Quantum theory of nonlinear propagation of Schrödinger solitons: squeezed states and sub-Poisson statistics

A. V. Belinskii and A. S. Chirkin

M. V. Lomonosov Moscow State University

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A quantum theory of propagation of solitons in a nonlinear medium is developed based on the nonlinear Schrödinger equation for the operators of the positive- and negative-frequency parts of the field. A derivation of this equation is given in the functional integration representation, which is convenient for the analysis of the dynamics of quantum field fluctuations. The propagation of a fundamental soliton, initially in a coherent state, is analyzed. It is shown that the statistics of the soliton photons in the nonlinear medium does not change. At the same time the fluctuations of one of the quadrature components of the field may be suppressed under certain conditions. Interference between the soliton and the coherent radiation alters the photon statistics of the resulting field. The conditions under which optimal suppression of the fluctuations of the number of photons is ensured and their sub-Poisson statistics is reached are elucidated and analyzed.

1. INTRODUCTIOI.

In recent times questions of production and utilization of quantum states of electromagnetic fields (squeezed states and states with sub-Poisson photon statistics) have attracted the increased attention of investigators: such fields are connected with prospects of solving a whole series of fundamental and applied problems of physics. Among these are attempts to detect gravitational waves and the perturbations of space due to the Earth's rotation predicted by the general theory of relativity, as well as the possibilities for substantial enhancement of limiting characteristics of various devices in interferometry, spectroscopy, optical coupling and information processing.[1–4]

For quantum fields in the squeezed state the reduction in the fluctuation of one of the quadrature components below the level of fluctuations corresponding to the coherent or vacuum state is characteristic. For fields with sub-Poisson photon statistics the dispersion of the photon number fluctuations is smaller than the average value.

In published papers, as a rule, treat the production of optical fields with markedly quantum statistics for the continuum radiation. The formation of quantum squeezed states of light pulses was analyzed only in Refs. 5–11, with Refs. 6–11 dealing with optical solitons. However the authors of Refs. 6–8 linearized the nonlinear operator equations describing the propagation of the solitons with respect to the quantum fluctuations. Their conclusions are therefore applicable only in the initial states of propagation. The results of Refs. 9–11 are not subject to this limitation and have more general character.

References 6, 7, 9–11 discuss the suppression of quantum fluctuations of Schrödinger solitons, formed in nonresonant cubic-nonlinear media,[12] while Ref. 8 discusses the suppression of resonant exciton solitons (the so-called 2π) pulses). It is shown in Ref. 8 that in the exciton region of the spectrum of semiconductors for pulses of 1–3 ps duration the cubic nonlinearity is $x^{(3)} \sim 10^{-6}$–$10^{-4.5}$ esu. Such a large nonlinearity permits the formation of solitons in relatively weak fields for maximum intensities on the order of a few milliwatts. On the other hand for Schrödinger solitons the power requirements are so far about one watt.

In this paper we develop the quantum theory of Schrödinger solitons. We first give the derivation of the operator nonlinear Schrödinger equation (NSE) in functional-integral form. This representation of the NSE makes possible the study of the evolution of quantum fluctuations outside the framework of the parametric approximation. We establish the regularity of the suppression of the quantum fluctuations of one of the quadrature components of the soliton in the process of its nonlinear propagation. We show that the interference between the soliton in the squeezed state and coherent pulse with special modulation of the envelope or the phase permits substantial suppression of the fluctuations of the number of soliton photons.

We note that the analysis of the quantum effects in the propagation of solitons in Ref. 9 and in the present paper is carried out in the Heisenberg picture. In Refs. 10 and 11 the operator NSE is solved in the Schrödinger picture; further in Ref. 11 it is also shown that quantum solitons are formed in the squeezed state. However the photon statistics was not investigated in Refs. 9–11.

2. THE NONLINEAR SCHRÖDINGER EQUATION AND ITS FUNCTIONAL-INTEGRAL FORM

We represent the electric field of the soliton, entering ($z = 0$) an optically nonlinear medium, in the form

$$\hat{E}_0(t) = E_0^{(+)}(t)e^{-i\omega_0 t} + E_0^{(-)}(t)e^{i\omega_0 t}, \tag{1}$$

where $E_0^{(+)}(t)$ is the slowly varying in time operator of the positive-frequency part of the field in the Heisenberg picture, $E_0^{(-)}(t)$ is the corresponding hermitian conjugate operator, and ω_0 is the carrier frequency.

