Electron acceleration by intense laser beam in a static magnetic field

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(Submitted 6 October 1989)
Zh. Eksp. Teor. Fiz. **97**, 1498–1509 (May 1990)

Possibilities for electron acceleration by intense laser light in a static magnetic field are examined. An acceleration mechanism is described in which the transverse magnetic field masks the oscillatory nature of the laser light. The latter is seen instead as a set of quasisteady crossed electric and magnetic fields. The conditions for the occurrence of this mechanism and the increment in the electron energy are found. Analytic expressions are derived, and numerical calculations are carried out. Under optimum conditions, the electron energy may be increased severalfold by a single passage through the focus.

1. INTRODUCTION

The literature reveals a significant recent interest in laser acceleration of electrons.[1-12] The interest stems from the presumed high efficiency of the corresponding devices. The laser acceleration schemes which have been proposed can be divided qualitatively into three main groups:

1) vacuum devices (the inverted free-electron laser,[1,2] the inverted Compton laser,[3] and certain others);

2) devices in which a plasma is to be used, and a longitudinal plasma wave is to cause the acceleration (these devices use plasma beat-wave acceleration,[4,5] acceleration in a plasma-wave field crossed with a perpendicular static magnetic field,[6] etc.);

3) devices in which a condensed medium is to be used as a retarding system, to reduce the phase velocity of the electromagnetic wave causing the acceleration (a waveguide,[7] a surface wave with total internal reflection,[8,9] acceleration on the basis of the inverse Cherenkov effect,[10-12] etc.).

Each of these schemes is interesting in itself, although each suffers from certain limitations. For example, the capabilities of an inverted free-electron laser would be limited by the large length and by nonlinearity (saturation) effects. These effects would prevent the use of an intense accelerating wave. In any scheme in which the properties of a medium are to be utilized, the field intensity is also limited by the condition that the medium should not be damaged by the field. For a condensed medium, this limitation means that there should be no breakdown, melting, etc. In a plasma, the field must not be so strong that the repulsion of plasma electrons from the focus by ponderomotive forces would become important in the strong nonuniform field.

All the schemes which we have listed are of a resonant nature in terms of the initial energy of the electrons, so it would be a complicated matter to achieve repeated acceleration in such devices.

In the present paper we consider another approach and some other schemes for laser acceleration of electrons—schemes which do not suffer from these severe intensity limitations and which are not of an explicitly resonant nature. Since the efficiency of these acceleration schemes can be quite high, their nonresonant nature raises the possibility in principle of cyclic acceleration. These new schemes are based on the use of the nonuniform field of focused intense laser light. With a static magnetic field having a suitable configuration, these are possibilities for efficient acceleration in this case.

2. TRANSVERSE MAGNETIC FIELD

2.1. Formulation of the problem and solution method

We consider an electron which, in the absence of a laser field, is moving in the xz plane in a static, uniform magnetic field H_0, which is directed along the y axis (Fig. 1). We assume that at the time $t = 0$ the orbit of the electron is tangent to the z axis at the point z_0. The classical equations of motion of an electron in the magnetic field are then

$$z=z_0+\frac{v_0}{\Omega}\sin\Omega t,\qquad x=\frac{v_0}{\Omega}(\cos\Omega t-1),\tag{1}$$

where $c = 1$, $\Omega = eH_0/\varepsilon_0$ is the Larmor frequency, ε_0 is the energy of the electron, and v_0 is its velocity. The radius of the Larmor orbit is $R = v_0/\Omega$.

We now assume that the electron is acted upon not only by the magnetic field H_0 but also by the field of the laser light. This light is polarized in the (x,z) plane, is focused, has a wavelength λ, and is propagating along the z axis. The focus of this laser beam has a diameter d and a length L (Fig. 1). We assume

$$\lambda\ll d\ll L\ll R.\tag{2}$$

By virtue of inequality (2), the longitudinal component E_z of the vector $\mathbf{E}$ is small; we will ignore it, assuming $E\parallel x$. The electric field ($\mathbf{E}\parallel x$) and the magnetic field ($\mathbf{H}\parallel y$) of the wave are chosen as follows:

$$H_y=E_x\equiv E=E_0(z)\cos[\omega(t-z)+\varphi_0],\tag{3}$$

where $\omega = 2\pi/\lambda$, $E_0(z)$ is the electric field amplitude, which depends weakly on z, and φ_0 is the phase of the electromagnetic field at the point $z = 0$ at the time $t = 0$. Actually, E_0 depends not only on z but also on the transverse coordinates x and y, but we will also ignore that dependence.

Near the point $z = z_0$, i.e., at small values of t, the first of Eqs. (1) becomes

$$z=z_0+v_0t-{}^1/_6v_0\Omega^2t^3.$$

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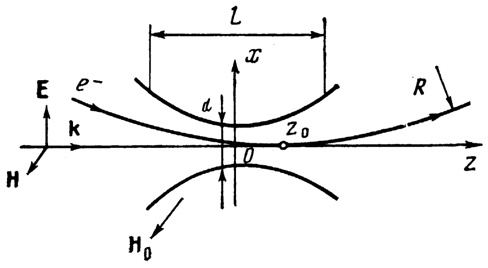

FIG. 1. Geometry of this acceleration method.

