Ultrasonic spectroscopy of rotating ^{3}He-*B*

Yu. M. Kyunaraĭnen, A. Yu. Manninen, K. Torizuka, A. V. Babkin,[1)] R. Kh. Salmelin, G. K. Tvalashvili,[2)] and Yu. P. Pekola

Low-Temperature Laboratory of the Helsinki Technological University, Finland

(Submitted 8 February 1990)
Zh. Eksp. Teor. Fiz. **98**, 516–531 (August 1990)

The effect of the rotating sample on the spectrum of the real squashing (RSQ) mode of ^{3}He-*B* at pressures below 12 bar and magnetic fields up to 40 mT was studied. The rotation makes it possible to resolve all five absorption lines. A pronounced asymmetry with respect to the sign of m_J was found in the intensities of the absorption lines. Doublet splitting of the central peak was recorded. The possibility of excitation of the RSQ mode by zero sound in the case of nonzero flow velocity of the superfluid component was studied; it is shown that in this case the efficiency of the excitation is at least four times less than predicted theoretically. The measurements are compared with the predictions of current theories.

A large number of collective excitations in ^{3}He-*A* and ^{3}He-*B* exist on account of the complexity of the order parameter of these superfluid phases.[1] In this paper we present the experimental results of a study of one such excitation—the real quadrupole mode [known in the literature as the real squashing (RSQ) mode] in rotating ^{3}He-*B*. All earlier experiments concerning the propagation of sound in superfluid ^{3}He were performed on samples at rest (see, for example, the reviews of Halperin[2] and Ketterson *et al.*[3]). We shall demonstrate how the spectroscopy of collective excitations can be employed to study the structure of the order parameter as well as to check experimentally the theoretical predictions regarding the character of the excitation of the RSQ mode by zero sound in superfluid ^{3}He-*B*. Some of our experimental results were briefly reported in Refs. 4 and 5.

1. COLLECTIVE EXCITATIONS IN ^{3}He-*B*

Ordinary hydrodynamic sound propagating in liquid ^{3}He at low temperatures is attenuated; the attenuation increases as T^{-2} as the temperature decreases. This means that at sufficiently low temperatures normal sound cannot exist in ^{3}He. However because there is a strong interaction between ^{3}He atoms, which results in the appearance of an effective molecular field, zero sound, i.e., waves which propagate without interatomic collisions so that there is not enough time for thermodynamic equilibrium to be established, can exist. It has been found that zero sound can propagate in both the normal Fermi liquid and in superfluid ^{3}He. In this paper, when we talk about excitation, absorption, etc., of ultrasound in our experiments, we mean precisely excitations of this type, i.e., zero sound.

An additional mechanism for attenuation of ultrasound appears at the λ point: breaking of Cooper pairs in the superfluid condensate by sound quanta, which is accompanied by a sharp increase in the attenuation. The necessary condition for pair breaking is $\hbar\omega \geqslant 2\Delta(T)$, where $\Delta(T)$ is the superfluid energy gap. If the temperature is close to T_λ, then the attenuation of the sound is weak, since only a small number of Cooper pairs in the condensate can be broken; as the temperature decreases the number of pairs increases, and hence the attenuation of ultrasound also increases until the condition for pair breaking is satisfied owing to a corresponding increase in the energy gap $\Delta(T)$. Thus at some temperature, below which the condition $\hbar\omega \geqslant 2\Delta(T)$ can no longer be satisfied, the attenuation of zero sound must vanish. In reality this does not happen. To explain this phenomenon an additional attenuation mechanism must be postulated, i.e., we assume that the excitation of Cooper pairs results in the appearance of some collective modes in the superfluid liquid.

The order parameter in superfluid ^{3}He is described in its general form by a 3×3 complex matrix, so that it has 18 degrees of freedom. The equilibrium order parameter, i.e., the order parameter corresponding to minimum free energy, is stable with respect to small disturbances, and therefore all 18 collective modes are possible.

The equilibrium order parameter in ^{3}He-*B* has the form[6] $\Delta \exp(i\varphi)\widetilde{R}(\hat{\mathbf{n}},\theta)$, where Δ is the isotropic energy gap, φ is the phase of the order parameter, and $\widetilde{R}(\hat{\mathbf{n}},\theta)$ is the rotation matrix describing the position of the orbital and spin coordinate systems relative to the magnetic anisotropy axis $\hat{\mathbf{n}}$, and $\theta = 104°$. At equilibrium in ^{3}He-*B* the total angular momentum of a Cooper pair vanishes, $\mathbf{J} = \mathbf{L} + \widetilde{R}\,\mathbf{S} = 0$, where $\mathbf{L}$ and $\mathbf{S}$ are the orbital and spin angular momentum, respectively.[7] Collective modes can be classified according to their total angular momentum quantum number J and also the projection m_J of J on the axis of quantization.[7,8] Thus J assumes the values 0, 1, or 2 and $m_J = -J,..., +J$.

The collective excitations can also be described differently. Suppose that near the Fermi surface the density of states does not depend on the energy, i.e., there exists a symmetry between quasiparticles and holes. Then the collective excitations can be separated into "density" excitations and spin excitations based on the character of their coupling with the external field.[8] The real components of the order parameter will describe density excitations and the imaginary components will describe spin density waves. Note that waves in which the density changes, i.e., sound waves, can be excited only if J is even. In these terms the imaginary mode with $J = 0$, corresponding to oscillations of the phase of the order parameter, is zero sound itself, which can be used for exciting other modes.

Many of the collective modes have a finite frequency, which is comparable in order of magnitude to the frequency corresponding to the energy gap, even in the limit as the wave vector vanishes, $\mathbf{q}\to 0$. Modes with some values of J have degenerate frequencies in zero magnetic field or for $\mathbf{q} = 0$. In the limit of total hole–quasiparticle symmetry only one imaginary mode with $J = 2$ can be excited by the sound.

 0038-5646/90/080287-09$03.00

Its frequency for $\mathbf{q} = 0$ is equal to approximately $\sqrt{(12/5)}\,\Delta(T)$. It is the existence of precisely this mode that results in the appearance of additional absorption of zero sound with frequency close to the limit at which Cooper pairs are broken by sound quanta, i.e., $\omega = 2\Delta/\hbar$.

In reality the density of energy levels in ^{3}He has a finite slope at the Fermi surface. The resulting asymmetry in the hole–quasiparticle system makes possible the excitation of additional collective modes by sound.[9] The best-studied mode, other than, of course, zero sound itself, is the real quadrupole mode with $J = 2$, the RSQ mode. The RSQ mode was first detected experimentally in 1980 independently by two groups.[10,11] The subsequent observation of five-fold splitting in a transverse magnetic field[12] completely corroborated the identification of this mode. Moreover, it was shown that the experimental angular dependence of the intensities of the absorption spectra can be described well by spherical harmonics of degree 2, as one would expect for a mode with $J = 2$.[8]

The splitting of the RSQ mode in perpendicular ($\hat{\mathbf{q}}\perp\mathbf{H}$) and longitudinal ($\hat{\mathbf{q}}\|\mathbf{H}$) external magnetic fields up to 180 mT was studied in Ref. 13. In the case $\hat{\mathbf{q}}\perp\mathbf{H}$ it was found that the dependence of the resonance frequencies of the sublevels on the magnitude of the field is strongly nonlinear for $H \geqslant 30$ mT. In addition, if the ^{3}He sample was cooled to a temperature below the λ point in the absence of an external magnetic field, then the splitting was observed only in fields exceeding 50 mT and did not depend on the field strength. Conversely, if the sample was cooled in an external field, then the splitting strongly depended on the field strength and diminished monotonically, vanishing at ≈ 20 mT. It is also of interest to observe the additional splitting of the peak $m_J = 0$ in the case $\hat{\mathbf{q}}\perp\mathbf{H}$. In a longitudinal magnetic field ($\hat{\mathbf{q}}\|\mathbf{H}$) splitting of the RSQ mode was not observed.

