

# Mesoscopics in metals with magnetic impurities

A. A. Bobkov, V. I. Fal'ko, and D. E. Khmel'nitskiĭ

*Institute of Solid-State Physics, Academy of Sciences of the USSR*

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It is shown that in mesoscopic junctions with a large number of magnetic impurities (such that the spin scattering time  $\tau_s$  is shorter than the diffusion time  $\tau_f$  through the sample) the mesoscopic part of the resistance varies with time in a random manner, retracing the evolution of a localized spin system caused by spin-lattice relaxation and Korringa relaxation (over a time  $\tau_K$ ). The fluctuations in magnetoresistance measured with a direct current become self-averaged in time, taking the value  $(e^2/h)^2 (\tau_s/\tau_f)^{2-d/2}$ , while the excess noise  $S_{\text{ex}}(\omega)$  arising in the system has a characteristic frequency dispersion  $\omega_* \sim \tau_f/\tau_s \tau_K$ .

The kinetic coefficients of conductors of small dimensions depend on the details of the actual distribution of scatterers.<sup>1</sup> If some of the scatterers have a free spin, the resistance  $R$  also depends on their spin state. The relaxation of the spins of magnetic impurities therefore leads to a time variation in the mesoscopic part of the resistance.<sup>2</sup>

The aim of the present work is the elucidation of the nature of the time dependence of the fluctuations in magnetoresistance and also the noise associated with them.

1. We will assume that a diffusing electron probes a system of impurity spins, the relaxation of which takes place either by interaction with other electrons or by spin-lattice relaxation, but not related to the probing electron. At low temperatures the time for the inversion of an electron spin  $\tau_s$  is then appreciably shorter than the relaxation time of an impurity spin  $\tau_K$  ( $\tau_s \ll \tau_K$ ). We will also consider that in the diffusion time  $\tau_f = L^2/D$  of an electron through the specimen ( $L$  is the dimension of the conductor,  $D$  is the electron diffusion coefficient) it can reverse its spin many times. The scale of the fluctuations in conductance  $G = 1/R$  is  $\delta G \sim e^2/h$ . During the time  $t$  the conductance changes due to relaxation of the spin system. In measuring the conductance over an extended time  $t > \tau_K$  an averaging of its fluctuations will take place. Each spin will then manage to reverse, determined by the characteristic dimension of the body with fluctuating resistance,  $L_s = (D\tau_s)^{1/2}$ , and the magnitude of the mean square of the fluctuations

$$\bar{K} = \lim_{\theta \rightarrow 0} \int_0^\theta \frac{d\theta'}{\theta} \langle \delta G(0) \delta G(\theta') \rangle \sim \left( \frac{e^2}{h} \right)^2 \left( \frac{\tau_s}{\tau_f} \right)^{2-d/2} \frac{V_d}{L^d}, \quad (1)$$

where  $d$  is the effective dimensionality of the specimen, and  $V_d = L_x L_y L_z$ .

The fluctuations in the conductance can thus be represented as

$$\delta G \sim \frac{e^2}{h} \left\{ \left( \frac{\tau_s}{\tau_f} \right)^{1-d/4} + f(t) \right\}, \quad (2)$$

where  $f(t)$  is a random function,  $|f| \sim 1$ . The characteristic time scale  $t$  over which the conductance changes appreciably is determined by whether even one of the  $\tau_f/\tau_s$  spin scatterers encountered by an electron in the diffusion time  $\tau_f$ , manages to change its state in the time  $t$ . The probability of such an elementary process for  $t < \tau_K$  is equal to  $t/\tau_K$ , and the characteristic correlation time of the fluctuations is

therefore  $\tau_*$ , which in order of magnitude is equal to  $\tau_s \tau_K / \tau_f$ . Thus, the fluctuation  $\delta G$  changes little over a time  $t < \tau_*$ . For  $\tau_K > t > \tau_*$  the ratio  $\tau_*/t$  determines the fraction of the specimen volume in which spin reversal has not occurred and which gives a contribution to the fluctuations in resistance. The correlation function  $\langle f(0)f(t) \rangle$  can, as a result, be expressed by the following interpolation function:

$$\langle f(0)f(t) \rangle = \left[ \frac{\tau_*}{\tau_* + t} \right]^{2-d/2}. \quad (3)$$

The random time dependence of conductance leads to an additional contribution to the spectral density  $S_{\text{ex}}(\omega)$  of the excess noise, we consider this dependence in §3.

