Mesoscopic fluctuations of the resistance of point contacts

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It is shown that the resistance of a point contact whose dimensions are smaller than the electron mean-free path undergoes mesoscopic fluctuations as a function of the magnetic field and voltage on the contact. In weak fields these fluctuations are determined by spatial scales much larger than both the dimensions of the contact and the electron mean free path. The spatial correlation functions of the conductance of a tunneling microscope with a contact of atomic size are calculated.

There is great interest in mesoscopic fluctuations of the resistance of small disordered metallic samples.[1-7] In the last few years there have appeared a number of works[5-7] in which the properties of metallic contacts whose dimensions a are not only less than the elastic electron mean-free path l but also of the order of the electron wavelength $\lambda = 2\pi\hbar/p_F$, where p_F the Fermi momentum. Another experimental possibility for studying mesoscopic fluctuations of the resistance is to investigate the resistance of a tunneling contact with dimensions $a \approx \lambda$ in a tunneling microscope. (Such investigations have already been performed,[8] but in Ref. 8 the resistance of the tunneling contact was not studied, but rather the tunneling microscope was employed as a potential contact for measuring voltage fluctuations.)

At first glance it appears that in the case $a \ll l$ the scattering of electrons by impurities does not affect the conductance of such contacts. We shall show below that in spite of the fact that the dimensions of the contact $a \ll l$ the dependence of the resistance of such a contact on the magnetic field is determined by spatial scales of the order $\min\{L_H, L_T\}$ which are much larger than l. Here $L_H = (\hbar c/eH)^{1/2}$ is the magnetic length, $L_T = (D\hbar/T)^{1/2}$ is the coherence length of the normal metal, $D = v_F l/3$ is the electron diffusion coefficient, v_F is the Fermi velocity of the electrons, H is the magnetic field strength, T is the temperature, and e is the electron charge.

The results obtained below are also applicable to metal-insulator-metal systems, in which the current is transported through the insulator with the help of resonance tunneling through localized states in the insulator.[9]

We shall start from the tunnel Hamiltonian

$$\hat{H}_T = \int_S d\mathbf{r}_1\, d\mathbf{r}_2 \{T(\mathbf{r}_1, \mathbf{r}_2)\, \Psi_1^+(\mathbf{r}_1)\, \Psi_2(\mathbf{r}_2) + \text{h.c.}\}. \tag{1}$$

Here $\Psi_{1,2}^+(\mathbf{r})$ and $\Psi_{1,2}(\mathbf{r})$ are operators that create and annihilate electrons in the "right-hand" and "left-hand" half-spaces, respectively, and the integration is performed over the plane of the tunneling contact S. We shall neglect the thickness of the tunneling contact, so that $T(\mathbf{r}_1, \mathbf{r}_2) = \hat{T}(\mathbf{r}_1)\delta(\mathbf{r}_1 - \mathbf{r}_2)$. In addition we shall assume that the dimensions of the contact are of the order of the electron wavelength λ, so that $\hat{T}(\mathbf{r}) = T_0(\mathbf{r}_0)\delta(\mathbf{r} - \mathbf{r}_0)$, where $\mathbf{r}_0$ is the coordinate of the contact on the surface of the sample.

The standard expression for the tunneling current follows from Eq. (1):

$$J(\mathbf{r}_0) = \pi e |T_0(\mathbf{r}_0)|^2 \times \int_{-\infty}^{\infty} d\varepsilon [n(\varepsilon + eV) - n(\varepsilon)]\, \nu_1(\varepsilon, \mathbf{r}_0)\, \nu_2(\varepsilon + eV, \mathbf{r}_0). \tag{2}$$

Here $\nu_i(\varepsilon, \mathbf{r}) = (2/\pi)\,\mathrm{Im} G_i^A(\varepsilon, \mathbf{r}, \mathbf{r})$ is the local density of states on the ith edge, $G_i^A(\varepsilon, \mathbf{r}, \mathbf{r}')$ is the advanced Green's function, $n(\varepsilon) = [\exp(\varepsilon/T) + 1]^{-1}$ is the Fermi distribution function, and V is the voltage on the contact.