Linear Zeemann splitting of the RSQ mode was predicted back in 1979.[14] The nonlinear dependence on the strength of the magnetic field, observed by Shivaram *et al.*,[13] was also explained.[15,16] It was attributed to two factors: 1) ellipsoidal distortion of the energy gap in ^{3}He-*B* and 2) the mixed nature of the RSQ mode, which in a magnetic field is no longer a pure ($J = 2$) mode, but contains admixtures of $J = 1$ and $J = 3$ modes.

In order that excitation of a collective mode by zero sound be possible the sound dispersion curve $\omega^2 = c_0 q$ must have at least one common point with the dispersion curve of the corresponding collective excitation, which in zero field can usually be written in the form $\omega^2 = \omega_{00}^2 + c_q^2 q^2$, where ω_{00} is the frequency of the mode with $H = q = 0$. The dispersion coefficients c_q, generally speaking, are different for each m_J, so that splitting of the collective modes is possible even in a zero magnetic field. Triple splitting with frequencies ω_0, ω_1, and ω_2, corresponding to $m_J = 0$, ± 1, and ± 2, was predicted for the RSQ mode.[17] These frequencies are related by the relation $(\omega_0 - \omega_1)/(\omega_0 - \omega_2) = 1/4$. Since the velocity of zero sound is quite high ($c_0 \approx 300$ m/s), the splitting is quite small, of the order of $10^{-4}\,\Delta(0)$. It has nonetheless been recorded experimentally.[13]

The splitting of the peak $m_J = 0$ remained puzzling until it was explained by Volovik,[18] who proposed that it is caused by the nonuniformity of the texture of the axis of quantization $\hat{\mathbf{h}} = \hat{R}\,\mathbf{H}/H$. Indeed, in the experimental cell employed in Ref. 13 the external magnetic field was oriented perpendicular to $\hat{\mathbf{q}}$ and parallel to the cell walls. Thus in the central part of the experimental cell $\hat{\mathbf{h}}$ is always oriented in the same direction as the magnetic field, but in some region directly adjacent to the cell walls the axis of quantization $\hat{\mathbf{h}}$ is perpendicular to the field.

In strong magnetic fields the frequency of the RSQ mode can be written in the general form $\omega^2 = \omega_{00}^2 + c_1^2 q^2 + c_2^2 (\hat{\mathbf{q}}\hat{\mathbf{h}})^2$, so that the frequency is determined by the relative orientation of $\hat{\mathbf{q}}$ and $\hat{\mathbf{h}}$. The coefficients c_1^2 and c_2^2 are of the order of v_F^2, where $v_F \approx 50$ m/s is the velocity at the Fermi surface. If we assume that the contributions to the total RSQ signal are independent of one another and add with relative weight proportional to the volume of the corresponding regions where the relative orientation of $\hat{\mathbf{q}}$ and $\hat{\mathbf{h}}$ does not change, then, the regions in which the orientation of $\hat{\mathbf{h}}$ does not change will make the main contribution to the attenuation of the sound. In the experimental cell employed in Ref. 13 there were two such regions: the central part of the cell and near the walls. In our very simple model the splitting of the peaks in the spectrum can be estimated as

$$\omega_w - \omega_c \approx q^2 c_2{}^2/2\omega_{00}.$$

The absence of the observed splitting of the components of the spectrum with $m_J \neq 0$ was later explained by Mineev[19] based on the angular dependence of their sound coupling coefficients (see Sec. 3).

The correctness of this phenomenological approach was later confirmed by Fishman and Sauls,[20] who, using the quasiclassical approximation, also found that the central peak was split. Moreover, they also explained the nonlinear behavior of the frequencies of the RSQ modes in weak fields and the smooth disappearance of the doublet splitting.

A different mechanism for the excitation of the RSQ mode by zero sound was proposed in Ref. 21. It was shown that even if the condition of total hole–quasiparticle symmetry is satisfied in ^{3}He at rest, any motion of the superfluid component will destroy this symmetry and allow zero sound to excite the RSQ mode by an independent mechanism. The efficiency of such excitation is found to be proportional to the quantity $(p_F|\mathbf{w}|\Delta)^2$, where $\mathbf{w} \equiv \mathbf{v}_s - \mathbf{v}_n$ is the difference of the velocities of the superfluid and normal components. The angular dependence of the sound attenuation in this case is also found to be completely different, so that even in the case $\mathbf{q}\|\mathbf{H}$ and when the flow velocity of the superfluid component is not zero, states with $m_J \neq 0$ should appear. Both mechanisms of excitation of the RSQ mode, i.e., the hole–quasiparticle asymmetry and flow of the superfluid component, are of the same magnitude for $w_0 \approx 1$ mm/s (Ref. 21).

It follows from what we have said above that spectroscopy of the collective modes is an exceptionally convenient tool for determining the Fermi-liquid parameters of ^{3}He, while the texture dependence of the RSQ mode spectrum gives a new method for studying ^{3}He-*B* experimentally. We note, however, that under the conditions of the real inhomogeneous texture it is extremely difficult to calculate the RSQ spectrum using the microscopic theory; moreover, exact calculations of the amplitudes of the sound absorption spectrum have not been performed even for the case of homogeneous texture.

The recently published theory of Salomaa and Volovik (SV) opens up comparatively large experimental possibilities.[22] Salomaa and Volovik proposed a comparatively simple phenomenological approach that reproduces the results of the microscopic theory with regard to obtaining the resonance frequencies of the RSQ mode and, importantly, makes it possible to obtain the amplitudes of the absorption spectrum. This information is in principle sufficient to determine completely the real texture in the experimental cell. The SV theory has three phenomenological parameters: 1) the Landé factor for splitting in a field, 2) the magnitude of the dispersion splitting in zero field, and 3) the ratio of the efficiencies for excitation of the RSQ mode by sound by the flow of the superfluid component and by the hole–quasiparticle asymmetry. The SV theory predicts that the absorption lines of the RSQ spectrum are asymmetric with respect to the sign of m_J and that there exists a "gyrosonic" effect in rotating ^{3}He-B, i.e., the amplitudes of the absorption lines of the RSQ spectrum depend on the direction of rotation of the sample with respect to the direction of propagation of sound in it.