In turn $E_0^{(+)}(t)$ can be written as

$$E_0^{(+)}(t) = \int_{-\omega_0}^{+\infty} \epsilon(\omega_0+\Omega)a(\Omega)e^{-i\Omega t}\,d\Omega = \epsilon(\omega_0)\int_{-\infty}^{+\infty} a(\Omega)e^{-i\Omega t}\,d\Omega, \tag{2}$$

where $a(\Omega)$ is the photon annihilation operator at frequency $\omega_0 + \Omega$, which satisfies the usual boson commutation relations, and $\epsilon(\omega_0 + \Omega)$ is the coefficient that determines the contribution of the various modes. The right side of (2) is

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valid for a narrow band spectrum.

Let the soliton modes be in coherent states. We denote the eigenstate and eigenvalue of the operator $a(\Omega)$ by $|\alpha(\Omega)\rangle$ and $\alpha(\Omega)$:

$$a(\Omega)|\alpha(\Omega)\rangle=\alpha(\Omega)|\alpha(\Omega)\rangle, \tag{3}$$

with

$$\alpha(\Omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\alpha(t)e^{i\Omega t}\,dt, \tag{4}$$

where $\alpha(t)$ is in general a complex function.

We write the coherent state of the soliton in the form

$$|\{\alpha\}\rangle=\prod_{\Omega}|\alpha(\Omega)\rangle, \tag{5}$$

where we assume for simplicity that Ω takes on a discrete set of values. It is not hard to verify that

$$\langle E_0^{(+)}(t)\rangle=\langle\{\alpha\}|E_0^{(+)}(t)|\{\alpha\}\rangle=\epsilon(\omega_0)\alpha(t)=A_0(t). \tag{6}$$

For the problem under consideration we have

$$A_0(t)=A_0\,\mathrm{sech}(t/\tau_c), \tag{7}$$

where τ_c is the soliton width.

In a nonlinear medium we write the field in the form

$$\hat{E}(t,z)=E^{(+)}(t,z)\exp[-i(\omega_0t-k_0z)]+E^{(-)}(t,z)\times\exp[i(\omega_0t-k_0z)], \tag{8}$$

where $k_0=c/\sqrt{\varepsilon_0}$ is the wave number and ε_0 is the linear part of the dielectric permeability of the medium. The nonliner induction operator will be taken in the normally-ordered form $\hat{D}_{nl}=\varepsilon_{nl}{:}\hat{E}^3(t,z){:}$, where ε_{nl} is the nonlinear part of the dielectric permeability.

Assuming that the operators $E^{(+)}(t,z)$ and $E^{(-)}(t,z)$ are slowly varying functions of their arguments we find in the second approximation of dispersion theory the following NSE for $E^{(+)}=E^{(+)}(t,z)$ (see also Refs. 6, 10, and 11):

$$i\frac{\partial E^{(+)}}{\partial z}+\frac{1}{2}g\frac{\partial^2E^{(+)}}{\partial t^2}+\frac{\mathrm{K}_0\mathrm{E}_{nl}}{2\varepsilon_0}E^{(-)}E^{(+)}E^{(+)}=0. \tag{9}$$

Here the z axis is parallel to the direction of propagation, t is the time in the comoving system of coordinates: $t\to t-z/u, u=(\partial k_0/\partial\omega_0)^{-1}$ is the group velocity, and the parameter $g=\partial^2k_0/\partial\omega_0^2$ characterizes the dispersion of the group velocity. The derivation of (9) is similar to the derivation of the classical NSE see, for example, Ref. 12).

We pass to the dimensionless variables

$$\zeta=\frac{z}{L_p},\quad \tau=\frac{t}{\tau_c},\quad \psi(\tau,\zeta)=\frac{E^{(+)}(\tau,\zeta)}{|A_0|},$$
$$\beta=\frac{L_p}{\mathrm{L}_{nl}},\quad L_p=\frac{\tau_c^2}{g},\quad L_{nl}=\frac{2\varepsilon_0}{\mathrm{K}_0\mathrm{E}_{nl}|A_0|^2}. \tag{10}$$

In the new notation the NSE (9) takes the form

$$i\frac{\partial\psi}{\partial\zeta}+\frac{1}{2}\frac{\partial^2\psi}{\partial\tau^2}+\beta\psi^+\psi^2=0 \tag{11}$$

with boundary condition

$$\psi(\tau,\zeta=0)=\psi_0(\tau). \tag{12}$$

We look for a solution of Eq. (11) in the form

$$\psi(\tau,\zeta)=\int_{-\infty}^{+\infty}G(\tau,\eta;\zeta)\psi_0(\eta)\,d\eta, \tag{13}$$

where the operator kernel $G(\tau,\eta;\zeta)$ satisfies the equation

$$\frac{\partial G}{\partial\zeta}-\frac{i}{2}\frac{\partial^2G}{\partial\tau^2}-i\beta\psi^+\psi G=0 \tag{14}$$

with boundary condition

$$G(\tau,\eta;0)=\hat{I}\delta(\tau-\eta), \tag{15}$$

and $\hat{I}$ is the unit operator.

