODUCTION

Experiments carried out at high energies on accelerators and storage rings are an important source of information about the properties of elementary particles and their interactions. At present, more powerful accelerators are being successfully constructed (by way of an example one can adduce the LEP complex at CERN just started in the summer of this year). However, the conventional means of accelerating particles will in the future, probably not be capable of sustaining the present rate of advance of experimental research towards higher and higher energies. Therefore some other means of obtaining information based on research into those characteristics of elementary particles that can be calculated theoretically with high accuracy and measured in relatively low-energy experiments will be of great value. A comparison of the results of such theoretical and experimental studies can be used to verify various theoretical models and to obtain bounds on the parameters of the theories.

A unique example of such a characteristic is provided by the magnitudes of the anomalous magnetic moments (AMM) of the electron a_e and the muon a_μ. The most accurate experimental value of the AMM of the electron (and positron) was recently obtained by the Dehmelt group at Washington University,[1] and of the muon, in experiments conducted at CERN more than twelve years ago.[2] If one compares the observed values of the AMM of the electron with the theoretical calculations, one can easily convince oneself that the main contribution to a_e is due to quantum electrodynamic processes,[3] while in the case of the muon contributions of the strong[4] and weak[5] interactions are substantial. The latter still remain completely outside the limits of experimental accuracy,[2] but with roughly a fivefold decrease in the error of measurement it will become possible to detect the contributions of the weak interactions to the AMM of the muon. A review of the theoretical and experimental aspects of the problem of the AMM of charged leptons is contained in Ref. 6.

As is well known,[6–9] at Brookhaven National Laboratory (BNL, USA) plans are underway to increase the accuracy of the measurement of the AMM of the muon by a factor of 20. If this does indeed take place, then a comparison of the experimental value a_μ^{exp} and theoretical value a_μ^{theo} will make it possible to validate not only quantum electrodynamics, but also the Weinberg–Salam theory of electroweak interactions.

At present the standard Glashow–Weinberg–Salam interaction model is adequate to describe all the available experimental data. However, the standard model is not free of defects (such as the large number of free parameters, the unanswered question of the number of generations of leptons and quarks, the absence of gravitation, ect.), and this fact has stimulated an active study of alternate models different from the standard model, but which predict the entire low-energy picture it gives.

In the various generalizations of the standard $SU(3)\times SU(2)\times U(1)$ model of the interaction that have been considered (such as, e.g., the technicolor theory,[10,11] grand unification,[12] superstrings,[13] etc.) a rich spectrum of new bosons is predicted (see also Ref. 14), and the virtual processes in which they participate lead to new contributions to the AMM of the charged lepton. In this connection it is of interest to study the contributions to the AMM of the lepton from different types of bosons B_i of arbitrary mass M_i (Ref. 15): 1) scalar neutral bosons, 2) pseudoscalar neutral bosons, 3) vector neutral bosons, 4) axial-vector neutral bosons, 5) vector charged bosons, and 6) axial-vector charged bosons.

In experiments carried out at Washington University on the measurements of the electron $(g-2)$-factor and experiments at CERN on the measurement of the muon $(g-2)$-factor, as well as in the experiments which are in the planning stage at BNL, the motion of the lepton is determined mainly by the action of the external magnetic field. In addition, in the case of the $(g-2)_\mu$ experiments the muons have relativistic energy, which, as was shown in the instance of the electrodynamic contribution[16–18] and the contribution of the weak interactions,[19–26] just like the presence of a relativistic field can lead to a substantial variation of the magnitude of the AMM of the lepton. Therefore in the discussion below of the AMM of a charged lepton we will take the action of the magnetic field into account, and also the possible influence of the motion of the lepton itself.

In Sec. 2 the contributions of various types of bosons to the AMM of a charged lepton moving in an external field are calculated. The dependence of the contributions on the strength of the field and the energy of the lepton are studied. Accurate expressions in the parameters $\lambda_i = M_i^2/m_l^2$ (m_l is the mass of the lepton, M_i is the mass of the boson) for the vacuum parts of the contributions to the AMM of the lepton, i.e., those parts of the contributions that are independent of the external field, are obtained, and errors in a number of

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published works are indicated. As an illustration of the application of the obtained formulas, bounds on the mass of the Higgs boson in the standard model of the electroweak interactions which can be obtained on the basis of a study of the AMM of the muon are discussed in Sec. 3.

