Effect of superconducting fluctuations on the thermoelectric force and thermal conductivity of a superconductor near the critical temperature

A. A. Varlamov and D. V. Livanov

Moscow Institute of Steel and Alloys
(Submitted 23 January 1990)
Zh. Eksp. Teor. Fiz. **98**, 584–592 (August 1990)

We discuss the effect of superconducting fluctuations on the thermoelectric force and thermal conductivity of a superconductor in the vicinity of the critical temperature. To do this, we first find an expression for the thermal flux operator of the system of interacting electrons in the Cooper channel. Because our intention is to compare our results with experimental data on high-temperature superconductors, our final expressions are formulated for the case of a quasi-two-dimensional electron spectrum, allowing us to investigate both the two-dimensional and three-dimensional regimes of fluctuation behavior, as well as the crossover between them. We find that including the contributions from superconducting fluctuations leads to the appearance of a characteristic peak in the temperature dependence of the differential thermoelectric force of a superconductor near T_c, which is in agreement with the results of recent experimental investigations of high-temperature superconductors.

1. INTRODUCTION

In connection with global investigations of the high-temperature superconductors (HTSC), researchers have become interested in the thermoelectric and thermal conductivity properties of these compounds, as witnessed by a number of experimental papers[1–5] on the temperature dependence of the thermoelectric force and thermal conductivity of high-temperature superconductors near their critical temperatures. These authors found a peak[1–3] in the temperature dependence of the thermoelectric force as the HTSC passed from normal to superconducting; this peak has not yet received a rigorous theoretical explanation.

It should be noted that the question of how superconducting fluctuations can influence the thermoelectric force of a superconductor near T_c was investigated earlier by Maki.[6] However, as we will show below, the principal contribution to the fluctuation-induced correction to the thermoelectric coefficient was missed in this paper. The authors of a recent experimental paper[1] relied on the theory in Ref. 6, but, finding it impossible to explain their experimental results within the framework of this theory, they had to settle for a semi-qualitative analysis of the phenomena and of their results.

Thus, the question of how to construct a microscopic theory from first principles that explains the anomalous behavior of the thermoelectric power of a superconductor near the critical temperature remains open. With regard to measurements of the temperature dependence of the thermal conductivity of the HTSC, Aliev *et al.*[4,5] noted an increase in the absolute value of the thermal conductivity as the temperature T_c was approached from above.

In this paper we will investigate the influence of superconducting fluctuations on the electronic part of the thermoelectric force and thermal conductivity of a superconductor at temperatures somewhat above the critical temperature. Because we intend to apply the theory we develop here to explain the properties of high-temperature superconductors, we will treat the electron spectrum as quasi-two-dimensional. In any calculation of the thermoelectric force it is very important to determine the role of electron scattering. To do this, we must compare the correlation length ξ with the mean free path l of the electrons. For the HTSC, both these quantities are extremely small, e.g., ξ in superconductors of BISCO type is estimated to be 15–20 Å, while l for the same single-crystal systems is estimated to be 80 to 90 Å from an extrapolation of the residual resistance down to $T = 0$. It would appear that these numbers give us a basis for treating single-crystals of BISCO as clean rather than dirty superconductors. However, noting that the resistance (and, consequently, the mean free path) of these materials is apparently strongly temperature-dependent, we should admit the possibility that near T_c the mean free path and the correlation length may turn out to be of the same order of magnitude.

As will be clear from what follows, calculations of the fluctuation-induced corrections to the thermoelectric force and thermal conductivity are greatly simplified for the case of a clean superconductor. For the most part, we will be investigating this type of superconductor in what follows; however, the estimate we arrive at here show that the observable effects (e.g., the growth in absolute magnitude of the thermoelectric force and thermal conductivity as the temperature T_c is approached) are qualitatively unchanged for the case of a dirty superconductor as well.

2. THE HEAT CURRENT OPERATOR IN THE PRESENCE OF ELECTRON-ELECTRON INTERACTIONS IN THE COOPER CHANNEL

In order to calculate the fluctuation-induced corrections to the coefficient of the thermoelectric force and thermal conductivity by the linear-response method, we must first find an expression for the heat flux operator that takes into account the presence in the system of electron–electron interactions in the Cooper channel.

