Current of thermalized spin-oriented photocarriers

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It is shown that, in a medium without a center of inversion, there arises in a system of thermalized photocarriers with nonequilibrium spin polarization a current whose direction is reversed when the spin is inverted. The photocurrent is generated as a result of spin relaxation and Larmor precession of polarized spins in a magnetic field owing to the spin splitting, which is linear in the wave vector, of the band states. It was found that there are two contributions to the current: relaxational, caused by the delay in the response of the carrier distribution function to the change in spin, and kinetic, caused by the appearance of a directed electron velocity and a displacement of the center of gravity of the electron wave packet accompanying scattering with flipping of the spin. The photocurrent in heterostructures with quantum wells and superlattices, in single crystals with the structure of the wurtzite and tellurium, and in deformed crystals with the structure of zincblende is calculated.

1. INTRODUCTION

In a brief report[1] we called attention to the possibility of generation of a photocurrent accompanying spin relaxation or Larmor precession of spins of optically oriented thermalized carriers in media without a center of inversion (noncentrosymmetric crystals, structures with quantum wells, or superlattices). In contrast to previously studied mechanisms of the circular photovoltaic effect (CPVE),[2-7] in this case the circular photocurrent decays after the light is switched off not during the momentum-relaxation time τ_p, but rather during the lifetime of the directed spin $T=\tau_0\tau_s/(\tau_0+\tau_s)$, where τ_0 is the lifetime and τ_s is the spin-relaxation time of the photoelectrons. In this paper we make a detailed calculation of the electric current arising in a system of carriers with a quasiequilibrium energy distribution and nonequilibrium spin distribution, for different mechanisms of spin relaxation. We shall first formulate and briefly clarify the final results. The subsequent sections are devoted to justifying these results.

We shall study single crystals or heterostructures whose symmetry admits in the effective electron Hamiltonian H, together with the usual parabolic contribution

$$E_{\mathbf{k}}^{0}=\sum_{j}\frac{\hbar^2k^2}{2m_{jj}} \tag{1.1}$$

(m is the effective mass of the electrons), a spin-dependent term that is linear in the wave vector $\mathbf{k}$

$$H^{(1)}\equiv{}^1/_2\hbar\boldsymbol{\sigma}\boldsymbol{\Omega}^{(1)}=\beta_{ij}\sigma_i k_j, \tag{1.2}$$

where σ_i are the Pauli matrices. The pseudotensor β is different from zero in a structure having a point group of symmetry transformations in which there exist axial- and polar-vector components that transform according to equivalent representations. Three-dimensional media with such symmetry exhibit natural optical activity (quartz, tellurium, crystals with the structure of wurtzite). In noncentrosymmetric nongyrotropic crystals, for example, in cubic crystals of A_3B_5 compounds, the term in $H^{(1)}$ linear in $\mathbf{k}$ can be induced by uniaxial deformation of a single crystal or size quantization in a structure with quantum wells or a periodic potential forming minibands in a superlattice. Thus in a symmetric quantum well $GaAs/Al_xGa_{1-x}As$ with the normal $\mathbf{z}\|(001)$ we have

$$H^{(1)}=(\beta/2)\,(-\sigma_x k_x+\sigma_y k_y), \tag{1.3}$$

i.e., $\boldsymbol{\Omega}^{(1)}=(\beta/\hbar)(-k_x,k_y,0)$, where $\mathbf{x}\|(100)$, $\mathbf{y}\|(010)$.

As will be shown in what follows, two independent mechanisms, one of which can be called relaxational (this mechanism is indicated in Ref. 1) and the other kinetic, contribute to the circular photovoltaic effect studied here. We shall clarify the nature of the relaxational mechanism: in a state of thermal equilibrium the spin density matrix of optically oriented electrons has the form

$$\rho^0=n\{f_0(E_{\mathbf{k}}^0+H^{(1)})({}^1/_2+\boldsymbol{\sigma}\mathbf{S})\}_{\text{sym}}, \tag{1.4}$$

where $f_0(E)$ is the equilibrium distribution function normalized to unity (in what follows, f_0 is a Maxwell function); n is the electron density; $\mathbf{S}$ is the average spin of the electrons; and,

$$\{MN\}_{\text{sym}}=(MN+NM)/2.$$

Here it is assumed that τ_0 and τ_s are much longer than the momentum relaxation time τ_p and the energy relaxation time τ_ε. In a state with the equilibrium distribution (1.4) there is no current:

$$\mathbf{j}=-e\sum_{\mathbf{k}}\text{Sp}\,(\rho^0\mathbf{v}_{\mathbf{k}})=en\sum_{\mathbf{k}}[f_0(E_{\mathbf{k}}^0)\Delta\mathbf{v}^{(1)}+\mathbf{v}_0\,\text{Sp}\,(\Delta\rho^0)]=0, \tag{1.5}$$

since the current contributions from the spin-dependent component of the velocity $\Delta\mathbf{v}^{(1)}=\nabla_{\mathbf{k}}(\mathbf{S}\boldsymbol{\Omega}^{(1)})$ and from the density-matrix component linear in $\mathbf{k}$ and $\mathbf{S}$

$$\Delta\rho^0=n\frac{df_0}{dE}\{H^{(1)},\boldsymbol{\sigma}\mathbf{S}\}_{\text{sym}}=n\frac{df_0}{dE}\mathbf{S}\boldsymbol{\Omega}^{(1)} \tag{1.6}$$

compensate one another. When $\mathbf{S}$ changes the component $\Delta\mathbf{v}^{(1)}$ changes instantaneously, while the component $\Delta\rho$ relaxes to the function (1.6) over the time τ_p. As a result, the two contributions in (1.5) no longer compensate one another and there arises the current

$$\mathbf{j}=-en(t)\,\tau_p\nabla_{\mathbf{k}}\left(\Omega_{\mathbf{k}}^{(1)}\frac{d\mathbf{S}}{dt}\right). \tag{1.7}$$

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In the derivation of this formula it was assumed that photogeneration of carriers does not occur at the time t. Under stationary conditions the quantity $\Delta\rho$ differs from $\Delta\rho^0$ by the factor $1-(\tau_p/\tau_0)$, which takes into account the indicated time lag, and the photocurrent is determined by the expression[1]

$$\mathbf{j}=en(\tau_p/\tau_0)\nabla_{\mathbf{k}}(\Omega_{\mathbf{k}}^{(1)},\ \mathbf{S}_0-\mathbf{S}). \tag{1.8}$$

Here $\mathbf{S}_0$ is the average spin of electrons whose energy relaxes to the bottom of the band. In the case when the spin depolarization occurring in the process of such relaxation can be neglected, $\mathbf{S}_0$ is the average spin of the electrons at the moment of their optical excitation.

The kinetic mechanism is connected with the spin-dependent scattering of the current carriers by impurities and phonons. It depends on the mechanism of spin relaxation. When the D'yakonov-Perel' (DP) mechanism predominates this contribution is, as a rule, less than the relaxational contribution (see Ref. 1).

If spin relaxation occurs primarily by means of the exchange interaction of electrons with paramagnetic ions or holes [the Bir-Aronov-Pikus (BAP) mechanism, see Ref. 8], then

$$\mathbf{j}=-\alpha en(\tau_p/\tau_s)\nabla_{\mathbf{k}}(\Omega_{\mathbf{k}}^{(1)},\mathbf{S}). \tag{1.9}$$

Here τ_s^{-1} is the corresponding contribution to the spin-relaxation rate and α is a coefficient of the order of unity that depends on the mechanism of momentum relaxation. For the Elliott-Yafet (EY) mechanism of spin relaxation, determined by **k-p** mixing of the conduction and valence bands,[8] the interference of transitions with and without spin flip (ballistic contribution) and the displacement of the electron in **r** space in the process of spin-dependent scattering (displacement contribution, see Refs. 9 and 10) play the main role in the kinetic photocurrent. The ballistic and displacement contributions are equal and the expression for the resulting current has the form

$$\mathbf{j}=-2\eta\delta\frac{k_BT}{E_g}n\,\mathrm{rot}_{\mathbf{k}}[\Omega_{\mathbf{k}}^{(1)}\mathbf{S}], \tag{1.10}$$

where k_BT is the thermal energy, E_g is the band gap, $\delta=\Delta/(E_g+\Delta)$, Δ is the spin-orbit splitting of the valence band, and η is a coefficient of the order of unity and depends on the details of the band structure and the momentum scattering mechanism. For the kinetic mechanism the current is determined by the instantaneous value of the spin **S** and the formulas (1.9) and (1.10) are valid in both the stationary and nonstationary regimes.