2. CALCULATION OF THE BOSON CONTRIBUTIONS TO THE AMM OF A CHARGED LEPTON

The most systematic way to find the contributions of various types of bosons to the AMM of the lepton (taking into account the dependence of the AMM on the energy of the lepton and the strength of the external field) is based on a calculation of the corresponding parts of the mass operator of the lepton $M(x',x)$ that enters into an equation analogous to the Schwinger equation in electrodynamics, which describes the motion of the lepton in the presence of a field, taking radiative boson effects into account

$$(\hat{p}-m_l)\Psi(x)=\int M(x',x)\Psi(x')dx',\qquad \hat{p}=i\hat{\partial}-e\hat{A}^{ext},\quad \hat{\partial}=\gamma_\nu\frac{\partial}{\partial x_\nu},\tag{1}$$

A^{ext} is the 4-potential of the external electromagnetic field. As a starting point, we choose the external field to be constant and crossed ($\mathbf{E}\perp\mathbf{H}$, $E=H$), which in the case of particles of relativistic energy is, within wide limits, a good approximation for any constant electromagnetic field (a more detailed discussion of this question can be found in Ref. 27). The results obtained here will depend on the characteristic parameter

$$\chi=[-(eF^{\mu\nu}p_\nu)^2]^{1/2}m_l^{-3},\tag{2}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor, and e and p_ν are the charge and the momentum of the lepton.

The mean value of the mass operator calculated to second order in perturbation theory $M^{(2)}$, averaged over the mass surface $p^2=m_l^2$ (we use the system of units with $c=\hbar=1$) between constant spinors

$$U=\frac{1}{(4npp_0)^{1/2}}\begin{pmatrix} m_l+np+(\sigma n)(\sigma p_1)\\ (m_l-np)(\sigma n)+(\sigma p_1)\end{pmatrix}V,$$

$$VV^+=1,\qquad np=n_\mu p^\mu,$$

where p_1 is the component of the momentum of the lepton transverse to the direction of the magnetic field $\mathbf{H}$, determines the contribution to the amplitude of elastic scattering of the lepton in an external field $T_{pp'}=-\overline{U}M^{(2)}(p',p)U$. The constant two-component spinor V describes the orientation of the spin of the lepton and obeys the equation

$$(\boldsymbol{\sigma}\mathbf{l})V=\xi V,$$

where $\xi=\pm1$ characterizes the two possible values of the projection of the particle spin on the direction of the unit vector $\mathbf{l}$. The mean value of the Pauli matrices σ is calculated according to the formula

$$\bar{\sigma}=V^+\sigma V=\xi\frac{1+\xi\xi'}{2}\mathbf{l}+\frac{1-\xi\xi'}{2}\left(\frac{i\xi[\mathbf{l}\mathbf{n}]+[\mathbf{l}[\mathbf{l}\mathbf{n}]]}{1-(\mathbf{l}\mathbf{n})^2}\right).$$

The part of the amplitude that depends on ξ is related to the addition to the AMM of the lepton a_l in the rest frame by the relation[26,27]

$$a_l=(VT\xi\mathbf{H})^{-1}\mathrm{Re}\,T_{pp'}.$$

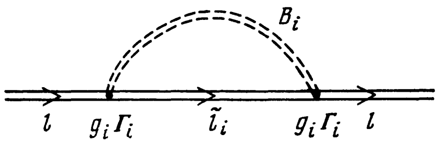

FIG. 1. Contribution of various types of bosons to the mass operator of a charged lepton in an external electromagnetic field.

To verify the calculations, it is necessary to specify the properties of the bosons. The characteristic Feynman diagram that describes the contribution to the mass operator of the charged lepton (for definiteness—negatively charged) is shown in Fig. 1. We will assume that neutral bosons (S^0, P^0, A^0. V^0) have interactions with leptons which are diagonal with respect to flavor and the neutral particle corresponding to the internal line in the diagram, which describes the contribution of the charged bosons (V^-, A^-), is a neutrino. The double lines in the figure mean that the wave functions and the propagators of the charged particles are the exact solutions of the corresponding equations taking into account the action of the external field.

Carrying out the calculations, the details of which are discussed in Appendix 1, for the contributions to the AMM of a charged lepton of mass m_l, moving with relativistic energy in a magnetic field, we obtain

$$a_l^{B_i}(\chi)=\frac{g_i^2}{(2\pi)^2}\int_0^\infty\frac{du}{(1+u)^3}\left(\frac{u}{\chi}\right)^{2/3}\Omega_i\Upsilon(z_i),\tag{3}$$

where

$$\Upsilon(z_i)=\int_0^\infty\sin\left(z_ix+\frac{x^3}{3}\right)dx,$$

and g_i is the coupling constant of the lepton with the boson B_i. We note that the analogous representation in terms of the function $\Upsilon(z)$ of the photon contribution to the AMM of the electron in electrodynamics was used in Ref. 17, and of the Z-, W-, and H-boson contributions was used in the theory of electroweak interactions in Refs. 19–26.

Table I displays the values of Γ_i, which characterize the structure of the coupling of the lepton and the bosons B_i, the type of the virtual fermion $\tilde{l}_i$, the functions Ω_i which determine the integrand in Eq. (3), and also the arguments z_i of the function $\Upsilon(z_i)$.