Let us divide the segment $[0,\zeta]$ into N small intervals $\Delta\zeta=\zeta_{j+1}-\zeta_j\,(j=0,1,2,...,N-1);\ \zeta_0=0, \zeta_N=\zeta$. The evolution of the operator G on the $(j+1)-st$ interval $\Delta\zeta$ with the nonlinearity neglected $(\beta=0)$ is described by the expression

$$G_l(\tau_{j+1},\eta;\Delta\zeta)=\left(-\frac{i}{2\pi\Delta\zeta}\right)^{1/2}\times\int_{-\infty}^{+\infty}\exp\left\{\frac{i(\tau_{j+1}-\tau_j)^2}{2\Delta\zeta}\right\}G_l(\tau_j,\eta;\Delta\zeta)\,d\tau_j. \tag{16}$$

In view of the condition (15) we have at the end of the first interval

$$G_l(\tau_1,\eta;\Delta\zeta)=(i2\pi\Delta\zeta)^{-1/2}\exp[i(\tau_1-\eta)^2/2\Delta\zeta]\hat{I}. \tag{17}$$

The effect of nonlinearity $(\beta\neq0)$ on the small segment $\Delta\zeta$ may be taken into account with the help of the infinitesimal operator, Ref. 13. Further

$$G(\tau_{j+1},\eta;\Delta\zeta)=\exp[i\beta\psi^+(\tau_j,\zeta_j)\psi(\tau_j,\zeta_j)\Delta\zeta]\times G_l(\tau_{j+1},\eta;\Delta\zeta). \tag{18}$$

The same expression can be obtained by solving (14) by the perturbations method.

Repeatedly using (16)–(18), we obtain for the "Green's function" of the entire segment $[0,\zeta]$

$$G(\tau,\eta;\zeta)=(i2\pi\Delta\zeta)^{-N/2}\hat{I}\int_{-\infty}^{+\infty}\dots\int_{-\infty}^{+\infty}\prod_{j=0}^{N-1}\exp\Big[\frac{i(\tau_{j+1}-\tau_j)^2}{2\Delta\zeta}+i\beta\psi^+(\tau_j,\zeta_j)\psi(\tau_j,\zeta_j)\Delta\zeta\Big]d\tau_j, \tag{19}$$

with $\eta=\tau_0$, $\tau=\tau_N$, and $\zeta_j<\zeta_{j+1}$. For $\Delta\zeta\to0$ but $\Delta\zeta N=\zeta$ the expression (19) may be written as

$$G(\tau,\eta;\zeta)=\hat{I}\int T\exp[i\mathscr{L}(\tau(x),x)]D\tau, \tag{20}$$

$$\mathscr{L}(\tau(x),x)=\int_0^{\zeta}S(\tau(x),x)\,dx, \tag{21}$$

$$S(\tau(x),x)={}^1\!/_2\dot{\tau}^2(x)+\beta\psi^+(\tau(x),x)\psi(\tau(x),x),$$

$$\dot{\tau}(x)=\frac{d\tau(x)}{dx},\quad D\tau=\prod_{x=0}^{\zeta}\frac{d\tau(x)}{(i2\pi dx)^{N/2}}.$$

Here T is the time-ordering operator:[1)]

$$T\exp[i\mathscr{L}(\tau(x),x)]=\prod_{x=0}^{\zeta}\exp[iS(\tau(x),x)dx].$$

In the case under consideration the role of time is played by the coordinate x. The differential $D\tau$ denotes integration over an infinity of trajectories connecting the points with the coordinates $(\eta, 0)$ and (τ, ζ).

Equations (13), (20), and (21) constitute a formal solution of the NSE (11) in functional-integral form. Replacing the operators in it by c-number functions leads to the corresponding classical equations.[13,14] We also note that in the limiting case of a nondispersive nonlinear medium ($g = 0$) these equations lead to the familiar result

$$\psi(\tau, \zeta) = \exp[i\beta\psi_0^+(\tau)\psi_0(\tau)\zeta]\psi_0(\tau).$$

The functional-integral form of NSE obtained here is convenient for the analysis of the behavior of the soliton fluctuations in the temporal representation.

