Characteristics of the Poiseuille flow in plane capillaries in an acoustically excited liquid crystal

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An investigation was made of the deformation of the director field in a homeotropically oriented nematic liquid crystal, induced by an acoustically excited oscillatory flow. The deformation of the liquid crystal director was investigated near the surface (in a layer of thickness less than 0.2 μm) by a specially developed method of modulation ellipsometry. Attenuation of a low-frequency acoustic wave in a thin layer of a nematic liquid crystal was observed for the first time. The attenuation was explained within the framework of the Ericksen–Leslie theory assuming that the nonlinear liquid crystal is compressible. The energy with which the liquid crystal is bound to the substrate was estimated for the first time from the dynamic experimental data.

1. INTRODUCTION

This investigation was carried out as a follow-up to our earlier experimental studies of the flexoelectric (or piezoelectric) effect which appeared as a result of acoustic excitation of liquid crystals in plane capillaries. The effect was first observed in the texture of the ferroelectric mesophase[1] and was then investigated in a synthetic hybrid structure formed from a nematic liquid crystal (NLC).[2] In the latter case the measurements were carried out on layers of thickness from a few to tens of microns at temperatures including the point when the phase transition to an isotropic liquid occurs. Observation of sharp temperature maxima of the flexoelectric effect in very thin cells made it desirable to investigate the nature of the distribution of the director across the thickness (coordinate z) and along the length (coordinate x) of the NLC layer. It was found that there were no published data on this topic (see, for example, Ref. 3).

Our aim was to determine the distributions, across the thickness and length of a layer, of deviations of the NLC director induced by low-frequency acoustic excitation. In these experiments we used the homeotropic orientation of an NLC on both surfaces (we recall that the flexoelectric effect was investigated for the hybrid orientation). The homeotropic orientation was selected to simplify theoretical interpretation of the experiments, since in the case of small deformations (strains) the equations for the velocity of motion of a liquid and for the orientation of the director become decoupled, as shown below. In this case the results of optical observations of the flow of liquid crystals can be utilized fully in a study of the flow of isotropic liquids. To the best of our knowledge, no experimental studies have yet been made of the flow of simple liquids in plane capillaries of micron thickness because of the difficulties encountered in the observation of flow in an optically isotropic medium. For liquid crystals the sensitivity increases by several orders of magnitude because of the strong optical anisotropy ($\Delta n = n_{\parallel} - n_{\perp} = 0.15$). This is why we were able to observe for the first time the attenuation of the amplitude of an oscillatory flow along a capillary, which could be only due to the influence of the compressibility of the liquid.

2. EXPERIMENTAL METHOD

We determined the following characteristics: a) the phase delay of a light beam transmitted normally across a homeotropically oriented NLC layer, measured as a function of the acoustic pressure and of the position along the capillary plane; b) angle of total internal reflection (TIR) at the interface between a homeotropically oriented NLC and TF-10 glass, accompanied by measurements of the ellipticity of light reflected by this interface at angles exceeding the TIR angle in the presence of an acoustic pressure. We used the apparatus shown schematically in Figs. 1a and 1b.

In all cases we found that periodic variation of the pressure along the x axis gave rise to Poiseuille flow of the liquid crystal with a displacement ξ_x and at a velocity $\dot{\xi}_x$, depending on z, and—as shown below—also on x. This flow altered in turn the orientation of the NLC director characterized by an angle $\theta(z,x)$ (Fig. 2).

The phase delay was measured using relatively thick (8 mm) optically polished glass plates of 56×24 mm dimensions with Teflon spacers parallel to the long sides, which were used to set the NLC capillary thickness between 5 and 60 μm. It was important to ensure that the capillary was horizontal, because otherwise we would expect a slow motion of the liquid that would distort the results of our measurements.