Using the rotation of the electron spin in a transverse magnetic field it is possible to choose experimental conditions such that there is no circular photocurrent at the moment of generation or it does not contribute to the recorded emf, so that only the current of thermalized carriers is measured.

To derive Eqs. (1.7)–(1.10) we require a general expression for the collision integral in the kinetic equation for the spin density matrix ρ, in which processes with spin flip and the linear in k splitting of the conduction band are taken into account. This expression will be derived in the next section.

2. COLLISION INTEGRAL TAKING INTO ACCOUNT THE SPLITTING OF THE SPIN BRANCHES

The spin density matrix ρ of the photoelectrons satisfies the kinetic equation

$$\frac{\partial\rho}{\partial t}+\frac{\rho}{\tau_0}+\frac{i}{\hbar}[H^{(1)}+H_{\mathbf{B}},\rho]=G+\mathrm{St}\,\rho. \tag{2.1}$$

Here G is the optical-generation matrix and $\mathrm{St}\rho$ is the collision integral. In addition to the characteristic splitting, which is linear in **k**, the Zeeman splitting of the spin branches in a magnetic field **B** is taken into account: $H_{\mathbf{B}}=\hbar\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}_{\mathbf{B}}/2$, where $\boldsymbol{\Omega}_{\mathbf{B}}=g\mu_0\mathbf{B}/\hbar$ is the Larmor precession frequency, μ_0 is the Bohr magneton, and g is the electron g-factor. Under stationary conditions the derivative $\partial\rho/\partial t$ is equal to zero.

It is convenient to write first a general expression for the collision integral in a basis of characteristic spin states $\alpha_{\mathbf{k}s}$ of the effective Hamiltonian

$$H_{\mathbf{k}}\alpha_{\mathbf{k}s}=E_{\mathbf{k}s}\alpha_{\mathbf{k}s}, \tag{2.2}$$

where

$$E_{\mathbf{k}s}=E_{\mathbf{k}}^0+\varepsilon_{\mathbf{k}s},\quad \varepsilon_{\mathbf{k}s}=\pm\hbar|\boldsymbol{\Omega}^{(1)}+\boldsymbol{\Omega}_{\mathbf{B}}|/2,$$

and the index $s=\pm 1/2$ indicates the values of the spin projection on the direction of the vector $\boldsymbol{\Omega}^{(1)}+\boldsymbol{\Omega}_{\mathbf{B}}$.

Using the standard methods for deriving a kinetic equation (using the diagrammatic technique of Konstantinov and Perel' or Keldysh's technique), it can be shown that the contribution of elastic scattering of electrons by static defects to the collision integral has the following form in the basis (2.2) (see Ref. 11):

$$\begin{aligned}(\mathrm{St}\,\rho)_{ss'}=N_i\sum_{\mathbf{k}_1s_1s_2}\Bigg\{&\frac{M_{\mathbf{k}s,\mathbf{k}_1s_1}M_{\mathbf{k}_1s_1,\mathbf{k}s_2}}{i(E_{\mathbf{k}_1s_1}-E_{\mathbf{k}s'})+\delta}\rho_{s_2s'}(\mathbf{k})\\&+\frac{M_{\mathbf{k}s_2,\mathbf{k}_1s_1}M_{\mathbf{k}_1s_1,\mathbf{k}s'}}{i(E_{\mathbf{k}s}-E_{\mathbf{k}_1s_1})+\delta}\rho_{ss_2}(\mathbf{k})-M_{\mathbf{k}s,\mathbf{k}_1s_1}\rho_{s_1s_2}(\mathbf{k}_1)\\&\times M_{\mathbf{k}_1s_2,\mathbf{k}s'}\left[\frac{1}{i(E_{\mathbf{k}_1s_1}-E_{\mathbf{k}s'})+\delta}+\frac{1}{i(E_{\mathbf{k}s}-E_{\mathbf{k}_1s_2})+\delta}\right]\Bigg\}.\end{aligned} \tag{2.3}$$

Here $\delta\to+0$, N_i is the concentration of defects, $M_{\mathbf{k}s,\mathbf{k}'s'}$ is the matrix element of the transition $\mathbf{k}'s'\to\mathbf{k}s$ in the basis (2.2)

$$M_{\mathbf{k}s,\mathbf{k}'s'}=\alpha_{\mathbf{k}s}^{+}\hat{M}_{\mathbf{k}\mathbf{k}'}\alpha_{\mathbf{k}'s'}, \tag{2.4}$$

and $\hat{M}_{\mathbf{k}\mathbf{k}'}$ is the spin matrix of scattering by one defect.

Neglecting the renormalization of the energy spectrum of the electrons owing to interaction with defects, we replace the energy denominators by delta functions according to the scheme

$$[i(E_{\mathbf{k}s}-E_{\mathbf{k}_1s_1})+\delta]^{-1}\to\pi\delta(E_{\mathbf{k}s}-E_{\mathbf{k}_1s_1}).$$

Since we are interested in corrections up to first order in $\varepsilon_{\mathbf{k}s}$, delta functions of the type $\delta(E_{\mathbf{k}s}-E_{\mathbf{k}_1s_1})$ can be replaced by the functions

$$\delta(E_{\mathbf{k}}^0-E_{\mathbf{k}_1}^0)+(\varepsilon_{\mathbf{k}s}-\varepsilon_{\mathbf{k}_1s_1})\frac{\partial}{\partial E_{\mathbf{k}}^0}\delta(E_{\mathbf{k}}^0-E_{\mathbf{k}_1}^0).$$

It is convenient to transform from the basis (2.2) to a fixed basis of spin states, which does not depend on the direction of **k**. To this end we multiply (2.3) by the matrix $\alpha_{\mathbf{k}s}\alpha_{\mathbf{k}s}^{+}$ and sum over s and s', taking into account the identities

$$\sum_s \alpha_{\mathbf{k}s}\alpha_{\mathbf{k}s}^{+}=\hat{I},$$
$$\sum_s \alpha_{\mathbf{k}s}\alpha_{\mathbf{k}s}^{+}\varepsilon_{\mathbf{k}s}=H_{\mathbf{k}}^{(1)}+H_B\equiv H_{\mathbf{k}}' \tag{2.5}$$

