Induced transition radiation

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Transition radiation produced when a stream of charged plasma crosses the interface between two media is considered. The induced emission is described by the Fresnel equations, i.e., wave reflection and refraction coefficients in substances consisting of dielectric media and a moving plasma. It is shown that the electromagnetic energy flux away from the interface is not equal to the incident flux. The energy excess is due to collective transition radiation and is drawn from the energy of plasma motion. The transition-radiation energy of one charged particle can be obtained from the Fresnel equations by using the Einstein relation that connects the probabilities of the induced and spontaneous emission processes.

Transition radiation of a charged particle is produced when the particle crosses the interface between two media having different dielectric properties. This radiation was predicted theoretically in 1944 by Ginzburg and Frank.[1] Let us consider a simple example in which a particle having a charge q and a mass M large enough so we can neglect the change of its velocity is normally incident on a boundary separating two isotropic dielectrics with dielectric constants ε_1 and ε_2. The electromagnetic waves propagate from the interface of the media both in the same direction as the particle and in the opposite direction. The energy radiated backwards $W^-(\omega, \theta^{(0)})$ in a frequency interval $d\omega$ and in a unit solid angle can be represented in the form given by Frank:[2]

$$W^-(\omega,\theta^{(0)}) = \frac{q^2\beta^2}{4\pi^2 c}\varepsilon_1^{1/2}\left|\frac{\sin\theta^{(0)}}{1-\mathbf{n}^{(0)}\beta}+\frac{r_1\sin\theta^{(0)}}{1-\mathbf{n}^{(1)}\beta}-\frac{t_1\sin\theta^{(2)}}{1-\mathbf{n}^{(2)}\beta}\right|^2 \qquad (1)$$

in Eq. (1), $\beta = \mathbf{v}/c$ is the dimensionless velocity of the particle and $\mathbf{n} = \mathbf{k}c/\omega$ is the dimensionless wave vector. We shall find it convenient to work here and below in a gauge in which $\mu = 1$.

Equation (1) for the transition-radiation energy has a simple physical interpretation: it is seen from (1) that the radiation consists of three parts: 1) a backward wave emitted by the particle prior to crossing the boundary, whose wave vector is $\mathbf{n}^{(0)}$ and makes an angle $\theta^{(0)}$ with the normal direction; 2) a wave with a vector $\mathbf{n}^{(1)}$ directed towards the boundary and then reflected from it, $\theta^{(1)} = \pi - \theta^{(0)}$; 3) a backward wave emitted by the particle after crossing the boundary; its wave vector is equal to $\mathbf{n}^{(2)}$ and makes an angle $\theta^{(2)}$ with the normal (see the figure). The coefficients r_1 and t_1 are the Fresnel coefficients that relate the amplitudes of the reflected and refracted waves with the amplitude of the wave incident on the boundary from the media ε_1 and ε_2, respectively:[4]

$$r_1 = \frac{\varepsilon_2^{1/2}\cos\theta^{(0)}-\varepsilon_1^{1/2}\cos\theta^{(2)}}{\varepsilon_2^{1/2}\cos\theta^{(0)}+\varepsilon_1^{1/2}\cos\theta^{(2)}}, \qquad \varepsilon_1^{1/2}\sin\theta^{(0)}=\varepsilon_2^{1/2}\sin\theta^{(2)},$$

$$t_1 = \frac{2\varepsilon_1^{1/2}\cos\theta^{(0)}}{\varepsilon_2^{1/2}\cos\theta^{(0)}+\varepsilon_1^{1/2}\cos\theta^{(2)}}. \qquad (2)$$

Expressions (2) correspond to Fresnel equations for waves so polarized that the electric-field vector lies in the plane of incidence. These are precisely the waves emitted by the particle as it crosses the interface. In this case the electric field of the wave performs work on the electric current carried by the charged particle.