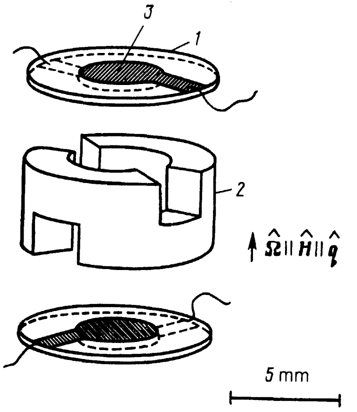

FIG. 1. The construction of the experimental cell: 1) quartz radiator, 2) quartz spacer, 3) gold-plated electric contact.

2. EXPERIMENTAL PROCEDURE AND RESULTS

All experimental results presented in this paper were obtained on the ROTA-2 nuclear demagnetization apparatus in the Low-Temperature Laboratory at Helsinki Technological University. The construction of the apparatus and its operation are discussed in detail in Ref. 23.

The construction of the experimental cell is shown in Fig. 1. Ultrasound was excited in superfluid ^{3}He by two quartz radiators, each of which could also simultaneously function as a receiver. The fundamental resonant frequency of the radiators was equal to 8.9 MHz, and the resonant frequencies of the two crystals differed by less than 5 kHz. It was found that sound could also be excited at the frequencies of the odd harmonics through the 13th harmonic (116 MHz). The cylindrical ampul had a diameter of 6 mm and was 4 mm high. The ultrasound propagated in the direction $\hat{\mathbf{q}}$ parallel to the rotational axis of the sample $\hat{\mathbf{\Omega}}$ and the external magnetic field **H**.

The ultrasound was emitted in the form of pulses 5 or 12 μs wide and usually with a repetition rate of 1 s^{-1}. The signal was recorded with a synchronous detector, from whose output two antiphase components of the received signal a_{in} and a_{out} were extracted simultaneously. Both components were displayed on a computer-controlled digital oscillograph. To minimize the signal processing time, the amplitude of the transmitted ultrasound was calculated using the simplified formula[10]

$$A(T)=\left[\left(\int_{\tau_1}^{\tau_2} a_{in}\,dt\right)^2+\left(\int_{\tau_1}^{\tau_2} a_{out}\,dt\right)^2\right]^{1/2}. \qquad (1)$$

The integration limits τ_1 and τ_2 were chosen so that the direct electrical interference from the excitation pulse and the reflected pulses would have no effect on the received signal. The numerical values obtained using the formula (1) were virtually identical to those obtained using the exact expression

$$A(T)=\int_{\tau_1}^{\tau_2} (a_{in}^2+a_{out}^2)^{1/2}\,dt,$$

The relative attenuation of the ultrasound $\Delta\alpha(T)=\alpha(T)-\alpha(T_\lambda)$ was calculated using the expression

$$\Delta\alpha(T)=-\frac{1}{L}\ln\left[\frac{A(T)}{A(T_\lambda)}\right]. \qquad (2)$$

The temperature of the sample was varied slowly during the experiment by means of weak demagnetization (magnetization) of a copper nuclear stage. In addition, the temperature interval in which the RSQ mode exists was usually swept within 10–15 min. The temperature of the sample was monitored with the help of a platinum NMR thermometer,[3)] which was thermally coupled directly with the superfluid ^{3}He. However, because of the significant spin-lattice relaxation time ($\approx$30 s) of ^{195}Pt in this temperature range, the time interval between two NMR pulses could not be less than 2 min. As a result, only several temperature measurements could be performed in the working temperature range of the RSQ mode; this substantially limited the use of the Pt-NMR-thermometer. Nonetheless it was found that in ^{3}He-B the zero sound itself can be used as a secondary thermometer and the resolution of the experimental spectra can thereby be increased significantly.

Indeed, since the phase velocity of zero sound near the RSQ mode depends strongly on temperature owing to the closeness of the imaginary quadrupole mode with $J=2$, the phase of the received signal Φ is also extremely sensitive to temperature changes. The phase Φ can be expressed as

$$\Phi=\tan^{-1}\left(\int_{\tau_1}^{\tau_2} a_{out}\,dt \Big/ \int_{\tau_1}^{\tau_2} a_{in}\,dt\right). \qquad (3)$$

In order to be able to compare the parameters of spectra obtained under different conditions each spectrum was approximated by a set of Lorentz functions with the temperature as the argument:

$$A\left(\frac{T}{T_\lambda}\right)=\sum_{i=1}^{N}\frac{A_i\delta_i^2}{[(T-T_i)/T_\lambda]^2+\delta_i^2}, \qquad (4)$$

where N is the number of lies in the spectrum and A_i, δ_i, and T_i determine the amplitude, width, and temperature, respectively, of the ith peak.

The spectra of the RSQ mode were measured for both directions of rotation of the cryostat and for both directions of sound propagation.

Figure 2 shows the typical experimental spectra for ro-

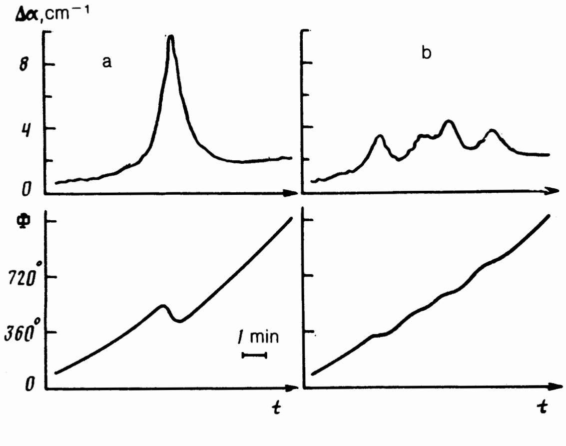

FIG. 2. Two spectra of the RSQ-mode of absorption of zero sound $\Delta\alpha$ and the phase of the signal Φ with $p = 3.2$ bar, $H = 32$ mT, $f = 44.7$ MHz, $\Omega = 0$ (a) and 1 rad/s (b); $\mathbf{H}\|\hat{\mathbf{q}}\|\hat{\mathbf{\Omega}}$.

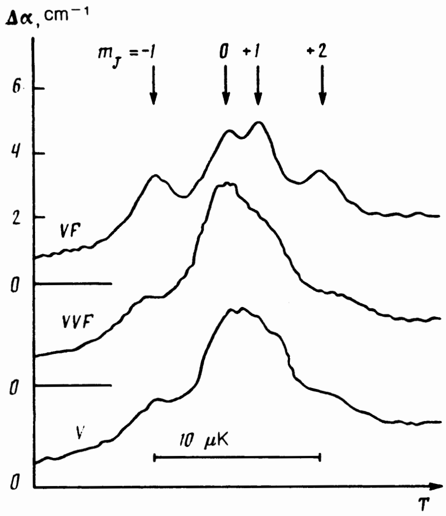

FIG. 3. Three types of spectra of the RSQ-mode of rotating ^{3}He-B with $\Omega = 0.87$ rad/s, $H = 25$ mT, and $f = 44.7$ MHz (see text for explanation).

tating ^{3}He-B and ^{3}He-B at rest. As the temperature changes resonances of the RSQ mode appear for the corresponding m_J, and this results in the appearance of sound attenuation peaks in the spectrum. The passage of each such peak also results in the appearance of characteristic features in the temperature dependence of the signal phase Φ.

