Theory of phase self-conjugation in unsteady stimulated backscattering in photorefractive crystals

V. I. Vinokurov and V. V. Shkunov

Institute of Problems of Mechanics, Academy of Sciences of the USSR
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A rigorous theory is derived for the space-time discrimination of uncorrelated spatial profiles obtained in stimulated backscattering of light in a photorefractive crystal in which oppositely directed waves are used to inscribe gratings by a diffusive mechanism. The physical reason for the discrimination is a preferential propagation of the conjugate configuration through regions where the time needed to inscribe the grating is shortest. The problem is solved through a numerical study of equations derived analytically for the conjugate wave with a speckle in the nondepleting-pump approximation. The optimum conditions for phase self-conjugation are found. The conditions for obtaining it are outlined. The theoretical results agree with experimental results.

The interaction of light waves in photorefractive crystals has attracted much interest recently because of a variety of physical effects and possibilities for controlling light beams.[1,2] In particular, photorefractive crystals are the most reliable media for phase self-conjugation of the beams from visible-range cw lasers.[3–7]

Phase self-conjugation of low-noise cw light was achieved in Refs. 5–7 in crystals in an arrangement similar to that used for phase self-conjugation in stimulated backscattering, which is the layout ordinarily used for phase conjugation of intense light from pulsed lasers.[8–10] In this method, the "pump" beam (the beam to be conjugated) is given a speckle by a phase plate and then sent into a medium in which the stimulated backscattering occurs and leads to nonlinear reflection of the beam. Under certain conditions—the entire caustic of the light beam is inside the nonlinear medium, and the number of speckle spots in this volume is sufficiently large—the reflected light called the "Stokes wave" is dominated by the component which is the conjugate of the incident beam. Although the schemes for phase self-conjugation in stimulated backscattering are the same, the physical mechanisms which result in the selection of the conjugate component in the reflected light are quite different for the different types of scattering and for the different regimes.

Underlying the physical mechanism for the selection of the conjugate component for stimulated Brillouin scattering is[8–10] the linear dependence of the local stimulated-Brillouin-scattering gain of the Stokes wave on the local intensity of the conjugated pump. The reason for the preferential amplification of the conjugate component of the Stokes wave is that its spatial profile matches that of the gain over the entire scattering volume: The locations of the intensity maxima of this configuration automatically fall at maxima of the gain.

A distinctive feature of stimulated diffusion backscattering, which is observed in $LiNbO_3$ crystals[6,11] and $BaTiO_3$ crystals,[5,7,11] is that the steady-state gain for the wave propagating opposite the pump is independent of the local pump intensity.

As was shown in Ref. 6, preferential amplification of the conjugate component occurs only in the stage in which the stimulated diffusive scattering builds up; it fades away as a steady state is approached. The reason for this preference here, in contrast with the case of stimulated Brillouin scattering, is that the conjugate configuration builds up at a rate high in comparison with that of the nonconjugate component, because the local relaxation time τ_M of the excitation grating in the medium is inversely proportional to the local pump intensity I: $\tau_M(R) \propto 1/I(R)$. The conjugate component propagates primarily through regions of pump maxima, where the process builds up relatively rapidly.

In this paper we derive a quantitative theory for the phase conjugation accompanying stimulated diffusive backscattering in photorefractive crystals. This study shapes the qualitative arguments of Ref. 6 into a rigorous theory.

1. DERIVATION OF RELAXATION EQUATION FOR THE PHOTOINDUCED ELECTRIC FIELD GRATING

Oppositely directed light waves of the same frequency and polarization, $E_L \exp(-i\omega t - ikz)$ and $E_s(-i\omega t + ikz)$, interact during stimulated diffusive scattering in photorefractive crystal on the grating of the quasistatic electric field $\mathscr{E} \exp(2ikz) +$ c.c. which is induced in the crystal when charge carriers which have been trapped by defects are redistributed (Fig. 1). This redistribution of charge in the volume of a crystal results from a spatially nonuniform diffusion current of photoexcited carriers, which arises at gradients of the carrier density in the field of the interference pattern $E_L^* E_s \exp(2ikz) +$ c.c. of the mixing waves. When the crystal has a photorefractive effect which converts the electric field profile into a refractive index profile

$$\delta n(\mathbf{R}) = {}^1/_2 n^3 r \mathscr{E} e^{2ikz} + \text{c.c.}$$

(r is the electrooptic coefficient, and n is the refractive index of the crystal), the light waves in turn sense the electric field grating which they have induced and are rescattered into each other by this grating.