We see thus that the picture for the onset of transition radiation is similar to the usual picture arising when an electromagnetic wave is incident on the interface of two media, i.e., to the problem of wave refraction and reflection. We shall show now that the Fresnel equations not only have a direct bearing on transition radiation, as seen from Eq. 1, but fully account for it. To this end we consider not the spontaneous emission of one particle, but the onset of transition radiation from a continuous wide beam of identically charged particles that cross the interface between two dielectrics. Clearly, if the particle density is high enough the interference between individual waves emitted by different charged particles will lead to total vanishing of the spontaneous emission (the only exceptions are waves propagating strictly perpendicularly to the interface, but from (1) it is seen that no such waves are excited, since in that case we have $\sin\theta = 0$). Now let an electromagnetic wave be incident from the left on an interface through which a beam of charged particles passes. This will produce also a reflected wave and refracted waves propagating from the boundary. Their amplitudes will be proportional to the amplitude of the incident wave. The connection between the wave amplitudes will be given by the Fresnel equations for media containing not only dielectrics but also a plasma stream. If it turns out that the energy flux of the waves leaving the boundary exceeds the incident flux, then the additional radiation, which is proportional to the incident radiation, can be interpreted as transition radiation induced by the incident electromagnetic waves. In other words, it can be stated that the induced transition radiation is described by the Fresnel equations.

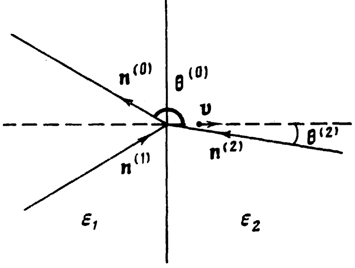

FIG. 1.

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The flux of charged particles constitutes a stationary plasma moving in the direction of the interface and filling uniformly both the left and right half-spaces. To describe only the transition radiation it is necessary to assume that the plasma particles do not interact with one another and with the material of the dielectric. In addition, the particle velocity does not change with time. The electromagnetic properties of the plasma are described by its dielectric tensor $\delta\varepsilon^p$

$$\delta\varepsilon_{\alpha\beta}{}^p=\frac{4\pi q^2}{\omega^2}\int\left(v_\alpha\frac{\partial f}{\partial p_\beta}+\frac{v_\alpha v_\beta}{\omega-\mathbf{kv}}\mathbf{k}\frac{\partial f}{\partial \mathbf{p}}\right)d\mathbf{p}. \tag{3}$$

Here $f(\mathbf{p})$ is the distribution function of the charged particles with respect to the momenta $\mathbf{p}$.

Thus, two different media are located on the left and the right of the interface, and their dielectric constants are $\varepsilon^{(1)}=\varepsilon_1+\delta\hat{\varepsilon}^p$ and $\varepsilon^{(2)}=\varepsilon_2+\delta\hat{\varepsilon}^p$, respectively. We examine first which normal electromagnetic waves exist in a medium with a dielectric constant $\varepsilon_{\alpha\beta}=\varepsilon\delta_{\alpha\beta}+\delta\varepsilon_{\alpha\beta}{}^p$. The solution of the dispersion equation for a monoenergetic momentum distribution of the plasma particles

$$f(\mathbf{p})=n_b\delta(\mathbf{p}-\mathbf{p}_{z_0})$$

yields dispersion equations for the electromagnetic waves. The oscillations so polarized that the electric vector of the wave is perpendicular to the direction of motion $(\mathbf{E}\cdot\beta)=0$ are actually not affected by the motion of the particles, and their refractive index is

$$n^2=\varepsilon-\omega_p{}^2/\gamma\omega^2. \tag{4}$$

In expression (4) $\omega_p{}^2=4\pi n_b q^2/M$ is the square of the plasma frequency, and $\gamma=(1+p_{z_0}{}^2/M^2c^2)^{1/2}$ is the Lorentz factor. Such waves are not excited by a moving flux and are therefore not considered here.