($\hat{I}$ is a 2×2 unit matrix). As a result we obtain for the collision integral

$$\begin{aligned}\mathrm{St}\,\rho=&\frac{2\pi}{\hbar}N_i\sum_{\mathbf{k}_1}\delta(E_{\mathbf{k}}^{0}-E_{\mathbf{k}_1}^{0})\\&\times[\hat{M}_{\mathbf{k}\mathbf{k}_1}\rho(\mathbf{k}_1)\hat{M}_{\mathbf{k}_1\mathbf{k}}-\{\hat{M}_{\mathbf{k}\mathbf{k}_1}\hat{M}_{\mathbf{k}_1\mathbf{k}},\rho(\mathbf{k})\}_{\mathrm{sym}}]\\+\frac{2\pi}{\hbar}N_i\sum_{\mathbf{k}_1}&-\frac{\partial}{\partial E_{\mathbf{k}}^{0}}\delta(E_{\mathbf{k}}^{0}-E_{\mathbf{k}_1}^{0})[\{\hat{M}_{\mathbf{k}\mathbf{k}_1}H_{\mathbf{k}_1}'\hat{M}_{\mathbf{k}_1\mathbf{k}},\rho(\mathbf{k})\}_{\mathrm{sym}}\\&+\{\hat{M}_{\mathbf{k}\mathbf{k}_1}\rho(\mathbf{k}_1)\hat{M}_{\mathbf{k}_1\mathbf{k}},H_{\mathbf{k}}'\}_{\mathrm{sym}}-\hat{M}_{\mathbf{k}\mathbf{k}_1}\{H_{\mathbf{k}_1}',\rho(\mathbf{k}_1)\}_{\mathrm{sym}}\hat{M}_{\mathbf{k}_1\mathbf{k}}\\&-{}^1/_2(\hat{M}_{\mathbf{k}\mathbf{k}_1}\hat{M}_{\mathbf{k}_1\mathbf{k}}\rho(\mathbf{k})H_{\mathbf{k}}'+H_{\mathbf{k}}'\rho(\mathbf{k})\hat{M}_{\mathbf{k}\mathbf{k}_1}\hat{M}_{\mathbf{k}_1\mathbf{k}})].\end{aligned} \tag{2.6}$$

It is convenient to represent the matrix $\hat{M}_{\mathbf{k}\mathbf{k}'}$ in Eq. (2.4) in the form of a linear combination of $\hat{I}$ (scattering without spin flip) and Pauli matrices (scattering with spin flip):

$$\hat{M}_{\mathbf{k}\mathbf{k}'}=A_{\mathbf{k}\mathbf{k}'}\hat{I}+\sigma\mathbf{B}_{\mathbf{k}\mathbf{k}'}, \tag{2.7}$$

where $A_{\mathbf{k}\mathbf{k}'}^{*}=A_{\mathbf{k}'\mathbf{k}}$, $\mathbf{B}_{\mathbf{k}\mathbf{k}'}^{*}=\mathbf{B}_{\mathbf{k}'\mathbf{k}}$ (hermiticity of the interaction operator) and $A_{\mathbf{k}\mathbf{k}'}=A_{-\mathbf{k}'-\mathbf{k}}$, $\mathbf{B}_{\mathbf{k}\mathbf{k}'}=\mathbf{B}_{-\mathbf{k}'-\mathbf{k}}$ (symmetry under time reversal). Neglecting processes with spin flip we have

$$\begin{aligned}\mathrm{St}\,\rho=&\frac{2\pi}{\hbar}N_i\\&\times\sum_{\mathbf{k}_1}|A_{\mathbf{k}\mathbf{k}'}|^2\{\delta(E_{\mathbf{k}}^{0}+H_{\mathbf{k}}'-E_{\mathbf{k}_1}-H_{\mathbf{k}_1}'),\ \rho(\mathbf{k}_1)-\rho(\mathbf{k})\}_{\mathrm{sym}}.\end{aligned} \tag{2.8}$$

For superlattices and deformed A_3B_5 crystals we shall take into account in the pseudovector $\mathbf{B}_{\mathbf{k}\mathbf{k}'}$, in addition to the main contribution $\mathbf{B}_{\mathbf{k}\mathbf{k}'}^{(0)}$, describing spin-dependent scattering in deformed single crystals, also the contribution $\mathbf{B}_{\mathbf{k}\mathbf{k}'}^{(1)}$, which is induced by size quantization or deformation and which is of the order of $\Omega_{\mathbf{k}}^{(1)}/E_{\mathbf{k}}^{(0)}$ as compared with $\mathbf{B}_{\mathbf{k}\mathbf{k}'}^{(0)}$.

Substituting the spin-dependent component of the equilibrium density matrix $\Delta\rho^0$ [see Eq. (1.6)] into the collision integral, we obtain three terms, which generate in the kinetic equation for the distribution function $\mathrm{Sp}\rho$ terms that are odd in $\mathbf{k}$ and lead to the appearance of a current

$$\begin{aligned}(\mathrm{St}\,\Delta\rho^0)_1=&2\pi N_i n\sum_{\mathbf{k}_1}\frac{\partial}{\partial E_{\mathbf{k}}^{0}}\delta(E_{\mathbf{k}}^{0}-E_{\mathbf{k}_1}^{0})\\&\times\{-\mathrm{Im}(A_{\mathbf{k}\mathbf{k}_1}^{*}[\mathbf{B}_{\mathbf{k}\mathbf{k}_1}^{(0)}\,\Omega_{\mathbf{k}_1}]\mathbf{S})f(E_{\mathbf{k}}^{0})\\&+\mathrm{Im}(A_{\mathbf{k}\mathbf{k}_1}^{*}[\mathbf{B}_{\mathbf{k}\mathbf{k}_1}^{(0)}\,\Omega_{\mathbf{k}}]\mathbf{S})f(E_{\mathbf{k}_1})\},\end{aligned} \tag{2.9}$$

$$\begin{aligned}(\mathrm{St}\,\Delta\rho^0)_2=&\pi N_i n\sum_{\mathbf{k}_1}\frac{\partial}{\partial E_{\mathbf{k}}^{0}}\delta(E_{\mathbf{k}}^{0}-E_{\mathbf{k}_1}^{0})[-2\mathbf{S}|\mathbf{B}_{\mathbf{k}\mathbf{k}_1}^{(0)}|^2\\&+(\mathbf{S}\mathbf{B}_{\mathbf{k}\mathbf{k}_1}^{(0)})\mathbf{B}_{\mathbf{k}_1\mathbf{k}}^{(0)}+(\mathbf{S}\mathbf{B}_{\mathbf{k}_1\mathbf{k}}^{(0)})\mathbf{B}_{\mathbf{k}\mathbf{k}_1}^{(0)}][f(E_{\mathbf{k}}^{0})\Omega_{\mathbf{k}_1}^{(1)}+f(E_{\mathbf{k}_1}^{0})\Omega_{\mathbf{k}}^{(1)}].\end{aligned} \tag{2.10}$$

$$\begin{aligned}(\mathrm{St}\,\Delta\rho^0)_3=&\frac{2\pi}{\hbar}N_i n\sum_{\mathbf{k}_1}\delta(E_{\mathbf{k}}^{0}-E_{\mathbf{k}_1})[f(E_{\mathbf{k}}^{0})+f(E_{\mathbf{k}_1}^{0})]\\&\times\mathrm{Im}(\mathbf{S}([\mathbf{B}_{\mathbf{k}\mathbf{k}_1}^{(0)}\mathbf{B}_{\mathbf{k}_1\mathbf{k}}^{(1)}]+[\mathbf{B}_{\mathbf{k}\mathbf{k}_1}^{(1)}\mathbf{B}_{\mathbf{k}_1\mathbf{k}}^{(0)}])).\end{aligned} \tag{2.11}$$

Formulas analogous to Eqs. (2.3), (2.6), (2.7), and (2.9)–(2.11) can also be derived for other mechanisms of scattering of photoelectrons. In the case of scattering by phonons, holes, or paramagnetic ions the corresponding contributions to the collision integral include summation over the initial and final states of the scattering system.

3. RELAXATIONAL MECHANISM

As pointed in Sec. 1, we assume that the following hierarchy of relaxation times holds:

$$\tau_p\ll\tau_\varepsilon\ll\tau_s,\ \tau_0. \tag{3.1}$$

In this case it is not necessary to study the relaxation of the energy of the hot photoelectrons to the bottom of the conduction band, and the generation matrix G in Eq. (2.1) can be written in the form

$$G=g\{f_0(E_{\mathbf{k}}^{0}+H_{\mathbf{k}}^{(1)}+H_B)({}^1/_2+\sigma\mathbf{S}_0)\}_{\mathrm{sym}}, \tag{3.2}$$

where g is the rate of optical excitation of electrons into the conduction band, while the quantity $\mathbf{S}_0$ was introduced in Eq. (1.8). We shall solve the kinetic equation (2.1) for the density matrix ρ by the method of iterations. For the zeroth-order approximation we employ the quasiequilibrium distribution ρ^0. We note that in a magnetic field the argument of the function f_0 in the expression (1.4) for ρ_0 also contains the operator H_B.