There exist several approaches to this problem. We will follow the approach developed in Ref. 7 to investigate the influence of electron–phonon interactions on the thermoelectric force of impure metals, as this approach seems to us

 0038-5646/90/080325-06$03.00

to be most closely tied to first principles.

We write the Lagrangian of the system of interacting electrons in the form

$$\mathscr{L}=i(\Psi^+\dot{\Psi}+\dot{\Psi}^+\Psi)-(1/2m)\Psi_{,i}^+\Psi_{,i}+\lambda\Psi^+\Psi'^+\Psi'\Psi, \quad (1)$$

where Ψ is a field operator, $\Psi = \Psi(\mathbf{x},t)$, and $\Psi' = \Psi(\mathbf{x}',t)$; m is the mass of an electron, $\Psi_{,i} = \partial\Psi/\partial x_i$, $i = 1, 2, 3$; $\dot{\Psi} = \partial\Omega/\partial t$, and λ is the interaction constant.

The Hamiltonian of the system H and the energy current operator $\mathbf{u}$ can be obtained from the energy momentum tensor T_μ^ν,

$$T_\mu^\nu=\frac{\partial\mathscr{L}}{\partial\Psi_{,\nu}}\Psi_{,\mu}+\Psi_{,\mu}^+\frac{\partial\mathscr{L}}{\partial\Psi_{,\nu}^+}-\delta_\mu^\nu\mathscr{L} \quad (2)$$

(δ_μ^ν is the Kronecker symbol) by integrating the corresponding components over the space variables:

$$H=\int T_0^0 d^3x d^3x', \quad (3)$$

$$u^i=\int T_0^i d^3x d^3x'. \quad (4)$$

Expressing the field operators $\Psi(\mathbf{x},t)$ in the usual way in terms of the second quantization operators $a_\mathbf{p}(t)$:

$$\Psi(x,t)=\sum_\mathbf{p} a_\mathbf{p}(t)e^{i\mathbf{p}\mathbf{x}}, \quad (5)$$

and substituting into Eqs. (1)–(3), we find

$$H=\sum_\mathbf{p} a_\mathbf{p}^+a_\mathbf{p}\varepsilon_\mathbf{p}-\lambda\sum_{\mathbf{p}\mathbf{p}'}a_\mathbf{p}^+a_{\mathbf{p}'}^+a_{\mathbf{p}'}a_\mathbf{p} \quad (6)$$

($\varepsilon_\mathbf{p}$ is the energy of an electron). Since we know the explicit form of the Hamiltonian, we can write the equation of motion for the second-quantized operators:

$$i\dot{a}_\mathbf{p}=[a_\mathbf{p},H]=\varepsilon_\mathbf{p}a_\mathbf{p}-\lambda\sum_{\mathbf{p}'}a_{\mathbf{p}'}^+a_{\mathbf{p}'}a_\mathbf{p}. \quad (7)$$

The energy current operator now can be obtained by substituting Eqs. (5) and (7) into Eqs. (1), (2), and (4). The thermal current operator that we require is connected with the energy current operator by the relation

$$\mathbf{J}^h=\mathbf{u}-(\mu/e)\mathbf{J}^e, \quad (8)$$

where e is the electron charge, μ is the chemical potential, and

$$\mathbf{J}^e=\frac{e}{m}\sum_\mathbf{p} a_\mathbf{p}^+a_\mathbf{p}\mathbf{p} \quad (9)$$

is the electric current operator.

After some computation, we obtain the final expression

$$\mathbf{J}^h=\sum_\mathbf{p}\xi_\mathbf{p}\mathbf{v}_\mathbf{p}a_\mathbf{p}^+a_\mathbf{p}-\frac{\lambda}{2}\sum_{\mathbf{p}\mathbf{p}'}(\mathbf{v}_\mathbf{p}+\mathbf{v}_{\mathbf{p}'})a_\mathbf{p}^+a_{\mathbf{p}'}^+a_{\mathbf{p}'}a_\mathbf{p}, \quad (10)$$

where $\xi_\mathbf{p} = \varepsilon_\mathbf{p} - \varepsilon_F$ is the energy of an electron measured from the Fermi level.