When this expression is substituted into expression (3) for the laser field, the phase of the field (the argument of the cosine) becomes

$$\varphi(t)\approx\varphi_0+\omega t/2\gamma^2+{}^1/_6\omega\Omega^2t^3. \tag{4}$$

The typical time scale or length scale over which the field E changes substantially (changes sign) along the trajectory of an electron is thus

$$t_0=l_0=\min\left\{\lambda\gamma^2,\ \left(\frac{6\pi}{\omega\Omega^2}\right)^{1/3}\right\}. \tag{5}$$

We assume that this length is small in comparison with the length of the focus:

$$l_0\ll L. \tag{6}$$

From the condition that the two terms inside the braces in Eq. (5) are comparable in magnitude, we can introduce the concept of a critical value of the relativistic factor, $\gamma_{\rm cr}$, which will be important for the analysis below. The critical value $\gamma_{\rm cr}$ is defined by $\gamma_{\rm cr}=(\omega/\Omega)^{1/3}$. Noting that the Larmor frequency is inversely proportional to γ, we can write

$$\gamma_{\rm cr}=(m\omega/eH_0)^{1/2}\gg1. \tag{7}$$

The equations of motion of an electron in a static magnetic field and in the field (3) are

$$\begin{aligned}&p_y=\text{const}=0,\\&dp_x/dt=eH_0v_z-eE(1-v_z),\\&dp_z/dt=-eH_0v_x-eEv_x,\\&d\varepsilon/dt=-eEv_x,\end{aligned} \tag{8}$$

where $\mathbf{p}=\varepsilon\mathbf{v}$ is the momentum of an electron, $\mathbf{v}$ is its velocity, and ε is the energy.

Taking account of the variation of the field (3), we will use perturbation theory to find an approximate solution of Eqs. (8), assuming that the field E is weak. In a zeroth-order theory, a solution of system (8) is given by the functions $x^{(0)}(t)$, $z^{(0)}(t)$, which are found from Eqs. (1) in the case $\varepsilon=\varepsilon_0=\text{const}$. In first order in E, the rate of change of the energy is given by

$$d\varepsilon^{(1)}/dt=-ev_x^{(0)}E(z^{(0)})\cos[\omega(t-z^{(0)})+\varphi_0]. \tag{9}$$

In Subsection 2.2 below, we derive the increment in the energy of the electron, $\Delta\varepsilon^{(1)}$, according to Eq. (9). If we are interested in a repeated interaction of the electron with a train of laser pulses, however, we should treat the field phase φ_0 as random and take an average over it. In this case $\Delta\varepsilon^{(1)}$ vanishes (more on this below), and we should go to second-order perturbation theory to calculate a nonzero increment in the energy. To do this, we must find the velocity $\mathbf{v}$ and the coordinates $\mathbf{r}$ of the electron to first order in E, i.e., the quantities $\mathbf{v}^{(1)}$ and $\mathbf{r}^{(1)}$. To calculate $v_x^{(1)}, v_z^{(1)}, x^{(1)}, z^{(1)}$ we write corresponding equations which follow from (8):

$$\begin{aligned}&\frac{dv_x^{(1)}}{dt}-\Omega v_z^{(1)}=-eE(1-v_z^{(0)}-(v_x^{(0)})^2)-\frac{\Omega}{\varepsilon_0}\varepsilon^{(1)}v_z^{(0)},\\&\frac{dv_z^{(1)}}{dt}+\Omega v_x^{(1)}=-eEv_x^{(0)}(1-v_z^{(0)})+\frac{\Omega}{\varepsilon_0}\varepsilon^{(1)}v_x^{(0)},\end{aligned} \tag{10}$$

where $\varepsilon^{(1)}\sim E$ is given by (9), and the frequency $\Omega=eH_0/\varepsilon_0$ is determined, as before, from the initial, E-independent energy of the electron.

The rate of change of the energy of the electron is determined by the last of Eqs. (8), which takes the following form to second order in E:

$$\begin{aligned}d\varepsilon^{(2)}/dt=&-eE_0(z^{(0)})[v_x^{(1)}\cos(\omega(t-z^{(0)})+\varphi_0)+\omega v_x^{(0)}z^{(1)}\\&\times\sin[\omega(t-z^{(0)})+\varphi_0]-eE_0'(z_0^{(0)})z^{(1)}v_x^{(0)}\cos[\omega(t-z^{(0)})+\varphi_0]].\end{aligned} \tag{11}$$

2.2. Electron acceleration in first-order perturbation theory

We first find the increment in the electron energy which is linear in the electric field of the wave. For this purpose it is sufficient to integrate Eq. (9) with the approximate expression (4) for the field phase $\varphi(t)$. As a result we find

$$\Delta\varepsilon^{(1)}=e\Omega\int_{-\infty}^{\infty}dt\,tE_0(z_0+t)\cos\left[\frac{\omega t}{2\gamma^2}+\frac{\omega\Omega^2t^3}{6}+\varphi_0+\omega z_0\right]. \tag{12}$$

We wish to stress that it is the initial, unperturbed energy of the electron which is used in determining the relativistic factor $\gamma=\varepsilon_0/m\gg1$.