The contributions (3) depend on the single dynamic field parameter χ. Note that Eqs. (3) give exact expressions for the boson contributions B_i to the AMM of a lepton moving in crossed electromagnetic fields. In this case the dynamic parameter takes the form

$$\chi=\frac{H}{H_0}\frac{p_0-p_3}{m_l},$$

where p_0 and p_3 are the energy of the lepton and the projection of its momentum on the vector $\mathbf{E}\times\mathbf{H}$. By virtue of the above-mentioned "universality" of the crossed fields, Eqs. (3) are also applicable in the case of the motion of a relativistic lepton in an arbitrary constant field. For a purely magnetic field the dynamic parameter has the form

$$\chi_H=\frac{H}{H_0}\frac{p_\perp}{m_l},$$

TABLE I.

Type and charge of boson	Γ_i	τ_i	Ω_i	z_i
1. Scalar, 0	1	l	$1+u/2$	$\left(\frac{u}{\chi}\right)^{2/3}\left(1+\lambda_i\frac{1+u}{u^2}\right)$
2. Pseudoscalar, 0	γ_5	l	$-u/2$	
3. Vector, 0	γ_μ	l	1	
4. Axial-vector, 0	$\gamma_\mu\gamma_5$	l	$-3-4/u-2u/\lambda_4$	
5. Vector, -1	γ_μ	ν	$2+1/u-1/u\lambda_i$	$\left(\frac{u}{\chi}\right)^{2/3}\left(-\frac{1}{u}+\lambda_i\frac{1+u}{u}\right)$
6. Axial-vector, -1	$\gamma_\mu\gamma_5$	ν		

where $p_\perp = (2\gamma n)^{1/2}$ is the projection of the momentum of the lepton on the plane perpendicular to the vector $\mathbf{H}$, n is the number of Landau levels in the magnetic field, and we have written $\gamma = eH$.

On the basis of the expression (3) we can obtain the asymptotic limits (see Appendix 2) of the various contributions to the AMM of a lepton in a magnetic field for small values of χ (relatively weak fields, $H \ll H_0 = m_l^2/e$, and not-too-large energies) and large χ (weak fields, $H < H_0$, and ultrarelativistic energies):

$$a_l^{B_1}(\chi)=\frac{g_1^2}{2\pi}\begin{cases}\dfrac{1}{2}\dfrac{1}{\pi\lambda_1}\left[\Delta_1+\dfrac{1}{3}\dfrac{\chi^2}{\lambda_1^2}+\dfrac{\chi^2}{\lambda_1^3}\left(\ln\lambda_1-\dfrac{257}{60}\right),\right. & \chi\ll\lambda_1,\\[2ex] \dfrac{7}{4}\dfrac{\Gamma(1/3)}{9\cdot 3^{1/2}(3\chi)^{2/3}}, & \chi\gg\lambda_1^{3/2},\end{cases}$$

$$a_l^{B_2}(\chi)=\frac{g_2^2}{2\pi}\begin{cases}-\dfrac{1}{2}\dfrac{1}{\pi\lambda_2}\left(\Delta_2+\dfrac{2}{3}\dfrac{\chi^2}{\lambda_2^2}\right), & \chi\ll\lambda_2,\\[2ex] -\dfrac{5}{4}\dfrac{\Gamma(1/3)}{9\cdot 3^{1/2}(3\chi)^{2/3}}, & \chi\gg\lambda_2^{3/2},\end{cases}$$

$$a_l^{B_3}(\chi)=\frac{g_3^2}{2\pi}\begin{cases}\dfrac{1}{2}\dfrac{1}{\pi\lambda_3}\left[\Delta_3+2\dfrac{\chi^2}{\lambda_3^3}\left(\ln\lambda_3-\dfrac{257}{60}\right)\right], & \chi\ll\lambda_3,\\[2ex] \dfrac{1}{2}\dfrac{\Gamma(1/3)}{9\cdot 3^{1/2}(3\chi)^{2/3}}, & \chi\gg\lambda_3^{3/2},\end{cases} \tag{4}$$

$$a_l^{B_4}(\chi)=\frac{g_4^2}{2\pi}\begin{cases}\dfrac{1}{2}\dfrac{1}{\pi\lambda_4}\left[\Delta_4+\dfrac{\chi^2}{\lambda_4^3}\left(-6\ln\lambda_4+\dfrac{691}{30}\right),\right. & \chi\ll\lambda_4,\\[2ex] -\dfrac{11}{2}\dfrac{\Gamma(1/3)}{9\cdot 3^{1/2}(3\chi)^{2/3}}, & \chi\gg\lambda_4^{3/2},\end{cases}$$

$$a_l^{B_{5,6}}(\chi)=\frac{g_{5,6}^2}{2\pi}\begin{cases}\dfrac{1}{2}\dfrac{1}{\pi\lambda_{5,6}}\left(\Delta_{5,6}+\dfrac{1}{10}\dfrac{\chi^2}{\lambda_{5,6}^3}\right), & \chi\ll\lambda_{5,6},\\[2ex] \dfrac{11}{2}\dfrac{\Gamma(1/3)}{9\cdot 3^{1/2}(3\chi)^{2/3}}, & \chi\gg\lambda_{5,6}^{3/2}.\end{cases}$$

In finding the terms the depend on χ it was assumed that the bosons are sufficiently heavy that $\lambda_i = M_i^2/m_l^2 \gg 1$ holds. The values of Δ_i, the characteristic vacuum contributions, are given below.