To derive a relaxation equation for the amplitude of the grating of the induced electric field, we use the standard equations of the band model[2,12,13] for the current $\mathbf{J}$, the charge density ρ, the density n_c of free carriers (for definiteness, we assume electrons), and the electric field $\vec{\mathscr{E}}(\mathbf{R})$, of

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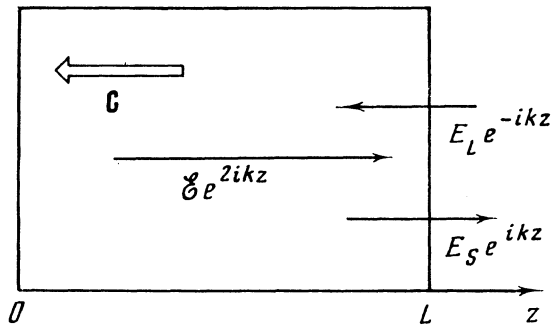

FIG. 1. Mixing of oppositely directed light waves $E_L \exp(-ikz)$ and $E_S \exp(ikz)$ at a grating of a diffusion electric field $\mathscr{E}\exp(2ikz)$ in an $LiNbO_3$:Fe crystal.

the light waves excited by the interference pattern $I(\mathbf{R})$ in the crystal:

$$\mathbf{J}=e\mu n_e\vec{\mathscr{E}}-eD\nabla n_e+\beta I, \tag{1a}$$

$$\frac{\partial\rho}{\partial t}+\nabla\mathbf{J}=0, \quad \nabla(\varepsilon\vec{\mathscr{E}})=4\pi\rho, \tag{1b}$$

$$\frac{\partial n_e}{\partial t}=\frac{\alpha I}{\hbar\omega}-\frac{n_e}{\tau}-\frac{\nabla\mathbf{J}}{e}=0. \tag{1c}$$

Here e, μ, and D are the charge, mobility, and diffusion coefficient of the free carriers; $\hat{\varepsilon}$ is the dielectric tensor; α is the optical absorption coefficient in the crystal; $\hbar\omega$ is the energy of the photon; τ is the carrier lifetime; and the term βI in Eq. (1a) describes the photogalvanic component of the current.[14,15] The zero on the right side of Eq. (1c) corresponds to the usual assumption that the free carriers are in a quasisteady state; that assumption is valid if τ is much shorter than the time required to establish the process, as it is, by a huge margin, for the beams from cw lasers.

For definiteness we consider the experimental conditions of Ref. 6, in which a linearly polarized pump beam enters an iron-doped $LiNbO_3$ crystal along the direction parallel to the **C** optic axis. For this case we can make the following simplifications. Since we have $\beta=\beta\mathbf{e}_z$ in this geometry ($\mathbf{e}_z$ is a unit vector along the **C** direction), and since the relations

$$\left|\frac{\partial I}{\partial z}\right|\sim\frac{2}{\lambda}I\gg|\nabla_\perp I|\sim\frac{\Delta\theta}{\lambda}I$$

hold for collimated laser beams, where $\Delta\theta$ is the divergence of the beams, and λ is the wavelength in the crystal, the redistribution of charges occurs along the z axis, which is the axis along which the beams propagate. Here $\mathbf{J}\sim\mathbf{J}e_z$. For this reason we can also ignore the transverse components of the photoinduced field: $\hat{\mathscr{E}}\sim\mathscr{E}\mathbf{e}_z$. The propagation of light beams near the direction of the optic axis of a uniaxial crystal can be assumed essentially isotropic. We thus ignore the depolarization of the light beams and assume

$$\mathbf{E}_{L,S}=E_{L,S}\mathbf{e}_x, \quad \varepsilon_{ik}=\varepsilon_0\delta_{ik}.$$

Finally, the high doping level of the crystal used Ref. 6 allows us to ignore the saturation of the space-charge field,[2,12,13] even in our case of a mixing of counterpropagating waves. In this approximation the absorption coefficient α and the linear recombination time τ are constants. As a result, our initial system of equations, (1), takes the simple form

$$J=e\mu n_e\mathscr{E}-eD\frac{\partial n_e}{\partial z}+\beta I, \tag{2a}$$

$$\frac{\partial\mathscr{E}}{\partial t}+\frac{4\pi J}{\varepsilon_0}=0, \quad \frac{\partial\rho}{\partial t}+\frac{\partial J}{\partial z}=0, \tag{2b}$$

$$n_e=\alpha\tau I+\left(\frac{\partial J}{\partial z}\right)\frac{\tau}{e}. \tag{2c}$$

The spatial distribution of the light intensity in the crystal in the course of stimulated diffusive scattering is

$$I(\mathbf{R})=|E_L|^2+E_L^*E_Se^{2ikz}+\text{c.c.}+|E_S|^2=I_0+I_1e^{2ikz}+\text{c.c.}$$

We thus seek a solution of Eqs. (2) in the form of a sum of the zeroth and first spatial harmonics of the grating:

$$J=J_0+J_1e^{2ikz}+\text{c.c.}$$
$$\mathscr{E}=\mathscr{E}_0+\mathscr{E}_1e^{2ikz}+\text{c.c.}$$
$$n_e=n_0+n_1e^{2ikz}+\text{c.c.}$$