An acoustic pressure was applied to such a capillary through a special tube connected through an acoustic guide to a 25GD-10 vibrator excited from an ac oscillator, which provided a voltage between 0 and 4 V. The acoustic guide (about 1.5 m long) and the oscillator frequency ($f = 57$ Hz) were selected to ensure the maximum acoustic pressure established with the aid of an acoustic resonance ($P = 12$ kPa, measured using a ROBOTRON-00017 noise meter). The other end of the capillary was open. The gap was made plane parallel after filling with a liquid crystal; this was done by using pressure screws and monitoring equal-thickness interference fringes in the light of a luminescent lamp. The cell was placed on a stage which could be moved horizontally by a micrometer screw linked to a potentiometer for automatic recording of the position of the light beam along the x axis. A

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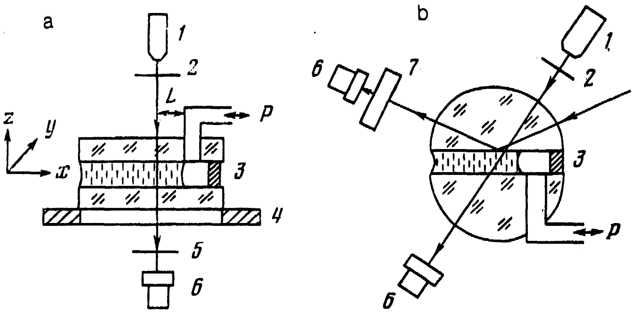

FIG. 1. Schematic diagram of the apparatus used in determination of the characteristics of nematic liquid crystals: a) determination of the amplitude of the phase delay as a function of an acoustic pressure P and the position L_x along a plane capillary; b) measurement of the angle of total internal reflection (a Senarmont compensator employed at angles higher than the total-internal-reflection made it possible to study the change in the ellipticity of the reflected light). 1) He–Ne laser of the LGN-207B type; 2) polarizer; 3) liquid-crystal cell; 4) movable stage connected to a potentiometer and used to record automatically the position of the light beam along the x axis; 5) analyzer; 6) photodetector; 7) Senarmont compensator.

beam from a helium–neon laser of the LGN-207B type passed through a system of rotating prisms and across a polarizer and an analyzer; the polarization vector of light incident on the sample made an angle of 45° with the direction of flow (x) axis.

The amplitude of the induced phase delay $\Delta\Phi_m$, needed in the subsequent determination of the amplitude of the angle of tilt of the director $\theta_m(z)$, was calculated using the expression

$$I=I_0 \sin^2 (\Delta\Phi_m/2), \tag{1}$$

where I is the effective intensity of the transmitted light at twice the modulation frequency ($2f$) obtained by placing the cell between crossed polarizers; I_0 is the effective value of the light intensity transmitted subject to a phase delay amounting to $\pi/2$ (in measurements on thin cells when the condition $\Delta\Phi_m < \pi/2$, was always satisfied, the quantity I_0 was taken to be the effective intensity of light modulated by a chopper when the analyzer was oriented parallel to the polarizer).

The TIR angle was measured by the usual method employing semicylindrical lenses made of TF-10 glass and characterized by a refractive index $N = 1.806$. The change in the optical transmission of a sample as the cell was rotated about its horizontal axis was recorded. Acoustic excitation was induced exactly as described above. At the highest pressure the change in the optical transmission was determined for several specific angles of incidence of light near the TIR angle of the extraordinary ray. The polarization vector of the incident beam was then within the plane of incidence.

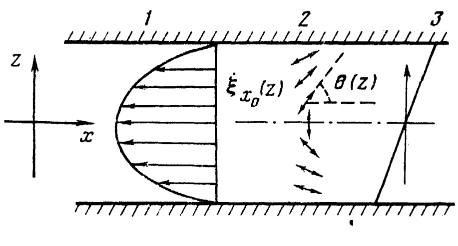

FIG. 2. Schematic representation of a parabolic profile of the velocities (1) and the distribution of the director across the thickness of a liquid crystal in the weak-coupling case (2, 3).

The angle of orientation of the director at the boundary with a wall θ^s was calculated from the TIR angle φ_e using the familiar expression[4]

$$\sin \varphi_e=(n_\parallel^2 \cos^2 \theta^s+n_\perp^2 \sin^2 \theta^s)^{1/2}/N, \tag{2}$$

where $n_\parallel$ and $n_\perp$ are the refractive indices of the investigated NLC along the directions parallel and perpendicular to the director, respectively, and the angle θ^s is measured from the normal to the layer.

In the absence of acoustic excitation we found $\theta^s = 0$ and $\varphi_e = \sin^{-1}(n_\parallel/N)$. Acoustic excitation rotated the director by an effective angle $\theta^s_{\rm eff}$, which reduced φ_e by an amount $\delta\varphi_e$. Equation (2) allowed us to calculate the value of $\theta_{\rm eff}$ from the new TIR angle $\theta_m{}^s = 2^{1/2}\theta^s_{\rm eff}$, and then to calculate the amplitude $\varphi_e - \delta\varphi_e$, needed in a comparison with the phase delay data.