The dispersion equation for the waves whose electric vector has a component along the velocity of motion is

$$n^2=\left[\varepsilon^2-\Omega_p{}^2-2\Omega_p{}^2\frac{(\varepsilon-1)}{(1-\mathbf{n}\beta)}-\Omega_p{}^2\frac{1-\Omega_p{}^2-(\varepsilon-\Omega_p{}^2)\beta^2}{(1-\mathbf{n}\beta)^2}\right]\Bigg/\left[\varepsilon-\Omega_p{}^2\frac{1-\varepsilon\beta^2}{(1-\mathbf{n}\beta)^2}\right], \tag{5}$$

$$\Omega_p{}^2=\omega_p{}^2/\gamma\omega^2.$$

In the general case Eq. (5) is of fourth order in n and has four roots. In the two cases $\Omega_p{}^2\ll\varepsilon$ and $\beta\ll1$ the solutions of (5) can be expressed compactly:

$$n^2=\varepsilon-\Omega_p{}^2\left[1+\beta^2\sin^2\theta\frac{(\varepsilon-1)}{(1-\mathbf{n}\beta)^2}\right], \tag{6}$$

$$(1-\mathbf{n}\beta)^2=\frac{\Omega_p{}^2}{\varepsilon}\frac{(1-\varepsilon\beta^2)(1-\beta^2\cos^2\theta)}{(1-\varepsilon\beta^2\cos^2\theta)}. \tag{7}$$

Equation (6) describes an ordinary electromagnetic wave whose refractive index as $n_b\to0$ is equal to $\varepsilon^{1/2}$. [In the case $\beta\ll1$ it is necessary to substitute $n=(\varepsilon-\Omega_p{}^2)^{1/2}$ in the right-hand side of (6).] Relation (7) describes two "drift" waves propagating together with the plasma—a "fast" one moving forward relative to the plasma, and a "slow" one moving backward and having a negative energy. In the limit $\beta\to0$ expression (7) describes electrostatic oscillations.

Let an electromagnetic wave with refractive index given by Eq. (6) be incident from the left on the interface of two media $\varepsilon^{(1)}$ and $\varepsilon^{(2)}$. Part of the wave is reflected from the boundary, and part is refracted and transformed into drift oscillations. The amplitudes of the reflected and refracted waves are obtained from the boundary conditions—they are the same Fresnel coefficients, but with a dielectric constant for anisotropic dielectrics given by the expression

$$\varepsilon_{\alpha\beta}^{(1,2)}=\varepsilon_{1,2}\delta_{\alpha\beta}-\Omega_p{}^2\left[\delta_{\alpha\beta}+\frac{n_\alpha\beta_\beta+n_\beta\beta_\alpha}{1-\mathbf{n}\beta}+\frac{(n^2-1)\beta_\alpha\beta_\beta}{(1-\mathbf{n}\beta)^2}\right]. \tag{8}$$

In what manner is the transition radiation contained in the Fresnel coefficients? If it turns out that the electromagnetic-energy flux departing from the boundary is not equal to the electromagnetic energy in the incident wave, then the difference is in fact the induced transition radiation. Indeed, the energy-flux conservation condition does not follow from the boundary conditions. If we have on the left of the boundary two waves, incident $\mathbf{E}^{(0)}\exp(-i\omega t+i\mathbf{k}^{(0)}\mathbf{r})$ and reflected $\mathbf{E}^{(1)}\exp(-i\omega t+i\mathbf{k}^{(1)}\mathbf{r})$, and on the right we have a refracted wave $\mathbf{E}^{(2)}\exp(-i\omega t+i\mathbf{k}^{(2)}\mathbf{r})$ and two drift waves $\mathbf{E}^{\pm}\exp(-i\omega t\pm i\mathbf{k}^{\pm}\mathbf{r})$, then the conditions on the amplitudes $\mathbf{E}^{(0)}$, $\mathbf{E}^{(1)}$, $\mathbf{E}^{(2)}$, $\mathbf{E}^{\pm}$ and on the wave vectors $\mathbf{k}^{(0)}$, $\mathbf{k}^{(1)}$, $\mathbf{k}^{(2)}$, $\mathbf{k}^{\pm}$ (here $\mathbf{k}^{(0)}$, $\mathbf{k}^{(1)}$, and $\mathbf{k}^{(2)}$ have directions opposite to those shown in the figure) are the following:

a) $\mathbf{k}_\perp=\text{const}=\mathbf{k}_\perp{}^{(0)}$—the transverse wave vectors for all five waves are equal. This condition follows from the homogeneity of all the quantities in the direction along the interface;

b) $\mathbf{E}_\perp{}^{(0)}+\mathbf{E}_\perp{}^{(1)}=\mathbf{E}_\perp{}^{(2)}+\mathbf{E}_\perp{}^{+}+\mathbf{E}_\perp$—the tangential component of the electric field is continuous. This follows from the Maxwell equation rot $\mathbf{E}=-(1/c)\partial\mathbf{B}/\partial t$;

c) the third condition

$$\left[\varepsilon_{z\beta}^{(1)}+n_{\perp\alpha}\left(\frac{\widetilde{\varepsilon_{\alpha\beta}(n_z)}}{n_z}\right)\right](E_\beta^{(0)}+E_\beta^{(1)})$$

$$=\left[\varepsilon_{z\beta}^{(2)}+n_{\perp\alpha}\left(\frac{\widetilde{\varepsilon_{\alpha\beta}(n_z)}}{n_z}\right)\right](E_\beta^{(2)}+E_\beta{}^{+}+E_\beta{}^{-})$$

is analogous to the continuity condition of the normal component of the induction vector $\mathbf{D}$ (the z axis is directed along the normal to the interface) when account is taken of the spatial dispersion. It follows from the equation div $\mathbf{D}=0$; $(\tilde{\ })$ denotes an operator acting on the coordinate dependence (along the z axis) of the wave vector $\mathbf{E}(\mathbf{r})$. In particular,

$$\tilde{n}_z\propto-i\frac{\partial}{\partial z},\quad \tilde{n}_z{}^{-1}\propto i\int dz.$$

For media without spatial dispersion $(d\varepsilon/d\mathbf{k}=0)$ the second term in the square brackets makes no contribution, since

$$\tilde{n}_z{}^{-1}\mathbf{E}\propto i\int_{-\delta}^{\delta}\mathbf{E}(z)\,dz\to0\quad\text{at}\quad\delta\to0.$$

For a plasma medium with a dielectric constant given by (8), the boundary condition (c) is also equivalent to the condition of continuity of the normal component of the induction vector $D_z{}^{(0)}+D_z{}^{(1)}=D_z{}^{(2)}+D_z{}^{+}+D_z{}^{-}$, since expression (8) can be recast in a form in which n_z is contained only in the denominators of $1/(1-n_z\beta)$ and $1/(1-n_z\beta)^2$. The boundary condition (c) for our problem is thus equivalent to the condition

$$\varepsilon_{z\beta}^{(1)}(E_\beta^{(0)}+E_\beta^{(1)})=\varepsilon_{z\beta}^{(2)}(E_\beta^{(2)}+E_\beta^{+}+E_\beta^{-}).$$

Two more conditions follow from the continuity of the electric charge and the normal component of the current carried by the moving particles:

d) $\rho^{(0)}+\rho^{(1)}=\rho^{(2)}+\rho^{+}+\rho^{-},\qquad \rho=\frac{1}{\omega}k_\alpha\sigma_{\alpha\beta}{}^{p}E_\beta;$

e) $j_z^{(0)}+j_z^{(1)}=j_z^{(2)}+j_z^{+}+j_z^{-},\qquad j_z=\sigma_{z\beta}{}^{p}E_\beta.$