We restrict our discussion to the case in which the pump suffers no depletion, $I_0\approx|E_L|^2\gg I_1$. We can then solve the system (2) by perturbation theory. For the amplitudes of the zeroth harmonics, we find in this manner

$$J_0=-\frac{\varepsilon_0}{4\pi}\dot{\mathscr{E}}_0, \quad \dot{\mathscr{E}}_0+\tau_M^{-1}\mathscr{E}_0=-\frac{4\pi}{\varepsilon_0}\beta I_0, \quad n_0=\alpha\tau I_0 \tag{3}$$

For a crystal in an open circuit the solution of the relaxation equation for the zeroth harmonics is

$$\mathscr{E}_0(t)=-\mathscr{E}_{\text{ph}}[1-\exp(-t/\tau_M)], \quad J_0=\beta I_0\exp(-t/\tau_M), \tag{4}$$

This solution describes cancellation of the photogalvanic current by a drift current in the field $\mathscr{E}_0(t)$, produced by charges at the boundary of the mixing region. Here $\mathscr{E}_{\text{ph}}=\beta/e\mu\theta\tau$ is the photogalvanic field, $\tau_M=4\pi\sigma_0/\varepsilon_0$ is the Maxwellian relaxation time in a crystal with a photoconductivity $\sigma_0=e\mu n_0$, and μ is the carrier mobility.

The relaxation equation in which we are interested, for the amplitude of the first harmonic of the electric field, $\mathscr{E}_1(t)$, can be found easily from Eqs. (2), with the help of the solution (4):

$$\frac{\partial\mathscr{E}_1}{\partial t}+\frac{\tau_M^{-1}}{1+2ikl_0+4k^2l_D^2}\left[\mathscr{E}_1+(\mathscr{E}_0(t)+\mathscr{E}_{\text{ph}}-i\mathscr{E}_D)\frac{I_1}{I_0}\right]=0. \tag{5}$$

Here we have introduced the following notation: $\mathscr{E}_D=2kD/\mu$ is the diffusive field of the grating, whose wave number is $2k$, and $l_{\text{ph},D,0}=\mu\tau\mathscr{E}_{\text{ph},D,0}$. are the drift lengths in the photogalvanic field, the diffusive field, and the field $\mathscr{E}_0(t)$.

The drift lengths in an $LiNbO_3$:Fe crystal are known to be extremely short ($\lesssim 10^{-6}$ cm), so terms of the type $kl_{\text{ph},D,0}$ can be ignored in (5). As a result the relaxation equation becomes the relation which was used in Ref. 6:

$$\frac{\partial \mathscr{E}_1}{\partial t}+\frac{\mathscr{E}_1}{\tau_M}=\left[i\mathscr{E}_D-\mathscr{E}_{ph}\exp\left(-\frac{t}{\tau_M}\right)\right]\frac{E_S}{E_L}\tau_M^{-1}. \quad (6)$$

It can be seen from Eq. (6) that in addition to the component of the electric field grating formed by the diffusive current, $\mathscr{E}_1 \propto i\mathscr{E}_D$, which is displaced with respect to the interference pattern $E_L{}^*E_S\exp(2ikz)$ by a quarter of a period, an unshifted component $\mathscr{E}_1 \propto \mathscr{E}_{ph}\exp(-t/\tau_M)$ appears in a crystal which exhibits a photogalvanic effect, such as lithium niobate. However, the photogalvanic current which excites this component disappears over the relaxation time $t\sim\tau_M$ of the inscribed grating because of a screening effect. It can be shown that the relative size of the correction for the photogalvanic effect decreases in accordance with $(\tau_0/t)^2$, where $\tau_0=\tau_M(|\mathscr{E}_{ph}/\mathscr{E}_D|)^{1/2}$, and is less than 10% for $t=5\tau_M$. Since the time scale for the onset of stimulated scattering is usually many times the grating relaxation time,[9,10] the unshifted component of the grating can be ignored over most of the time required to establish the stimulated diffusive scattering, and the relaxation equation for excitations of the medium can be put in a form typical of stimulated scattering:

$$\frac{\partial Q}{\partial t}+\frac{Q}{\tau_M}=i\frac{E_L{}^*E_S}{\tau_M|E_L|^2}, \quad Q=\frac{\mathscr{E}_1}{\mathscr{E}_D}. \quad (7)$$

In this equation, unlike equations for other types of stimulated scattering, the relation time τ_M depends on the pump intensity: $\tau_M \propto n_0^{-1}\propto |E_L|^{-2}$