An analysis of the experimental results showed that the angle θ^s_m measured by this method was equal to the average angle of the director within a layer of thickness $\sigma^{-1} = (K_{33}/\omega\gamma_1)^{1/2} \approx 0.7$ μm (here K_{33} and γ_1 are the elastic constant and viscosity of the NLC, and ω is the angular frequency of the acoustic excitation).

We investigated thinner surface layers ($d \approx 0.2$ μm) by modulation ellipsometry involving determination of the degree of ellipticity of light reflected from an interface between the TF-10 glass and a vibrating NLC layer (Fig. 1b). Independent published data on the refractive index and the measured values of the amplitudes of the azimuthal angles enabled us to calculate the angle θ^s_m from the expression

$$2\delta_m(2\omega)=A\,\frac{n_\parallel^2-n_\perp^2}{4n_\parallel}(\theta_m{}^s)^2, \tag{3}$$

where $\delta_m(2\omega)$ is the measured amplitude of the azimuthal angle (the azimuth of the incident wave was 45°),

$$A=\frac{\partial(\delta_p-\delta_s)}{\partial n_{\rm eff}}\bigg|_{n_{\rm eff}=n_\parallel}, \qquad n_{\rm eff}=(n_\perp^2 \sin^2 \theta_m{}^s+n_\parallel^2 \cos^2 \theta_m{}^s)^{1/2},$$

and ω is the frequency of the acoustic excitation; $\delta_p - \delta_s$ is the relative difference between the phases of the p and s waves, and the value of A for the case when the director oscillates in the place of incidence is

$$A=\frac{\partial}{\partial n_{\rm eff}}\,2\,\mathrm{arctg}\left[N\frac{(N^2\sin^2\varphi-n_{\rm eff}^2)^{1/2}}{n_\parallel n_\perp\cos\varphi}\right]_{n_{\rm eff}=n_\parallel}$$

(φ is the angle of incidence exceeding the TIR angle).

All the measurements were carried out on the nematic liquid crystal p-amyl-p′-cyanobiphenyl (5CB) exhibiting a transition from the nematic (N) to the isotropic (I) phase at $T_{NI} = 34.5$ °C. The refractive indices $n_\parallel$ and $n_\perp$ were taken from Ref. 5.

The homeotropic orientation was set by a special treatment of the glass half-cylinders with chromium distearyl chloride.

3. EXPERIMENTAL RESULTS

Figure 3 shows the phase delay of a light beam transmitted by cells of different thickness as functions of the ampli-

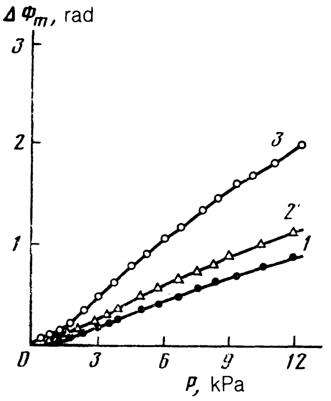

FIG. 3. Amplitudes of the phase delay of a light beam transmitted by cells of different thickness: 1) $d = 9\,\mu$m; 2) 17 μm; 3) 32 μm.

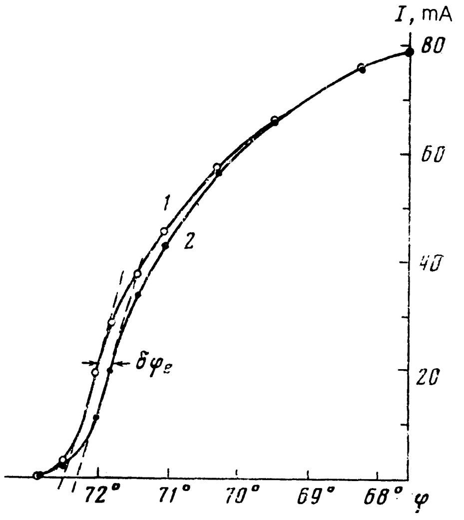

FIG. 4. Dependence of the intensity of light transmitted by a nematic crystal layer of thickness 32 μm on the angle of incidence close to the total-internal-reflection angle: 1) in the absence of acoustic excitation $\varphi_e = 72.5°$; 2) at the maximum possible acoustic excitation ($P = 12$ kPa), $\varphi_e - \delta_\varphi = 72.3°$.

tude of the acoustic excitation. The light beam passed near the zone of excitation of shear flow ($x \approx 0$).