From Eqs. (4) it follows that for any type of boson ($B = S^0, P^0, V^0, A^0, V^-, A^-$) the dependence of the contributions on the parameter χ is the same: at small χ ($\chi \ll \lambda_i$) small corrections arise, quadratic in χ, to the vacuum contributions $a_l^{B_i}(0)$ (these latter are determined by the quantities Δ_i). At large χ ($\chi \gg \lambda^{3/2}$) all of the contributions explicitly demonstrate the dynamic nature of the AMM of the lepton, varying with the growth of χ as $\chi^{-2/3}$. For the magnetic field strengths and the lepton energies in the measurements of the $(g-2)$-factors of the electron and the muon, the parameter χ was small and, consequently, the dynamic field effects were also small. However, in the case of the motion of relativistic leptons in the vicinity of astrophysical objects, where according to present estimates the magnetic fields can reach values like $10^{-1}H_0 = 4.4\cdot 10^{12}$ Oe, it is necessary to take into account the dependence on the parameter χ in Eqs. (4) in computing the contributions of the various bosons to the AMM of the lepton.

Let us now turn to the vacuum contributions to the AMM of a charged lepton, for which we obtain

$$a_l^{B_i}(0)=\frac{g_i^2}{4\pi^2}\frac{\Delta_i}{\lambda_i},\qquad i=1,2,3,4,5,6,$$

$$\begin{aligned}
\Delta_1 &= \left(\frac{\lambda_1^4}{4}-\frac{5}{4}\lambda_1^3+\lambda_1^2\right)\varepsilon_1^{-1}\ln K_1 + \left(\frac{\lambda_1^3}{4}-\frac{3}{4}\lambda_1^2\right)\ln\lambda_1-\frac{\lambda_1^2}{2}+\frac{3}{4}\lambda_1,\\
\Delta_2 &= \frac{1}{2}\left[\left(\frac{\lambda_2^3}{2}-\frac{3}{2}\lambda_2\right)\varepsilon_2^{-1}\ln K_2 + \frac{1}{2}(\lambda_2^3-\lambda_2^2)\ln\lambda_2-\lambda_2^2-\frac{\lambda_2}{2}\right],\\
\Delta_3 &= \left(\frac{\lambda_3^4}{2}-2\lambda_3^3+\lambda_3^2\right)\varepsilon_3^{-1}\ln K_3 + \left(\frac{\lambda_3^3}{2}-\lambda_3^2\right)\ln\lambda_3-\lambda_3^2+\frac{\lambda_3}{2},\\
\Delta_4 &= \left(\frac{\lambda_4^4}{2}-3\lambda_4^3+4\lambda_4^2\right)\varepsilon_4^{-1}\ln K_4 + \left(\frac{\lambda_4^3}{2}-2\lambda_4^2+\lambda_4\right)\ln\lambda_4-\lambda_4^2+\frac{5}{2}\lambda_4-1,\\
\Delta_5(\lambda_5=\lambda) &= \Delta_6(\lambda_6=\lambda)=\left(\lambda^3-\frac{5}{2}\lambda^2+\frac{3}{2}\lambda\right)\ln\left|\frac{\lambda}{\lambda-1}\right| - \lambda^2+2\lambda+\frac{1}{4},
\end{aligned} \tag{5}$$

$$K_i=|(\lambda_i-\varepsilon_i)/(\lambda_i+\varepsilon_i)|,\qquad \varepsilon_i=|\lambda_i(\lambda_i-4)|^{1/2}.$$

Note that in the derivation of these relations no restrictions of the values of the boson mass M_i were imposed, i.e., these formulas describe the dependence of the vacuum contributions to the AMM of the lepton on the corresponding boson masses exactly. When specific values are substituted for the constants g_i and the boson masses M_i, expressions (5) make it possible to find the contributions to the AMM of the lepton of mass m_l that arise in the various theories. Thus, using these formulas, we can find the vacuum contributions of the Z, W, and H bosons to the AMM of the lepton in the standard Weinberg–Salam model of the electroweak interactions, the main terms in the expansion of which for $\lambda_i \gg 1$ were determined in Ref. 28. The combined use of Eqs. (4) and (5) allow one to investigate how the contributions of the Z, W, and H bosons[19-26] to the AMM of the lepton vary.