3. LINEAR RESPONSE OPERATOR AND DIAGRAMMATIC ANALYSIS

In the method of linear response, the coefficients of the thermoelectric force and thermal conductivity are determined by the relations

$$\beta_{ik}=\frac{1}{-i\omega}Q_{ik(e-h)}^R(\omega), \quad \varkappa_{ik}=\frac{1}{-i\omega}Q_{ik(h-h)}^R(\omega), \quad (11)$$

where Q_{ik}^R is the analytic continuation into the upper half of the complex frequency plane of the Fourier transforms of the following correlators: for the thermoelectric force, the correlator of the electron current and the thermal flux operators, and for the thermal conductivity, the correlator of two thermal flux operators:

$$\begin{aligned} Q_{ik\,(e-h)}(X-X')&=-\theta(t-t')\langle[\tilde{J}_i^h(X),\ \tilde{J}_k^e(X')]\rangle, \\ Q_{ik\,(h-h)}(X-X')&=-\theta(t-t')\langle[\tilde{J}_i^h(X),\ \tilde{J}_k^h(X')]\rangle, \end{aligned} \quad (12)$$

here $\tilde{\mathbf{J}}^h(X)$ and $\tilde{\mathbf{J}}^e(X)$ are operators in the Heisenberg representation, $X = (\mathbf{x},t)$, $\theta(x)$ is the Heaviside step function, and the angle brackets denote averaging over a grand canonical ensemble.

Equation (10) implies that the heat flux operator must be associated with two vertices in the diagram technique (Fig. 1):

$$\Gamma_i^{h(0)}=\xi_\mathbf{p}v_i, \quad \Gamma_i^{h(int)}=-q_i/2m, \quad (13)$$

the vertex corresponding to the electric current operator is written as usual: $\Gamma_i^e = ev_i$. Note that taking into account the interaction of the electrons in the Cooper channel leads to the appearance of an additional vertex $\Gamma_i^{h(int)}$ and to additional diagrams over and above those included in calculating the fluctuation-induced conductivity.

The diagrams that appear in the course of calculating the thermoelectric force in first-order perturbation theory with superconducting fluctuations taken into account are shown in Fig. 2, while for the thermal conductivity they are shown in Fig. 3. Investigation of diagrams containing the vertex $\Gamma_i^{h(int)}$ (Fig. 2d and Figs. 3d–3e) shows that these diagrams are less singular as the temperature approaches T_c than the other diagrams; this is because this vertex contains the small momentum $q \ll p$. This implies, as in the case of the conductivity, that the principle contribution near T_c to the thermoelectric force coefficient and the thermal conductivity of a clean superconductor comes from diagrams of the Aslamazov–Larkin type (Fig. 2c and Fig. 3c), which we will also discuss in the subsequent sections.

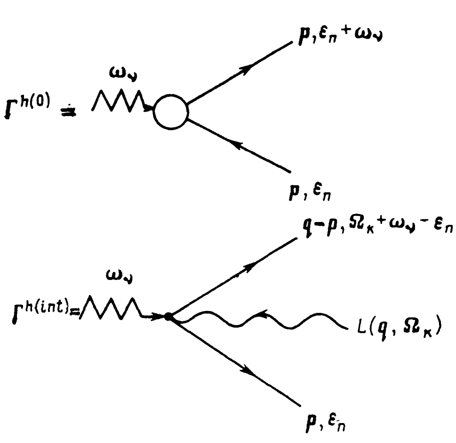

FIG. 1. Field-theoretic vertices corresponding to the thermal current operator.

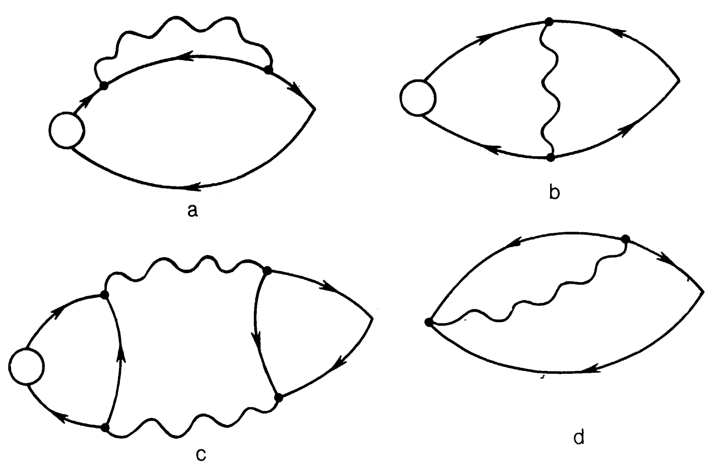

FIG. 2. Diagrams which arise in computing the thermoelectric power. For symmetry-related graphs we show only one diagram.