By virtue of definition (5) and condition (6), the envelope of the field, $E_0(z_0+t)$, varies only slightly over the interval t_0, so we can replace $E_0(z_0+t)$ by $E_0(z_0)$ in Eq. (12). As a result, the solution of (12) takes the following form:[13]

$$\Delta\varepsilon^{(1)}=2^{5/3}\pi^{1/2}\sin\varphi_0\frac{eE_0(z_0)}{(\omega^2\Omega)^{1/3}}\frac{d}{d\xi}\Phi(\xi), \tag{13}$$

where $\Phi(\xi)$ is the Airy function,[14]

$$\xi=2^{-2/3}(\gamma_{\rm cr}/\gamma)^{4/3}, \tag{14}$$

and $\gamma_{\rm cr}$ is given by (7). We can draw some conclusions from (13) and (14).

1. The change in the energy of the electron, $\Delta\varepsilon^{(1)}$, given by (13) is proportional to the electric field amplitude of the wave at the point z_0 at which the unperturbed orbit of the electron is tangent to the axis of the laser beam.

2. The quantity $\Delta\varepsilon^{(1)}$ is proportional to the sine of the field phase at the point z_0 at $t=0$; taking an average over φ_0 causes $\Delta\varepsilon^{(1)}$ in (13) to vanish.

3. By virtue of the properties of the Airy function, the value of $\Delta\varepsilon^{(1)}$ is very small for $\gamma<\gamma_{\rm cr}$ in (7), i.e., for $\xi>1$.

Under the condition $\gamma \gg \gamma_{cr}$ ($\xi < 1$), expression (13) can be approximated by

$$\frac{\Delta\varepsilon^{(1)}}{\varepsilon_0} \sim \frac{eE_0\lambda}{m}\left(\frac{\gamma_{cr}}{\gamma}\right)^{2/3} \propto \gamma^{-2/3}, \qquad \Delta\varepsilon^{(1)} \propto \gamma^{1/3}. \tag{15}$$

The absolute increase in energy, $\Delta\varepsilon^{(1)}$, for $\gamma > \gamma_{cr}$ increases with γ, in proportion to $\gamma^{1/3}$, while the relative increase falls off as $\gamma^{2/3}$. From the standpoint of the relative increase, the optimum situation thus corresponds to $\gamma \approx \gamma_0$.

2.3. Quadratic acceleration at low energies

As was shown above, taking an average over φ_0 causes the increment in the electron energy found in the first-order perturbation theory in E_0 to vanish. To find a nonvanishing average change in the energy of the electron we need to calculate $\Delta\varepsilon^{(2)}$. To do this, we need to solve Eqs. (10) and then integrate (11). These calculations are simple but laborious. Skipping the details, we write the result averaged over φ_0 (we omit the averaging sign from $\Delta\varepsilon^{(2)}$):

$$\Delta\varepsilon^{(2)} = \frac{e^2}{2\varepsilon_0}\operatorname{Re}\left\{\int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt'\, E_0(z_0+t)E_0(z_0+t') \times\left[F_1(t,t') + \frac{i\omega}{\Omega}\sin\Omega t F_2(t,t')\right] \times\exp\left[i\omega\left(t-t' - \frac{v_0}{\Omega}(\sin\Omega t - \sin\Omega t')\right)\right]\right\}, \tag{16}$$

where

$$F_1 = \frac{1-\sin^2\Omega t\cos\Omega(t-t')}{\cos^2\Omega t} + \frac{\sin\Omega t\sin\Omega(t-t')}{\cos\Omega t} - \frac{v_0\cos\Omega t'}{\cos^2\Omega t},$$

$$F_2 = \frac{1}{\cos\Omega t}(1-\cos\Omega(t-t')) + v_0(\cos\Omega t - \cos\Omega t') - v_0^2\Omega(t'-t)\sin\Omega t'. \tag{17}$$

Expressions (16) and (17) can be simplified considerably by virtue of the small values of the parameters $|\Omega t| \ll 1$ and $1/\gamma \ll 1$. The methods for approximately evaluating the integrals, and the final results found, differ substantially for the regions of low and high energies, which are defined by the conditions $\gamma < \gamma_{cr}$ and $\gamma > \gamma_{cr}$, respectively. We first consider the case $\gamma < \gamma_{cr}$. In this energy region the exponential in the integrand in (16) is a rapidly oscillating function of t'. After multiplying and dividing the integrand by $-i\omega(1 - v_0\cos\Omega t')$, we can integrate over t' by parts. In the terms which contain the functions F_1 and F_2 in (16), the first nonvanishing results (which are on the same order of magnitude) are found through a double and triple, respectively, integration by parts. The condition under which this calculation procedure is valid is that the corrections found through further integration by parts be small. One can show that this condition is $\gamma < \gamma_{cr}$, as has been assumed. The result of the calculation of $\Delta\varepsilon$ in (16) by this method is

$$\Delta\varepsilon = \frac{e^2\Omega v_0}{4\varepsilon_0\omega^2}\int_{-\infty}^{\infty} dt \frac{\sin\Omega t E_0^2(z_0+t)}{\cos^2\Omega t(1-v_0\cos\Omega t)^3}\,\frac{z_0+v_0\sin\Omega t}{\Omega} \times(1-3v_0\cos\Omega t + 2\cos^2\Omega t). \tag{18}$$