The vacuum contributions of neutral massive scalar bosons, pseudoscalar bosons, and vector and axial-vector bosons to the AMM of the electron and the muon were recently considered using a method different from the one we have used.[29,30] The authors of Ref. 30 obtained analytic formulas for the corresponding vacuum contributions to the AMM for two cases: $\lambda_i < 4$ and $\lambda_i > 4$. If certain transformations are made in the formula of Ref. 30, then it becomes possible to obtain common expressions, suitable simultaneously for both cases, which should coincide with our result (5) for the neutral bosons ($\lambda_i \neq 4$). However, this agreement does not obtain, because in Ref. 30 the contributions of the scalar and pseudoscalar bosons are given with incorrect sign, which is also reflected in the sign of the corresponding limiting values for $\lambda_{1,2} \ll 1$ and $\lambda_{1,2} \gg 1$. In addition to this, the contribution obtained in Refs. 29 and 30 of the neutral axial-vector boson differs from our result (5) and the expression given in Ref. 31, which coincides with it. Also note that the treatment in Ref. 29 mistakenly uses an estimate of the mismatch between theory and experiment $a_e^{\text{theo}} - a_e^{\text{exp}}$ that is 10 times too small, which led the authors of this reference to bounds on the squares of the coupling constants of the electron with the bosons are too low.

For the cases of light ($\lambda_i \ll 1$) and heavy ($\lambda_i \gg 1$) bosons it is possible to obtain from Eqs. (5) the following limiting values for the contributions to the AMM of the negatively charged lepton (cf the results of Ref. 30):

$$a_l^{B_i}(0) = \frac{g_i^2}{8\pi^2} k_i, \quad B_i = S^0,\ P^0,\ V^0,\ A^0,\ V^-,\ A^-. \qquad (6)$$

Here for $\lambda_i \ll 1$

$$k_1 = \frac{3}{2}, \quad k_2 = -\frac{1}{2}, \quad k_3 = 1,$$

$$k_4 = -\frac{2}{\lambda_4} \to -\infty, \quad k_5 = k_6 = \frac{2}{\lambda_{5,6}} \to \infty,$$

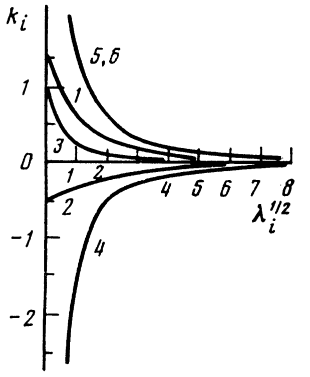

FIG. 2. Dependence of the functions k_i, which determine the vacuum contributions of the various types of bosons to the AMM of the lepton, on the ratio $\lambda_i^{1/2} = M_i/m_l$: 1) S^0, 2) P^0, 3) V^0, 4) A^0, 5) V^-, 6) A

and for $\lambda_i \gg 1$

$$k_1 = \lambda_1^{-1} \ln \lambda_1 - \lambda_1^{-1} \frac{7}{6}, \quad k_2 = -\lambda_2^{-1} \ln \lambda_2 + \lambda_2^{-1} \frac{11}{6},$$

$$k_3 = \lambda_3^{-1} \frac{2}{3} - \lambda_3^{-2} 2 \ln \lambda_3, \quad k_4 = -\frac{10}{3} \lambda_4^{-1} + \lambda_4^{-2} 2 \ln \lambda_4,$$

$$k_5 = k_6 = \lambda_{5,6}^{-1} \frac{10}{3} + \lambda_{5,6}^{-2} \frac{2}{3}.$$

Note the clear separation of the contributions of the scalar and the pseudoscalar bosons in the region $\lambda_i \gg 1$, which is due to the presence, in the main terms of the expansion, of the factor $\ln \lambda_i$, which can be large in magnitude. Figure 2 shows the dependence of the functions k_i on the ratio of the boson masses to the lepton mass $\lambda_i^{1/2}$ for the types of bosons under consideration.

3. CONCLUSION

The overall conclusion that can be drawn from the present study is that the contributions of the various bosons to the AMM of a charged lepton depend substantially not only on the magnitude of the coupling constants g_i between the bosons and the leptons, but also on the type of boson (and on the magnitude of the boson charge), and on its mass. The formulas for the vacuum contributions $a^{B_i}(0)$, which take the dependence on the parameter $\lambda_i = (M_i/m_l)^2$ into account exactly, can be used to obtain bounds on the values of the coupling constants g_i and the masses of the bosons. As an example let us consider the contribution to the AMM of the muon from the scalar Higgs bosons in the standard model of the electroweak interactions. The constant g_1 in this case is related to the Fermi constant:

$$g_1^2/8\pi^2 = 2^{1/2} G_F m_\mu^2/8\pi^2 = 2.3 \cdot 10^{-9},$$

and we may consider the possibility of obtaining bounds on the mass of the Higgs boson using the explicit expression for the function $k_1(\lambda_1)$, where $\lambda_1 = (M_H/m_\mu)^2$.