To study the growth of the Stokes wave E_S as a function of position and time from a seed noise scattered by crystal defects near the rear surface of the crystal, $z=0$, we use the parabolic wave equation[9,10]

$$\frac{\partial E_S}{\partial z}-\frac{i}{2k}\Delta_\perp E_S=-\frac{i}{2}\Gamma Q E_L, \quad (8)$$

which is valid for describing the diffraction of light beams for which the angular divergence is not large, $\Delta\theta\ll 1$. In this equation, $\Gamma=(\omega/c)n^3|r_{13}|\mathscr{E}_D$ is the steady-state gain for the mixing of the counterpropagating waves in the crystal, r_{13} is the electrooptic coefficient, Q obeys Eq. (7), and the diffraction of the pump wave E_L is described by the parabolic equation

$$\frac{\partial E_L}{\partial z}+\frac{i}{2k}\Delta_\perp E_L=0, \quad (9)$$

which corresponds to the propagation of a nondepleting pump along the $-z$ direction.

2. EQUATIONS FOR THE CORRELATED AND UNCORRELATED COMPONENTS OF THE STOKES WAVE

The relaxation equation (7) can be solved easily for each point in space by expressing the time evolution $Q(\mathbf{R},t)$ in terms of the (as yet unknown) functional dependence $E_S(\mathbf{R},t)$:

$$Q(\mathbf{R},t)=i\int_0^t \frac{E_S(\mathbf{R},\eta)}{E_L}\exp\left(\frac{t-\eta}{\tau_M}\right)d\left(\frac{\eta}{\tau_M}\right). \quad (10)$$

As a result, the equation for the Stokes wave becomes

$$\frac{\partial E_S}{\partial z}-\frac{i}{2k}\Delta_\perp E_S=\frac{\Gamma}{2}\int_0^t E_S(\mathbf{R},\eta)\exp\left(\frac{t-\eta}{\tau_M}\right)d\left(\frac{\eta}{\tau_M}\right). \quad (11)$$

The solution of this equation depends strongly on the spatial structure of the pump wave, which determines the spatial structure of the function $\tau_M(\mathbf{R})\propto I_L^{-1}$. Let us examine and compare three versions of the spatial structures of the incident wave E_L and the reflected wave E_S.

1) If the pump wave is a plane wave, with $E_L=$ const, then τ_M is also independent of the spatial coordinates, and all spatial configurations of the wave E_S are equally probable by virtue of the linearity of Eq. (11). It is thus sufficient to solve the equation for only one plane-wave component, e.g.,

$$E_S(z,t)=a_0(z,t)e^{ikz},$$

which is directed strictly opposite the pump wave,

$$\frac{\partial a_0(z,t)}{\partial z}=\frac{\Gamma}{2}\int_0^t a_0(z,\eta)\exp\left(\frac{t-\eta}{\tau_M}\right)d\left(\frac{\eta}{\tau_M}\right). \quad (12)$$

2) If the pump wave has a speckle, and if the length scales of the variations in the speckle field satisfy

$$l_\perp\sim\lambda/\Delta\theta, \quad l_\parallel\sim\lambda/\Delta\theta^2$$

(λ and $\Delta\theta$ are the wavelength and divergence of the beam) are much shorter than the corresponding longitudinal and transverse dimensions of the mixing region (for a focused beam, this condition corresponds to complete immersion of the caustic within the crystal), the grating relaxation time $\tau_M(\mathbf{R})$ is a function of position. In this case, we would expect differences in the amplification of the conjugate and nonconjugate configurations of the Stokes wave. Making use of the linearity of Eq. (11) we seek a solution of this equation in the form[9]

$$E_S(\mathbf{r},z,t)=a_1(z,t)E_L{}^*(\mathbf{r},z)/I_L{}^{1/2}+a_2(z,t)N(\mathbf{r},z), \quad (13)$$

where $a_1(z,t)$ and $a_2(z,t)$ are amplitudes which vary slowly over length scales $l_\parallel$. They describe the growth as a function space and time of the conjugate and nonconjugate configurations excited when the pump wave is spontaneously scattered by crystal defects in the opposite direction. In addition, $I_L=\langle|E_L|^2\rangle$ is the average pump intensity, and $N(\mathbf{r},z)$ is the spatial distribution of the uncorrelated wave. This distribution satisfies a propagation equation which is the complex conjugate of (9), the orthogonality condition $\langle NE_L\rangle=0$, and (for simplicity) the normalization condition $\langle|N|^2\rangle=1$.

Substituting the conjugate part of the solution (13) into (11), averaging the resulting equation over the ensemble of realizations of the speckle field E_L after first multiplying the right and left sides of the equation by E_L^* (Ref. 9), and making the assumption that the amplitude $a_1(z,t)$ is deterministic, we find

$$\frac{\partial a_1(z,t)}{\partial z}=\Gamma\int_0^t\frac{a_1(z,\eta)}{[1+(t-\eta)/\langle\tau_M\rangle]^3}d\left(\frac{\eta}{\langle\tau_M\rangle}\right), \quad (14)$$

where the angle brackets mean an average over the ensemble of realizations.