The singularity of the integrand at $\Omega t = \pm\pi$ plays no role here, since there is no field ($E_0 = 0$) in this region, by virtue of condition (2). Making use of the small values of the parameters $|\Omega t| \ll 1$ and $1/\gamma \ll 1$, we specify a model for the envelope E_0 in order to carry out some explicit calculations:

$$E_0(z) = E_0\exp(-z^2/L^2). \tag{19}$$

As a result, Eq. (18) becomes

$$\Delta\varepsilon = \frac{e^2E_0^2}{\varepsilon_0\omega^2}\gamma^2\exp\left(-\frac{z_0^2}{L^2}\right)\int_{-\infty}^{\infty}\frac{x\,dx}{(1+x^2)^3}(3-x^2) \times\exp\left[\frac{-x^2}{(\gamma\Omega L)^2} - \frac{2z_0x}{\gamma\Omega L^2}\right]. \tag{20}$$

In the case $z_0 = 0$ we have $\Delta\varepsilon = 0$, since the integrand is odd. At small values of $|z_0|$ ($|z_0| \ll L$), Eq. (20) simplifies and can be approximated by

$$\Delta\varepsilon^{(2)} = -\frac{2e^2E_0^2\gamma z_0}{\varepsilon_0\omega^2L^2\Omega}\int_{-\infty}^{\infty}\frac{x^2(3-x^2)}{(1+x^2)^3}\exp\left[-\left(\frac{x}{\gamma\Omega L}\right)^2\right]dx \approx \frac{-e^2E_0^2z_0}{\varepsilon_0\omega^2L^3\Omega^2}, \tag{21}$$

Here we have used $\gamma\Omega L \gg 1$ or

$$L \gg R/\gamma. \tag{22}$$

By virtue of condition (22), the integral in (21) can be written in the form

$$\int_{-\gamma\Omega L}^{\gamma\Omega L} dx\frac{x^2(3-x^2)}{(1+x^2)^3} = -2\int_{\gamma\Omega L}^{\infty} dx\frac{x^2(3-x^2)}{(1+x^2)^3} \approx 2\int_{\gamma\Omega L}^{\infty}\frac{dx}{x^2} = \frac{2}{\gamma\Omega L},$$

so we find estimate (21) for $\Delta\varepsilon^{(2)}$. The function $\Delta\varepsilon^{(2)}(z_0)$ is of odd parity in its argument (Fig. 2). It reaches a maximum

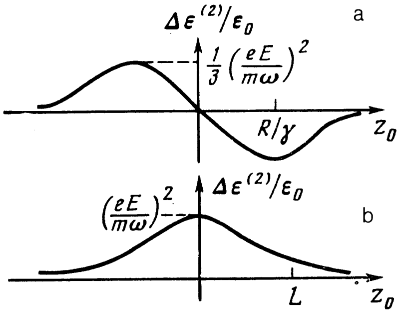

FIG. 2. The relative change in the average energy of the electrons, $\varepsilon^{(2)}/\varepsilon_0$, versus the position z_0 at which the electron trajectory is tangent to the axis of the laser beam. a—Low energies, $\gamma < \gamma_{cr}$, $l \sim R/\gamma$; b—high energies, $\gamma > \gamma_{cr}$.

at $z_0 \sim -R/\gamma$, so the maximum energy increase can be estimated to be

$$\Delta\varepsilon^{(2)}_{max} \sim \frac{e^2E_0^2}{\varepsilon_0\omega^2(\Omega L)^3\gamma}, \quad \left(\frac{\Delta\varepsilon}{\varepsilon_0}\right)_{max} \sim \left(\frac{eE\lambda}{2\pi m}\right)^2 \frac{1}{(L\Omega\gamma)^2}. \tag{23}$$

We stress that this acceleration effect is not associated with gradient forces, although $\Delta\varepsilon$ is proportional to z_0 [see (18)]. Acceleration occurs for $z_0 < 0$. In this case, the gradient force is a retarding force at the point z_0 at which the electron orbit is tangent to the axis of the laser beam. In general, whenever an electron passes through the focal region acceleration by gradient forces would be impossible because these forces are potential forces. The acceleration effect described here is determined not by the average ponderomotive potential of a relativistic electron[15] but by oscillations of an electron in the field of the wave. At the applicability limit of estimate (23), i.e., for $R \sim \gamma L$, the maximum relative increment in the energy is $(eE\lambda/2\pi m)^2$.

2.4. Acceleration in quasisteady crossed field

For $\gamma > \gamma_{cr}$, the procedure of evaluating the integral in (16) through an integration by parts cannot be used. Since the condition $\gamma > \gamma_{cr}$ corresponds to the case of high energies, the general expressions (16) and (17) simplify in this approximation:

$$\Delta\varepsilon^{(2)} = -\frac{e^2\Omega^4}{4\varepsilon_0}\mathrm{Re}\int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt'\, E_0(z_0+t)E_0(z_0+t')$$
$$\times t'\left[2t-t'+i\frac{\omega\Omega^2}{6}t(t-t')(4t^2+tt'+t'^2)\right]\exp\left[\frac{i\omega\Omega^2}{6}(t^3-t'^3)\right]. \tag{24}$$

The characteristic value t_0 of the values of $t - t'$ which contribute significantly to the integrals in (24) is determined by the general expression (5) for t_0. It follows from that expression that the curvature of an electron orbit in a static magnetic field for $\gamma > \gamma_{cr}$ disrupts the interaction with the laser field before the latter can change sign. Over the time t_0 given by (5) the electron is acted upon by the crossed electric and magnetic fields of the laser wave, which are steady, in a sense. There is no averaging over many oscillations. In this case the acceleration mechanism is quite different in nature from that for $\gamma < \gamma_{cr}$.