Let us dwell for a moment on the bounds that follow from the AMM of the muon. Since at present the discrepancy $\Delta_\mu \lesssim 9 \cdot 10^{-9}$ between the theoretical value a_μ^{theo} and the experimental value a_μ^{exp} (see Ref. 6) exceeds the maximum possible value attained in the limit $M_H/m_\mu \to 0$) of the contribution of the H boson [consider the asymptotic limit (6)]

$$a_\mu^H|_{max} = \frac{3}{2} \frac{g_1^2}{8\pi^2} = 3.5 \cdot 10^{-9},$$

the bounds on the mass of the Higgs boson will come into view as the mismatch between the theoretical predictions and the experimental values of the AMM of the muon decreases. If after the planned 20-fold increase in the accuracy of the measurement of a_μ the discrepancy Δ_μ decreases by a factor of 5, then from the magnitude of the AMM of the muon, the bound on the mass of the Higgs boson will be

$$M_H \gtrsim 0.5 m_\mu = 53 \text{ MeV}, \qquad (7)$$

which is more restrictive condition then the presently existing requirement $M_H \geq 14$ MeV, which follows from experimental investigations of the decay of excited states of ^4He (Ref. 32). If success is in fact attained in decreasing the discrepancy Δ_μ by a factor of 20, then the bound on the mass of the Higgs boson will be increased to

$$M_H \gtrsim 2.5 m_\mu = 265 \text{ MeV}. \quad (8)$$

As a result, this could lead to the bound

$$M_H \gtrsim 700 \text{ MeV}, \quad (9)$$

since even now for a sufficiently large mass of the t quark ($m_t > 80$ GeV) the range $2m_\mu < M_H < 700$ MeV is excluded, thanks to an analysis of the data on the decay of B mesons.[32]

The possibility of obtaining bounds on the mass of the Higgs boson from the magnitude of the AMM of the electron in the foreseeable future is problematic since success would require a decrease in the magnitude of the discrepancy between the theoretical and experimental values by several orders of magnitude.

APPENDIX 1

Let us enlarge on the main steps in the calculations leading to Eqs. (3). Consider for definiteness the calculated magnitudes of $a_l^{S^0}(\chi)$, i.e., the contribution of the scalar neutral massive boson to the AMM of the charged lepton. First we find the contribution $M_{S^0}^{(2)}(x',x)$ to the mass operator of the lepton in the presence of an external crossed field, using the $E(p,x)$ representation:[27]

$$M_{S^0}^{(2)}(x',x) = \int \frac{dp'\,dp}{(2\pi)^8} E(p',x') M(p',p) \bar{E}(p,x),$$

$$\bar{E}(p,x) = \gamma_0 E^+(p,x),$$

where

$$E(p,x) = R(p,x) \exp[-ipx + i\eta(p,\varphi)],$$

$$R(p,x) = 1 + e\frac{\hat{n}\hat{A}}{2np},$$

$$\eta(p,\varphi) = \int_0^\varphi \left[\frac{e^2A^2(\rho)}{2np} - \frac{epA(\rho)}{np}\right] d\rho,$$

$$\varphi = nx,\quad n^2 = nA = 0,\quad A_\mu = A_\mu(\varphi) = b_\mu\varphi,\quad b_\mu = (0,0,b,0)$$

and $A_\mu(\varphi)$ is the 4-potential of the crossed field and we have written $n_\mu = (1,0,0,1)$. The propagator of the charged lepton in the external crossed field can be represented in the form

$$G_l(x',x) = -\int \frac{dk}{(2\pi)^4} \frac{E(k',x')(\hat{k}+m_l)\bar{E}(k,x)}{k^2 - m_l^2 + i\varepsilon}$$

$$= \frac{1}{(4\pi)^2} \int_0^\infty \frac{ds}{s^2} \left[m_l + m_l s\hat{n}\hat{b} + \frac{\hat{\Delta}}{2s} + \frac{b^2 s\hat{n}}{3}\Delta_- + \frac{1}{4}(\hat{\Delta}\hat{n}\hat{b} + \hat{n}\hat{b}\hat{\Delta}) \right]$$

$$\times \exp\left\{ i\left[-sm_l - \frac{\Delta^2}{4s} - \frac{b\Delta}{2}(\varphi'+\varphi) + \frac{b^2 s}{12}(\varphi'-\varphi)^2 \right]\right\},$$

where we have introduced the notation $\Delta_\mu = x'_\mu - x_\mu$, $\Delta_\pm = \Delta n = \Delta_0 \pm \Delta_3$ and discarded the terms that disappear upon integration of the expression for $M_{S^0}^{(2)}(p',p)$. The propagator of the neutral boson S^0 has the form

$$D_{S^0}(x',x) = -\int \frac{d^4q}{(2\pi)^4} \frac{\exp[-iq(x'-x)]}{q^2 - M_1^2 + i\varepsilon}$$

$$= \frac{1}{(4\pi)^2} \int_0^\infty \frac{dt}{t^2} \exp\left[-i\left(tM_1^2 + \frac{\Delta^2}{4t}\right)\right].$$