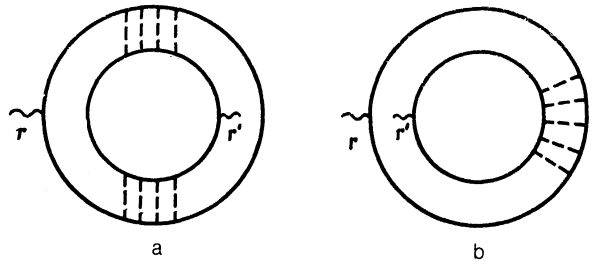
Mesoscopic fluctuations of $J(\mathbf{r})$ are determined by the corresponding fluctuations $\delta\nu(\varepsilon, \mathbf{r}) = \nu(\varepsilon, \mathbf{r}) - \langle \nu(\varepsilon, \mathbf{r}) \rangle$, which arise as a result of the random interference of electron waves propagating along different diffusion trajectories.[2] Here the angular brackets $\langle\ \rangle$ denote averaging over the impurities.

1. We shall begin by studying the mesoscopic fluctuations of the local density of states. To calculate the correlation function $\langle \delta\nu(\varepsilon, \mathbf{r}) \cdot \delta\nu(\varepsilon', \mathbf{r}') \rangle$ for $|\mathbf{r} - \mathbf{r}'| > l$ it is necessary to sum the diagrams shown in Fig. 1a,[10] where the solid lines correspond to the electron Green's functions and the dashed lines correspond to scattering by impurities. Summation of the Cooper and diffusion diagrams leads to the expression

$$\langle \delta\nu(\varepsilon, \mathbf{r}, \mathbf{H})\, \delta\nu(\varepsilon', \mathbf{r}', \mathbf{H}') \rangle = \frac{2}{\pi^2} \mathrm{Re}\{P_{\varepsilon-\varepsilon'}(\mathbf{r}, \mathbf{r}', \mathbf{H}+\mathbf{H}')\, P_{\varepsilon-\varepsilon'}(\mathbf{r}', \mathbf{r}, \mathbf{H}+\mathbf{H}') + P_{\varepsilon-\varepsilon'}(\mathbf{r}, \mathbf{r}', \mathbf{H}-\mathbf{H}')\, P_{\varepsilon-\varepsilon'}(\mathbf{r}', \mathbf{r}, \mathbf{H}-\mathbf{H}')\}, \tag{3}$$

where $P_{\varepsilon-\varepsilon'}(\mathbf{r}, \mathbf{r}', \mathbf{H})$ satisfies the equation[4,11]

$$\left\{-D\left(\nabla - \frac{ie}{c}\mathbf{A}\right)^2 + \tau_\varphi^{-1} - i\hbar^{-1}(\varepsilon - \varepsilon')\right\} P_{\varepsilon-\varepsilon'}(\mathbf{r}, \mathbf{r}', \mathbf{H}) = \delta(\mathbf{r} - \mathbf{r}'). \tag{4}$$



a b

FIG. 1.

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Here A is the vector potential of the magnetic field and τ_φ is the disruption time of the phase of the electron wave functions.

It follows from Eqs. (3) and (4) that for $\mathbf{H}=\mathbf{H}'=0$, $|\mathbf{r}-\mathbf{r}'|>l$

$$\langle \delta\nu(\varepsilon,\mathbf{r})\,\delta\nu(\varepsilon',\mathbf{r}')\rangle = \frac{1}{(2\pi^2 D\hbar)^2}\,\mathrm{Re}\begin{cases}\left[\dfrac{\exp\left\{-\dfrac{|\mathbf{r}-\mathbf{r}'|}{L(\varepsilon-\varepsilon')}\right\}}{|\mathbf{r}-\mathbf{r}'|}\right]^2, & |\mathbf{r}-\mathbf{r}'|<L_z,\\ \left[\dfrac{2}{L_z}K_0\left(\dfrac{|\mathbf{r}-\mathbf{r}'|}{L(\varepsilon-\varepsilon')}\right)\right]^2, & L_z<|\mathbf{r}-\mathbf{r}'|<L_x, L_y.\end{cases} \tag{5}$$

Here L_z is the thickness of the film, L_x and L_y are the dimensions in the plane of the film

$$L(\varepsilon)=\{D[\tau_\varphi^{-1}+i\hbar^{-1}\varepsilon]^{-1}\}^{1/2},$$

and $K_0(x)$ is a Macdonald function.