By virtue of condition (6), the amplitudes $E_0(z_0 + t)$ and $E_0(z_0 + t')$ vary only slightly over intervals on the order of t_0 in (5). This can be utilized to simplify expression (24) further. The integrals over t and t' in (24) are evaluated by replacing the variables $t - t' = u$ and $t + t' = v$ with the help of expression (19) for $E_0(z)$ in the limit $L \to \infty$. Skipping the details of the calculation, we write the final result:

$$\Delta\varepsilon^{(2)}_{crs} = 2\cdot 3^{-7/6}\pi^{-1/2}\Gamma\left(\frac{5}{6}\right)\frac{e^2E_0^2(z_0)}{\varepsilon_0\omega^{4/3}\Omega^{8/3}},$$
$$\left(\frac{\Delta\varepsilon^{(2)}}{\varepsilon_0}\right)_{crs} \approx \left(\frac{eE\lambda}{m}\right)^2\left(\frac{\gamma_{cr}}{\gamma}\right)^{4/3}, \tag{25}$$

where $\Gamma(x)$ is the gamma function.

At the applicability limit of this calculation method, with $\gamma \sim \gamma_{cr}$ and $eE\lambda \sim m$, we have $\Delta\varepsilon^{(2)} \sim \varepsilon_0$, according to (25); i.e., the electron acceleration is an extremely large effect.

While the function $\Delta\varepsilon^{(2)}$ is odd for $\gamma < \gamma_{cr}$ [see (20)], the functional dependence $\Delta\varepsilon^{(2)}(z_0)$ is qualitatively different for $\gamma > \gamma_{cr}$: The acceleration is proportional to the square of the field at the point z_0, and the function $\Delta\varepsilon^{(2)}(z_0)$ [see (25)] is an even function, which reaches a maximum at $z_0 = 0$ (Fig. 2). Under the optimum conditions for each of the acceleration mechanisms, estimates (23) and (25) agree in order of magnitude.

Since the laser field does not have time to change sign over a time interval $\sim t_0$, expression (25) can be interpreted qualitatively as an acceleration in steady-state crossed fields in the presence of an additional static magnetic field H_0. We are led to ask which of the fields has the greater effect and should be considered in lowest order. To answer this question we estimate the transverse electron velocity v_x at small values of t in two extreme cases: in a static magnetic field H_0 alone and in a static magnetic field crossed with a static electric field of equal magnitude E, without the static field H_0. Using Eqs. (1) and the results derived in Ref. 16, we easily find

$$v_x^{H_0} = v_0\Omega t, \quad v_x^{crs} = \frac{eEt}{m\gamma^3}, \quad \frac{v_x^{crs}}{v_x^{H_0}} = \frac{eE\lambda\gamma_{cr}^2}{m\gamma^2} = \frac{E}{\gamma^2 H_0}. \tag{26}$$

Under the conditions $eE\lambda < m$ and $\gamma > \gamma_{cr}$, the inequality $v_x^{crs} < v_x^{(H_0)}$, holds; i.e., the curvature of the electron trajectory in the field H_0 is more substantial than that in crossed fields. This is the justification for the use of perturbation theory in the laser field. The criterion in this case is, according to (26),

$$E < \gamma^2 H_0. \tag{27}$$

This condition may be satisfied even though the condition $E \ll H_0$ does not necessarily hold.

If condition (27) does hold, we cannot make direct use of the solution, which we mentioned above, of the problem of acceleration in steady-state crossed fields.[16] As before, we must consider the effect of the field H_0 exactly in the zeroth approximation. We do this in Eqs. (1) and then by iterating in E incorporate the interaction with the laser field. Under the approximation (6), the field E can be assumed constant in Eqs. (8): $E = E_0\cos\varphi_0$. This assumption simplifies the problem greatly and makes it a rather simple matter to derive the changes in the electron energy in first and second orders in E:

$$\Delta\varepsilon^{(1)} = eE\Omega t^2\cos\varphi_0, \quad \overline{\Delta\varepsilon^{(2)}} = \frac{e^2E^2}{6\varepsilon_0}\Omega^2 t^4. \tag{28}$$

Substituting the value $t = t_0$ from (5) into these equations for the case $\gamma > \gamma_{cr}$, we find results which are the same (to within coefficients of order unity) as results which have already been derived, (15) and (25).

2.5. Estimates and numerical calculations

The results derived above show that there are two important parameters in this problem: γ/γ_{cr} and

$$\eta = \lambda e E_{0\,max}/m. \tag{29}$$

The latter parameter is a measure of the acceleration efficiency. It is related to the parameter used in the power-series

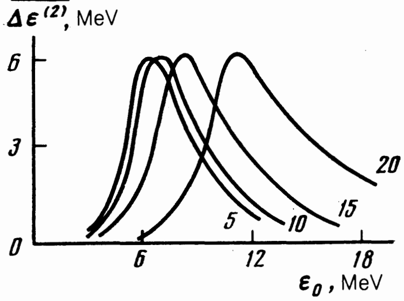

FIG. 3. Increment in the energy of the electrons versus their initial energy for various magnetic field strengths. The curves are labeled with the value of H_0 in kilogauss. The light intensity is $I = 10^{15}$ W/cm^2.