In this section scattering with spin flip is neglected and the collision integral is reduced to expressions of the type (2.8). When the distribution ρ^0 is substituted into them these expressions vanish. We shall substitute into Eq. (2.1) the density matrix in the form $\rho=\rho^0+\delta\rho$ and we shall find the trace of all matrix terms appearing in this equation. As a result we obtain for the component $\delta\rho^{(-)}$, which is odd in $\mathbf{k}$ and determines the photocurrent,

$$\delta\rho^{(-)}=f_0(E_{\mathbf{k}}^{0})\tau_1\left[g(\mathbf{S}_0-\mathbf{S})\Omega_{\mathbf{k}}^{(1)}-n\frac{d\mathbf{S}}{dt}\Omega_{\mathbf{k}}^{(1)}\right] \tag{3.3}$$

when the inequalities (3.1) are satisfied. Here τ_l is the relaxation time of the electron distribution in $\mathbf{k}$-space, described by the polynomial $P_l(\cos\theta)$ of order l. In the notation of the preceding section, for scattering by static defects

$$\frac{1}{\tau_i(E)}=\frac{2\pi}{\hbar}N_i\sum_{\mathbf{k}_1}|A_{\mathbf{k}\mathbf{k}_1}|^2[1-P_l(\cos\theta)]\delta(E_{\mathbf{k}}^{0}-E_{\mathbf{k}_1}^{0}), \tag{3.4}$$

where θ is the angle between $\mathbf{k}$ and $\mathbf{k}_1$, $E=E_{\mathbf{k}}^{0}$. The expression (3.3) is applicable under both stationary and nonstationary conditions, when the quantities g, n, $\mathbf{S}^0$, and $\mathbf{S}$ change over characteristic times longer than τ_1 and τ_s.

Substituting Eq. (3.3) into the expression for the ballistic current

$$\mathbf{j}=-e\sum_{\mathbf{k}}\mathrm{Sp}(\rho\mathbf{v}_{\mathbf{k}}) \tag{3.5}$$

and carrying out the summation over $\mathbf{k}$, we find, finally,

$$\mathbf{j}=eg\tau_p\nabla_{\mathbf{k}}(\Omega_{\mathbf{k}}^{(1)},\mathbf{S}_0-\mathbf{S})-en\tau_p\nabla_{\mathbf{k}}\left(\Omega_{\mathbf{k}}^{(1)},\frac{d\mathbf{S}}{dt}\right). \tag{3.6}$$

Here τ_p is the transport time, i.e., the average relaxation time τ_1 appearing in the expression for the mobility. In limiting

cases Eq. (3.6) goes over into the formulas (1.7) and (1.8) presented in the Introduction.

4. KINETIC CONTRIBUTION

D'yakonov-Perel' mechanism

If the spin relaxation is determined by the D'yakonov-Perel' mechanism, then the kinetic contribution appears when the term

$$H^{(3)}={}^1/_2\hbar(\sigma\Omega^{(3)}), \tag{4.1}$$

which is cubic in $\mathbf{k}$, is included in the Hamiltonian $H_k = 1/2\hbar(\sigma\Omega_\mathbf{k})$, determining the spin splitting of the conduction band. This component is also different from zero in undeformed crystals in the absence of spatial quantization. In quantum wells only the terms containing k_x and k_y remain in Eq. (4.1). The terms linear in k_z vanish and the terms quadratic in k_z give the term (1.3) linear in $\mathbf{k}$. Correspondingly, the velocity operator $\mathbf{v}$ contains a component that is quadratic in $\mathbf{k}$

$$\mathbf{v}_2={}^1/_2\nabla_\mathbf{k}(\sigma\Omega^{(3)}), \tag{4.2}$$

which contributes to the current if the density matrix contains the component ρ_2 which includes a second-order polynomial and relaxes with a time τ_2.

To calculate ρ_2 Eq. (2.1) must be iterated to second order in $\Omega_\mathbf{k}$, including in it $\Omega^{(3)}$ aside from $\Omega^{(1)}$. The corresponding contribution to ρ contains two terms, ρ_0 and ρ_2. The first term does not depend on $\mathbf{k}$ and determines the rate $1/\tau_{s,\mathrm{DP}}$ of spin relaxation in the D'yakonov-Perel' mechanism.

The second term is equal to

$$\rho_2=-\tau_2 N f_0(E_\mathbf{k}^0)\,\tau^* S_i\sigma_j[(\Omega_\mathbf{k}^2-\overline{\Omega_\mathbf{k}^2})\delta_{ij}-(\Omega_{\mathbf{k}i}\Omega_{\mathbf{k}j}-\overline{\Omega_{\mathbf{k}i}\Omega_{\mathbf{k}j}})], \tag{4.3}$$

where the overbar indicates averaging over the directions of $\mathbf{k}$. This component makes the following contribution to the current:

$$\mathbf{j}=-e\sum_\mathbf{k}\mathrm{Sp}(\mathbf{v}_2\rho_2). \tag{4.4}$$

For quantum wells the terms in Eq. (4.3) with $\Omega_\mathbf{k}=\Omega^{(1)}$ contribute a nonzero current, and in this case it is necessary to set $\tau^*=\tau_1$. For deformed crystals only the terms in Eq. (4.3) containing products of $\Omega^{(1)}$ and $\Omega^{(3)}$ make a nonzero contribution. In this case τ^* must be set equal to τ_1 if $\Omega^{(1)}$ is the first term on the right-hand side, and to τ_3 if the first term is $\Omega^{(3)}$.

For comparison we also present an expression for $1/\tau_{s,\mathrm{DP}}$

$$(1/\tau_{s,\mathrm{DP}})_{ij}=\tau^*[\overline{\Omega_\mathbf{k}^2}\delta_{ij}-\overline{\Omega_{\mathbf{k}i}\Omega_{\mathbf{k}j}}]. \tag{4.5}$$

On averaging over the directions of $\mathbf{k}$ only the terms quadratic in $\Omega^{(1)}$ and $\Omega^{(3)}$ remain in Eq. (4.5), and correspondingly τ^* is equal to τ_1 or τ_3.

BAP mechanism and scattering by paramagnetic ions

If the spin relaxation is determined by the exchange interaction with holes or paramagnetic ions, then $\mathbf{B}_{\mathbf{kk}'}$ in Eq. (2.7) does not depend on $\mathbf{k}$ and $\mathbf{k}'$ and the only term in $\mathrm{St}\Delta\rho^0$ in Eqs. (2.9)–(2.11) that contributes to the current is $(\mathrm{St}\Delta\rho^0)_2$. The corresponding contribution to the component of the density matrix $\rho_1 = -\tau_1(\mathrm{St}\Delta\rho^0)_2$ for scattering by bound holes or ions is equal to

$$\rho_1^{(2)}=-2\pi\tau_1\sum_{\mathbf{k}_i}\frac{\partial}{\partial E_\mathbf{k}^0}\delta(E_\mathbf{k}^0-E_{\mathbf{k}_i}^0)\frac{N_h}{g_h}$$
$$\times\mathrm{Sp}^h[\Omega_\mathbf{k}\mathbf{S}|\mathbf{B}^0|^2-(\mathbf{S}\mathbf{B}^0)(\Omega_\mathbf{k}\mathbf{B}^0)]. \tag{4.6}$$

Here N_h is the density of paramagnetic ions or holes, g_h is the spin degeneracy, and the trace Sp^h is taken over the spin indices of the holes (ions).