We note that for $|\mathbf{r}-\mathbf{r}'|>|L(\varepsilon-\varepsilon')|$ the quantity $\langle\delta\nu(\varepsilon,\mathbf{r})\delta\nu(\varepsilon',\mathbf{r}')\rangle$ undergoes, aside from exponential decay, oscillations as a function of $|\mathbf{r}-\mathbf{r}'|$.

Multiplying Eq. (5) by $n(\varepsilon)$ and $n(\varepsilon')$ and integrating over $\mathbf{r}$ and $\mathbf{r}'$ we arrive at an expression for the correlation function of the electron densities in the disordered metal[12].

$$\langle\delta n(\mathbf{r})\,\delta n(\mathbf{r}')\rangle \approx \frac{1}{|\mathbf{r}-\mathbf{r}'|^4}\begin{cases}|\mathbf{r}-\mathbf{r}'|^{-2}, & l<|\mathbf{r}-\mathbf{r}'|<L_z, L_T,\\ L_z^{-2}, & L_z<|\mathbf{r}-\mathbf{r}'|<L_T.\end{cases} \tag{6}$$

Here $\delta n(\mathbf{r})=n(\mathbf{r})-\langle n(\mathbf{r})\rangle$ are the fluctuations of the density of the electron gas. The correlation function of the densities depends on the magnetic field, so that, for example, for $l<|\mathbf{r}-\mathbf{r}'|<L_H<L_T, L_z$ the relation

$$\frac{\langle\delta n(\mathbf{r},\mathbf{H})\,\delta n(\mathbf{r}',\mathbf{H})\rangle}{\langle\delta n(\mathbf{r},0)\,\delta n(\mathbf{r}',0)\rangle}=1-\left(\frac{|\mathbf{r}-\mathbf{r}'|^2}{3\pi L_H^2}\right)^2$$

is satisfied.

It is also interesting that the quantity (5) is directly related to the fluctuations of the density of states in the mesoscopic sample[10]

$$\langle\delta\nu(\varepsilon)\,\delta\nu(\varepsilon')\rangle\sim\mathrm{Re}\,L(\varepsilon-\varepsilon')\,[D^2\hbar^2L_xL_yL_z]^{-1}$$

by integration over $\mathbf{r}$ and $\mathbf{r}'$ over the volume of the sample ($|L(\varepsilon-\varepsilon')|<L_x, L_y, L_z$).

On the other hand, the quantity Eq. (3) can be measured by scanning the surface of the sample with a tunneling microscope. In this case

$$\frac{\langle\delta G_0(\mathbf{r})\,\delta G_0(\mathbf{r}')\rangle}{\langle G\rangle^2}=\frac{1}{\langle\nu\rangle^2}\int d\varepsilon\,d\varepsilon'\,n'(\varepsilon)n'(\varepsilon')\langle\delta\nu(\varepsilon,\mathbf{r})\,\delta\nu(\varepsilon',\mathbf{r}')\rangle. \tag{7}$$