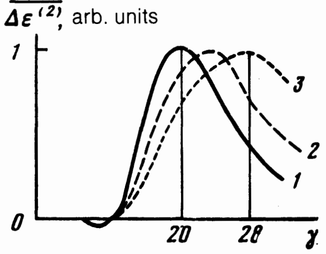

FIG. 5. Increment in the energy of the electrons versus the factor γ for various laser beam intensities. 1—$I = 10^{14}$ W/cm^2 ($\overline{\Delta\varepsilon} = 0.25$ MeV); 2—$I = 3\cdot 10^{15}$ W/cm^2 ($\overline{\Delta\varepsilon} = 6$ MeV); 3—$I = 10^{16}$ W/cm^2 ($\overline{\Delta\varepsilon} = 14$ MeV).

expansion in the field E. The physical meaning of the parameter η is the work performed by an electric field of strength E_0 over a distance on the order of the wavelength of the light. The parameter η can also be thought of (as in Ref. 17) as the ratio of the amplitude of the electron velocity oscillation in the field of the wave to the velocity of light.

Let us look at some estimates. We take $\lambda = 10^{-3}$ cm, $\omega = 2\cdot 10^{14}\ \mathrm{s}^{-1}$ (the beam from a CO_2 laser), $L = 0.3$ cm, $d = 0.01$ cm, $\gamma = 10$, and $H_0 = 2\cdot 10^4$ Oe. In this case we have $\Omega = 3\cdot 10^{10}\ \mathrm{s}^{-1}$ and $R = 1$ cm, and conditions (2) and (6) hold.

The parameter η in (29) has a value of 0.5 at a light intensity $I = 10^{14}$ W/cm^2 and a value $\eta = 5$ at $I = 10^{16}$ W/cm^2. The ratio $\gamma/\gamma_{\rm cr}$ for these values of H_0, ω, and γ is roughly 1/2. We thus see that these parameter values correspond to the transition region between low and high energies. An estimate of the increase in the electron energy from (25) shows that the quadratic acceleration becomes comparable to the linear acceleration at the light intensity $I \sim 10^{15}$–10^{16} W/cm^2, so we are justified in using a perturbation theory in the field E at intensities I up to these values. At larger values of E and I, the problem will have to be solved numerically.

Numerical calculations which we have carried out confirm the basic qualitative predictions of the theory which were given above. Figure 3 shows the energy of accelerated electrons, averaged over the field phase φ_0, as a function of the initial energy of the electrons for various strengths of the transverse magnetic field H_0. The error of these calculations is $\sim 15\%$. The intensity of the laser light is $3\cdot 10^{15}$ W/cm^2.

A basic feature of the curves in Fig. 3 is a sharp increase in the energy increase with increasing initial electron energy ε_0. This effect corresponds to a transition from the case $\gamma < \gamma_{\rm cr}$ to the case $\gamma > \gamma_{\rm cr}$ for acceleration in crossed fields [see (25)].

The maximum energy increase does not depend on the magnetic field H_0 (within the errors). The position of the maximum, ε_{0m} (Fig. 4), has a magnetic-field dependence $H_0^{-0.4}$ (from the condition $\gamma \approx \gamma_{\rm cr}$ we conclude $\varepsilon_{0m} \propto H_0^{-0.5}$). Figure 5 shows the same curves as in Fig. 3, but here the parameter is the intensity I, and the magnetic field H_0 is fixed at $H_0 = 5$ kHz. The curves in Fig. 5 coincide up to $I = (4\text{–}7)\cdot 10^{14}$ W/cm^2. This result corresponds to the value $\eta = 0.5$, which is the limiting value from the perturbation-theory standpoint. The change in the shape of the curves and the shift of the maximum at high values of the intensity I cannot, of course, be explained by the theory above, which is based on a method of iterations in the field E. The maximum increase in the energy and the dependence on the intensity (Fig. 6) agree well with the theory within the range of applicability of the theory, i.e., at $I \lesssim 10^{15}$ W/cm^2.

It is interesting to note the "phase focusing" of the electrons: The phase distribution of the electrons as they pass through the region near $z = 0$ differs from the uniform distribution specified at $t = -\infty$. The electrons become bunched in phase near $\varphi|_{z=0} = -\pi/2$, i.e., specifically under those conditions which maximize the acceleration effect in first order in the field. This phenomenon is reflected by Fig. 7.

We have not studied such characteristics of the accel-

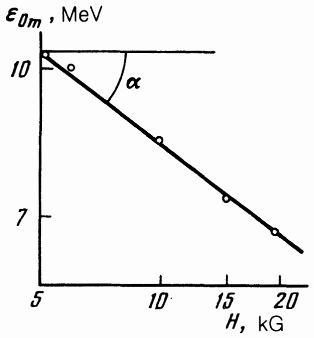

FIG. 4. Optimum initial energy of the electrons versus the magnetic field for $I = 10^{15}$ W/cm^2 and $\tan\alpha = 0.4$.