The dependences $\Delta\Phi_m(p_{ac})$ obtained at low acoustic pressures could be approximated, as expected, by a parabolic law. The amplitudes of the phase delay readily yielded the average value of the birefringence amplitude across the thickness:

$$\Delta\Phi_m = \frac{2\pi d}{\lambda}\langle (n_e - n_0)_m \rangle_z$$

$$= \frac{2\pi d}{\lambda}\left\langle \left[\frac{n_\parallel n_\perp}{(n_\perp^2 \sin^2\theta_m(z) + n_\parallel^2 \cos^2\theta_m(z))^{1/2}} - n_\perp \right] \right\rangle \quad (4)$$

where in the case of small tilts of the angle from the homeotropic orientation we should have

$$(n_e - n_0)_m(z) = \frac{n_\perp (n_\parallel^2 - n_\perp^2)}{2n_\parallel^2}\theta_m^2(z).$$

For a known (e.g., linear) law of variation of θ_m with the thickness z, the absolute values of the angle θ_m can be readily calculated using the data on $\Delta\Phi_m$. However, the linear dependence $\theta(z)$ is expected in the case of an infinitesimally small binding energy W_0 between the NLC and the solid wall of the capillary. For finite values of W_0, we required the special calculation reported below.

In accordance with the experimental geometry shown in Fig. 1, the maximum tilt of the director should occur in the region of the maximum velocity gradient $\partial\dot{\xi}/\partial z$, i.e., near the walls. However, this was prevented by the nonzero energy of coupling of the director to the wall. Therefore, it was very important to determine independently the tilt angle θ_m^s near the wall. The method used to measure the TIR angle at a wavelength λ (Refs. 4 and 6) allowed us to estimate the angle θ_m^s in a surface layer of depth of order $\sigma^{-1} \approx 0.7\,\mu$m.

Figure 4 shows how the intensity of the light transmitted by an NLC layer of thickness 32 μm depended on the angle of incidence near the TIR angle of reflection $\varphi_e = 72.5°$. Curve 1 was obtained in the absence of any acoustic excitation, whereas curve 2 was obtained in the presence of the maximum (in our experiments) acoustic pressure, as mentioned above. There was a clear change in the angle φ_e by an amount $\delta\varphi_e$. The difference $\delta\varphi_e$ could be used to calculate the amplitude of oscillations of the angle θ_m^s, which for a cell of thickness 32 μm was 8.4°. A similar experiment, carried out on a thinner cell ($d = 17\,\mu$m) gave $\theta_m^s = 4°$.

These values of the angles near the surface were less than those calculated from the phase delay ignoring the finite energy of binding of an NLC to walls [deduced from the linear profile of the angle $\theta(z)$]. However, they were considerably greater than the surface angles we deduced using modulation ellipsometry, because in the latter case a thinner layer near the walls was probed (Fig. 5). Matching these values with the aid of a rigorous calculation should make it possible to estimate the energy of coupling of the director to the walls directly under dynamic conditions (at the frequency ω), which had not been done before.

Figure 6 shows the dependence of the phase delay on the coordinate x along the plane of a layer measured from the edge, where acoustic modulation took place. The curves are plotted for layers of different thickness and are normalized to the phase delay in the limit $x \to 0$. The results presented in Fig. 6 were at first sight unexpected: periodic deformation of the director became damped out with distance and the damping effect was stronger for thinner layers. The characteristic distances in the x direction over which the phase delay decreased by a factor of e^2 were 10, 30, and 150 mm for

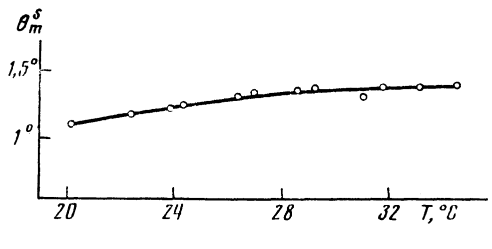

FIG. 5. Temperature dependence of the angle θ_m^s obtained from modulation ellipsometry experiments (homeotropic orientation, $d = 32\,\mu$m).