Here

$$G_0(\mathbf{r})=\frac{dJ(\mathbf{r})}{dV}\bigg|_{V=0},\qquad \langle G\rangle=\pi^2 e|T_0|^2\nu_0\langle\nu\rangle,$$

and ν_0 is the density of states in the microscope tip. We assume that the density of states in the tip of the tunneling contact does not depend on ε. It should be kept in mind that the formula (4) for $|\mathbf{r}-\mathbf{r}'|<L_z$ corresponds to a three-dimensional sample. In the case of a semi-infinite sample, when $\mathbf{r}$ and $\mathbf{r}'$ are located on the surface, an extra factor of four appears in the corresponding expression. As a result, for $l<|\mathbf{r}-\mathbf{r}'|<L_T$ we have

$$\frac{\langle\delta G_0(\mathbf{r})\,\delta G_0(\mathbf{r}')\rangle}{\langle G\rangle^2}=\frac{1}{(\hbar D\langle\nu\rangle\pi^2)^2}\begin{cases}|\mathbf{r}-\mathbf{r}'|^{-2}, & |\mathbf{r}-\mathbf{r}'|<L_z,\\ \left[\dfrac{1}{L_z}\ln\dfrac{|\mathbf{r}-\mathbf{r}'|}{L_T}\right]^2, & |\mathbf{r}-\mathbf{r}'|>L_z.\end{cases} \tag{8}$$

2. We shall now study mesoscopic conductance fluctuations due to variation of the magnetic field.

It is important that in order for the Friedel oscillations to decay rapidly the impurities closest to $\mathbf{r}_0$ must make the main contribution to $\langle[\delta\nu(\varepsilon,\mathbf{r}_0)]^2\rangle$. This makes it impossible to calculate this quantity with the help of the diagrammatic technique of Ref. 13, which presupposes that the potential generated by the impurities in the sample has a white-noise spectrum. It is significant, however, that the dependence of the quantity $\delta\nu(\varepsilon,\mathbf{r}_0)$ [and hence also $\delta J(\mathbf{r}_0)$] on the magnetic field is determined by the large spatial scales of the order of $\min\{L_H, L_T\}\gg l$. For this reason we assume that the magnetic field dependences obtained below for the resistance of the contacts are universal in the magnetic fields.

Using the standard diagrammatic technique[13] and summing the diagrams shown in Fig. 1b we obtain

$$\frac{\langle[\delta G_0(\mathbf{H})-\delta G_0(0)]^2\rangle}{\langle G\rangle^2}=B\int d\varepsilon_1\,d\varepsilon_2 n'(\varepsilon_1)n'(\varepsilon_2)\times[3P_{\varepsilon_{12}}(0)+P_{\varepsilon_{12}}(2\mathbf{H})-4P_{\varepsilon_{12}}(\mathbf{H})], \tag{9}$$

where

$$P_{\varepsilon_{12}}(\mathbf{H})=\mathrm{Re}\,P_{\varepsilon_1-\varepsilon_2}(\mathbf{r}_0,\mathbf{r}_0,\mathbf{H}),\qquad B=\frac{2}{\pi\langle\nu\rangle}.$$

The expression (9) is reminiscent of the analogous formula obtained for the dependence of the activation energy of impurities in a disordered metal on $\mathbf{H}$.[14] The formal difference lies in the fact that $n'(\varepsilon)$ is replaced by $n(\varepsilon)$ and in the quantity B.

In weak magnetic fields, when $L_H>L_T$, we obtain from Eq. (9) (the magnetic field is oriented along the normal to the surface of the film)

$$\frac{\langle[\delta G_0(\mathbf{H})-\delta G_0(0)]^2\rangle}{\langle G\rangle^2} \approx \frac{1}{D\hbar\langle\nu\rangle}\left(\frac{\omega_H}{2T}\right)^4 \begin{cases} 0{,}3L_T^{-4}, & L_T, \quad L_H<L_z, \\ 0{,}9L_z^{-4}, & L_z<L_T, \quad L_H. \end{cases} \tag{10}$$

Here $\omega_H = 4DeH/c$.

It follows from Eqs. (9) and (10) that in weak magnetic fields the magnetoresistance of the contact depends quadratically on $\mathbf{H}$, $G_0(\mathbf{H}) = G_0(0) + \beta_{ik}\mathbf{H}_i\mathbf{H}_k$, where β_{ik} is a random tensor which depends on the realization of the scattering potential near the contact.