Bearing in mind that the spin-relaxation time τ_s for the BAP mechanism or in the case of scattering by ions is determined by the formula

$$\frac{1}{\tau_{sij}}=\frac{4\pi}{\hbar}\frac{N_h}{g_h}\sum_{\mathbf{k}_i}\delta(E_\mathbf{k}^0-E_{\mathbf{k}_i}^0)\mathrm{Sp}^h(|\mathbf{B}|^2\delta_{ij}-B_iB_j), \tag{4.7}$$

the expression (4.6) can be rewritten in the form

$$\rho_1^{(2)}=-\tau_1\frac{\hbar}{2}\Omega_{\mathbf{k}i}S_i\frac{\partial}{\partial E_\mathbf{k}^0}\frac{f_0(E_\mathbf{k}^0)}{\tau_{sij}(E_\mathbf{k}^0)}. \tag{4.8}$$

For the mechanisms studied τ_s^{-1} is a scalar and, in accordance with Eq. (2), it is proportional to the velocity of the electrons, i.e., $E^{1/2}$.

Correspondingly the current is equal to

$$\mathbf{j}=-e\sum_\mathbf{k}\mathrm{Sp}(\rho_1^{(2)}\mathbf{v}_0)=-e\alpha n\nabla_\mathbf{k}(\Omega^{(1)}\mathbf{S})\frac{\tau_p}{\tau_{s,\mathrm{BAP}}}. \tag{4.9}$$

Here $\tau_{s,\mathrm{BAP}}^{-1}$ is the inverse spin-relaxation time $\tau_s^{-1}(E)$ averaged over the Maxwellian distribution. For $\tau_1\propto E^\nu$ for electrons in the three-dimensional case

$$\alpha=\frac{\pi^{1/2}}{2}\frac{\Gamma(\nu+3)-{}^1/_2\Gamma(\nu+2)}{\Gamma(\nu+{}^5/_2)}, \tag{4.10}$$

where $\Gamma(x)$ is the gamma function.

For free holes N_h in Eqs. (4.6) and (4.7) is replaced by $\Sigma_{n_h}f(p_h)[1-f(p_h)]$, where $f(p_h)$ is the equilibrium momentum distribution function of the holes, normalized to N_h. Here, as in Ref. 12, it is assumed that the momentum of the holes $\mathbf{p}_h$ is much greater than the momentum of the electrons $\mathbf{k}$. In this approximation and in the case of annihilation interaction with holes, the matrix element $\mathbf{B}_{\mathbf{k}\mathbf{k}_i}$ determined by the total momentum $\mathbf{K}\approx\mathbf{p}_h$ does not depend on $\mathbf{k}$ and $\mathbf{k}_i$ and the formulas (4.6)–(4.10) remain in force.

Elliott-Yafet mechanism

In the case of scattering by piezoelectric acoustic oscillations or polar optical phonons, Coulomb centers, or impurities with a short-range potential, the matrix element $\mathbf{B}_{\mathbf{k}\mathbf{k}_i}$ in Eq. (2.7) is determined by the expression

$$\mathbf{B}_{\mathbf{k}\mathbf{k}_i}=A_{\mathbf{k}\mathbf{k}_i}\mathbf{b}_{\mathbf{k}\mathbf{k}_i}=A_{\mathbf{k}\mathbf{k}_i}(\mathbf{b}_{\mathbf{k}\mathbf{k}_i}^{(0)}+\mathbf{b}_{\mathbf{k}\mathbf{k}_i}^{(1)}), \tag{4.11}$$

where

$$\mathbf{b}_{\mathbf{k}\mathbf{k}_i}^{(0)}=-i[\mathbf{k}\mathbf{k}_i]\frac{\hbar^2}{2mE_g}\eta\delta. \tag{4.12}$$

For A_3B_5 crystals we have $\eta=(2-\delta)/(3-\delta)$. In Eq. (4.12) it is assumed that in calculating $\mathbf{b}_{\mathbf{k}\mathbf{k}_i}^{(0)}$ and m only the $\mathbf{k}\cdot\mathbf{p}$ interaction of the nearest conduction band Γ_6 with the nearest valence band $\Gamma_{15}=\Gamma_8+\Gamma_7$ is taken into account.

In calculating $\mathbf{b}^{(0)}_{\mathbf{k}\mathbf{k}_i}$ terms linear in $\mathbf{k}$ and $\mathbf{k}_i$ are taken into account in the interband matrix elements.[8] To calculate $\mathbf{b}^{(1)}_{\mathbf{k}\mathbf{k}_i}$ it is also necessary to take into account in the interband matrix elements the terms quadratic in $\mathbf{k}$ (for quantum wells) or linear in the deformation. Since these same terms are responsible for the linear in $\mathbf{k}$ splitting of the conduction band $\mathbf{b}^{(1)}_{\mathbf{k}\mathbf{k}_i}$ can be expressed in terms of $\mathbf{\Omega}_{\mathbf{k}}$ and $\mathbf{\Omega}_{\mathbf{k}_i}$:

$$\mathbf{b}^{(1)}_{\mathbf{k}\mathbf{k}_i}=\frac{\hbar}{E_g}(2-\delta)\,(\mathbf{\Omega}^{(1)}_{\mathbf{k}}+\mathbf{\Omega}^{(1)}_{\mathbf{k}_i}). \tag{4.13}$$

Ballistic current. It can be shown that of the three terms in $\mathrm{St}\Delta\rho^0$, determined in Eqs. (2.9)–(2.11), $(\mathrm{St}\Delta\rho^0)_i$ makes the main contribution. The contributions of the components $(\mathrm{St}\Delta\rho^0)_2$ and $(\mathrm{St}\Delta\rho^0)_3$ is less than the contribution of $(\mathrm{St}\Delta\rho^0)_1$ in respect to the parameter k_BT/E_g. We shall present expressions, confirming these estimates, for the contributions of these components to the current in the case of quantum wells.[1)]

In the case of the scattering by acoustic or optical phonons, determined by the interband constants of the deformation potential, the matrix element is $\mathbf{B}^{(0)}_{\mathbf{k}\mathbf{k}_i}\propto\mathbf{k}+\mathbf{k}_i$ while $\mathbf{B}^{(1)}_{\mathbf{k}\mathbf{k}_i}$ does not depend on $\mathbf{k}$ and $\mathbf{k}_i$ (except for the explicit dependence of ε_{ij} on $\mathbf{q}=\mathbf{k}-\mathbf{k}_i$). In this case $(\mathrm{St}\Delta\rho^0)_1$ and $(\mathrm{St}\Delta\rho^0)_2$ do not contribute to the current.

For scattering by impurities with a short-range potential, when $|A_{\mathbf{k}\mathbf{k}_i}|^2$ does not depend on $\mathbf{q}$, the component $(\mathrm{St}\Delta\rho^0)_i$, according to Eqs. (2.9) and (4.10), contributes the current determined by the expression

$$\mathbf{j}=-en\eta\delta\frac{k_BT}{E_g}[(\mathbf{S}\nabla_{\mathbf{k}})\mathbf{\Omega}_{\mathbf{k}}-\mathbf{S}(\nabla_{\mathbf{k}}\mathbf{\Omega}_{\mathbf{k}})]. \tag{4.14}$$

In those cases when $|A_{\mathbf{k}\mathbf{k}_i}|^2$ depends on $\mathbf{q}$, Eq. (4.13) contains an additional dimensionless factor Z. If $|A_{\mathbf{k}\mathbf{k}_i}|^2$ is written in the form $F(E,E_i,\mu)$, where $E=E^0_{\mathbf{k}}$ and $E_1=E^0_{\mathbf{k}_1}$, then for three-dimensional electrons

$$Z=\int_0^\infty Z(E)E^{1/2}f_0(E)\,dE\Big/\int_0^\infty E^{1/2}f_0(E)\,dE, \tag{4.15}$$

where

$$Z(E)=\int_{-1}^{1}d\mu(1-\mu^2)\left(\frac{3}{2}F+E\frac{\partial F}{\partial E_1}\right)_{E_1=E}\Big/\int_{-1}^{1}d\mu(1-\mu)F(E,E_1,\mu).$$

As the calculation shows, in the case of scattering by piezoelectric oscillations or polar optical phonons, when $|A_{\mathbf{k}\mathbf{k}_1}|^2\propto q^{-2}$ and also in the case of scattering by a coulomb potential, when $|A_{\mathbf{k}\mathbf{k}_1}|^2\propto(q^2+L_D^{-2})^{-1}$, the factor $Z=1$ (L_D is the Debye screening length). In the case of scattering by acoustic phonons, which is determined by the deformation (short-range) interaction, we have

$$Z=\frac{a}{c}+\frac{1}{5(2-\delta)}\frac{(2b+3^{1/2}d)}{c}. \tag{4.16}$$

Here a, b, d, and c are constants in the deformation potential of the valence band or the conduction band. It is obvious that in this case the sign of the effect depends on the sign of the constant of the deformation potential.