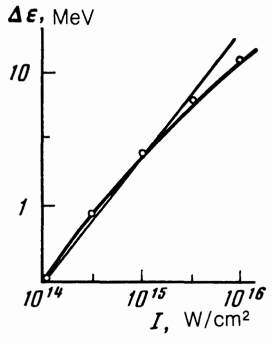

FIG. 6. Maximum increase in energy versus the light intensity for $H = 5$ kHz.

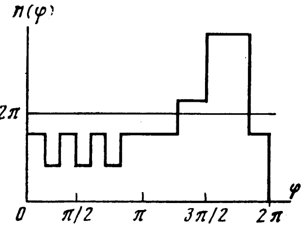

FIG. 7. Phase distribution of electrons as they pass the point $z = 0$ ($H = 5$ kHz, $I = 10^{15}$ W/cm^2). The horizontal line shows a uniform phase distribution.

eration as the average energy buildup and the phase distribution of the electrons for the case in which the initial electron beam is not monoenergetic.

In considering the possibility of cyclic acceleration, we need to recall that the electron energy spectrum is determined by the preceding acceleration processes, so it is only in the first stage of the acceleration that we can talk in terms of a monoenergetic electron beam, as we have done in the present paper. Figure 8 shows the evolution of the electron energy spectrum in this case. We see that many of the electrons are in a high-energy tail; in other words, the acceleration process may be though of as a transformation of the original spectrum consisting of a shift $\Delta\varepsilon'$ up the energy scale, a broadening Γ, and a loss of some electrons, δ. For the conditions in Fig. 4 we would have $\overline{\Delta\varepsilon^{(2)}} = 2.2$ MeV, $\Delta\varepsilon' = 6$ MeV, $\Gamma = 2$ MeV, and $\delta = 67\%$.

3. LONGITUDINAL MAGNETIC FIELD

In all the schemes discussed above the static magnetic field $H_0 \| y$ was directed perpendicular to the direction of the basic motion of the electrons and to the axis of the laser beam (i.e., the z axis). What are the possibilities for electron acceleration in other field configurations? Let us consider the case of a longitudinal magnetic field $H_0 \| z$. In this case the role of the magnetic field and the very formulation of the electron acceleration problem are quite different from those in the case of a transverse field. The transverse magnetic field serves as a factor which limits the duration of and spatially localizes the interaction between the electron and the laser field. A longitudinal magnetic field cannot play such a role: It curves the trajectories of the electrons, twisting them into a spiral around the z axis. Under these conditions, the nonuniformity of the focused laser light emerges as a major factor. It is more or less clear at the outset that the acceleration

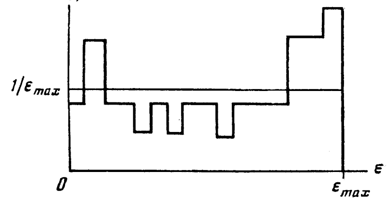

FIG. 8. Electron energy spectrum after the electrons have passed through the focus ($H_0 = 5$ kHz, $I = 10^{15}$ W/cm^2). The horizontal line shows a uniform energy spectrum.

effect (if it exists at all) should be optimized when the length of the caustic is roughly equal to one turn of the spiraling electron orbit in the magnetic field, i.e., under the condition $L \sim v_{z0}/\Omega$, where (as before) Ω is the cyclotron frequency, and v_{z0} is the longitudinal component of the initial velocity of an electron. Under the condition $L \gg v_{z0}/\Omega$ the electromagnetic wave is approximately a plane wave, and we know that the energy of an electron does not change in a plane wave in a uniform magnetic field in the absence of a cyclotron resonance.

Under the condition $L \ll v_{z0}/\Omega$ the region in which the electron interacts with the laser field is itself very small, and the change in the electron energy is correspondingly small.

If an electron spiraling in a magnetic field H_0 is to experience approximately the maximum effect of the laser field, the transverse dimension of the caustic must be quite large: $d \gtrsim R \sim v_{\perp 0}/\Omega$, $v_{\perp 0}$ is the component of the initial velocity of the electron in the plane perpendicular to the magnetic field.

Finally, over a wide range of the values of γ, for all feasible magnetic fields H_0, the conditions $\gamma^2\lambda \ll R \ll L \sim v_{z0}/\Omega \sim \Omega^{-1}$ hold. These conditions state that there is no cyclotron resonance; a cyclotron resonance would occur in a longitudinal field at $\omega\gamma^{-2} \sim \Omega$.

The calculations of the change in the energy of an electron as it passes through the focus are largely similar to the calculations above. We will accordingly skip the details, focus on certain basic points, and write the final result. As in the preceding section of this paper, the interaction of the electron with the magnetic field should be taken into account exactly in the zeroth approximation. The solution of the corresponding equations of motion is analogous to (1) and can be written

$$\begin{aligned} z^{(0)} &= z_0 + v_{z0}t, \\ x^{(0)} &= x_0 + (v_{x0}/\Omega)\sin\Omega t + (v_{y0}/\Omega)(\cos\Omega t - 1), \\ y^{(0)} &= y_0 + (v_{y0}/\Omega)\sin\Omega t - (v_{x0}/\Omega)\cos\Omega t, \end{aligned} \tag{30}$$

where x_0, y_0, z_0 and v_{x0}, v_{y0}, v_{z0} are, respectively, the coordinates and velocities of the electron at the time $t = 0$. The interaction with the laser field is dealt with by perturbation theory. To first order in E we find an exponentially small value of $\Delta\varepsilon^{(1)}(t \to \infty)$ by virtue of the condition $\gamma^2\lambda \ll L$.