3. SOLITON SQUEEZED STATES

We shall show that a soliton propagating in a nonlinear medium, which starts from a coherent state, ends up in a squeezed state. We write the operator $\psi_0(\tau)$ in the form

$$\psi_0(\tau) = a_0(\tau) + \xi_0(\tau), \qquad (22)$$

where the function $a_0(\tau)$ is classical, $\xi_0(\tau)$ is an operator, and

$$a_0(\tau) = \langle\psi_0(\tau)\rangle = \operatorname{sech}\tau, \qquad (23a)$$

$$\langle\xi_0(\tau)\rangle = 0. \qquad (23b)$$

We solve (13), (20) by iteration. To the propagation regime of a single soliton coresponds the value $\beta = 1$ of the nonlinear parameter. If the quantum fluctuations of the soliton are ignored $[\xi_0(\tau) \equiv 0]$ then the problem reduces to the well-studied classical one.[12] In that case the NSE with the boundary condition (23a) has the fundamental solution

$$a(\tau, \zeta) = \exp(-i\zeta/2)\operatorname{sech}\tau. \qquad (24)$$

Consequently, in the framework of the functional-integral NSE the formal Greens's function (20) corresponds to the operator

$$G_0(\tau, \eta; \zeta) = \hat{I}\int \exp[i\mathscr{L}_0(\tau(x))]D\tau = \hat{I}\exp\left(-\frac{i\zeta}{2}\right)\delta(\tau - \eta), \qquad (25)$$

$$\mathscr{L}_0(\tau(x)) = \int_0^\zeta S_0(\tau(x))\,dx,$$

$$S_0(\tau(x)) = {}^1/_2\dot{\tau}^2(x) + a_0^2(\tau(x)).$$

We now analyze the behavior of the quantum fluctuations. Let us replace the operator ψ in (21) by $\psi_0(\tau)$. In the linear approximation in the fluctuations we have

$$\mathscr{L}(\tau(x)) = \mathscr{L}_0(\tau(x)) + \delta\mathscr{L}(\tau(x)),$$

$$\delta\mathscr{L}(\tau(x)) = \int_0^\zeta S_\xi(\tau(x))\,dx, \qquad (26)$$

$$S_\xi(\tau(x)) = a_0(\eta)[\xi_0^+(\tau(x)) + \xi_0(\tau(x))].$$

We neglect the effect of the fluctuations on the trajectories over which we are integrating in (20), i.e., we replace $\xi_0[\tau(x)]$ by $\xi_0(\eta)$. Then

$$S_\xi(\eta) = a_0(\eta)[\xi_0^+(\eta) + \xi_0(\eta)], \quad \delta\mathscr{L}(\eta) = S_\xi(\eta)\zeta,$$

and

$$T\exp[i\mathscr{L}(\tau(x))] = \exp[i\mathscr{L}_0(\tau(x)) + i\delta\mathscr{L}(\eta)].$$

Thus, in this approximation, the integral (20) takes the form

$$G(\tau, \eta; \zeta) = \hat{I}\delta(\tau - \eta)\exp[-i\zeta/2 + i\delta\mathscr{L}(\eta)],$$

where relation (25) was taken into account. As a result the solution of Eq. (13) takes the form

$$\psi(\tau, \zeta) = \exp[-i\zeta/2 + i\delta\mathscr{L}(\tau)]\psi_0(\tau). \qquad (27)$$

To analyze the dispersion of the quantum fluctuations of the soliton $\xi(\tau, \zeta) = \psi(\tau, \zeta) - \langle\psi_0(\tau)\rangle$ in the nonlinear medium it is sufficient to consider the terms linear in $\xi_0(\tau)$ and $\xi_0^+(\tau)$. As a result we obtain

$$\xi(\tau, \zeta) = [1 - iH(\tau, \zeta)]\xi_0(\tau)\exp(-i\zeta/2) - iH(\tau, \zeta)\xi_0^+(\tau)\exp[i(2\varphi - \zeta/2)]. \qquad (28)$$

where $H(\tau, \zeta) = a_0^2(\tau)\zeta$ and φ is the initial phase of the soliton.

We introduce the quadrature components

$$X = \xi^+ + \xi, \quad Y = i(\xi^+ - \xi),$$

their averages are $\langle X\rangle = \langle Y\rangle = 0$. For the relative dispersions of the quadratures we have

$$\left.\begin{matrix}\langle x^2(\tau, \zeta)\rangle \\ \langle y^2(\tau, \zeta)\rangle\end{matrix}\right\} = 1 \pm 2H(\tau, \zeta)\sin 2\phi + 4H^2(\tau, \zeta)\sin^2\phi. \qquad (29)$$

Here

$$\langle x^2(\tau, \zeta)\rangle = \langle X^2(\tau, \zeta)\rangle / \langle X^2(\tau, 0)\rangle, \quad \phi = \varphi - \zeta/2,$$

$$\langle X^2(\tau, 0)\rangle = \langle Y^2(\tau, 0)\rangle = \langle\xi_0(\tau)\xi_0^+(\tau)\rangle = \text{const}.$$

It is seen that the dispersion of one of the quadratures can grow with increasing ϕ, while that of the other decreases. The initially uniform relative distribution of the fluctuations over the soliton becomes nonuniform (see curve 1 in Fig. 1).