4. FLUCTUATION-INDUCED CORRECTION TO THE THERMOELECTRIC FORCE

Keeping in mind that the majority of HTSC studied to date have a layered structure, we assume that the electron spectrum is quasi-two-dimensional. In the limiting cases of strong and weak coupling between the layers, this also allows us to automatically obtain results for the two-dimensional and three-dimensional cases.

Thus, for a Fermi surface in the form of a fluted cylinder, the electron energy can be written in the form

$$\xi_{\mathbf{p}}=\varepsilon_{\mathbf{p}}-\varepsilon_F=v_0(|\mathbf{p}_{\parallel}|-p_0)+w\cos(p_{\perp}a), \tag{14}$$

where v_0 is the Fermi velocity in the plane of the layer, w is the overlap integral that characterizes the probability for an electron to hop between layers, a is the spacing between layers, and $\mathbf{p}=(\mathbf{p}_{\parallel},p_{\perp})$ is the quasimomentum of an electron.

As we have already noted in the Introduction, we will be interested in the case of a clean superconductor, so that we can ignore impurity scattering of the electrons. Then the single-electron Green's function for temperatures above the critical temperature can be written in the usual form with the spectrum (14):

$$G(\mathbf{p},\ \varepsilon_n)=1/(i\varepsilon_n-\xi_{\mathbf{p}}), \tag{15}$$

while the fluctuation-induced propagator[8] is

$$L(\mathbf{q},\Omega_k)=-\frac{1}{\rho}\left[\ln\frac{T}{T_c}+\psi\left(\frac{1}{2}+\frac{|\Omega_k|}{4\pi T}+\alpha_{\mathbf{q}}\right)-\psi\left(\frac{1}{2}\right)\right]^{-1}, \tag{16}$$

where

$$\alpha_{\mathbf{q}}=\frac{6\eta}{\pi^2 v_0^2}\langle(\mathbf{vq})^2\rangle,\quad \eta=\frac{7\zeta(3)}{48\pi^2}\frac{v_0^2}{T_c^2},$$

η is the parameter of the Landau–Ginzburg theory for the case of a clean superconductor near T_c, $\zeta(x)$ is the Riemann zeta function, $\psi(x)$ is the logarithmic derivative of the gam-

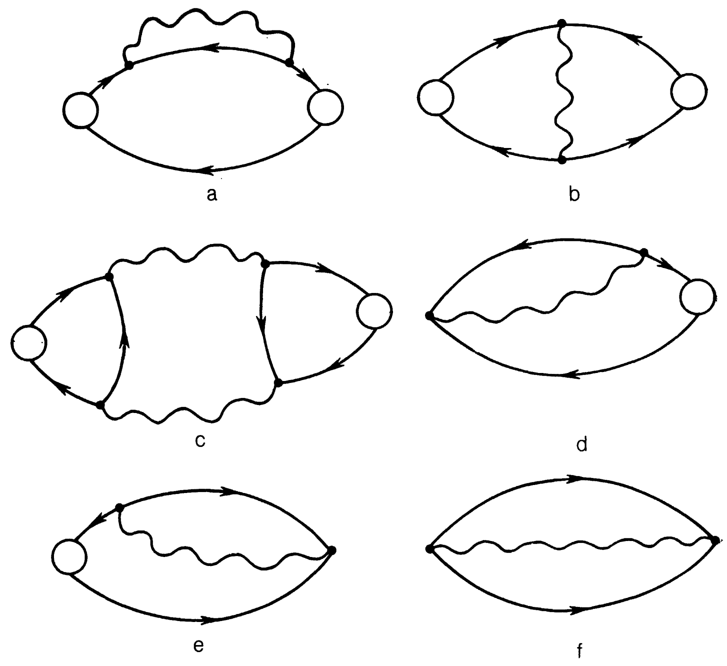

FIG. 3. Diagrams which arise in computing the thermal conductivity. For symmetry-related graphs we show only one diagram.

ma function, and ρ is the density of state at the Fermi level.