We thus obtain for $M_{S^0}^{(2)}(p',p)$

$$M_{S^0}^{(2)}(p',p) = \frac{ig_1^2}{(2\pi)^8} \int \frac{d^4x'\,d^4x\,d^4q\,d^4k}{(q^2 - M_1^2)(k^2 - m_l^2)} \{\ldots\}$$

$$= \frac{ig_1^2}{(4\pi)^2} \int \frac{ds\,dt\,d^4x'\,d^4x}{s^2 t^2}$$

$$\times E(p',x') S(x',x) E(p,x) \exp\left\{ i\left[-sm_l^2 - \frac{\Delta^2}{4s} - \frac{b\Delta}{2}(\varphi+\varphi') + \frac{b^2 s}{12}(\varphi'+\varphi)^2 - tM_1^2 - \frac{\Delta^2}{4t} - px - \frac{pb}{2np}\varphi^2 + \frac{b^2\varphi^3}{6np} + p'x' + \frac{p'b}{2np}\varphi'^2 - \frac{b^2\varphi'^3}{6np'} \right]\right\}. \quad (10)$$

After the following transformation of the variables of integration:

$$d^4x'\,d^4x \to d\Delta_1\,d\Delta_2\,d\Delta_+\,d\Delta_-\,d\nabla_1\,d\nabla_2\,d\nabla_+\,d\nabla_-,$$

where

$$\nabla_\mu = x_\mu + x_\mu',\quad \nabla_\pm = \nabla_0 \pm \nabla_3,\quad \mu = 0, 1,$$

several of the integrations in formula (10) fall out (see also Ref. 27) and we arrive at a representation for $M_{S^0}^{(2)}(p',p)$ in the form of a double integral over the variable t and s. Introducing the new variables

$$u = \frac{s}{t},\quad \rho = -\frac{(e^2b^2u^4)^{1/3}}{1+u} t,$$

after a few transformations we obtain

$$M_{S^0}^{(2)}(p',p) = -\frac{g_1^2(2\pi)^4\delta^4(p'-p)}{8\pi^2} \int_0^\infty \frac{du}{(1+u)^2} \left\{ \left(m_l + \frac{\hat{p}}{1+u}\right) \times \left[f_1(z) + \int_0^\infty \frac{\exp(-iz\rho)}{\rho} d\rho \right] + if(z)\left(\frac{\chi}{u}\right)^{1/3} \left(\frac{\sigma F e}{2m\chi} u + \frac{e}{4} \frac{\hat{p}\sigma F + \sigma F\hat{p}}{m^2\chi} \frac{u}{u+1} \right) - \frac{2}{3} \frac{\gamma F F p e^2}{m^4\chi^2} u f'(z) \left(\frac{\chi}{u}\right)^{2/3} \right\},$$

$$\chi = \frac{e|b|pn}{m^3},\quad |b|^2 = -b^2 = b_\mu b^\mu,\quad b_\mu = (0,0,b,0).$$

Here

$$f(z) = i\int_0^\infty d\rho \exp\left[-i\left(\rho z + \frac{\rho^3}{3}\right)\right] = \Upsilon(z) + i\Phi(z),$$

$$f'(z) = \int_0^\infty \rho\,d\rho \exp\left[-i\left(\rho z + \frac{\rho^3}{3}\right)\right],$$

$$f_1(z) = \int_0^\infty \frac{d\rho}{\rho} \exp(-iz\rho)\left[\exp\left(-i\frac{\rho^3}{3}\right) - 1\right],$$

$$\sigma_{\mu\nu} = i(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)/2.$$

The part of $M_{S^0}^{(2)}(p',p)$ that depends on the orientation of the spin of the lepton and determines the magnitude of the contributions $a_l^{S^0}(\chi)$ is proportional to the function $f(z)$ and contains no divergent terms.

Similarly we can calculate the contributions to the AMM of the lepton from other types of bosons. In calculating the contributions of charged bosons to the mass operator of the lepton it is necessary to consider boson propagators which allow for the action of external crossed field:[22]

$$D_{\mu\nu}(x',x)=(g_{\mu\nu}+M_B^{-2}D_\mu'D_\nu')\Pi_\nu^{\beta}(x',x),$$

$$D_\mu'=\frac{\partial}{\partial x'^\mu}+ieA_\mu(\varphi'),$$

$$\Pi_\mu^{\beta}(x',x)$$

$$=-\int\frac{d^4p}{(2\pi)^4}\frac{1}{(p^2-M_B^2)}\Big\{g_\mu^{\beta}+\frac{e}{pn}\int_{nx}^{nx'}F_\mu^{\beta}\,d\varphi-\frac{e^2}{2(pn)^2}$$

$$\times(A^2+A'^2)n^\beta n_\mu-\frac{e^2}{(pn)^2}\Big(\int_0^{nx}F_\lambda^{\beta}\,d\varphi\Big)\Big(\int_0^{nx'}F_\mu^{\lambda}\,d\varphi\Big)\Big\}$$

$$\times\exp\Big[-ip(x'-x)+\frac{e}{pn}\int_{nx}^{nx'}\Big[pA-\frac{eA^2}{2}\Big]d\varphi\Big].$$