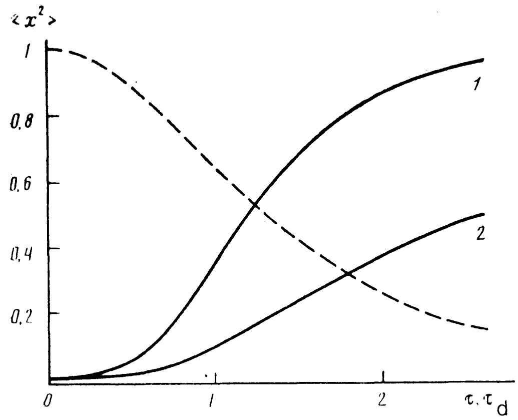

FIG. 1. Plots of the dependence of the reduced dispersion of the quantum fluctuations of one of the optical soliton quadratures: instantaneous (curve 1) and averaged over the detection time $2\tau_d$ (curve 2) values in the neighborhood of the soliton peak. The dashed curve gives the soliton profile. Unity corresponds to the level of vacuum fluctuations. The distance traversed, normalized to the length of the dispersive spreading, is $\zeta = 5$, the time τ is normalized to the width of the soliton.

The quadrature components have extreme values under the condition

$$\operatorname{tg}\phi=-1/H(\tau,\ \zeta). \tag{30}$$

Then

$$\langle x^2\rangle_{\substack{min\\max}}=[(1+H^2(\tau,\ \zeta))^{1/2}\mp H(\tau,\ \zeta)]^2. \tag{31}$$

For $H\gg 1$ we have $\langle x^2\rangle_{\min}\approx(2H)^{-2}$, $\langle x^2\rangle_{\max}\approx(2H)^2$.

If the soliton phase is optimized for its peak then at the edges the squeezing is decreased. For ultrashort pulses, in view of the finite response time of the detector (which we denote as $2\tau_d$) an averaging of the squeezing takes place, i.e.

$$\langle\langle x^2(\tau,\zeta)\rangle\rangle=\frac{1}{2\tau_d}\int_{\tau-\tau_d}^{\tau+\tau_d}\langle x^2(\tau_1,\zeta)\rangle\,d\tau_1. \tag{32}$$

The curve 2 in Fig. 1 illustrates the degradation of the squeezing due to detection for the case $\tau=0$. Nevertheless, as can be seen from Fig. 1, a rather deep squeezing of quantum oscillations can be achieved in optical solitons.

It should be noted that the form of the squeezed state under discussion differs from the squeezed state formed under degenerate parametric amplification (see, for example, Ref. 5), in which the suppression of the fluctuations as a function of distance proceeds according to the exponential law. At the same time, for both cases the regularities of the behavior of the quadrature fluctuations at short distances turn out to be the same. Therefore the results of the Refs. 6–8, where the parametric approximation was used to analyze the quantum fluctuations of the soliton, are only valid for small values of $H(\tau,\zeta)$ (in our notation). For $H(\tau,\zeta)<1$ the results of the parametric approximation and the approximation we have developed coincide [see Eq. (31)].

4. SUPPRESSION OF INTENSITY FLUCTUATIONS; SUB-POISSON PHOTON STATISTICS

The question of intensity fluctuations is important first of all from the point of view of the direct detection of the soliton. Conclusions about the nature of the statistics of the fluctuations can be deduced on the basis of an analysis of the moments of the photon number. Below we confine ourselves to the calculation of just the dispersion

$$\sigma^2(\tau,\ \zeta)=\langle(n-\langle n\rangle)^2\rangle. \tag{33}$$

The photon number operator

$$n\equiv n(\tau,\ \zeta)=\psi^+(\tau,\ \zeta)\psi(\tau,\ \zeta)$$

is defined at the cross section ζ of the medium at time τ.

The quantum averaging in (33) can be carried out over the initial coherent state of the solitons or over the initial vacuum fluctuations after subtracting off from $\psi(\tau,\zeta)$ the nonzero expectation value as, for example, in (22). We shall utilize the latter variant. Then in accordance with (27) the dispersion of the photon number equals

$$\sigma^2(\tau,\ \zeta)=\sigma_0^2(\tau)=a_0^2(\tau)\langle\xi_0(\tau)\xi_0^+(\tau)\rangle. \tag{34}$$

It follows that the photon statistics in the propagation of the soliton does not change, remaining Poisson.