Let us begin with a calculation of the thermoelectric coefficient. We will write the contribution to the linear response operator corresponding to diagram 2c in the form

$$Q_{ij(e-h)}(\omega_\nu)=-2eT\sum_{\Omega_k}\int\frac{d^3q}{(2\pi)^3}B_{1i}(\mathbf{q},\Omega_k,\omega_\nu)$$
$$\times B_{2j}(\mathbf{q},\Omega_k,\omega_\nu)L(\mathbf{q},\Omega_k+\omega_\nu)L(\mathbf{q},\Omega_k), \qquad (17)$$

where the block Green's functions $\mathbf{B}_1$ and $\mathbf{B}_2$ have the form

$$\mathbf{B}_1(\mathbf{q},\Omega_k,\omega_\nu)=T\sum_{\varepsilon_n}\int\frac{d^3p}{(2\pi)^3}\mathbf{v}G(\mathbf{p},\varepsilon_n+\omega_\nu)$$
$$\times G(\mathbf{p},\varepsilon_n)G(\mathbf{q}-\mathbf{p},\Omega_k-\varepsilon_n), \qquad (18)$$

$$\mathbf{B}_2(\mathbf{q},\Omega_k,\omega_\nu)=T\sum_{\varepsilon_n}\int\frac{d^3p}{(2\pi)^3}\xi_{\mathbf{p}}\mathbf{v}G(\mathbf{p},\varepsilon_n+\omega_\nu)$$
$$\times G(\mathbf{p},\varepsilon_n)G(\mathbf{q}-\mathbf{p},\Omega_k-\varepsilon_n). \qquad (19)$$

Near T_c we can neglect the dependence of the blocks $\mathbf{B}_1$ and $\mathbf{B}_2$ on the frequency Ω_k. The expression for the block $\mathbf{B}_1$ is known:

$$\mathbf{B}_1(\mathbf{q},0,\omega_\nu)=-\frac{1}{4\pi^2}\langle\mathbf{v}(\mathbf{vq})\rangle\frac{\rho}{T_c^2}\psi''\left(\frac{1}{2}\right), \qquad (20)$$

where $\langle\mathbf{v}(\mathbf{vq})\rangle = 1/2v_0^2\mathbf{q}_\parallel$. For the block $\mathbf{B}_2$, after some uncomplicated calculations, we find

$$\mathbf{B}_2(\mathbf{q},0,\omega_\nu)=\frac{i\rho}{32T_c}\langle\mathbf{v}(\mathbf{vq})\rangle-\frac{1}{8\pi}\langle\mathbf{v}(\mathbf{vq})\rangle\left(\frac{\partial\rho}{\partial\varepsilon}\right)_{\varepsilon_F}\ln\frac{\omega_D}{2\pi T_c}, \qquad (21)$$

where ω_D is the Debye frequency. Substitution of the first term on the right side of Eq. (21) into Eq. (17) gives a contribution to the thermoelectric force that cancels the contributions of the remaining diagrams in Fig. 2, which are calculated under the condition $\rho=$ constant. From this we see that the basic contribution to the thermoelectric force is given by the second term on the right side of Eq. (21).

Subsequent substitution of the blocks (20) and (21) into Eq. (17) gives

$$Q_{\parallel(e-h)}(\omega_\nu)=-\frac{e}{64\pi^3}\psi''\left(\frac{1}{2}\right)\frac{v_0^4}{T_c}\rho\left(\frac{\partial\rho}{\partial\varepsilon}\right)_{\varepsilon_F}\ln\frac{\omega_D}{2\pi T_c}$$
$$\times\sum_{\Omega_k}\int\frac{d^3q}{(2\pi)^3}\mathbf{q}_\parallel^2L(\mathbf{q},\Omega_k+\omega_\nu)L(\mathbf{q},\Omega_k). \qquad (22)$$