The displacement current is caused by the displacement of the center of gravity of the wave packet of the electrons in the process of spin-dependent scattering. In accordance with Refs. 9 and 10 this current is determined by the expression

$$\mathbf{j}=\frac{2\pi}{\hbar}e\sum_{\mathbf{k}\mathbf{k}_1}\mathrm{Im}\,\mathrm{Sp}\{\rho(\mathbf{k})M_{\mathbf{k}\mathbf{k}_1}(\nabla_{\mathbf{k}}+\nabla_{\mathbf{k}_1})M_{\mathbf{k}_1\mathbf{k}}\}\delta(E_{\mathbf{k}}{}^0-E_{\mathbf{k}_1}{}^0). \tag{4.17}$$

The expression (4.17) is an extension of the formulas of Ref. 9 to the case when the density matrix $\rho(\mathbf{k})$ is not diagonal. It can be obtained both by the method developed in Ref. 9 and by taking into account the renormalization of the current operator by the scattering.[13] The density-matrix component

$$\rho_1(\mathbf{k})=-\frac{i}{\hbar}[H^{(1)},\rho^0]\tau_1=nf_0(E_{\mathbf{k}}{}^0)\,(\sigma[\mathbf{\Omega}^{(1)}\mathbf{S}]), \tag{4.18}$$

which is the solution of Eq. (2.1) to first order in $\Omega^{(1)}$, makes a nonzero contribution to the current in Eq. (4.17).

Substituting Eqs. (4.18), (2.7), (4.11), and (4.12) into Eq. (4.9) we obtain for the current an expression that is identical to Eq. (4.14) and remains valid for all mechanisms of scattering for which $\mathbf{M}_{\mathbf{k}\mathbf{k}_i}$ and $\mathbf{B}_{\mathbf{k}\mathbf{k}_i}$ are determined by the formulas (2.7), (4.11), and (4.12). In the case of scattering by acoustic phonons, which is determined by the deformation interaction, there is an additional factor Z identical with (4.16).

5. QUANTUM WELLS AND SUPERLATTICES

As indicated above, for symmetric quantum wells $H^{(1)}$ is determined by the formula (1.3), where $\beta=2\gamma_c\overline{k_z^2}$.[14,15,2)] Here γ_c is a coefficient in the expression for $\Omega^{(3)}$, which for A_3B_5 crystals has the form

$$\Omega^{(3)}_i=\frac{2\gamma_c}{\hbar}k_i(k^2_{i+1}-k^2_{i-1}). \tag{5.1}$$

In accordance with Eqs. (1.8) and (3.3), the stationary current due to the relaxational mechanism is determined for $\mathbf{S}_\perp=0$ by the formula[1]

$$j_{\perp i}=\pm en\frac{\beta}{\hbar}\frac{\tau_p}{\tau_0}S_i. \tag{5.2}$$

The $+$ sign corresponds to $i=x$ and the $-$ sign corresponds to $i=y$. The kinetic contribution in the D'yakonov-Perel' mechanism in accordance with Eqs. (4.3) and (4.4) is determined by the expression

$$\Delta j_i=\mp\frac{e\gamma_c}{\hbar^3}\frac{nS_i}{\tau_{s\perp,\mathrm{DP}}}m\frac{\tau_2}{\tau_1}\frac{\langle E^2\tau_1\rangle}{\tau_pk_BT} \tag{5.3}$$

(the brackets indicating averaging over energy). Here

$$\frac{1}{\tau_{s\parallel,\mathrm{DP}}}=\frac{2}{\tau_{s\perp,\mathrm{DP}}}=\frac{2k_BT\tau_p\beta^2m_e}{\hbar^4}. \tag{5.4}$$

The ratio of the contributions (5.3) and (5.2) is equal to

$$\frac{\Delta j}{j}=\frac{\tau_0}{2\tau_{s\perp,\mathrm{DP}}}\frac{k_BT}{\Delta E}\frac{\tau_2}{\tau_1}. \tag{5.5}$$

Here $\Delta E=\hbar^2\overline{k_z^2}/2m\approx\hbar^2\pi^2/2md^2$, where d is the size of the well. One can see that the contribution (5.3) can become

significant for quite wide wells for not very low temperatures and for $\tau_s \ll \tau_0$.

For scattering by heavy holes, spin flip does not occur for $k_h = 0$. For this reason the BAP mechanism can become significant only if the hole density is high or the temperatures are high, when the Fermi energy of the holes or $k_B T$ are comparable to the separation of the levels of the light holes E_{lh} and heavy holes E_{hh}. In these cases the mixing of the states E_{hh} and E_{lh} as well as the possible filling of the E_{lh} level must be taken into account. In accordance with Eq. (4.14), the term $(\mathrm{St}\Delta\rho^0)_i$ makes the main contribution to the current determined by the Elliott-Yafet mechanism. This contribution is equal to

$$j_i^{(1)} = \pm 2en \frac{\beta}{\hbar} \frac{k_B T}{E_g} S_i \eta\delta = \pm 2en \frac{\beta}{\hbar} S_i \left(\frac{3\tau_p}{\tau_{s\perp,\mathrm{EY}}} \right)^{1/2}. \quad (5.6)$$

Here

$$\frac{1}{\tau_{s\perp,\mathrm{EY}}} = \frac{1}{3} \frac{1}{\tau_p} \eta^2 \delta^2 \left(\frac{k_B T}{E_g} \right)^2. \quad (5.7)$$

The Elliott-Yafet mechanism does not contribute to the components $\tau_{s\parallel}^{-1}$. In accordance with Eqs. (3.10) and (3.11) the contributions of $(\mathrm{St}\Delta\rho^0)_2$ and $(\mathrm{St}\Delta\rho^0)_3$ to the current are equal to

$$j_i^{(2)} = \mp 8en \frac{\beta}{\hbar} S_i \left(\frac{k_B T}{E_g} \eta\delta \right)^2 = \mp 24en \frac{\beta}{\hbar} S_i \frac{\tau_p}{\tau_{s\perp,\mathrm{EY}}}, \quad (5.8)$$

$$j_i^{(3)} = \pm 2en \frac{\beta}{\hbar} S_i \left(\frac{k_B T}{E_g} \eta\delta \right)^2 \frac{3-\delta}{\delta} = \pm en \frac{\beta}{\hbar} S_i \frac{3-\delta}{\delta} \frac{\tau_p}{\tau_{s\perp,\mathrm{EY}}}. \quad (5.9)$$

These contributions, as indicated above, are less than $j_i^{(1)}$ in terms of the parameter $k_B T/E_g$. As regards the ratio of the contributions (5.6) and (5.2), it depends strongly on the temperature and on τ_p. It is obvious that the contribution (5.6) predominates if

$$\frac{k_B T}{E_g} \eta\delta > \frac{\tau_p}{\tau_0}.$$

If the light is oriented along the normal to the surface, then the current in the quantum wells arises only in a transverse magnetic field. In the field $\mathbf{B}$ the spin $\mathbf{S}(t)$ is determined by the equation

$$n \left(\frac{\partial S_i}{\partial t} + \frac{1}{\tau_{sij}} S_j + [\mathbf{S}\Omega_\mathbf{B}]_i \right) = g(S_{0i} - S_i). \quad (5.10)$$

In the stationary state, when $g = n/\tau_0$ with $\mathbf{S}_0 \| \mathbf{z}$,

$$\mathbf{S}_\perp = \frac{T_\parallel T_\perp}{\tau_0} \frac{[\Omega_\mathbf{B} \mathbf{S}_{0z}]}{1 + \Omega_\mathbf{B}^2 T_\parallel T_\perp}, \quad (5.11)$$

where $T_{\parallel,\perp}^{-1} = \tau_0^{-1} + \tau_{s\parallel,\perp}^{-1}$.