The boundary conditions a)–e) suffice to determine the amplitudes and wave vectors of the reflected and refracted waves. We are interested now, however, in the electromagnetic energy flux away from the boundary and normalized to the incident-flux density:

$$K=(-S_z^{(1)}+S_z^{(2)}+S_z^{+}+S_z^{-})/S_z^{(0)}.$$

Calculations for the case $\Omega_p{}^2\ll\varepsilon_{1,2}$ yield

$$\begin{aligned}K=1&+\beta^2\frac{\omega_p{}^2}{\gamma\omega^2}(1-\beta^2)\sin^2\theta^{(0)}\left\{\frac{1}{(1-\mathbf{n}^{(0)}\beta)^3}+\frac{t_1(\varepsilon_2/\varepsilon_1)^{1/2}}{2(1-\mathbf{n}^{(0)}\beta)^2}\right.\\&\times\left[1-2\frac{\varepsilon_1t_1}{\varepsilon_2(t_1-t_2)}\right]+\frac{1}{(1-\mathbf{n}^{(0)}\beta)}\left[r_1-2\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^{1/2}\frac{t_1^2}{t_1-t_2}\right]\\&+\frac{r_1^2}{(1-\mathbf{n}^{(1)}\beta)^3}+\frac{r_1t_1(\varepsilon_2/\varepsilon_1)^{1/2}}{2(1-\mathbf{n}^{(1)}\beta)^2}\left[1-2\frac{\varepsilon_1t_1}{\varepsilon_2(t_1+t_2)}\right]\\&+\frac{1}{(1-\mathbf{n}^{(1)}\beta)}\left[1-2\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^{1/2}\frac{t_1^2}{t_1+t_2}\right]\\&+\frac{(\varepsilon_1/\varepsilon_2)t_1^2}{(1-\mathbf{n}^{(2)}\beta)^3}+\frac{t_1(\varepsilon_1/\varepsilon_2)}{2(1-\mathbf{n}^{(2)}\beta)^2}\\&\times\left[t_1+2\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^{1/2}t_2\left(\frac{1}{t_1-t_2}-\frac{r_1}{t_1+t_2}\right)\right]\\&\left.+\frac{2(\varepsilon_1/\varepsilon_2)^{1/2}t_1t_2}{(1-\mathbf{n}^{(2)}\beta)}\left(\frac{1}{t_1-t_2}-\frac{r_1}{t_1+t_2}\right)\right\}.\end{aligned}\tag{9}$$

For $\beta\ll1$ we have

$$\begin{aligned}K&=1+\frac{3}{2}\frac{\omega_p{}^2}{\omega^2}\beta^2\sin^2\theta^{(0)}\left[1+\bar r_1-\left(\frac{\varepsilon^{(1)}}{\varepsilon^{(2)}}\right)^{1/2}\bar t_1\right]^2\\&=1+6\frac{\omega_p{}^2}{\omega^2}\beta^2\sin^2\theta^{(0)}\cos^2\theta^{(0)}\\&\times\frac{(\varepsilon_2-\varepsilon_1)^2}{\varepsilon^{(1)}[(\varepsilon^{(2)})^{1/2}\cos\theta^{(0)}+(\varepsilon^{(1)})^{1/2}\cos\theta^{(2)}]^2}.\end{aligned}\tag{10}$$

In Eq. (9) t_2 is the transmission coefficient of the wave from medium ε_1 to ε_2, and is similar to t_1 with the substitutions $\varepsilon_1\to\varepsilon_2$, $\varepsilon_2\to\varepsilon_1$, $\theta^{(0)}\to\theta^{(2)}$, $\theta^{(2)}\to\theta^{(0)}$. The quantities $\bar r_1$ and $\bar t_1$ in (10) are the same Fresnel coefficients, but for the media $\varepsilon^{(1,2)}=\varepsilon_{1,2}-\omega_p{}^2/\omega^2$. We see, first, that the difference $K-1$ is proportional to the density n of the moving flux. Second, for a quiescent plasma ($\beta=0$) the waves are not amplified by incidence on the interface. Finally, if the media on the right and left are identical ($\varepsilon_1=\varepsilon_2$), the electromagnetic-energy flux is conserved ($r_1=\bar r_1=0$; $t_1=t_2=\bar t_1=\bar t_2=1$). The energy-flux nonconservation is in fact the manifestation of induced transition radiation.[3] It differs from the spontaneous radiation in that near the Cherenkov resonance (at $1-\mathbf{n}\beta\approx0$) the amplification effect is larger, proportional to $(1-\mathbf{n}\beta)^{-3}$.