Reducing the sum in Eq. (22) over Ω_k to a contour integral, carrying out the analytic continuation in frequency ω_ν in the usual way,[9] and integrating over frequency, we find

$$Q_{\parallel(e-h)}(\omega)=-i\omega\frac{e}{2^{10}\pi^2}\psi''\left(\frac{1}{2}\right)\frac{v_0^4}{T_c^2}\rho^{-1}\left(\frac{\partial\rho}{\partial\varepsilon}\right)_{\varepsilon_F}$$
$$\times\ln\frac{\omega_D}{2\pi T_c}\int\frac{d^3q}{(2\pi)^3}\mathbf{q}_\parallel^2\left[\frac{T-T_c}{T_c}+\frac{3}{2}\eta\mathbf{q}_\parallel^2+\delta_0^2\sin^2\frac{q_\perp a}{2}\right]^{-3}, \qquad (23)$$

where

$$\delta_0^2=7\zeta(3)w^2/8\pi^2T_c^2$$

is the quasi-two-dimensional parameter, which determines the effective dimension of the fluctuations. Finally, carrying out the integration over momentum $\mathbf{q}$ in Eq. (3) over a single period of the fluted cylinder and making use of Eq. (11), we obtain the required correction to the thermoelectric coefficient:

$$\frac{\beta_{fl}}{\beta_n}=\frac{9\pi^2}{14\zeta(3)}\frac{T_c}{\varepsilon_F}\frac{1}{T_c\tau}\ln\frac{\omega_D}{2\pi T_c}\left[\frac{T-T_c}{T_c}\left(\frac{T-T_c}{T_c}+\delta_0^2\right)\right]^{-1/2}, \qquad (24)$$

where

$$\beta_n=-\frac{e}{3}v_0^2T\tau\left(\frac{\partial\rho}{\partial\varepsilon}\right)_{\varepsilon_F}$$

is the thermoelectric force of a normal layered metal in the plane of the layers.

Making use of the result Eq. (9) for the fluctuation-induced conductivity of a layered superconductor,

$$\frac{\sigma_{fl}}{\sigma_n}=\frac{\pi^4}{56\zeta(3)}\frac{T_c}{\varepsilon_F}\frac{1}{T_c\tau}\left[\frac{T-T_c}{T_c}\left(\frac{T-T_c}{T_c}+\delta_0^2\right)\right]^{-1/2}, \qquad (25)$$

we can also write down an expression for the differential thermoelectric force calculated to first order and including the superconducting fluctuations:

$$Q=Q_n+Q_{fl}=\frac{\beta_n+\beta_{fl}}{\sigma_n+\sigma_{fl}},\quad \frac{Q_{fl}}{Q_n}=\frac{\beta_{fl}}{\beta_n}-\frac{\sigma_{fl}}{\sigma_n}. \qquad (26)$$

It is clear from a comparison of Eqs. (24) and (25) that the relative correction to the thermoelectric coefficient due to the fluctuations exceeds by the large logarithm the fluctuation-induced contribution to the conductivity and gives the primary contribution to the differential thermoelectric force:[1]

$$\frac{Q_{fl}}{Q_n}=\frac{9\pi^2}{14\zeta(3)}\frac{T_c}{\varepsilon_F}\frac{1}{T_c\tau}\ln\frac{\omega_D}{2\pi T_c}\times\begin{cases}\dfrac{1}{\delta_0}\left(\dfrac{T_c}{T-T_c}\right)^{1/2}, & \delta_0^2\gg\dfrac{T-T_c}{T_c},\\[2ex] \dfrac{T_c}{T-T_c}, & \delta_0^2\ll\dfrac{T-T_c}{T_c}.\end{cases} \qquad (27)$$

As is clear for $T^*-T_c\sim T_c\delta_0^2$ (where T^* is the temperature at which the size of the fluctuation-induced Cooper pair transverse to the layer is comparable to the spacing between the layers), the character of the temperature dependence changes from three-dimensional to two-dimensional.