At the same time the interference of the soliton that has passed through the nonlinear medium with the coherent radiation permits the photon statistics of the total radiation to become sub-Poisson. In the simplest case, when the coherent radiation is mixed with the soliton with the help of a light-splitting plate, introducing negligible losses of the soliton, the resultant radiation may be described by the "shifted" operator[1]

$$\psi_\Sigma(\tau,\ \zeta)=\psi(\tau,\ \zeta)+C(\tau), \tag{35}$$

where $C(\tau)$ is the normalized amplitude of the coherent radiation.

The dispersion of the photon number of the total field equals

$$\begin{aligned}\sigma_\Sigma^2(\tau,\ \zeta)=\langle\xi_0(\tau)\xi_0^+(\tau)\rangle\{&a_0^2(\tau)\\ &+2|C(\tau)|a_0(\tau)\cdot(\cos\nu+2H\sin\nu)\\ &+|C(\tau)|^2[1+2H\sin 2\nu-4H^2\sin^2\nu]\},\end{aligned} \tag{36}$$

where

$H=H(\tau,\zeta),\nu=-(\theta+\zeta/2),\theta=\theta(\tau)=\arg C(\tau)$.

The ratio

$$F(\tau,\zeta)=\frac{\sigma_\Sigma^2(\tau,\zeta)}{\sigma^2(\tau,\zeta)}=1+4H\sin\nu\frac{(H\sin\nu+\cos\nu+p)}{(1+2p\cos\nu+p^2)}, \tag{37}$$

where $p=p(\tau)=a_0(\tau)/|C(\tau)|$, characterizes the change in the statistics of the photon number fluctuations.

Under certain conditions one may obtain the value $F<1$, which corresponds to sub-Poisson statistics for the photons. Below we shall exhibit possible forms of the function $C(\tau)$, corresponding to optimal suppression of quantum noise.

It can be shown that $F(\tau)$ [the relation (37)] assumes its minimum value at every instant of time τ if the condition

$$p(\tau)=-\sin[\varphi(\tau)+\nu(\tau)]/\sin\varphi(\tau), \tag{38}$$

is satisfied, where $\varphi(\tau)=\frac{1}{2}\operatorname{arctg}H^{-1}(\tau,\zeta)$. The derivation of (38) is rather laborious and will be omitted. We only note that its validity was verified by numerical modelling. In this way the form of the auxiliary pulse $C(\tau)$ differs from the soliton form $a_0(\tau)$ for achieving optimal suppression of the fluctuations. The condition (38) may be satisfied under phase, amplitude or amplitude-phase modulation of the pulse $C(\tau)$. We shall discuss the first two cases.

Let the form of the envelope of the soliton and the auxiliary pulse be the same, i.e., $p(\tau)=\text{const}$. In that case condition (38) is satisfied for phase modulation (PM) of the form

$$\nu(\tau)=-\pi/2-\varphi(\tau)\pm\arccos\{p[1+\operatorname{tg}^{-2}\varphi(\tau)]^{-1/2}\}. \tag{39}$$

Since we have $|\operatorname{tg}\varphi(\tau)|\leqslant 1$, PM may be effective over the duration of the entire pulse (for arbitrary τ) only if $p\leqslant\sqrt{2}$. In the opposite case the value of arccos {...} in (39) does not always exist and the fulfillment of (38) is not realized.

The form of the function (39) is represented in Fig. 2a (curve 1) for $p=\sqrt{2}$ and the " + " sign in front of the arccosine. The " − " sign corresponds to interference of $a_0(\tau)$ and $C(\tau)$ with the phase difference, leading to a lowering of the amplitude of the resultant pulse. The τ dependence of the Fano factor for the case under consideration is depicted by curve 1 in Fig. 3.

One way to produce PM in an initially unmodulated soliton consists in making use of the phase self interaction in a medium with cubic nonlinearity in the absence of disper-