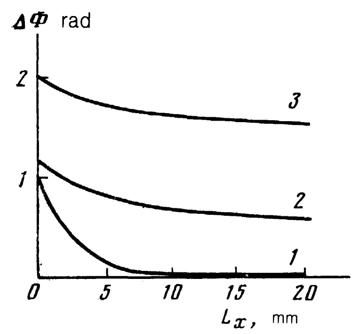

FIG. 6. Dependence of the phase delay on the coordinate x along the plane of the liquid crystal layer: 1) $d = 9\ \mu$m; 2) 17 μm; 3) 32 μm.

the cells 9, 17, and 32 μm thick. This effect could be explained allowing for the compressibility of the liquid, which was important in the adopted flow geometry when the ratio of the length of the capillary to its thickness was of order 10^3. It has been known for a long time that this effect occurs in gases.[7] Figure 7 shows the coordinate dependences of θ_m^s determined by modulation ellipsometry, which were in good qualitative and quantitative agreement with the experimental data on the phase delay reduction.

4. THEORETICAL ANALYSIS

We set ourselves two theoretical tasks: a) to account for the observed attenuation of the acoustically induced deformation of the director along the coordinate x; b) to calculate the profile of the angle $\theta(z)$ across the thickness of the layer in the limit $x \to 0$, which would be in agreement with all the available experimental data, and to find the energy with which a homeotropically oriented NLC binds to a solid wall. The latter was of considerable importance because the $\theta(z)$ profile obtained under acoustic excitation conditions differed considerably from that, for example, associated with the Fréedericksz transition, when the maximum tilt of the director occurred in the center of the layer. Clearly, our experiments should be more sensitive to the properties of the surface and the values of the binding energy W_0 obtained by the shear flow method should supplement significantly our information on this topic.

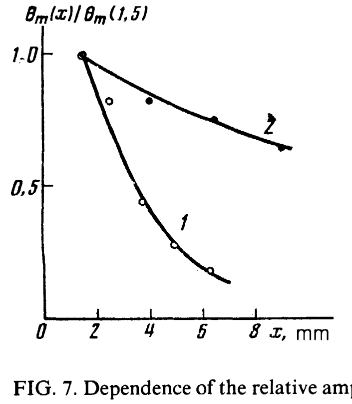

FIG. 7. Dependence of the relative amplitude of oscillations of the director along the plane of the layer at the boundary on the coordinate x: 1) $d = 9\ \mu$m; 2) 32 μm.

The linearized system of equations describing the radiation shown in Fig. 2 is as follows:

$$\rho\ddot{\xi}_x = -P_{,x} + 0{,}5(\alpha_4+\alpha_5)\dot{\xi}_{x,z,z} + 0{,}5\alpha_2(\dot{\theta} - \dot{\xi}_{x,z})_{,z},$$
$$\gamma_1\dot{\theta} + \alpha_2\dot{\xi}_{x,z} + K_{33}\theta_{,z,z} = 0, \qquad (5)$$
$$P = -\rho c^2 \langle \xi_{x,x} \rangle_z.$$

Here, ρ is the density of the liquid crystal and c is the velocity of sound, α_2, α_4, and α_5 are the Leslie viscosity coefficients, ξ and P are the displacement and pressure in the layer, and the symbol $U_{,x}$ denotes the partial derivative of U with respect to x. The system (5) is derived on the assumption that $\theta^2 \ll 1$, $\xi_{x,y} = 0$, $\theta_{,y} = 0$, $\xi_y = 0$, $P(x,y,z) = P(x)$, $\xi_{x,z} \gg \xi_{x,x}$, $\theta_{,z} \gg \theta_{,x}$, $\xi_x \gg \xi_z$.

In the range of frequencies ω where the wavelength of a viscous wave in an NLC is much greater and the wavelength of an orientational wave is much less than the layer thickness, i.e., when

$$10(\alpha_4+\alpha_5)/\rho d^2 \gg \omega \gg K_{33}/\gamma_1 d^2, \qquad (6)$$

the above system of equations simplifies greatly and the problem can be subdivided into the hydrodynamic problem of the flow of a viscous compressible liquid in a plane capillary, and the orientational problem of the distortion of the director field in a given hydrodynamic stream. The first reduces to solving the system of equations

$$P_{,x} = 0{,}5(\alpha_4+\alpha_5)\dot{\xi}_{x,z,z},$$
$$P = -\rho c^2 \langle \xi_{x,x} \rangle_z \qquad (7)$$

subject to the boundary conditions

$$\dot{\xi}_x|_{z=\pm d/2} = 0,$$
$$P|_{x=0} = P_0 \exp(j\omega t), \qquad (8)$$
$$P|_{x=\infty} = 0.$$