5. FLUCTUATION-INDUCED CORRECTIONS TO THE THERMAL CONDUCTIVITY

The calculations of the fluctuation-induced correction to the thermal conductivity are analogous to those described in the previous section. The linear response operator corresponding to the diagram 3c has the form

$$Q_{ij(h-h)}(\omega_\nu)=-2T\sum_{\Omega_k}\int\frac{d^3q}{(2\pi)^3}B_{2i}(\mathbf{q},\Omega_k,\omega_\nu)$$
$$\times B_{2j}(\mathbf{q},\Omega_k,\omega_\nu)L(\mathbf{q},\Omega_k+\omega_\nu)L(\mathbf{q},\Omega_k). \qquad (28)$$

Here, in contrast to the calculation of the thermoelectric coefficient, the primary contribution is determined by the first term involving the block $\mathbf{B}_2$. Substituting this term into Eq. (28) we calculate the sum over frequencies Ω_k as before; carrying out the analytic continuation and integrating over $\mathbf{q}$, we find

$$\frac{\varkappa_{fl}}{\varkappa_n}=\frac{9\pi^5}{128[7\zeta(3)]^2}\frac{T_c}{\varepsilon_F}\frac{1}{T_c\tau}\left[\frac{T-T_c}{T_c}\left(\frac{T-T_c}{T_c}+\delta_0^2\right)\right]^{-1/2}, \tag{29}$$

where $\varkappa_n = 1/3v_0^2\rho T\tau$ is the coefficient of thermal conductivity of a normal layered metal in the plane of the layers.

It is clear that the temperature dependence of this fluctuation-induced correction is entirely analogous to that of the conductivity and the thermoelectric force, and in the limiting cases of two-dimensional and three-dimensional behavior the fluctuations have the form

$$\frac{\varkappa_{fl}}{\varkappa_n}=\frac{9\pi^5}{128[7\zeta(3)]^2}\frac{T_c}{\varepsilon_F}\frac{1}{T_c\tau}\begin{cases}\dfrac{1}{\delta_0}\left(\dfrac{T_c}{T-T_c}\right)^{1/2}, & \delta_0^2\gg\dfrac{T-T_c}{T_c},\\[2ex] \dfrac{T_c}{T-T_c}, & \delta_0^2\ll\dfrac{T-T_c}{T_c}.\end{cases} \tag{30}$$

6. DISCUSSION OF RESULTS

As we already mentioned in the Introduction, in their investigations of HTSC near T_c the authors of a number of papers[1-3] reported observing a peak in the temperature dependence of the thermoelectric force. The temperature dependence of the excess thermoelectric force obtained in Ref. 1 is well described by the function $(T-T_c)^{-1/2}$, and the authors attributed this to the influence of superconducting fluctuations.

It is clear from our microscopic treatment of the fluctuation-induced contributions to the thermoelectric force for the case of fluctuations with three-dimensional behavior that exactly this type of growth of the thermoelectric power correction should be observed as we approach the temperature T_c from above. Since this result is obtained in first-order perturbation theory with the superconducting fluctuations taken into account, the power-law growth in the immediate vicinity of T_c is limited by the contributions of succeeding orders. Furthermore, we cannot ignore the inhomogeneity of the sample, which leads to a washing-out of the transition temperature and to smearing of fluctuation-induced effects. Clearly, all this implies that the increase in the thermoelectric force observed near T_c must be cut off; therefore, its temperature dependence passes through a maximum and then, after passing through T_c, the thermoelectric force rapidly decreases to zero, as it should in the superconducting phase. An estimate of the magnitude of the effect based on Eq. (27) shows that for compounds of the form YBCO the relative contribution of fluctuations near T_c can reach 20 to 30%.

With regard to experimental studies of the fluctuation-induced part of the electronic thermal conductivity, matters become much more complicated. First of all, the important thing here is the phonon thermal conductivity, which forms the background against which the fluctuation contributions $\varkappa_{fl}$ appear. Secondly, the electronic thermal conductivity itself does not change very sharply on passing through the critical temperature (in contrast to the behavior of the thermoelectric force), so that evidence of the fluctuation-induced effects is not so clear. Nevertheless, the authors of Refs. 4 and 5 noted the beginning of a growth in the thermal conductivity as the temperature was decreased down to 125–130 K; this temperature exceeds by 5–10 K the transition temperature measured by nulling the electrical resistivity. This growth can be associated with an observed fluctuation-induced contribution to the electron part of the thermal conductivity which, according to estimates based on Eq. (30), can reach a few percent near T_c.