In ^{3}He-B at rest only the ground state with $m_J = 0$ can be excited by sound. However rotation of the sample completely changes this picture: the efficiency with which the other states can be excited by sound increases sharply and peaks with $m_J \neq 0$ appear in the spectrum even for $\mathbf{H}\|\hat{\mathbf{q}}$. We also point out that rotation appreciably reduces the intensity of the peak with $m_J = 0$.

The texture of the vector $\hat{\mathbf{n}}$ in the experimental volume is determined by minimizing the total free energy.[24] In the case of a long cylindrical cell at rest, placed in a longitudinal magnetic field, the so-called "flare-out" texture is obtained.[25,26] In this case, in the central part of the cell the vector $\hat{\mathbf{n}}$ is oriented parallel to the magnetic field. As one moves away from the center the vector $\hat{\mathbf{n}}$ tilts monotonically away from $\mathbf{H}$ and near the wall it makes with $\mathbf{H}$ an angle equal to approximately 60°. Correspondingly the quantization axis $\hat{\mathbf{h}}$, likewise parallel to $\mathbf{H}$ at the center, is oriented perpendicular to $\mathbf{H}$ near the cell wall.

When the sample is rotated the "flare-out" texture is deformed by vortices, arising in the superfluid liquid.[24] If, however, vortices do not form easily,[27] then at some critical rotational velocity the texture changes abruptly from "flare-out" to "flare-in,"[28,29] i.e., to a texture in which at the wall $\hat{\mathbf{n}}$ makes an angle of 90° with $\mathbf{H}$. The character of the change in the texture itself exhibits all the characteristics of a first-order phase transition.

The form of the experimental curves makes it possible to determine which of the two possibilities is realized under our experimental conditions. Figure 3 shows three different spectra corresponding to samples prepared under different conditions. The spectrum denoted by V is characterized by a dominant central and weak side peaks, while in the VF spectrum the side peaks are much sharper. A spectrum of the type V was always observed when cooling of the rotated sample was started at a temperature above the λ point, while the VF spectrum was usually recorded when the rotation started at a temperature below T_λ. Sometimes, though, intermediate VVF spectra characterized by highly nonreproducible amplitudes of the peaks were observed.

We explain this behavior by noting that the spectra of the type V and VF, respectively, are observed when vortices are present and absent in the liquid. Indeed, since the average velocity of the counterflow of the superfluid liquid in which there are no vortices is higher than in a liquid in which there are vortices, in the first case the tilting of $\hat{\mathbf{h}}$ away from $\hat{\mathbf{q}}$ as one moves away from the center of the cell is large. As a result, in the VF case an additional mechanism appears that increases the excitation efficiency of states with $m_J \neq 0$. The VVF spectra can be regarded as a case in which a nonequilibrium number of vortices is present in the experimental cell.

We also observe doublet splitting of the central ($m_J = 0$) peak in the spectrum of the RSQ mode caused by the presence of nonuniform texture at high rotational velocities $\Omega \geqslant 1.5$ rad/s. An example of a spectrum confirming this phenomenon is shown in Fig. 4. As the sound frequency increases the resonance frequencies of the RSQ modes shift in the direction of lower temperatures T/T_λ and the absorption lines become narrower. For this reason we observed the side peaks corresponding to $m_J = \pm 1$ even in the stationary case. The nonzero efficiency of excitation of these states by sound in ^{3}He-B at rest can probably be explained by the nonuniformity of the texture of $\hat{\mathbf{h}}$ (see Sec. 3).

3. DISCUSSION

Our experimental results can be compared with current theoretical descriptions of the RSQ mode in the presence of both an external magnetic field and rotation of the sample. In order to make a quantitative comparison accurate calculations that take into account the real texture determined by the geometry of our experimental cell must be performed, but we found that a qualitative analysis is possible even without such calculations.

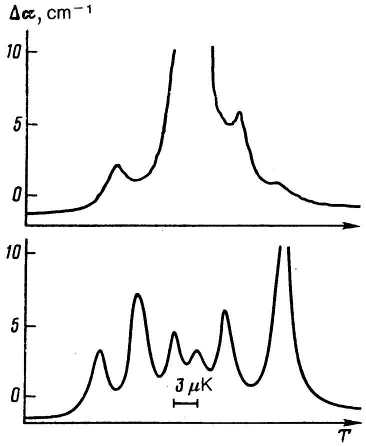

FIG. 4. Two spectra of the RSQ-mode with $p = 6.5$ bar, $H = 25$ mT, and $f = 62.5$ MHz, demonstrating the rotation-induced splitting of the $m_J = 0$ peak: $\Omega = 0$ (top) and $\Omega = 4$ rad/s (bottom).

According to our measurements the resonance frequency of the RSQ mode agrees well with the theoretical estimate $\hbar\omega = a\Delta(T)$, where $a = \sqrt{8/5} \approx 1.265$, if Greywall's temperature scale[30] and the BCS energy gap are employed. The pressure dependence of a is shown in Fig. 5 together with data obtained by other authors.[10,13,31] The small decrease in a with increasing pressure agrees well with the results of Ref. 32. We note that in Ref. 31 the Helsinki temperature scale[33] was employed and the value $a \approx 1.104$ was obtained, but conversion to Greywall's scale gives good agreement with the other results. We also note that in the case when the BCS gap is replaced by the somewhat more complicated gap of Ref. 34 the quantity a, in all cases, is equal to approximately 1.18.

As we have already mentioned in Sec. 1, the phenomenological theory of Salomaa and Volovik gives the most complete description of the spectra of the RSQ mode.[22] This theory makes a number of predictions that can be checked experimentally.

According to Salomaa and Volovik the main frequencies of the RSQ mode in the presence of a strong magnetic field ($H > 50$ mT) are given by the expression

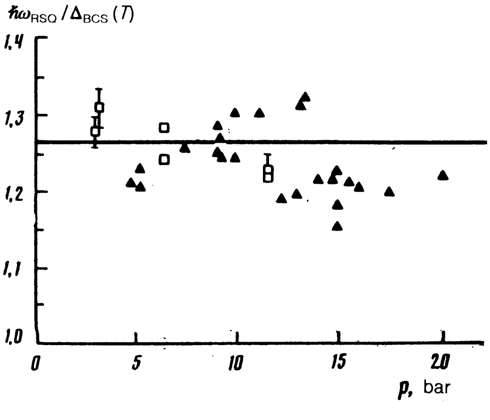

FIG. 5. The pressure dependence of the quantity $\hbar\omega/\Delta_{BCS}(T)$ obtained as a result of our measurements (squares) and the measurements performed by other groups[10,13,31] (triangles). Greywall's temperature scale is used.[30] The straight line corresponds to the value $a = \sqrt{(8/5)} \approx 1.265$ (see text).

$$\omega_{m_J}^2 = \omega_{00}^2 + 2\omega_{00} H m_J + c_1^2 q^2 + c_2^2 q^2 [{}^1/_3 (1 + {}^1/_2 m_J^2) + {}^1/_2 (\hat{\mathbf{q}}\hat{\mathbf{h}})^2 (1 - {}^1/_2 m_J^2)], \quad (5)$$

where c_1^2 and c_2^2 are constants of order v_F^2, the term proportional to H describes the linear Zeemann splitting, and the last term determines how efficiently the RSQ mode is excited by zero sound.