3) Accordingly, we find the following equation for the slowly varying deterministic amplitude of the uncorrelated component, $a_2(z,t)$, amplified in the pump field $E_L(\mathbf{r},z)$:

$$\frac{\partial a_2(z,t)}{\partial z}=\frac{\Gamma}{2}\int_0^t\frac{a_2(z,\eta)}{[1+(t-\eta)/\langle\tau_M\rangle]^2}d\left(\frac{\eta}{\langle\tau_M\rangle}\right). \quad (15)$$

The integrodifferential equations (14) and (15) describe the evolution as a function of position in the mixing region of the envelopes for the conjugate and nonconjugate components of the reflected wave. Their solution for $a_1(L,t)$ and $a_2(L,t)$ at the crystal face where the laser beam enters makes it possible to see the time evolution of the nonlinear reflection coefficient,

$$R(t) \propto |a_1|^2 + |a_2|^2 ,$$

and the quality of the conjugation,

$$H(t) = |a_1|^2/(|a_1|^2 + |a_2|^2).$$

3. ANALYTIC SOLUTION OF THE EQUATIONS FOR THE SLOWLY VARYING AMPLITUDES

It is convenient to solve Eqs. (12), (14), and (15) by Laplace transforms in the variable t. The Laplace transforms

$$\tilde{a}_i(x,s) = \int_0^\infty a_i(x,t)\, e^{-st}\, dt$$

of the envelopes $a_i(z,t)$ obey the equation

$$\partial \tilde{a}_i(x,s)/\partial x = \mathscr{L}_i(s)\, \tilde{a}_i(x,s), \tag{16}$$

where $\mathscr{L}_{0,1,2}(s)$ are the Laplace transforms of the kernels of the integral operators on the right sides of Eqs. (12), (14) and (15), respectively. In (16) we have used the substitution of variables $x = \Gamma z/2$, $\tau = t/\langle \tau_M \rangle$. Since $\mathscr{L}_i(s)$ does not have an x dependence, the solution of Eq. (16) is

$$\tilde{a}_i(x,s) = \frac{\alpha_i}{s} \exp[\mathscr{L}_i(s)x], \tag{17}$$

where $\alpha_i = s\tilde{a}_i(0,s)$ corresponds to boundary and initial conditions

$$a_i(0,t) = a_i(z,0) = \alpha_i.$$

It is customary to choose the conditions in this form for stimulated-scattering problems:[9] α_i corresponds to the effective amplitude of the corresponding component which is excited in a small region near the rear of the crystal, $0 < z \lesssim \Gamma^{-1}$, as a result of spontaneous scattering of the pump wave into this component by crystal defects. Since the amplification of the Stokes wave in the mixing volume is exponential, it is this region which dominates the amplitude of the component at the exit, at $z = L$ (since the signal from this region travels farthest in the crystal).

For the exponential kernel of Eq. (12), the Laplace transform of the function $a_0(z,t)$ reduces to the sum of two tabulated transforms:

$$\tilde{a}_0(x,s) = \alpha_0 \left[\frac{1}{1+s} \exp\left(\frac{x}{1+s}\right) + \frac{x}{s(1+s)} \exp\left(\frac{x}{1+s}\right) \right]. \tag{18a}$$

We then find

$$a_0(z,t) = \alpha_0 \left\{ \exp\left(-\frac{t}{\langle \tau_M \rangle}\right) I_0\left(\left(\frac{2\Gamma z t}{\langle \tau_M \rangle} \right)^{1/2} \right) + \int_0^{t/\langle \tau_M \rangle} e^{-\xi} I_0((2\Gamma z \xi)^{1/2})\, d\xi \right\}, \tag{18b}$$

where I_0 is the modified Bessel function.

After transforming the kernel $(1+\tau)^{-3}$ of the operator of Eq. (14) and substituting into (17), we find the following expression for the Laplace transform of the function $\tilde{a}_1(x,s)$:

$$\tilde{a}_1(x,s) = \frac{\alpha_1}{s} \exp\{[(1-s) + s^2 e^s E_1(s)]x\}, \tag{19a}$$

where

$$E_1(s) = \int_1^\infty e^{-su} \frac{du}{u}.$$

Correspondingly, for $\tilde{a}_2(x,s)$ we find

$$\tilde{a}_2(x,s) = \frac{\alpha_2}{s} \exp\{[1 - se^s E_1(s)]x\}. \tag{20a}$$

Unfortunately, these functions do not reduce to tabulated functions. For the functional dependence of interest here,

$$a_i(x,t) = (2\pi i)^{-1} \int_{+0-i\infty}^{+0+i\infty} ds\, e^{st} \tilde{a}_i(x,s),$$

we need to use the contour shown in Fig. 2, with $\mathrm{Re}\, s < |\varepsilon|, |\varepsilon| \to 0$, to carry out the integration in the complex plane. This integration contour is chosen because of the pole in the integrand at the point $s = 0$ and because the integrand is an analytic function throughout the plane of the variable s, with a cut $(-\infty, 0)$, as can be seen from the representation

$$E_1(s) = -\left[C + \ln s + \sum_{k=1}^{\infty} \frac{(-1)^k s^k}{k!k} \right]$$

where C is Euler's constant.[16]