If at the moment $t = 0$, the carriers are excited by a narrow pulse, whose width is small compared with τ_s, τ_0, and Ω_B^{-1}, then

$$\mathbf{S}_\perp = \exp(-t/\bar{\tau}_s) [\Omega_\mathbf{B} \mathbf{S}_{0z}] \tau_s^* \sinh(t/\tau_s^*) \quad \text{for } \Omega_\mathbf{B} \tilde{\tau}_s < 1/2, \quad (5.12)$$

$$\mathbf{S}_\perp = \exp(-t/\bar{\tau}_s) [\Omega_\mathbf{B} \mathbf{S}_{0z}] \omega^{-1} \sin \omega t \quad \text{for } \Omega_\mathbf{B} \tilde{\tau}_s > 1/2. \quad (5.13)$$

Here

$$\bar{\tau}_s^{-1} = 1/2 (\tau_{s\parallel}^{-1} + \tau_{s\perp}^{-1}), \quad \tilde{\tau}_s^{-1} = |\tau_{s\parallel}^{-1} - \tau_{s\perp}^{-1}|,$$
$$(\tau^*)^{-1} = 1/2 \tilde{\tau}^{-1} [1 - (2\Omega_\mathbf{B} \tilde{\tau}_s)^2]^{1/2}, \quad \omega = 1/2 \tilde{\tau}_s^{-1} [(2\Omega_\mathbf{B} \tilde{\tau}_s)^2 - 1]^{1/2}.$$

For $2\Omega_\mathbf{B} \tilde{\tau}_s \gg 1$ we have $\omega = \Omega_\mathbf{B}$. As follows from Eq. (1.9) and (3.6), in the case that measurements are performed in the nonstationary regime with $\Omega \gg \tau_{s\parallel,\perp}^{-1}$ the relaxation contribution is $\Omega\tau_s$ times greater than the kinetic contribution.

In accordance with the symmetry requirements the direction of the current in a transverse magnetic field does not depend on the mechanism of current flow. Depending on the direction of $\mathbf{B}$, the current in a symmetric quantum well can be both perpendicular to $\mathbf{B}$ (for $\mathbf{B} \| (100)$ and (010)) and parallel to $\mathbf{B}$ (for $\mathbf{B} \| (110)$ and $(1\bar{1}0)$). For arbitrary direction of $\mathbf{B}$ the angle between j and the (010) axis is equal to the angle between $\mathbf{B}$ and the (100) axis. The current contribution associated with the asymmetry of the quantum well, i.e., with $\Omega^{(1)} \propto [\mathbf{nk}]$ ($\mathbf{n}$ is a unit vector normal to the plane of the well), is always parallel to $\mathbf{B}$.

We shall estimate the expected contributions to the current which are caused by the different mechanisms. Spin relaxation, determined by the linear-in-$\mathbf{k}$ splitting of the conduction band, was studied in Ref. 16 for the superlattice GaAs–$Al_{0.35}Ga_{0.65}$–As (15 Å + 15 Å). The measured value is $\beta = 10^{-2}$ eV·Å^{-1}. For $T_e = 5$ K, $\tau_p = 8 \cdot 10^{-12}$ s, $\tau_0 = 10^{-9}$ s, and $\tau_{s\parallel} = 1.2 \cdot 10^{-10}$ s the relaxational mechanism (5.2) makes the main contribution to the current, and the kinetic contributions (5.3) and (5.6) are equal to 15 and 1%, respectively, of the contribution (5.2).

In an optimal magnetic field, i.e., for $\Omega_\mathbf{B} = (T_\parallel T_\perp)^{-1/2}$ and $S_0 = 1/2$, at high light intensities when the photoconductivity exceeds the dark conductivity, the open-circuit field intensity is $E_{xx}^{\max} = 0.5 \cdot 10^{-3}$ V/cm. For the quantum wells of size $d = 120$ Å, studied in Ref. 16, at $T = 77$ K and with $\tau_p = 8 \cdot 10^{-13}$ s, $\tau_0 = 4 \cdot 10^{-11}$ s, and $\tau_{s\parallel} = 3 \cdot 10^{-10}$ s the relaxational contribution (5.2) also predominates. The contributions (5.3) and (5.6) are equal to 1 and 5%, respectively, of the contribution (5.2). The quantity $E_{xx}^{\max}$ under the conditions indicated above is equal to $1.3 \cdot 10^{-2}$ V/cm.

6. DEFORMED A_3B_5 CRYSTALS

In deformed A_3B_5 crystals

$$\Omega^{(1)} = -\frac{C_3 \varepsilon}{\hbar} [\mathbf{lk}]. \quad (6.1)$$

Here $\mathbf{l}$ is a unit vector with the components $l_z = \varepsilon_{xy}/\varepsilon$, etc., and $\varepsilon = (\Sigma_{i>j} \varepsilon_{ij}^2)^{1/2}$. In accordance with Eqs. (1.8) and (3.6) the contribution of the relaxational mechanism to the current is determined by the expression

$$\mathbf{j} = -\frac{en\tau_p \varepsilon C_3}{\hbar \tau_0} [\mathbf{l}, \mathbf{S}_0 - \mathbf{S}]. \quad (6.2)$$

The kinetic contribution in the D'yakonov-Perel' mechanism, according to Eqs. (4.3) and (4.4), is equal to

$$\Delta\mathbf{j} = -\frac{en\tau_p \varepsilon C_3}{4\hbar \tau_s^{(3)}} \frac{\tau_2(\tau_1 + 2\tau_3)}{\tau_1 \tau_3} [\mathbf{lS}]. \quad (6.3)$$

Here $(\tau_s^{(3)})^{-1}$ is the contribution to τ_s^{-1} in Eq. (4.5) determined solely by $\Omega^{(3)}$:

$$\frac{1}{\tau_{s,\mathrm{DP}}^{(3)}} = 8Q\tau_{j,} \gamma_c^2 \frac{(k_B T)^3 m^3}{\hbar^8}. \quad (6.4)$$

The coefficient Q depends on the mechanism of scattering.

This contribution to $\tau_{s,\mathrm{DP}}^{-1}$ predominates for small deformations, while for large deformations $\Omega^{(1)}$ makes the main contribution to $\tau_{s,\mathrm{DP}}^{-1}$. This contribution is equal to

$$\frac{1}{\tau_{s\,ij,\mathrm{DP}}}=\tau_p\frac{(C_3\varepsilon)^2 m k_B T}{\hbar^4}(\delta_{ij}+l_i l_j). \tag{6.5}$$

The contribution to the current from the BAP mechanism or from scattering by paramagnetic ions is equal to, according to Eq. (4.9),

$$\mathbf{j}=-en\alpha\frac{\tau_p}{\tau_{s,\mathrm{BAP}}}\frac{C_3\varepsilon}{\hbar}[\mathbf{l}\times\mathbf{S}]. \tag{6.6}$$

According to Eq. (4.14) the Elliott-Yafet mechanism makes the contribution

$$\mathbf{j}^{(4)}=2en\frac{C_3\varepsilon}{\hbar}\frac{k_B T}{E_g}\eta\delta\,[\mathbf{l}\times\mathbf{S}]. \tag{6.7}$$

Under tension or compression along the (111) axis, when $\mathbf{l}\|(111)$, and when the light propagates in the same direction, current arises only in a transverse magnetic field, and

$$[\mathbf{l}\times\mathbf{S}]=\frac{T_\| T_\perp}{\tau_0}\frac{\Omega_B S_0}{1+\Omega_B^2 T_\| T_\perp}. \tag{6.8}$$

In this case the current is always parallel to the magnetic field.