The cause of the energy-flux nonconservation after reflection and refraction of the electromagnetic wave is that the moving plasma is an anisotropic medium with spatial dispersion. The energy flux density $\mathbf{S}$ is given for such a medium by the expression[4]

$$\mathbf{S}=\frac{c}{4\pi}\left\{[\mathbf{EB}]-\frac{1}{2}\frac{\partial\varepsilon_{\alpha\beta}}{\partial\mathbf{n}}E_\alpha E_\beta\right\}.\tag{11}$$

The first term in (11) conserves the normal flux component, since $[\mathbf{EB}]_z=[\mathbf{E}_\perp\mathbf{B}_\perp]$. From the conservation conditions on the tangential components of the electric field [condition (b)] and of the magnetic induction [condition (c)] follows a conservation condition on the energy flux in ordinary media without spatial dispersion. For plasma-like media, however, the dielectric tensor (8) depends on the wave vector, so that the contribution of the second term in (11) is not equal to zero. For a moving plasma this leads to nonconservation of the electromagnetic-energy flux. Clearly, the additional energy is drawn from (or delivered to) the energy of translational motion of the charged particles.

As seen from (9) and (10), wave amplification on reflection from the boundary is a maximum for the frequencies $\omega\sim\omega_p/\gamma^{1/2}$. At higher frequencies the gain decreases like ω^{-2}, and at lower ones the expression for the square of the refractive index (6) becomes negative and electromagnetic waves no longer propagate.

At what plasma densities n_b does the induced transition radiation begin to prevail over the spontaneous radiation? Since transition radiation is formed in a region near the interface between the media, at a distance from the interface on the order of the radiation-formation length[5]

$$l_f\approx c/(\omega-\mathbf{kv})\approx\gamma^2c/\omega$$

($n\approx1$) it is necessary that the average distance $n_b^{-1/3}$ between the plasma particles be less than l_f, i.e.,

$$n_b>\left(\frac{\omega}{c}\right)^3\gamma^{-6}.\tag{12}$$

For optical frequencies $\lambda\sim5000$ Å the plasma density $n_b>10^{13}\gamma^{-6}$ cm^{-3} is not so high. For plasma frequencies, when the amplification is a maximum, the collective mechanism almost always works, since inequality (12) yields for $\omega\approx\omega_p/\gamma^{1/2}$

$$n_b<\left(\frac{Mc^2}{q^2}\right)^3\gamma^{18},$$

which means $n_b<6\cdot10^{37}\gamma^{18}$ cm^{-3} for electrons and $n_b<4\cdot10^{48}\gamma^{18}$ cm^{-3} for protons.

We know that the induced and spontaneous emission processes are related. The connection between the probabilities of the induced and spontaneous emissions is given by the Einstein formula. We show now that this connection is valid also for transition radiation. To this end it is necessary to go to the limit of low plasma density $n_b\to0$ and use the Fresnel equations that describe the induced process. It is necessary here to reformulate the problem. Consider not the usual triplet of waves, incident $\mathbf{n}^{(0)}$, reflected $\mathbf{n}^{(1)}$, and refracted $\mathbf{n}^{(2)}$ (and also the two drift waves), but a triplet reversed in time: two incident waves $\mathbf{n}^{(1)}$ and $\mathbf{n}^{(2)}$ and one refracted $\mathbf{n}^{(0)}$ (see the figure) since we are interested in a wave that moves away backwards from the interface, for comparison with expression (1). The wave amplitudes $E^{(1)}$ and $E^{(2)}$ must be related in such a way that no wave moving away from the interface be produced in the medium ε_2. It is necessary also to smear out somewhat the flux-particle distribution function $f(\mathbf{p})$, $\Delta p \ll p_{z_0}$. In this case the expression for the gain K takes the form