In an external magnetic field the vector $\hat{\mathbf{h}}$ tends to become oriented parallel to $\mathbf{H}$, as a result of which the conditions $\hat{\mathbf{q}} \| \mathbf{H} \| \hat{\mathbf{h}}$ are satisfied in the entire experimental volume except for a region near the cell walls. If the superfluid ^{3}He-B is rotated, then the counterflow $\mathbf{w}$ arising in it tends to reorient $\hat{\mathbf{h}}$ parallel to $\mathbf{w}$. If there are no vortices in the liquid, then the angle γ between $\hat{\mathbf{h}}$ and $\mathbf{w}$ changes monotonically from zero to $\pi/2$ as the rotational velocity is increased. In the ideal case, in the absence of rotation and with $\mathbf{H} \| \hat{\mathbf{q}}$ the spectrum of the RSQ mode can contain only one absorption peak corresponding to $m_J = 0$. The remaining four lines with $m_J = \pm 1$ and ± 2 can be observed only if $\mathbf{H}$ makes a nonzero angle with $\hat{\mathbf{q}}$.

As we have already mentioned in Sec. 1, the existence of counterflow in superfluid ^{3}He-B results in an additional mechanism of excitation of the RSQ mode by zero sound.[21] The efficiency of this excitation is proportional to $(p_F w/\Delta)^2$, though the exact numerical value of the proportionality constant is not accurately known. In this sense we hoped to obtain from the experimental data the necessary new information about some parameters characterizing ^{3}He-B, such as, in particular, the energy gap.

Figures 6a and b show, respectively, the positions and intensities of the lines in the spectrum of the RSQ mode as a function of the rotational velocity Ω. These dependences were obtained by approximating our primary experimental spectra by a collection of Lorentz functions [see Eq. (4)]. Since the absolute temperature could not be measured accurately enough, we assume that for $\Omega = 0$ the positions of the peaks with $m_J = \pm 1$ in Fig. 6a correspond to the values ± 1. It is obvious from the expression (5) that these peaks can be resolved only owing to linear Zeemann splitting. One can see that the relative positions of the lines in the spectrum exhibit a strongly nonlinear dependence on the rotational velocity. In particular, the different dependences of the amplitudes of different peaks on the rotational velocity can be explained by the difference in the angular dependence of the sound excitation coefficients of states with different $|m_J|$, but on the basis of earlier theories[8,12] it is impossible to explain the observed striking asymmetry of the behavior of the peaks with positive and negative values of m_J.

Aside from the resonant frequencies of the RSQ mode, the SV theory makes specific predictions regarding the dependence of the amplitudes of the m_J peaks on the rotational velocity. This theory contains the phenomenological parameter $H_0 = c_2^2 q^2 / 4g\omega_{00}$, which determines the magnetic field in which the dispersion is equal to the splitting caused by this field. The quantity H_0 can be calculated from the experimentally observed shift of the resonance frequencies (Fig. 6) for low and high rotational velocities Ω ($\gamma \approx 0$ and $\gamma \approx \pi/2$). In these cases

$$\frac{H_0}{H} \approx \frac{2[(\omega_{-1} - \omega_{-2}) - (\omega_{+2} - \omega_{+1})]}{\omega_{+2} - \omega_{-2}} \quad (\gamma = 0), \quad (6a)$$

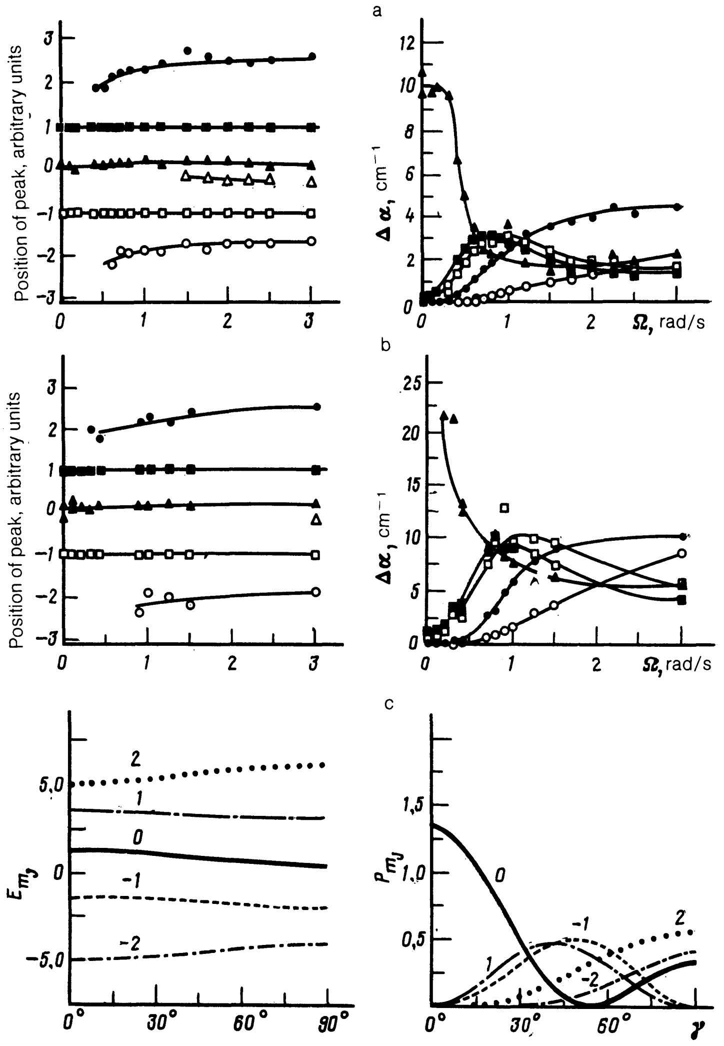

FIG. 6. a, b) The dependence of the position of the absorption peaks of the RSQ-mode and the intensity of the lines on the rotational velocity of the sample: a) $p = 3.0$ bar, $H = 40$ mT, and $f = 44.7$ MHz; b) $p = 6.5$ bar, $H = 40$ mT, and $f = 62.5$ MHz; ●—$m_J = +2$, ○—$m_J = -2$, ■—$m_J = +1$, □—$m_J = -1$, △, ▲—$m_J = 0$; c) the position E_{m_J} and the amplitude P_{m_J} of the peaks according to the SV theory;[21] $H/H_0 = 2.5$, $E_{mJ} \approx (\omega_{mJ} - \omega_{00})/gH_0 - 2(c_1/c_2)^2$.

$$\frac{H_0}{H} \approx \frac{2[(\omega_{+2}-\omega_{+1})-(\omega_{-1}-\omega_{-2})]}{(\omega_{+2}-\omega_{+1})-(\omega_{-1}-\omega_{-2})} \quad (\gamma=\pi/2), \qquad (6b)$$

where ω_{m_J} is the frequency of each m_J-th peak. These relations are difficult to use directly because for $\gamma = 0$ sound can excite only the state with $m_J = 0$ and, conversely, for $\gamma = \pi/2$ there is no resonant absorption for $m_J = \pm 1$.[19] Moreover, the nonlinear corrections to the resonant frequencies[13] (not described by the SV theory) become quite large for the value of the external magnetic field employed in the experiment. All this makes it difficult to obtain a numerical value of H_0 from the formulas (6a) and (6b). By using the experimental data obtained in the weakest magnetic field we were nonetheless able to estimate H_0, $H_0 \approx 15$ mT, which is somewhat less than the theoretically predicted[20] value of 25 mT. Figure 6c shows the theoretically predicted γ dependence of the amplitudes of the absorption peaks.[22]

In order to be able to make comparisons with the predictions of the SV theory a correspondence between the effective value γ_{eff} and the rotational velocity Ω of the sample must be established. Such a relation was obtained by comparing the theoretical and experimental dependence of the amplitudes of the absorption peaks in the limit $\mathbf{w} = 0$. The result is presented in Fig. 7: at first γ_{eff} increases linearly as Ω increases, but then it has a tendency to saturate. Since the energy of the flow decreases with the pressure the threshold value of the transition to saturation should increase as the pressure increases, and this was indeed observed in the experiment.