We shall estimate the contributions made by different mechanisms to the current. As an example we shall study p-type GaAs with hole density $p=4\cdot10^{16}\,\mathrm{cm}^{-3}$ and compensation $K=0.3$. For this material the spin-relaxation rate was found to be increased by uniaxial deformation under compression along the (111) axis.[17-19] At $T=77$ K, according to Ref. 20, $\tau_p=3\cdot10^{-13}$ s, $\tau_0=10^{-8}$ s, and $\tau_s=1.5\cdot10^{-9}$ s. Under these conditions the main contribution to the current is the kinetic one (6.7) due to the contribution of the Elliott-Yafet mechanism. We underscore the fact that the Elliott-Yafet mechanism does not make an appreciable contribution to the spin-relaxation rate in GaAs, which is determined by the DP and BAP mechanisms.[21] The contributions (6.2) and (6.3) are equal to 3 and 5%, respectively, of the contribution (6.7). Because τ_s^{-1} increases as the pressure increases, the optimal current is reached when $(\tau_{s\perp}^{(1)})^{-1}=3^{-1/2}[\tau_0^{-1}+(\tau_s^{(3)})^{-1}]$. Under the indicated deformation and with photoconductivity predominating over the dark conductivity, at $\tau_0=10^{-9}$ s the quantity $E_{xx}^{\max}=10^{-5}$ V/cm. If under the same conditions τ_0 is reduced to $2\cdot10^{-10}$ s, the quantity $E_{xx}^{\max}$ increases by a factor of 12.

For the same samples at $T=300$ K the times are $\tau_p=1.5\cdot10^{-13}$ s, $\tau_0=10^{-8}$ s, and $\tau_s^{(3)}=10^{-10}$ s. In this case the contribution (6.7) is the main one, and the contributions (6.2) and (6.3) are equal to 0.3 and 28%, respectively, of the contribution (6.7). Under the maximum pressure reached in Ref. 20 ($P=4$ kbar) we have $E_{xx}^{\max}=5\cdot10^{-5}$ V/cm. When τ_0 decreases to $5\cdot10^{-10}$ s the value of $E_{xx}^{\max}$ increases to 10^{-3} V/cm.

The contribution (6.5) can predominate in crystals with paramagnetic impurities. Since τ_0 and τ_s decrease simultaneously when the hole density increases, in strongly doped p-type samples the relaxational contribution (6.2) will apparently be greater than the contribution (6.5).

7. CRYSTALS WITH WURTZITE STRUCTURE

The band structure of A_3B_6 and A_3B_5 crystals with a lattice of the wurtzite type is similar to that of cubic A_3B_5 crystals, oriented along the (111) axis and having C_{3v} symmetry.[8] The higher symmetry of wurtzite C_{6v} is not manifested in the effects studied here. For this reason all formulas from the preceding section with $\mathbf{l}\|(111)$ are applicable to wurtzite, if $C_3\varepsilon/2$ is replaced in them by a corresponding constant α_c determining the spin-dependent splitting of the conduction band:

$$H_{\mathbf{k}}=\alpha_c(\boldsymbol{\sigma}\cdot[\mathbf{n}\times\mathbf{k}]). \tag{7.1}$$

Here $\mathbf{n}$ is a unit vector oriented along the principal axis C_6.

If the crystal-field splitting of the valence band Δ_{cr} is small compared with the spin-orbit splitting Δ, then the expression (4.12) for η remains valid. In the opposite limiting case $\Delta_{\mathrm{cr}}\gg\Delta$ we have $\eta=1$. In the case of excitation with light only out of the upper valence band the quantity S_0 is equal to 1/2 instead of 1/4 for A_3B_5 crystals. For this geometry, like in III-V crystals deformed along the (111) axis, a current arises only in a transverse magnetic field. In the absence of a magnetic field the current due to the circular photovoltaic effect arises only if the light propagates in a direction perpendicular to the C_6 axis. In this case the strong birefringence makes it difficult to observe the effect. For this reason no quantitative studies of the circular photovoltaic effect have yet been performed on wurtzite crystals.

8. TELLURIUM

Unlike wurtzite, in tellurium excited by circularly polarized light propagating along the principal axis C_3 there arises a current due to the circular photovoltaic effect in the same direction. In a transverse magnetic field there appears, as a result of the Hall effect, a transverse current which reaches a maximum for $\Omega_B=\tau_p^{-1}$. In addition to this current, a transverse current, which is generated by the mechanism studied here and which reaches a maximum in weaker fields, $\Omega_B=(T_\| T_\perp)^{-1/2}$, should also be observed.

The spin-orbital splitting of the conduction band of tellurium is determined by the expression

$$H_{\mathbf{k}}=\alpha_c\boldsymbol{\sigma}\mathbf{k}, \tag{8.1}$$

i.e.,

$$\boldsymbol{\Omega}^{(1)}=(\alpha_c/2\hbar)\mathbf{k}. \tag{8.2}$$

For this reason, in accordance with Eqs. (3.6), (1.10) and (4.14), the contributions to the current from the relaxational and kinetic mechanisms, due to the spin relaxation in the Elliott-Yafet mechanism, are determined by the expressions

$$\mathbf{j}_{\mathrm{rel}}=-2en\frac{\tau_p}{\tau_0}\frac{\alpha_c}{\hbar}\mathbf{S}_\perp, \tag{8.3}$$

$$\mathbf{j}_{\mathrm{kin}}=2en\frac{k_B T}{E}\eta\delta'\mathbf{S}_\perp. \tag{8.4}$$

Here

$$\eta=[2(2-\delta')-(2-\delta)\delta/\delta']/(4-\delta-2\delta'),$$
$$\delta=2\Delta/(E_g+2\Delta),\quad \delta'=\Delta'/(E_g'+\Delta'),$$

E_g is the band gap, i.e., the separation between the top of the

valence bands M'_{1v} or M'_{2v} and the conduction band M'_{3c}; 2Δ is the separation between the bands M'_{1v} and M'_{3v}; E'_g is the separation between the lower valence band M'_{3v} and M'_{3c}; and, Δ' is the separation between the uppermost of the bands M'_{1v} or M'_{2v} and M'_{3v}. In contrast to wurtzite crystals, in tellurium the current is oriented along $\mathbf{S}_\perp$, i.e., perpendicular to the magnetic field, like the Hall current. Since the constant β in the term linear in $\mathbf{k}$ in the spectrum of the valence band, a constant determining the current due to the circular photovoltaic effect, is approximately two orders of magnitude larger than the constant α_c in Eq. (8.1), the contribution studied here can be separated from the Hall contribution only for $T/\tau_p \gtrsim 10^2$. For this reason it may be more convenient to excite two-dimensional electrons located in a quantum well at the surface of the sample; these electrons make no Hall contribution, while the effect under study remains.

[1)] For inelastic scattering $\mathrm{St}\Delta\rho^0$ has an additional term, containing the product $\mathbf{SA}_{\mathbf{kk}}\mathbf{B}^{(1)}_{\mathbf{kk}_1}$, that contributes to the current: $(\mathrm{St}\Delta\rho^0)_4 \sim f_0(E^0_k) - f_0(E^0_{k_1})$. Estimates show that the contribution of this term to the current is small in the parameter $(\tau_p/\tau_\varepsilon)(T_e - T)/T$, where T_e is the temperature of the electrons and T is the temperature of the lattice, compared with the contribution of $(\mathrm{St}\Delta\rho^0)_1$.

[2)] In asymmetric quantum wells $\mathbf{\Omega}^{(1)}$ can also contain the contribution $\mathbf{\Omega} \sim \mathbf{n} \times \mathbf{k}$, where $\mathbf{n}$ is a unit vector normal to the surface.

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