In second order, after taking an average over the field phase φ_0 [see (3)], we find

$$\Delta\varepsilon^{(2)} = -\frac{e^2\Omega v_{\perp 0}^2}{2\varepsilon_0\omega^2(1-v_{z0})^2}\int_{-\infty}^{\infty} dt\, E_0^2(z_0+t)\sin 2(\Omega t+\alpha), \tag{31}$$

where $v_{\perp 0} = (v_{x0}^2 + v_{y0}^2)^{1/2}$ and $\alpha = \mathrm{arctg}(v_{x0}/v_{y0})$. Using the explicit expression for $E_0(z)$ in (19), we can rewrite (31) as

$$\Delta\varepsilon^{(2)} = -\frac{2^{-1/2}\pi^{1/2}}{2\varepsilon_0\omega^2}e^2E_0^2\frac{v_{\perp 0}^2}{v_{z0}(1-v_{z0})^2}\Omega L \times \exp\left[-\frac{\Omega^2L^2}{2v_{z0}^2}\right]\sin 2(\Omega z_0+\alpha). \tag{32}$$

Acceleration occurs if $\sin 2(\Omega z_0 + \alpha) < 0$. The acceleration effect reaches a maximum under the condition $\Omega z_0 + \alpha = -\pi/4 + \pi n$, $n = 0, \pm 1, \pm 2$. As a function of

L, the quantity $\Delta\varepsilon^{(2)}$ reaches a maximum at $\Omega L = v_{z0}$; this value corresponds to the condition $L \approx R / v_{\perp 0}$.

The energy increment $\Delta\varepsilon^{(2)}$ given by (32) depends strongly on the relation between $v_{\perp 0}$ and v_{z0}, i.e., on the relation between the transverse and longitudinal components of the initial energy of the electron. The restriction $d \gtrsim R = v_{\perp 0}/\Omega$ should be kept in mind here; because of this restriction, $v_{\perp 0}$ cannot be too large. Assuming $R \sim d = (\lambda L)^{1/2}$ and using $L \approx v_{z0}/\Omega$, we find the maximum permissible value to be $v_{\perp 0} = (2\pi\Omega/\omega)^{1/2}$. The corresponding maximum increment in the energy of an electron is

$$\Delta\varepsilon_{max}^{(2)} = \frac{(2\pi)^{3/2} e^2 E_0^2 \gamma^4 \Omega}{\varepsilon_0 \exp(1/2)\,\omega}, \quad \left(\frac{\Delta\varepsilon^{(2)}}{\varepsilon}\right)_{max} = \left(\frac{eE\lambda}{m}\right)^2 \frac{\gamma^2\Omega}{\omega}. \tag{33}$$

Comparison with (25) with $\gamma \sim \gamma_{cr}$ shows that the efficiency of the acceleration in a longitudinal magnetic field is comparable to that of acceleration in a transverse field only at extremely large values of γ: $\gamma \sim (\omega/\Omega)^{1/2} \sim 10^2$ (in the case $\omega = 2\cdot 10^{14}\ s^{-1}$, $\Omega = 2\cdot 10^{10}\ s^{-1}$). Note, however, that in the case of acceleration in a longitudinal field the efficiency of the process increases with increasing γ: $\Delta\varepsilon^2, \Delta\varepsilon/\varepsilon \sim \gamma$. This feature could potentially make the case of a longitudinal magnetic field attractive for a study of acceleration in the case of high energies.

4. CONCLUSION

We have studied certain schemes for the acceleration of electrons in a focused laser field in the presence of a static transverse or static longitudinal magnetic field. In evaluating these schemes from the standpoint of efficiency and feasibility, we should give preference to the case of the transverse uniform magnetic field under conditions such that the regime of quasisteady crossed fields is realized (Subsections 2.4 and 2.5). According to the results found, the acceleration efficiency can be quite high in this case: The electron energy can increase severalfold during a single passage through the focus. This possibility of arranging conditions such that the oscillatory nature of the laser field is masked by the static magnetic field, and the acceleration occurs in quasisteady crossed fields, is a new physical result, which has not been described in the literature previously, to the best of our knowledge. In order to make practical use of this result, it will be necessary to analyze the possibilities for cyclic acceleration. In order to maintain optimum acceleration conditions $[\gamma = \gamma_{cr}(z)]$ it will be necessary to reduce the magnetic field in synchronization with the increasing value of γ, specifically, in accordance with $H_0 \propto \gamma^{-2}$. The primary limitation of this acceleration mechanism is the condition $t_0 < L$ which we adopted above. Whether this is a fundamental limitation and whether the acceleration mechanism in quasisteady crossed fields can be achieved for $t_0 \gg L$ are questions which go beyond the scope of the present paper.

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Translated by Dave Parsons