The final expressions for the pressure and velocity in such a layer are[7]

$$P = P_0 \frac{1-\exp[2k(z-x)]}{1-\exp(2kz)} \exp(j\omega t + kx),$$
$$\dot{\xi}_x = \frac{P_0 d^2 k}{4(\alpha_4+\alpha_5)}\left(\frac{4z^2}{d^2} - 1\right)\frac{1+\exp[2k(z-x)]}{1-\exp(2kz)} \exp(j\omega t + kx), \qquad (9)$$

where $k = [6\omega(\alpha_4+\alpha_5)/j\rho c^2 d^2]^{1/2}$.

The real part of the wave vector k is

$$\mathrm{Re}\, k = -\left[\frac{3\omega(\alpha_4+\alpha_5)}{\rho c^2 d^2}\right]^{1/2} < 0,$$

so that the oscillatory motion of the liquid is damped out along the layer. A reduction in the velocity by a factor of e occurs over a distance

$$\delta = d\left[\frac{\rho c^2}{3\omega(\alpha_4+\alpha_5)}\right]^{1/2}. \qquad (10)$$

Under experimental conditions we found $\delta \approx (3\text{–}5)10^3 d$ for the attenuation of the velocity and angle, and we also found that the phase delay decreased in this distance by a factor e^2.

The second task can be reduced to solving the equations

$$\gamma_1\dot{\theta}+\alpha_2\xi_{x,z}+K_{33}\theta_{,z,z}=0,$$

$$\xi_x=\xi_{x_0}(x)\,(1-4z^2/d^2)\exp(j\omega t) \qquad (11)$$

subject to the boundary conditions

$$(W_0\theta=\pm K_{33}\theta_{,z})\,|_{z=\pm d/2}. \qquad (12)$$

The solution to this problem can be represented in the form

$$\theta=\left(\frac{\alpha_2}{\gamma_1}\frac{8z\xi_{x_0}}{d^2}+A\,\mathrm{sh}(\sigma z)+B\,\mathrm{ch}(\sigma z)\right)\exp(j\omega t),$$

where $\sigma=(-j\omega\gamma_1/K_{33})^{1/2}$, so that we finally obtain

$$\theta_m=\frac{\alpha_2}{\gamma_1}\frac{4\xi_{x_0}}{d}\left(\frac{2z}{d}-\frac{\mathrm{sh}(\sigma z)}{\mathrm{sh}(\sigma d/2)}\,\frac{1+2K_{33}/dW_0}{1+\sigma K_{33}/W_0\,\mathrm{cth}(\sigma d/2)}\right).$$

Since in the investigated range of frequencies we have $|\sigma d|\gg 1$, it follows that

$$\theta_m\approx\frac{\alpha_2}{\gamma_1}\frac{4\xi_{x_0}}{d}\left(\frac{2z}{d}-\frac{\mathrm{sh}(\sigma z)}{\mathrm{sh}(\sigma z/2)}\,\frac{1+2K_{33}/dW_0}{1-\sigma K_{33}/W_0}\right). \qquad (13)$$

The ratio of the angle θ_m, deduced by extrapolation at the boundary using the approximation of the linear dependence $\theta(z)$, to the value of the angle θ_m^s measured from TIR or by modulation ellipsometry, is given by

$$\frac{\theta_m^{extr}}{\theta_m^{s}}\approx\left|1-\frac{W_0}{\sigma K_{33}}\right|\approx\left[1+\frac{2^{1/2}W_0}{(\omega\gamma_1K_{33})^{1/2}}+\frac{W_0^2}{\omega\gamma_1K_{33}}\right]^{1/2}. \qquad (14)$$

We determine the phase delay of a layer deformed in this way and identify the conditions under which a correction associated with the finite coupling or binding energy W_0 is important:

$$\Delta\Phi_m=\frac{2\pi}{\lambda}\frac{n_\perp(n_\parallel^2-n_\perp^2)}{2n_\parallel^2}\int_{-d/2}^{+d/2}\theta_m^2(z)\,dz.$$

If $\theta_m(z)$ is replaced with Eq. (13), we obtain

$$\Delta\Phi_m\approx\frac{2\pi}{\lambda}\frac{n_\perp(n_\parallel^2-n_\perp^2)}{2n_\parallel^2}\times\left(\frac{\alpha_2}{\gamma_1}\frac{4\xi_{x_0}}{d}\right)^2\frac{1}{3}d\left(1+\frac{12}{\sigma d}\,\frac{1+2K_{33}/dW_0}{1-\sigma K_{33}/W_0}\right), \qquad (15)$$

i.e., if $|\sigma d|\gg 10$, we can calculate the phase delay ignoring the influence of W_0, as was done in the calculation of $\Delta\Phi_m$.