Let us say a few more words about Ref. 6, in which the fluctuation-induced contribution to the thermoelectric force of a superconductor was calculated for temperatures above the critical temperature. In our opinion, this work contains several inaccuracies, which lead to the impossibility of explaining the available experimental material within the framework of theory presented there. First of all, the author of Ref. 6 took into account some necessarily small corrections $\sim T/\varepsilon_F$, which arise from the fluctuation propagator; however, he ignored similar terms which occur in the Green's function blocks and which also give important contributions to the thermoelectric coefficient β in the calculation carried out above. Secondly, in the transition to the differential thermoelectric force according to Eq. (26) it was taken for granted in Ref. 6 that the inequalities $\beta_{fl}\gg\beta_n$ and $\sigma_{fl}\gg\sigma_n$ hold near T_c; however, this assumption was completely contradicted in calculating the fluctuation-induced corrections only to first order in perturbation theory. As a result, the differential thermoelectric force obtained in Ref. 6 decreases monotonically to zero as the temperature T_c is approached.

In conclusion, let us say a few words about the "dirty" case, where $1\ll\xi$ holds. In this case, the diagrams for the linear response operator should be averaged over the positions of the impurities, which leads to a renormalization of the vertices and the Green's functions. Since it is necessary to take into account terms of order T/ε_F in order to obtain a nonzero result for the thermoelectric force, the corresponding expansions must be carried out for all quantities which depend on energy—the density of states, the relaxation time, etc. Within the framework of our approach, this is an extremely difficult problem. We have calculated only the correction associated with expanding the density of states, and we are convinced that the contribution from processes of the Aslamazov–Larkin type lead to the same temperature dependence that we have found in our discussion of clean superconductors given here. That is, following the usual theory of superconducting fluctuations, the small parameter T/ε_F in the coefficient in Eq. (26) is replaced by the corresponding parameter $1/\varepsilon_F\tau$ for the dirty superconductor case. The contribution from Maki–Thompson processes, in which only an expansion of the density of states is taken into account, turns out to equal zero.

In conclusion, the authors express their deep gratitude to A. I. Buzdin, M. Yu. Reizer, and A. V. Sergeev for valuable discussions and remarks

[1)] We note here that the logarithmic approximation $\ln(\omega_0/2\pi T_c)\gg1$, which is usual in the case of conventional superconductors, ceases to be valid for the high-temperature superconductors. This implies that for the case of a sufficiently high T_c we are not permitted to omit the contribution in Eq. (26) to the differential thermoelectric power, which comes from fluctuation-induced corrections to the conductivity and which can change not only the coefficient but in principle even the sign of the correction.

[1]M. A. Howson, M. B. Salamon, T. A. Friedmann *et al.*, Phys. Cond. Mat. **1**, 465 (1989).
[2]V. V. Moshchalkov, I. G. Mutik, N. A. Samarin *et al.*, Fiz. Nizk. Temp. **14**, 988 (1988) [Sov. J. Low Temp. Phys. **14**, 543 (1988)].
[3]S. Yan and Lu Qi Li, Solid State Commun. **65**, 355 (1988).
[4]G. F. Aliev, N. V. Brandt, and V. V. Moshchalkov, *Superconductivity: Physics, Chemistry and Engineering*, V. 2, Sect. 5, p. 29 (1989).
[5]G. F. Aliev, V. V. Pryadun, and L. I. Leonyuk, Preprint No. 26, Moscow State Univ. Physics Faculty, 1989.
[6]K. Maki, J. Low Temp. Phys. **14**, 419 (1974).
[7]A. Vilenkin and P. L. Tailor, Phys. Rev. B **10**, 5280 (1978).
[8]L. G. Aslamazov and A. A. Varlamov, J. Low Temp. Phys. **38**, 223 (1980).
[9]L. G. Aslamazov and A. I. Larkin, Fiz. Tverd. Tela (Leningrad) **10**, 1104 (1968) [Sov. Phys. Solid State **10**, 875 (1968)].

Translated by Frank J. Crowne