A relatively weak magnetic field  $H > T/\mu_{\text{imp}}$  leads to polarization of the impurity spins. Under these conditions spin relaxation is suppressed,  $\tau_{s,K} \sim \tau_{s,K}^0 \exp(\mu_{\text{imp}} H/T)$ . The scale of the mesoscopic fluctuations averaged over long time intervals increases, which must be manifest in the growth in amplitude of random oscillations of magnetoresistance to the level  $e^2/h$  in fields

$$H > \frac{T}{\mu_{\text{imp}}} \ln \left( \frac{\tau_f}{\tau_s^0} \right).$$

The characteristic time  $\tau_*$  is also extended

$$\tau_* \sim \tau_*^0 \exp(2\mu_{\text{imp}} H/T), \quad (4)$$

which leads to the depression of the low-frequency maximum of the spectral density of excess noise.

2. In order to describe these properties of a mesoscopic conductor more rigorously, we calculate the correlation function of conductances at different times

$$K(t) = \langle G(0)G(t) \rangle - \langle G(0) \rangle \langle G(t) \rangle \quad (5)$$

with the help of the impurity diagram technique.<sup>3</sup> The corresponding diagrams agree with those given by Al'tshuler *et al.*<sup>4</sup> As a result, the correlation function  $K(t)$  can be expressed through two-particle Green's functions, a cooperon  $C(x_1, x_2, t_1, t_2)$  and a diffuson  $D_i(x_1, x_2, t_1 - t_2)$  in a time representation, with the help of a relation which is derived in a way analogous to that used by Fal'ko and Khmel'nitskiĭ.<sup>5</sup>

In a metal with magnetic impurities a cooperon and a diffuson are conveniently represented in the form of a sum of triplet and singlet parts:<sup>6</sup>

$$\begin{aligned} C_{\alpha\beta\gamma\delta} &= C^0 \delta_{\alpha\beta} \delta_{\gamma\delta} + C_{ij}^4 \sigma_{\alpha\beta}^i \sigma_{\gamma\delta}^j, \\ D_{\alpha\beta\gamma\delta} &= D^0 \delta_{\alpha\delta} \delta_{\gamma\beta} + D_{ij}^4 \sigma_{\alpha\delta}^i \sigma_{\gamma\beta}^j. \end{aligned} \quad (6)$$

Without taking into account the relaxation of the impurity spins,  $D^{1,0}$  and  $C^{1,0}$  have the form<sup>6</sup>

$$\begin{aligned} D^0(\mathbf{q}, \omega) &= [-i\omega + D\mathbf{q}^2]^{-1}, \\ D_{ij}^1(\mathbf{q}, \omega) &= \delta_{ij}[-i\omega + D\mathbf{q}^2 + 4/3\tau_s]^{-1}, \\ C^0(\mathbf{q}, \omega) &= [-i\omega + D\mathbf{q}^2 + 2/\tau_s]^{-1}, \\ C_{ij}^1(\mathbf{q}, \omega) &= \delta_{ij}[-i\omega + D\mathbf{q}^2 + 2/3\tau_s]^{-1}. \end{aligned} \quad (7)$$

The characteristic momenta in Eq. (7) satisfy  $q \sim 1/L$ , so we have  $Dq^2 \sim \tau_f^{-1}$ , and the characteristic frequencies are of the order of the temperature  $T$ . In accord with the assumption that  $\tau_s \ll \tau_f$  and  $T\tau_s/\hbar \ll 1$ , the main contribution to the fluctuation in conductance is described by an expression containing only  $D^0$ .

The relaxation of impurity spins is treated in deriving Eq. (7) by taking the correlation of impurity spins  $\langle S_i(0)S_j(t) \rangle$  in the form

$$\langle S_i(0)S_j(t) \rangle = S(S+1) \exp(-|t|/\tau_K). \quad (8)$$

Summing the ladder diagrams under these conditions leads to the expression

$$D^0(\mathbf{q}, \omega) = \{-i\omega + D\mathbf{q}^2 + \tau_s^{-1} [1 - \exp(-|t|/\tau_K)]\}^{-1}, \quad (9)$$

where  $D_i^0(\mathbf{q}, \omega) = d\theta D_i^0(\theta, \mathbf{q}) \exp(-i\omega\theta)$ . The correlator of conductances [Eq. (5)] then has the form