The "spin" terms in the expressions for $M_{B_i}^{(2)}$ $(i=2,3,4,5,6)$ are also free of divergences.

APPENDIX 2

In order to evaluate the integral in Eqs. (3) with respect to the variable u for large values of the argument $(z>1)$, we will use the asymptotic series expansion of the function $\Upsilon(z)$ (Ref. 33)

$$\Upsilon(z)=1/z+2/z^4+O(1/z^7), \qquad (11)$$

and in the case of small values $(z<1)$ we will use the Taylor series expansion

$$\Upsilon(z)=\Upsilon(0)+z\Big[\frac{d}{dz}\Upsilon(z)\Big]\Big|_{z=0}+\dots.$$

For given values of the parameter χ we find the region of variation of the variable u in which it is possible to use one of the above representations of the function $\Upsilon(z)$. On the basis of the form of the argument z (see Table I) we can distinguish three characteristic regions of the values of the parameter χ: 1) $\chi\ll\lambda$, 2) $\lambda\ll\chi\ll\lambda^{3/2}$, and 3) $\chi\gg\lambda^{3/2}$. We will be interested in the first and third of these regions. In the first region the minimum value of the argument at the point $u=u_0\gtrsim\lambda/2$ is $z_{\min}=3(\lambda/2\chi)^{2/3}$. Thus in this region we can replace the function $\Upsilon(z)$ by the first few terms of its series (11). Note that two terms of the series suffice everywhere to determine the main term in the external field parameter χ.

In the region $\chi\gg\lambda^{3/2}$ we can distinguish three regions of variation of the variable u, which require different approximate expressions for the function $\Upsilon(z)$:

$$\Upsilon(z)=\begin{cases}\dfrac{1}{z_1}+\dfrac{2}{z_1^4}+\dots, & z_1=\dfrac{\lambda}{\chi^{2/3}}\dfrac{u+1}{u^{4/3}},\quad 0\leqslant u<\dfrac{\lambda^{3/4}}{\chi^{1/2}}, \qquad (12a)\\[2ex] \Upsilon(0)+z\Big[\dfrac{d}{dz}\Upsilon(z)\Big]\Big|_{z=0}+\dots, & \dfrac{\lambda^{3/4}}{\chi^{1/2}}\leqslant u\leqslant\chi, \qquad (12b)\\[2ex] \dfrac{1}{z_2}+\dfrac{2}{z_2^4}+\dots, & z_2=\dfrac{u^{2/3}}{\chi^{2/3}},\quad u>\chi. \qquad (12c)\end{cases}$$

However, as was explained in the calculations, the main contribution to the contributions (3) comes from the interval (12b).

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Note added in proof (27 March 1990): The presented approach can be used to determine the contributions to the AMM of the lepton that arise in the supersymmetric theories. The contribution of the photino–slepton loop to the mass operator of the lepton in an external field (see Ref. 34) is described by the diagram shown in Fig. 1 if the lepton l_i is identified with the supersymmetric partner of the photon—the photon $\tilde\gamma$ of mass $m_{\tilde\gamma}$, and the boson B_i is identified with the partner of the lepton—the slepton $\tilde l$ of mass $m_{\tilde l}$ and it is assumed that $g_i\Gamma_i=e(1\pm\gamma_5)$. As a result, Eq. (3) will again be valid for the photino–slepton contribution $\tilde a_l(\chi)$ to the AMM of the lepton if in the given case we set $\Omega=-1$ and $z=(u/\chi)^{2/3}[-1/u+(m_{\tilde l}/m_l)^2(u+1)/u+(m_{\tilde\gamma}/m_l)^2(u+1)/u^2]$. The requirement that the vacuum part $\tilde a_\mu(0)$ of the considered contribution to the AMM of the muon be less than $\Delta_\mu\simeq 9\cdot 10^{-9}$ leads for $m_{\tilde\mu}\gg m_{\tilde\gamma}$ to the following bound on the mass of the smuon $\tilde\mu$: $m_{\tilde\mu}>22$ GeV, and, for a discrepancy Δ_μ 20 times smaller, we arrive at the condition $m_{\tilde\mu}>99$ GeV. This same requirement for fixed mass $m_{\tilde\mu}$ can lead to a lower bound on the mass of the photino.