5. COMPARISON OF THE THEORETICAL AND EXPERIMENTAL RESULTS

5.1. Characteristic attenuation lengths of acoustically induced strains

Equation (1) allows us to calculate the characteristic attenuation lengths of the oscillatory flow in 5CB. Assuming that $\alpha_4+\alpha_5=0.16$ Pa, $\rho=10^3$ kg/m^3, and $c=1.5\times10^3$ m/s (Refs. 5, 8, and 9), we found that the attenuation lengths were 36, 70, and 130 mm for cells of thicknesses 9, 17, and 32 μm, respectively. The experimental values of these lengths were 10, 30, and 150 mm (Fig. 6), i.e., our theory agreed with these results.

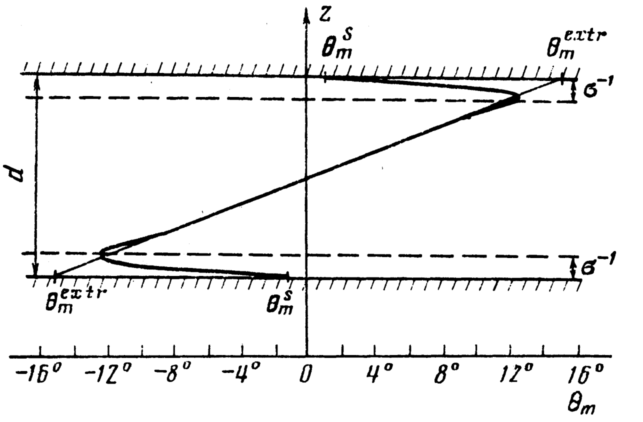

FIG. 8. Typical distribution of the amplitude of the angle of tilt of the director for a binding energy $W_0=2\times10^{-1}$ erg/cm^2 and a layer of thickness $d=32$ μm. Here, $\theta_m^s=\pm1°$, $\theta_m^{extr}=\pm15°$, $\sigma^{-1}=0.7$ μm.

We then determined the temperature dependence of the attenuation length $L(T)$ for a cell of thickness 9 μm. It was found that L depended weakly on T even near the phase transition to an iostropic liquid. This was to be expected because Eq. (10) does not contain any parameters critically dependent on temperature, so that the attenuation given above is typical of any (including isotropic) liquid.

Equations (9) and (13) allow us to calculate θ_m^{max} in the limit $x\to0$ if we know the pressure and the capillary length $l0$. Our estimates indicate that for $P=12$ kPa and $x_0=25$ mm, we should have θ_m^{max} amounting approximately to 10° and 20° for cells of thicknesses 17 and 32 μm, whereas the experimental values are 10° and 15°, respectively.

5.2. Profile of the tilt angle of the director across the layer thickness

Equation (14) allows us to calculate the angles found from the phase delay and modulation ellipsometry by variation of just one independent parameter, which is the energy with which a homeotropically oriented NLC binds to the solid surface of a glass coated with chromium distearyl chloride.

The typical distribution of $\theta_m(z)$ for a cell of thickness 32 μm is shown in Fig. 8. The value of the energy W_0 needed to ensure the agreement between the theory and experiments is 2×10^{-1} erg/cm^2, which is in agreement with the latest experimental data[10] where larger values of the binding energy are obtained from measurements involving the Fréedericksz transition. It should be stressed once again that this gives the "dynamic" binding energy, representing the deformation of the director at a frequency of about 50 Hz.

6. CONCLUSIONS

Experimental and theoretical evidence was obtained of the need to allow for the compressibility of a liquid in a study of its Poiseuille flow in a thin capillary. Calculations were made of the flow profile and of the angles of tilt of the director across the thickness of a layer allowing for the finite energy of its binding to a wall.

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