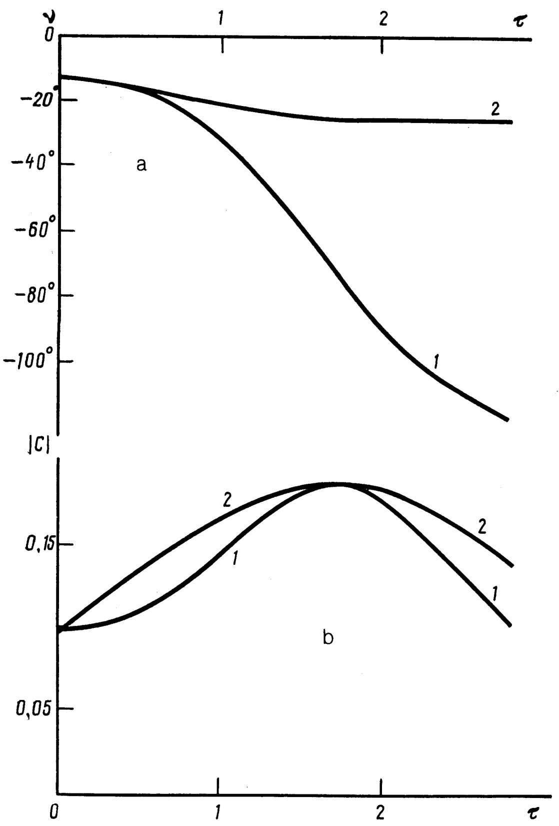

FIG. 2. Plots of the dependence of the phase (a) and the envelope (b) of the auxiliary pulse on the normalized time τ for $\zeta = 5$; curve 1 is optimal, curve 2 is approximate.

sion effects. Then the phase increase equals

$$\nu(\tau) = \varphi + B\,\mathrm{sech}^2\,\tau, \qquad (40)$$

where φ and B are real constants, the first of which is the initial phase and the second is determined by the nonlinearity of the medium, the maximum intensity of the soliton and the traversed distance.

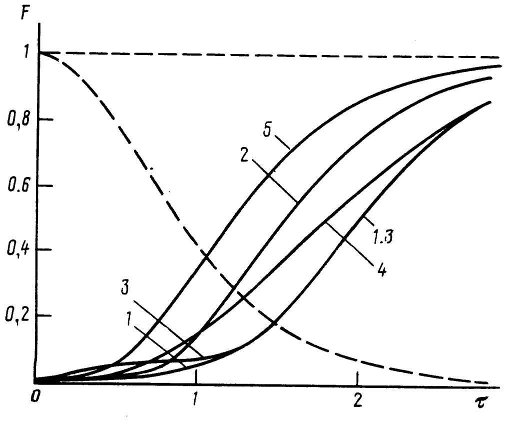

FIG. 3. Plots of the time dependence of the Fano factor for $\zeta = 5$: the dashed straight line corresponds to coherent radiation, the dashed curve is the envelope of the fundamental soliton, curve 1 corresponds to optimal amplitude of phase modulation, curve 2—to the approximating phase modulation, curve 3—to the approximating amplitude modulation, curve 4—to mixing with continuous radiation, curve 5—to mixing with a soliton of the same form.

An example of the approximation of the dependence (39) by the function (40) is shown by the curve 2 in Fig. 2a, and the dependence of the Fano factor corresponding to it by curve 2 in Fig. 3 ($p = \sqrt{2}$). As was to be expected, near the soliton peak the suppression of the intensity fluctuations is practically optimal. Differences arise on the wings of the pulse in view of significant discrepancies between the required and the approximating curves (Fig. 2).

We consider now the possibility of realizing the condition (38) for amplitude modulation of the auxiliary pulse $C(\tau)$. In the case $\zeta = 5, \nu = -95.655\,°$ (for this phase the amplitude $|C(\tau)|$ is minimal) we have calculated the optimal form of the auxiliary pulse, which is shown as curve 1 in Fig. 2b. We note that it corresponds to the same τ dependence of the Fano factor as in the optimal PM, i.e., curve 1 in Fig. 3. It is seen that to obtain optimal suppression of the quantum fluctuations the auxiliary pulse should lag the fundamental one (in the present case by approximately a time $\tau \approx 1.7$).

An attempt to approximate this curve by some retarded soliton (curve 2 in Fig. 2b) leads to a barely noticeable increase of the Fano factor (curve 3 in Fig. 3, $\nu = -95.655\,°$) in the region $0.1 < \tau < 1.2$. For other τ curve 3 almost exactly coincides with the curve 1.

To obtain a better feeling for the results under discussion we also calculate the τ dependence of the Fano factor for the following two interesting cases. The first constitutes the result of mixing the soliton with continuous radiation, having constant amplitude and phase chosen to be optimal for the soliton peak ($|C| = 0.099$, $\nu = -95.655\,°$). This case is illustrated by curve 4 in Fig. 3. It is seen that this mixing variant is fully competitive with that discussed above.

The second case turns out to be most unfavorable (curve 5 in Fig. 3). It is the result of mixing two identical solitons, differing only in amplitude, i.e., p and ν are constants chosen to be optimal for the point $\tau = 0$. But even in this case the photon statistics in the soliton is essentially sub-Poisson.