$$\begin{aligned} K(t) &= \left(\frac{4e^2}{h}\right)^2 \tau_f^{-2} \sum_{\mathbf{q}} \int d\epsilon_1 d\epsilon_2 \frac{\partial n(\epsilon_1)}{\partial \epsilon_1} \frac{\partial n(\epsilon_2)}{\partial \epsilon_2} \\ &\times \left\{ |D_i^0(\epsilon_1 - \epsilon_2, \mathbf{q})|^2 \cdot \right. \\ &+ \frac{1}{2} \int dt_1 dt_2 D_i^0(t-t_2, \mathbf{q}) D_{i_0}^0 \\ &\times (t_1 - t, \mathbf{q}) \exp[i(\epsilon_1 - \epsilon_2)(t_1 - t_2)] \left. \right\}, \end{aligned} \quad (10)$$

where  $q_i = \pi n_i/L_i$ ,  $L_i$  is the linear dimension of the specimen. At low temperatures satisfying the condition  $T\tau_f/\hbar \ll 1$  the derivative satisfies  $\partial n(\epsilon)/\partial \epsilon \sim -\delta(\epsilon)$ , and Eq. (10) can be simplified:

$$\begin{aligned} K(t) &= \frac{24}{\pi^4} \left(\frac{e^2}{h}\right)^2 \sum_{\substack{n_x=1, \\ n_y, z=0}}^{\infty} \left[ n_x^2 + \left(\frac{L}{L_y}\right)^2 n_y^2 + \left(\frac{L}{L_z}\right)^2 n_z^2 + \lambda(t) \right]^{-2} \\ &= \frac{6}{\pi^4} \left(\frac{e^2}{h}\right)^2 \\ &\times \frac{V_d}{L^d} \begin{cases} -2\lambda^{-2} + \pi\lambda^{-3/2} \operatorname{cth}(\pi\lambda^{1/2}) + \pi^2\lambda^{-1} \operatorname{ch}^{-2}(\pi\lambda^{1/2}), & d=1 \\ \pi/(1+\lambda), & d=2, \\ -2\lambda^{-1} + 2\pi\lambda^{-1/2} \operatorname{cth}(\pi\lambda^{1/2}), & d=3 \end{cases} \end{aligned} \quad (11)$$

where

$$\lambda(t) = \frac{\tau_f}{\tau_s} \left[ 1 - \exp\left(-\frac{|t|}{\tau_K}\right) \right].$$

The dimensionality  $d$  is determined by the relation between the lengths  $L_i$  and the value of the parameter  $\lambda(t)$ . Over long

time, where we have  $\lambda \gg 1$ , the effective dimensionality can increase. For  $|t| < \tau_K$

$$\lambda(t) = \frac{\tau_f |t|}{\tau_K \tau_s},$$

and Eq. (11) agrees with the interpolation formula (3).

Over large times  $t > \tau_K$ , as can be seen from Eq. (9), the contributions from all components  $D^{1,0}$  and  $C^{1,0}$  are of the same order of magnitude. Therefore Eq. (11) which correctly describes the time range  $t < \tau_K$ , should be changed to take account of these contributions.

We have, finally

$$K(t \rightarrow \infty) = \left(\frac{e^2}{h}\right)^2 \xi_d \left(\frac{\tau_s}{\tau_f}\right)^{2-d/2} \frac{V_d}{L^d}, \quad (12)$$

$$\xi_1 = \xi_2 = 1.54, \quad \xi_3 = 2.44.$$

3. The random time dependence of the conductance leads to an extra contribution to the spectral density of current noise

$$S(\omega) = \int dt I(0)I(t) \exp(i\omega t). \quad (13)$$

The noise can be separated into an equilibrium part, determined by the Nyquist theorem, and excess noise, with intensity proportional to the square of the voltage. Under conditions when the resistance experiences time variations, the Nyquist theorem gives a noise intensity proportional to the time average of the value of the conductance. The spectral density of the excess noise  $S_{\text{ex}}(\omega)$  is determined by the correlation function  $K(\omega)$ , i.e.,  $S_{\text{ex}}(\omega) = V^2 K(\omega)$  and is of the form