$$K=1+\frac{2\pi q^2 c}{\omega^2 \varepsilon_1^{1/2}\cos\theta^{(0)}}\int d\mathbf{p}\frac{\partial f}{\partial \mathbf{p}}\mathbf{n}^{(0)}\beta^3 \times\left[\frac{\sin\theta^{(0)}}{1-\mathbf{n}^{(0)}\beta}+\frac{r_1\sin\theta^{(0)}}{1-\mathbf{n}^{(1)}\beta}-\frac{t_1\sin\theta^{(2)}}{1-\mathbf{n}^{(2)}\beta}\right]^2. \quad (13)$$

We introduce the optical thickness τ corresponding to K:

$$K=e^{-\tau}=1-\tau.$$

Here we have $|\tau|\ll 1$, since $\tau \propto n_b$.

The Einstein relations for a homogeneous medium connect the reabsorption coefficient, which describes the wave enhancement by the induced process, with the spontaneous-emission power $\mathscr{P}(\omega,\theta)$ of one particle:[6,7]

$$\mu_r=-\frac{(2\pi)^3 c}{\varepsilon\omega^2}\int \mathscr{P}(\omega,\theta)\,\mathbf{n}\frac{\partial f}{\partial \mathbf{p}}\,d\mathbf{p}. \quad (14)$$

In the case of transition radiation the amplification of the electromagnetic waves takes place in a narrow region near the interface. The width of the amplification region is of the order of the length of radiation formation. We therefore integrate (14) in the vicinity of the boundary

$$\tau=\int \mu_r\,dl=-\frac{(2\pi)^3c^2}{\varepsilon\omega^2\cos\theta}\int W(\omega,\theta)\,\mathbf{n}\frac{\partial f}{\partial \mathbf{p}}\beta\,d\mathbf{p}. \quad (15)$$

In the derivation of (15) we have recognized that $dl\cos\theta = v_z dt$, and $\int\mathscr{P}(\omega,\theta)dt = W(\omega,\theta)$ is the radiation energy. The transition radiation is produced in two media, ε_1 and ε_2, it is therefore necessary to introduce in (15) the sum over the three waves:

$$\tau=-\frac{(2\pi)^3c^2}{\omega^2}\sum_{i=0,1,2}\int\frac{W^{(i)}}{\varepsilon_i\cos\theta^{(i)}}\beta\mathbf{n}\frac{\partial f}{\partial \mathbf{p}}\,d\mathbf{p}. \quad (16)$$

Since $W(\omega,\theta)$ is the spectral energy of the radiation per unit solid angle, the backward radiation energy W^- is equal to

$$W^-=\sum_{i=0,1,2}W^{(i)}\frac{d\Omega^{(i)}}{d\Omega}, \quad (17)$$

where $\Omega^{(i)}$ is the solid angle for different waves. From the refraction and reflection law it follows that

$$d\Omega^{(2)}/d\Omega=\varepsilon_1\cos\theta^{(0)}/\varepsilon_2\cos\theta^{(2)},\quad d\Omega^{(1)}=d\Omega^{(0)}=d\Omega.$$

Expressions (16) and (17) are thus a generalization of the Einstein relation (14) for transition radiation. Comparing (16) and (17) with (13) we obtain a value of W^- that coincides exactly with expression (1) known from the theory of transition radiation.

We have shown that collective induced transition radiation is described by Fresnel formulas for media containing a moving plasma. An expression for the spontaneous-emission energy can be obtained from the Fresnel equations with allowance for the Einstein relation between spontaneous and induced emission. We have here an analogy between Cherenkov and transition radiation. Induced Cherenkov radiation is described by inverse Landau damping, and induced transition radiation by the Fresnel coefficients.

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