It is easy to see that the character of the dependence obtained agrees qualitatively with that predicted theoreti-

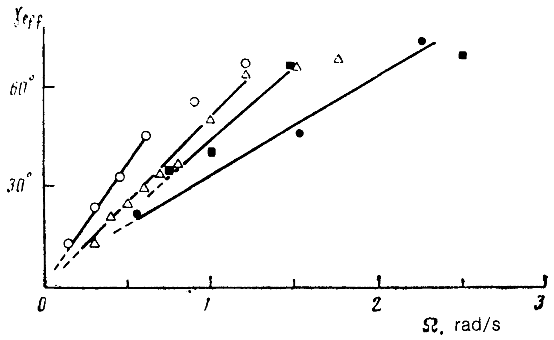

FIG. 7. γ_{eff} versus Ω for $H = 40$ mT: ○—$p = 2.3$ bar, $f = 44.7$ MHz; △—$p = 3.0$ rad, $f = 44.7$ mT; ■-$p = 3.2$ bar, $f = 44.7$ MHz; ●—$p = 11.5$ bar, $f = 80.6$ MHz.

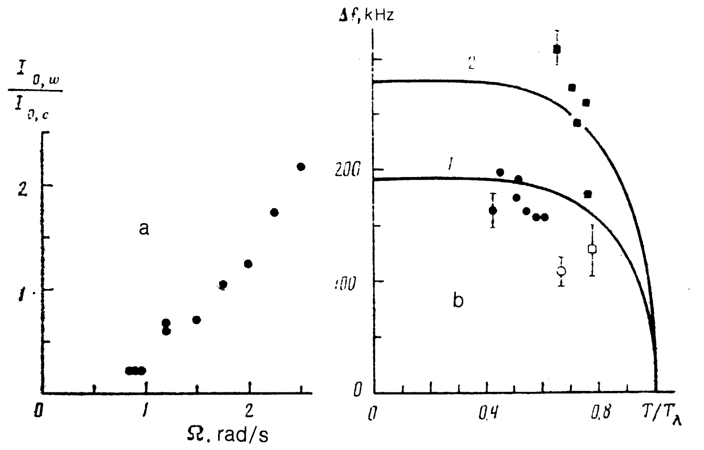

FIG. 8. The splitting of the $m_J = 0$ peak: a) ratio of the components of the doublet splitting as a function of Ω with $p = 3.0$ bar, $H = 40$ mT, and $f = 44.7$ MHz (see text for explanation); b) splitting of the peak according to our measurements at $p = 3.0$ bar (□) and 6.5 bar (○) as well as the measurements of Shivaram *et al.*[13] at $p = 1$ bar (■) and $p = 5$ bar (●). The theoretical curves are plotted following Ref. 20 for $p = 5$ (1) and 1 (2) bar.

cally. The quantitative discrepancy, however, can possibly be explained by the nonuniformity of the real texture of $\hat{\mathbf{h}}$ in the experimental cell.

The texture-induced doublet splitting of the peak $m_J = 0$ (see Fig. 4) probably arises for two reasons. First, the experimental cell always contains a region in which the $\hat{\mathbf{h}}$ axis is approximately constant; in such regions ^{3}He makes the main contribution to the absorption of sound. In the central part of the cell we have $\hat{\mathbf{h}} \| \hat{\mathbf{q}}$ ($\gamma \approx 0$), while near the walls we have $\hat{\mathbf{h}} \perp \hat{\mathbf{q}}$ ($\gamma \approx \pi/2$). The different relative orientations of the vectors $\hat{\mathbf{h}}$ and $\hat{\mathbf{q}}$ in these regions give different resonance frequencies for one and the same value of m_J; this follows from Eq. (5). Second, a necessary condition for splitting is that the amplitude of the absorption peak corresponding to a given value of γ in one or another region must not vanish. One can see from Fig. 6 that this condition is satisfied only for the $m_J = 0$ peak: the amplitude of the $m_J = \pm 1$ peaks vanishes for both $\gamma = 0$ and $\gamma = \pi/2$ and the amplitude of the $m_J = \pm 2$ peaks vanishes for $\gamma = 0$. This explains why peaks with $m_J = \pm 1$ and $m_J = \pm 2$ are not split. Doublet splitting of all five states is possible only in a texture of the "flare-in" type,[29] which can probably be realized only if vortices are present and if the rotational velocities Ω are high enough. We also note that when $\hat{n}$-solitons are present[35] triplet and even quadrupole splitting of all lines of the spectrum of the RSQ mode is possible,[36] and the multiplet splitting can reach approximately 1/3 of the splitting of the central peak.

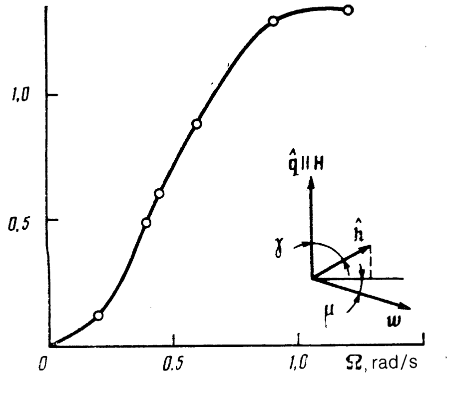

FIG. 9. Q versus the rotational velocity Ω. See text for explanation.

However no indications of splitting of lines with $m_J \neq 0$ were observed up to rotational velocities of 4 rad/s.

As Ω increases the relative amplitude of the signal from the central part of the cell decreases, while the amplitude of the signal from the region near the walls increases. The ratio of the amplitudes of these two components as a function of Ω is shown in Fig. 8a. Figure 8b shows the doublet splitting measured at pressures of 3.0 and 6.5 bar. Here the splitting is given in frequency units; this was obtained by comparing the splitting with the separations from peaks with $m_J = \pm 1$ and $m_J = \pm 2$, which are determined by the Zeemann splitting and are equal to $\Delta\omega_{\pm 1} = 2gH$ and $\Delta\omega_{\pm 2} = 4gH$, respectively. The values of the constant g were taken from Ref. 13. Figure 8b also shows the experimental data taken from Ref. 13 and the values predicted theoretically in Ref. 20. As one can see, our data agree well with earlier data, though the temperature dependence appears to be more linear than that predicted theoretically.