$$\begin{aligned} S_{\text{ex}}(\omega) &= \left(\frac{e^2 V}{h}\right)^2 \frac{\tau_K \tau_f}{\tau_s} \\ &\times \frac{V_d}{L^d} \begin{cases} 0.81 - 0.94(\omega \tau_K \tau_s / \tau_f)^{1/2}, & d=1 \\ 0.19 \ln \{ (\tau_f / \tau_s)^2 [1 + (\omega \tau_K)^2]^{-1} \}, & d=2 \\ 0.82 (\tau_f / \tau_s)^{1/2} (1 + 1.3 \omega \tau_K)^{-1/2}, & d=3 \end{cases} \end{aligned} \quad (14)$$

for  $\omega < \omega_* = \tau_s^{-1}$  and

$$S_{\text{ex}}(\omega) \sim \left(\frac{e^2 V}{h}\right)^2 \frac{\tau_K \tau_f}{\tau_s} \frac{1}{(\omega \tau_s)^2}$$

for  $\omega > \omega_*$ .  $V$  here signifies the constant driving voltage in the contact. In magnetic fields with  $\mu_{\text{imp}} H/T > 1$ , the times  $\tau_s$  and  $\tau_K$  grow exponentially, i.e.,  $\tau_{s,K} \sim \tau_{s,K}^0 \exp(\mu_{\text{imp}} H/T)$ , as a result of which a narrowing of the low-frequency maximum in the density of the excess noise takes place. Time variations of resistance in mesoscopic systems have recently<sup>7</sup> been studied intensely. Here we want to draw attention to the contribution to the noise from relaxation of impurity spins with the characteristic dependence on magnetic field described above. It should be remarked that in some sense the noise described here is a variant of the noise arising as a result of electron correlation throughout the medium.<sup>8</sup>

4. For a low concentration of magnetic impurities  $\tau_s \gg \tau_f$ , the spin for most electrons which have diffused through the specimen has not been reversed. In this case the diffusion and cooperon can be expanded in powers of  $\tau_s^{-1}$ , for example:

$$D^0(\omega, \mathbf{q}) = \frac{1}{-i\omega + D\mathbf{q}^2} - \frac{1 - \exp(-|t|/\tau_K)}{\tau_s(-i\omega + D\mathbf{q}^2)^2}. \quad (15)$$

Therefore

$$\lim_{t \rightarrow \infty} \frac{\langle \delta G(0) \delta G(t) \rangle}{\langle [\delta G]^2 \rangle} = 1 - \zeta(7-d) c_d \frac{V_d \tau_f}{L^d \tau_s},$$

$$c_1=3,9, \quad c_2=2,3, \quad c_3=1,6. \quad (16)$$

The intensity of the excess noise associated with relaxation of impurity spins is, under these conditions, equal to

$$S_{ex}(\omega) = B_d \left( \frac{e^2 V}{h} \right)^2 \frac{\tau_K \tau_f V_d}{\tau_s L^d} \frac{1}{1 + (\omega \tau_K / 2)^2}, \quad (17)$$

where  $B_1 = 2$ ,  $B_2 = \pi/6$ ,  $B_3 = 5\pi/16$ . Equations (15)–(17) were obtained on the assumption that the number of magnetic impurities in the specimen is large. If, however, the specimen contains a few or even a single magnetic impurity, then Eqs. (15)–(17) describe correctly only the order of magnitude of the fluctuations in conductance and of the excess noise. The values of these quantities in actual structure depend on the distribution of magnetic impurities in the specimen.

5. In conclusion we consider the conditions for the applicability of the theory presented.

We assume that the temperature  $T$  is higher than the Kondo temperature  $T_K$  for an isolated impurity. At lower temperatures the theory is applicable in principle if  $\tau_s$  and  $\tau_K$  are replaced by quantities found by taking the Kondo effect into account. The condition that  $T$  is greater than the transition temperature to a spin glass state  $T_{SG}$  is more limiting. Since  $T_{SG} \sim \hbar/4\pi\tau_s S(S+1)$  [Eqs. (9,10)], the inequality

$$\hbar/\tau_s \gg T \gg T_{SG} \quad (18)$$

is only satisfied over a narrow temperature range.

We confined ourselves in this note to the evaluation of

the square of the fluctuations of conductance  $G$ . In principle this is sufficient if the fluctuations are Gaussian. It is well known that the distribution function of the fluctuations departs from Gaussian for large  $G - \langle G \rangle$  (Ref. 11). The time dependences  $G(t)$  in a metal with magnetic impurities and non-Gaussian static fluctuations require special investigations.

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