Since the integrand satisfies the conditions of Jordan's lemma in the left half-plane, we can write

$$a_i(\tau, x) = \frac{i}{2\pi} (J_1 + J_2 + J_3),$$

where $J_{1,2,3}$ are the values of the integrals along the corresponding trajectories (Fig. 2). As a result, using the relation[16]

$$E_1(y \pm i0)\,|_{y<0} = -\mathrm{Ei}(-y) \mp i\pi,$$

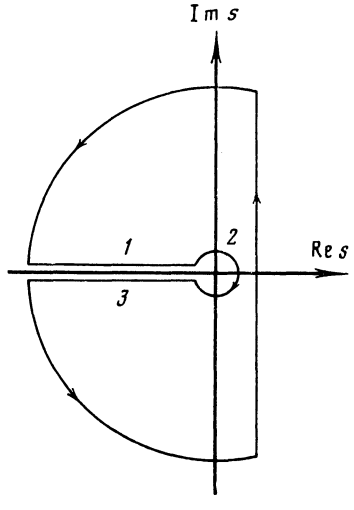

FIG. 2. Integration contour used in the derivation of (19b) and (20b).

where $\mathrm{Ei}(y)$ is the integral exponential function, we find

$$a_1(z,t)=\alpha_1\exp\left(\frac{\Gamma z}{2}\right)\left\{1-\frac{1}{\pi}\int_0^\infty\frac{dp}{p}\sin\left(\frac{\Gamma z}{2}\pi p^2e^{-p}\right)\right.$$
$$\times\exp\left[p\left(\frac{\Gamma z}{2}-\frac{t}{\langle\tau_M\rangle}\right)\right.$$
$$\left.\left.-\frac{\Gamma z}{2}p^2e^{-p}\,\mathrm{Ei}(p)\right]\right\}, \qquad (19b)$$

$$a_2(z,t)=\alpha_2\exp\left(\frac{\Gamma z}{2}\right)\left\{1-\frac{1}{\pi}\int_0^\infty\frac{dp}{p}\sin\left(\frac{\Gamma z}{2}\pi pe^{-p}\right)\right.$$
$$\left.\times\exp\left[-p\frac{t}{\langle\tau_M\rangle}-\frac{\Gamma z}{2}pe^{-p}\,\mathrm{Ei}(p)\right]\right\}. \qquad (20b)$$

4. DYNAMICS OF THE STIMULATED DIFFUSIVE SCATTERING

Using these results, we can describe the evolution of the reflection process for the conjugate and nonconjugate components of the Stokes wave and thus their competition in time. Figure 3 shows the results of numerical calculations from (19b) and (20b), in units of ΓL, for the behavior of the total gain

$$g_iL=\ln|a_i(L,t)/\alpha_i|^2$$

over a distance L for the situations considered. The independent variable is the normalized time $\Theta=t/\langle\tau_M\rangle(\Gamma L/2)^{1/2}$. Examining Fig. 3, we first note that these curves are "essentially self-similar" in this normalization of the coordinate axes. The self-similarity is not perfect, but the total gain values for each spatial configuration change by less than 2% as the parameter ΓL—the overall steady-state gain—varies over the range 10–20. The second point we note is that the conjugate component, being diffracted to a relatively great extent in regions in the mixing region with a minimum local rise time

$$\tau_M(\mathbf{R})\propto|E_L(\mathbf{R})|^{-2}=|E_L^*(\mathbf{R})|^2,$$

overtakes the nonconjugate wave in growth. The overall gain values for these configurations move asymptotically close together only at times dozens of times longer than the average Maxwellian time $\langle\tau_M\rangle$. The third point we note is that the time over which the nonlinear reflection coefficient stabilizes can be estimated to be $t_{\mathrm{rel}}\approx\langle\tau_M\rangle(\Gamma L)^{1/2}$. This result is close to the estimate found for this parameter for other types of stimulated scattering.[9,10]

Also shown in Fig. 3, by curve 3, is a plot of the logarithm of the discrimination coefficient, $\ln K$, where

$$K(t)=\left|\frac{a_1}{\alpha_1}\right|^2\Big/\left|\frac{a_2}{\alpha_2}\right|^2=\exp[(g_1-g_2)L]$$