The observation of the so-called "gyrosonic" effect, predicted by Salomaa and Volovik[37] based on the results of Ref. 21, could serve as a direct confirmation of the excitation of the RSQ mode owing to the existence of counterflow. Namely, when the sample is rotated the $\mathbf{w}$ and $\hat{\mathbf{q}}$ vectors always have some definite orientation relative to $\hat{\mathbf{h}}$ at each point in the sample. The absorption of sound is determined by the quantity $Q = (\hat{\mathbf{q}}\cdot\hat{\mathbf{h}})(\mathbf{w}\cdot\hat{\mathbf{h}})$, i.e., it depends on the orientation of $\mathbf{w}$ and $\hat{\mathbf{q}}$. In other words, the spectrum of the RSQ mode must depend on the sign of $\hat{\boldsymbol{\Omega}}$ and $\mathbf{q}$ even when the sound propagates along the rotational axis of the sample.

To determine the best conditions for observing the gyrosonic effect we write Q in the form

$$Q = w \cos\gamma \sin\gamma \cos\mu,$$

where μ is the angle between the vector $\mathbf{w}$ and the plane containing $\hat{\mathbf{q}}$ and $\hat{\mathbf{h}}$ (see Fig. 9). We shall first study the dependence of μ on Ω.

In the first approximation we shall assume that the $\hat{\mathbf{n}}$-texture in the geometry of our experimental cell is of the type "flare-out." We write the components of $\hat{\mathbf{h}}$ in cylindrical coordinates:

$$h_r = \sin\beta\left(-{}^1/_4\cdot\sqrt{15}\sin\alpha - {}^5/_4\cos\alpha\cos\beta\right), \tag{7a}$$

$$h_\varphi = \sin\beta\left(-{}^1/_4\cdot\sqrt{15}\cos\alpha + {}^5/_4\sin\alpha\cos\beta\right), \tag{7b}$$

$$h_z = -{}^1/_4 + {}^5/_4\cos^2\beta, \tag{7c}$$

where α and β are the azimuthal and polar angles, respectively. Using the computational results obtained by Salomaa[29] and the experimental data of Ref. 24 we can obtain three pairs of average values of α and β: 1) $\langle\alpha\rangle = 57°$, $\langle\beta\rangle = 31°$ for $\Omega = 0$; 2) $\langle\alpha\rangle = 114°$, $\langle\beta\rangle = 46°$ for $\Omega = 0.3$ rad/s; 3) $\langle\alpha\rangle = 132°$, $\langle\beta\rangle = 60°$ for $\Omega = 1$ rad/s. Since $\mu = \tan^{-1}(h_r/h_\phi)$ it is easy to verify that $\cos\mu$ is close to unity even for $\Omega \approx 0.5$ rad/s.

Figure 9 shows the dependence of Q on Ω; the values of γ obtained from measurements performed at a pressure of 2.3 bar were used (see Fig. 7). It is easy to see that the rotational velocity $\Omega \approx 1$ rad/s corresponds to the maximum expected gyrosonic effect. Of course, there is still some doubt that the texture under our experimental conditions is indeed of the "flare-out" type, since, as one can see from Fig. 7, $\gamma_{\rm eff}$ approaches zero for small values of Ω, while for the true texture of this type this quantity must be of the order of 50° [see the expression (7c)].

We employed two methods to observe the gyrosonic effect. In the first method we fixed the direction of propagation $\hat{\mathbf{q}}$ of the sound wave and varied the direction of rotation of the cryostat. In the second method we reversed the procedure, i.e., we periodically changed the direction of propagation of the sound during one rotation cycle. In principle both methods should be equivalent, but we found that the second method is preferable at high pressures, when the reproducibility of the texture from one rotation cycle to another left much to be desired.

Figure 10 shows the amplitudes of the peaks of the RSQ mode as a function of the rotational velocity Ω of the cryostat for different combinations of the orientations of $\hat{\mathbf{\Omega}}$ and $\hat{\mathbf{q}}$. Because, for reasons we do not understand, the quartz radiators (receivers) or the coaxial feed lines were not identical we found that the absolute values of the amplitudes of the peaks for two orientations of $\hat{\mathbf{q}}$ differ systematically by $\approx 30\%$. For this reason we employed for the analysis only the data obtained by the first method. The existence of the gyrosonic effect in this case would indicate that when the direction of rotation is changed the relative order of the peaks in the recorded spectrum would change correspondingly. However we did not observe any difference, outside of the experimental error, in the obtained spectra.

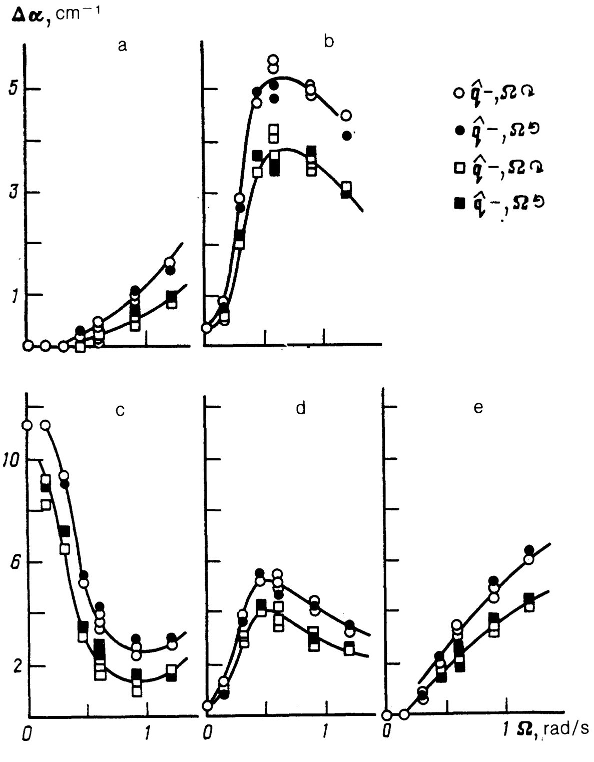

FIG. 10. The amplitudes of the absorption peaks of the RSQ mode for different orientations of $\hat{\mathbf{q}}$ and $\hat{\mathbf{\Omega}}$ with $p = 2.3$ bar, $H = 40$ mT, $f = 44.7$ MHz: a–$m_J = -2$, b–$m_J = -1$, c–$m_J = 0$, d–$m_J = +1$, e–$m_J = +2$; the amplitude of the $m_J = 0$ peak was taken to be equal to the sum of the amplitudes of the doublet components: circles for $\hat{\mathbf{q}}^+$ and squares for $\hat{\mathbf{q}}^-$.