Beside the Fano factor it is of interest to estimate the dispersion of the photon number fluctuations, which directly determines the detection shot noise. In the form normalized to the maximal initial value the dispersion may be written as

$$\sigma^2(\tau,\zeta) = a_0{}^2(\tau)\{1 + 2p^{-1}(\tau)\cos\nu(\tau) + p^{-2}(\tau) + 4Hp^{-1}(\tau)\sin\nu(\tau)\,[1 + p^{-1}(\tau)\cos\nu(\tau) + p^{-1}(\tau)H\sin\nu(\tau)]\}. \qquad (41)$$

It is not hard to obtain this expression by making use of Eq. (36).

In accordance with (41) we have calculated the dispersion for the above mixing variants. The results of the calculation are shown in Fig. 4. The best noise suppression is achieved by the optimal amplitude modulation (curve 1 Fig. 4). In the case of optimal PM (curve 2) the dispersion turns out somewhat larger than in the case of amplitude modulation, due to the fact that the minimal value $p = \sqrt{2}$ which it requires corresponds to a larger amplitude of the auxiliary pulse and, consequently, to an increased intensity of the resultant pulse.

To avoid excessive crowding of Fig. 4 we omit the curves for the approximating amplitude and phase modula-

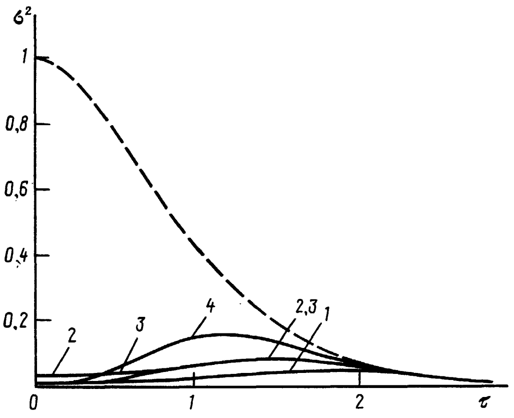

FIG. 4. Plots of the time dependence of normalized dispersion for $\zeta = 5$: the dashed curve corresponds to the coherent soliton, curve 1—to optimal amplitude modulation, curve 2—to optimal phase modulation, curves 3 and 4—to mixing with continuous radiation and with a soliton of the same form (there is an error in the labeling of curve 3: for small τ it is below curve 2).

tions: they are very close to curves 1 and 2. Also curve 3, illustrating mixing with continuous radiation ($|C| = 0.099$; $\nu = -95.655°$), differs little from them.

Just as before, the interference of solitons identical in form (curve 4 in Fig. 4, $p = 10.15$, $\nu = -95.655°$) turns out to be least productive. But also here the suppression of the photon number fluctuations in comparison with the initial coherent soliton turns out to be quite significant. Thus, even when the soliton has comparatively short propagation length ($\zeta = 5$) significant suppression of the quantum intensity fluctuations is possible, i.e., one may reach significantly sub-Poisson photon statistics.

5. CONCLUSION

It follows from the results of this work that the initially coherent optical soliton goes over under nonlinear propagation into a definite quantum state with suppressed fluctuations of the field quadrature or the photon number. The latter is realized in the interference of the soliton in the squeezed state with coherent radiation. We determine forms of amplitude and phase modulations of the coherent radiation that result in sub-Poisson statistics of the photons. The peculiarities in the detection of ultrashort light pulses by the photon counting method were recently discussed in Ref. 15.

The main parameter that determines the suppression of the quantum fluctuations of the soliton is $H(0,\zeta) = a_0^2(0)\zeta = \zeta = Z/L_p$. We estimate its value under typical conditions.[12] In single-mode fiber lightguides the propagation of the fundamental soliton at wavelength $\lambda_0 = 1.5$ μm for $D = -2\pi cg/\lambda_0^2 = 15$ ps/(nm·km) may be realized for a pulse power of 1 W for width $\tau_c \approx 4$ ps. Here $L_p = L_{nl} \approx 800$ m. For a lightguide length of a few kilometers the parameter ζ will be equal to several units. Consequently a quite realistic possibility exists of obtaining power pulses with suppressed quantum fluctuations. Naturally one should keep in mind that our calculations were performed without taking into account absorption and distributed medium noise, which could lead to definite corrections of the results here obtained. Without a doubt this circumstance deserves special discussion.

The method for analyzing the behavior of the quantum fluctuations of the fundamental optical soliton developed here leads in essence to a quasistatic approximation for the fluctuations; at each instant of time their size is determined by the value of the envelope at that same instant.

This method may be applied to solitons of higher order as well as to the so-called dark solitons.[12] In the framework of the approximation developed here the answer to the question of how the quantum fluctuations would behave themselves in their case is quite obvious—the temporal dynamics of the fluctuations dispersion will be connected with the soliton envelope.

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[1)] In Ref. 9 the necessity for the presence of the T operator was not noted.

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