From this curve we get an idea of the level of the discrimination of the nonconjugate components and thus the quality of the conjugation. This curve again is essentially self-similar, as is demonstrated by Fig. 4, from which we see that $(g_1-g_2)_{\max}/\Gamma=\ln K_{\max}/\Gamma L$ and $\Theta_{\max}$, where $K_{\max}$ is the maximum value of the discrimination coefficient, and $\Theta_{\max}$ is the corresponding time, are essentially independent of ΓL. The curves in Fig. 3 agree well qualitatively with the experimental curves from Ref. 6. They demonstrate that the reflection coefficient stabilizes at a steady-state level, the extent of conjugation attains a maximum (a maximum of the discrimination gain) in a very unsteady region, $t_{\max}\lesssim 0{,}5\langle\tau_M\rangle(\Gamma L)^{1/2}$, and the quality of the conjugation is gradually degraded at $t>\langle\tau_M\rangle(\Gamma L)^{1/2}$. It is difficult to make an exact quantitative comparison of the experimental and theoretical results, however, because the experiments were carried out in a focused pump beam, in an absorbing crystal, and in the pump saturation regime, and none of these factors were considered in the theory. In particular, the experiments of Ref. 6 revealed a decrease not only in the extent of conjugation in a nearly steady state but also in the absolute strength of the conjugate component. This effect would appear to be a consequence of pump depletion, and could not be predicted by a linear theory.

5. ESTIMATES OF THE CONJUGATION QUALITY

The quality of the conjugation in stimulated backscattering of a speckle pump wave is known[9] to be deter-

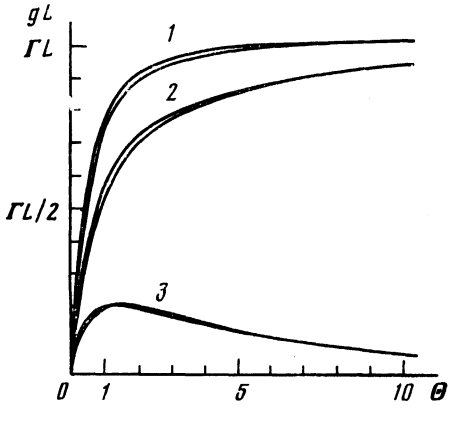

FIG. 3. The overall gain values (1) g_1L and (2) g_2L and (3) the discrimination gain $(g_1-g_2)L$ versus the dimensionless time Θ.

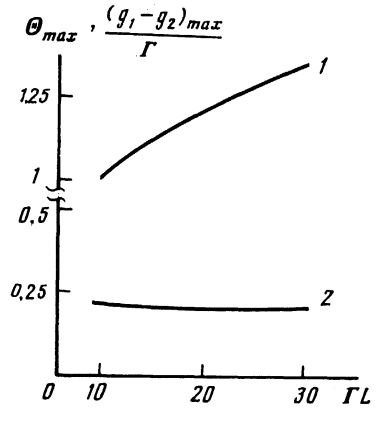

FIG. 4. 1—The dimensionless time $\Theta_{\max}$ at which the discrimination is at a maximum; 2—the relative discrimination gain $(g_1-g_2)_{\max}/\Gamma$ at these times. The independent variable is the gain of the overall steady-state amplification, ΓL.

mined by the following factors: the degree of discrimination of the nonconjugate components, K; the fraction of the conjugate component in the spontaneous seed noise which is concentrated in the solid angle occupied by the reflected wave; and the level of the "serpentine" distortions due to the scattering (through small angles) of the self-conjugate component by spatial variations in the local stimulated-scattering gain.

For a speckle pump beam which is focused in a crystal in which the caustic is completely immersed, the solid angle occupied by the Stokes wave is determined by a geometric factor, specifically, the ratio of the diameter of the caustic to its length: $\Delta\Omega \sim \Delta\theta_L^2 \sim (d/L)^2$, where $\Delta\theta_L^2$ is the divergence of the speckle pump wave in the mixing region. For this reason, the fraction of the power of the seed sources which corresponds to the conjugate component, $\zeta = |\alpha_1/\alpha_2|^2$, can be estimated from

$$\zeta \sim (\theta_d/\Delta\theta_L)^2 \sim N_0^{-1},$$

where θ_d is the diffraction angle, which corresponds to the diameter of the caustic, and N_0 is the number of speckle spots in the conjugate beam. If we then ignore the serpentine distortions, we can easily work from (13) to find an estimate of the extent of conjugation, H: $H \approx [1 + N/K(L)]^{-1}$, where $K(L)$ is the discrimination coefficient in the exit cross section, $z = L$.