With the help of the SV theory we estimate the upper limit of the ratio w/w_0, where w_0 is the velocity of the counterflow for which the two mechanisms excite the RSQ mode with equal efficiency: asymmetry of the hole–quasiparticle system and flow of the superfluid component (see Sec. 1). For $\gamma = 45°$, taking the maximum spread of the experimental points ($\approx 30\%$ for $m_J = \pm 2$ peaks and $\Omega = 0.6$ rad/s), we obtain $w/w_0 \leqslant 0.14$. Since for this rotational velocity the maximum value of w in the experimental cell is equal to 1.8 mm/s we obtain $w_0 \geqslant 13$ mm/s, which is significantly larger than the theoretical estimate $w_0 \approx 1$ mm/s of Sauls and Serene.[21] Nevertheless, as we recently learned, the authors of Ref. 21 recently refined the calculation presented there, after which their estimate of w_0 increased to 6 mm/s. This already practically eliminates the discrepancy between experiment and theory. We also note that the nonuniformity of the texture of ^{3}He under the conditions of the experiment could also cause the observed gyrosonic effect to be absent.

In conclusion we can say that measurements of the spectra of the RSQ mode in rotating ^{3}He-B have made it possible to establish how the amplitudes of the absorption peaks depend on the rotational velocity of the sample. The observed dependence is in good qualitative agreement with the predictions of the phenomenological theory of Salomaa and Volovik.[22] To make a quantitative comparison between the experimental results and the theory it is probably necessary to perform calculations taking into account the nonuniformity of the real texture. We also found that because of the effect of the counterflow of the superfluid component the RSQ mode is not excited by the sound as efficiently as the theory predicts.

We thank P. Bergland, G. E. Volovik, M. Krusius, O. V. Lounasmaa, V. P. Mineev, M. M. Salomaa, G. A. Kharadze, and Z. Yan for their contribution to this work and for valuable discussions. Two of us (A.V.B. and G.K.T.) are sincerely grateful to the personnel at the Low-Temperature Laboratory for their hospitality. This work was performed under the auspices of the joint scientific project ROTA of the Academy of Finland and the Academy of Sciences of the USSR.

[1)] P. N. Lebedev Physical Institute of the Academy of Sciences of the USSR, Moscow.
[2)] Institute of Physics of the Academy of Sciences of the Georgian SSR, Tbilisi, USSR.
[3)] PLM-3 NMR thermometer, Instruments for Technology, Finland. A new model of this thermometer is being produced by the firm RV-Elektroniikka Oy, Veromiehentie 14, SF-015110 Vantaa, Finland.

[1]W. P. Wolfe, Physica B **90**, 96 (1977).
[2]W. P. Halperin, Proceedings of the 16th International Conference on Low Temperature Physics, edited by W. G. Clark, North-Holland, Am-

sterdam (1981), P. III, p. 1596.
[3]J. B. Ketterson, B. S. Shivaram, M. V. Meisel *et al.*, *Quantum Fluids and Solids–1983,* edited by E. D. Adams and G. G. Ihas, American Institute of Physics, New York (1983), p. 288.
[4]R. H. Salmelin, J. P. Pekola, A. J. Manninen *et al.*, Phys. Rev. Lett. **63**, 620 (1989).
[5]J. P. Pekola, R. H. Salmelin, A. J. Manninen *et al.*, *Quantum Fluids and Solids 1989,* edited by G. G. Ihas and Y. Takano, American Institute of Physics, New York (1989), p. 63.
[6]D. M. Lee and R. C. Richardson, *Physics of Liquid and Solid Helium,* edited by K. H. Benneman and J. B. Ketterson, Wiley, New York (1978), p. 287.
[7]K. Maki, J. Low Temp. Phys. **24**, 755 (1976).
[8]N. Schopohl and L. Tewordt, *ibid.* **45**, 67 (1981).
[9]V. E. Koch and P. Wolfe, Phys. Rev. Lett. **46**, 486 (1981).
[10]R. W. Giannetta, A. Ahonen, E. Polturak *et al., ibid.* **45**, 262 (1980).
[11]D. B. Mast, B. K. Sarma, J. R. Owers-Bradly *et al., ibid.* **45**, 266 (1980).
[12]O. Avenel, E. Varoquaux, and H. Ebisawa, *ibid.* **45**, 1952 (1980).
[13]B. S. Shivaram, M. W. Meisel, B. K. Sarma *et al., ibid.* **49**, 1646 (1982).
[14]L. Tewordt and N. Schopohl, J. Low Temp. Phys. **37**, 421 (1979).
[15]N. Schopohl, M. Warnke, and L. Tewordt, Phys. Rev. Lett. **50**, 1066 (1983).
[16]R. S. Fishman and J. A. Sauls, Phys. Rev. B **33**, 6068 (1986).
[17]P. N. Bryusov and V. N. Popov, Zh. Eksp. Teor. Fiz. **78**, 2419 (1980) [Sov. Phys. JETP **51**, 1217 (1980)].
[18]G. E. Volovik, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 365 (1984) [JETP Lett. **39**].
[19]V. P. Mineev, Zh. Eksp. Teor. Fiz. **88**, 507 (1985) [Sov. Phys. JETP **61**(2), 297 (1985)].
[20]R. S. Fishman and J. A. Sauls, Phys. Rev. Lett. **61**, 2871 (1988).
[21]J. A. Sauls and J. W. Serene, Proceedings of the 17th International Conference on Low Temperature Physics, edited by U. Eckern, A. Schmid, W. Weber, and H. Wuhl, Elsevier, Amsterdam (1984), p. 775.
[22]M. M. Salomaa and G. E. Volovik, J. Low Temp. Phys. **77**, 17 (1989).
[23]R. H. Salmelin, J. M. Kyynardinen, M. P. Berglund, and J. P. Pekola, *ibid.* **76**, 83 (1989).
[24]P. J. Hakonen, M. Krusius, M. M. Salomaa *et al., ibid.* **76**, 225 (1989).
[25]H. Smith, W. F. Brinkman, and S. Engelsberg, Phys. Rev. B **15**, 199 (1977).
[26]G. F. Spencer and G. G. Ihas, Phys. Rev. Lett. **48**, 1118 (1982).
[27]P. J. Hakonen and K. K. Nummila, *ibid.* **59**, 1006 (1987).
[28]M. M. Salomaa, J. Phys. C (1990) (in press).
[29]A. D. Gongadze, Z. Yanu, Y. Kondo *et al.*, Phys. Rev. Lett. (1990) (in press).
[30]D. S. Greywall, Phys. Rev. B **27**, 7520 (1986).
[31]S. Adenvwalla, Z. Zhao, J. B. Ketterson, and B. K. Sarma, J. Low Temp. Phys. **76**, 1 (1989).
[32]P. N. Fraenkel, R. Keolian, and J. D. Reppy, Phys. Rev. Lett. **62**, 1126 (1989).
[33]T. A. Alvesalo, T. Haavasoja, M. T. Manninen, and A. T. Soinne, *ibid.* **44**, 1076 (1980).
[34]J. W. Serene and D. Rainer, Phys. Rep. **101**, 221 (1983).
[35]K. Maki and P. Kumar, Phys. Rev. B **16**, 4805 (1977).
[36]V. N. Kiviladze and G. A. Kharadze, Fiz. Nizk. Temp. **14**, 1235 (1988) [Sov. J. Low Temp. Phys. **14**, 683 (1988)].
[37]M. M. Salomaa and G. E. Volovik, in *Quantum Fluids and Solids,* 1989, edited by G. G. Ihas and Y. Takano, American Institute of Physics, New York (1989), p. 115.

Translated by M. E. Alferieff