Since the absolute values of the discrimination coefficient are small in comparison with other types of stimulated scattering in our situation, specifically, since we have $K_{\max} \approx \exp(0.21\Gamma L)$ (this quantity would not exceed 10^2 for the typical values $\Gamma L \sim 20$), we would have to reduce the number of speckle spots in the conjugate beam in order to achieve a high conjugation quality. For a fixed value of the total gain ΓL (fixed by the crystal), however, a decrease in the number of speckle spots in the pump beam gives rise to a sharp increase in the serpentine distortions. The fraction of the energy represented by the serpentine noise is determined by the average fluctuation in the local stimulated-scattering gain, $g = \langle g\rangle + \delta g$, over the longitudinal correlation length of the pump field, $l_\parallel \sim \lambda/\Delta\theta_L^2$ (Refs. 9, 10):

$$(1-H)_{\text{Ser}} \approx \mu_1 \langle \delta g^2\rangle^{1/2} l_\parallel = \mu_1 \frac{\langle \delta g^2\rangle^{1/2}}{\langle g\rangle} \langle g\rangle l_\parallel.$$

The dimensionless coefficient $\langle \delta g^2\rangle^{1/2}/\langle g\rangle$ here is determined by the dependence of the local gain on the pump intensity, and the coefficient μ_1 is determined by its angular spectrum. For stimulated Brillouin scattering we would have $g = G|E_L|^2$, so that for a speckle field with Gaussian statistics we would have

$$(1-H)_{\text{Ser}} \approx \mu_1 \langle g\rangle l_\parallel = \mu_1 \langle g\rangle L/N_0^{1/2}.$$

The last equality holds for focused beams, since we would have $L/l_\parallel \approx N_0^{1/2}$ for them. The typical values of μ_1 for stimulated Brillouin scattering, $\mu_1 \sim 0.1$–0.2, would make it possible to experimentally observe a conjugation quality (at $K \gg N_0$) on the order of 80–90% for the typical values $\langle g\rangle L \approx 25$ and $N_0^{1/2} \approx 20$ (Ref. 9).

In the case of stimulated diffusion scattering, the local gain is a more complicated function of the pump intensity, but the results of this study show that one can estimate the relative scale of the spatial fluctuations of the gain, which would confer an advantage on the amplification of the conjugate component under maximum-discrimination conditions:

$$\frac{\langle \delta g^2\rangle^{1/2}}{\langle g\rangle} \approx \frac{\ln K}{\Gamma L}.$$

The level of the serpentine distortions during stimulated diffusion scattering,

$$(1-H)_{\text{ser}} \approx \mu_2 \Gamma L/N_0^{1/2},$$

is thus smaller by a factor of roughly $\mu_2/\mu_1 \approx \Gamma L/\ln K_{\max} \approx 5$ than during stimulated Brillouin scattering for a given overall gain and for a given pump angular spectrum.

We adopt the following expression, which incorporates both the serpentine noise and the limitation on the discrimination coefficient, for the maximum attainable extent of conjugation, $H_{\max}$:

$$H_{max} = \left(1 + \frac{N_0}{K_{max}} + \mu_2 \frac{\Gamma L}{N_0^{1/2}}\right)^{-1}. \qquad (21)$$

At small values of $N_0/K_{\max}$ and $\mu_2 \Gamma L/N_0^{1/2}$, this expression becomes the corresponding limiting expressions. Figure 5 shows, in the coordinates ΓL, $N_0^{1/2}$, "quality contour curves," i.e., lines corresponding to a fixed extent of conjugation, according to (21) with $\mu_2 = 10^{-2}$. The values $H_{\max} \approx 70\%$ found experimentally at $N_0 \sim 30$ agree well with the theoretical predictions of this formula for the value $\Gamma L \approx 20$.

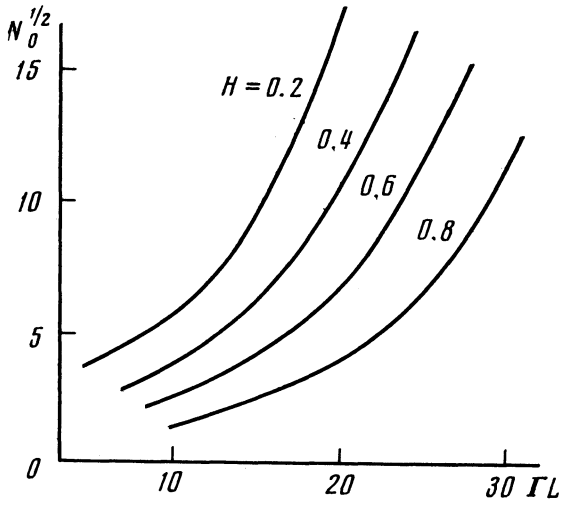

FIG. 5. Contour curves of the phase conjugation quality for the value $\mu_2 = 10^{-2}$.

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