In strong magnetic fields, when $L_H < L_T$, the quantity $G_0(\mathbf{H})$ undergoes random oscillations. From Eq. (9) for $L_T > L_H > l$ we obtain (the magnetic field is oriented along the normal to the surface)

$$\frac{\langle[\delta G_0(\mathbf{H})-\delta G_0(0)]^2\rangle}{\langle G\rangle^2} = \frac{1}{2\pi^2\hbar D\langle\nu\rangle} \times \begin{cases} (6-5\cdot 2^{1/2})\zeta\left(\frac{1}{2}\right)\left(\frac{\omega_H}{\hbar D}\right)^{1/2}, & L_H<L_T<L_z, \\ \frac{3}{L_z}\ln\frac{\omega_H}{T}, & L_z<L_H<L_T, \end{cases} \tag{11}$$

where $\zeta(x)$ is the Riemann zeta function.

It follows from Eq. (11) that the amplitude of the oscillations $\delta G_0(\mathbf{H})$ increases as H increases. The characteristic period of these oscillations ΔH also increases as H increases, so that $\Delta H \sim H$. This assertion can be verified by calculating the function $\langle[\delta G_0(\mathbf{H}) - \delta G_0(0)][\delta G_0(\mathbf{H}+\Delta\mathbf{H}) - \delta G_0(0)]\rangle$.

The qualitative explanation of the results is that the contribution of the impurities to $\delta J(\mathbf{r}_0)$ and $\delta\nu(\varepsilon_i\mathbf{r}_0)$ on account of the Friedel oscillations of the electron density decays in a power-law fashion as a function of the distance from the impurity to the point of observation,[12] i.e., distant impurities make a small contribution. This contribution, however, is much more sensitive to the magnetic field than the contribution of the nearest impurities. As a result at $T = 0$ impurities located at a distance L_H from the observation point make the main contribution to Eq. (11), and the contribution of impurities to δJ increases in a power-law fashion as H increases.

Another possibility for studying fluctuations of the local density of states[15] is to measure the fluctuations of the conductance as a function of v. In this case

$$\frac{\langle[\delta G_V-\delta G_0]^2\rangle}{\langle G\rangle^2} = \begin{cases} \frac{2(eV)^{1/2}}{\pi^{5/2}D^{3/2}\langle\nu\rangle\hbar^{3/2}}\int_0^\infty \frac{dx(1-\cos x)}{x^{3/2}}, & l<L_V<L_T, \quad L_z, \\ \frac{2}{\pi^2 DL_z\langle\nu\rangle\hbar}\ln\frac{eV}{T}, & l<L_z<L_V<L_T. \end{cases} \tag{12}$$

Here $L_v = (D\hbar/eV)^{1/2}$.

The quantity G_V as a function of V (at $T = 0$) contains all characteristic periods of the oscillations. In addition, the amplitudes of the corresponding harmonics increase in a power-law fashion as V increases, and the important characteristic spatial scale of the problem is of the order of $L_V > l$. From the experimental viewpoint this means that the more accurately $J(V)$ is measured the finer the structure that can be observed.

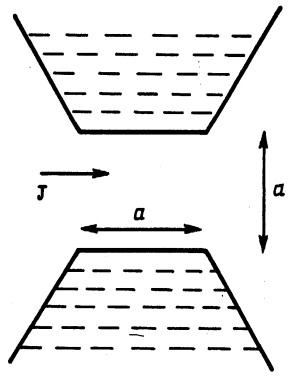

FIG. 2.

It should also be noted that in the general case the density of states of the tip of a tunneling microscope can also undergo mesoscopic fluctuations as a function of the magnetic field or energy. This leads to an additive contribution to the correlation functions of the conductance, given by the relations (10)–(12). The magnitude of this correction is also determined by the equalities Eqs. (10)–(12), and in addition the values for the tip must be used for the diffusion coefficient, the density of states, the phase disruption time, and the geometric dimensions.

As already mentioned, we assume that relations (10)–(12) are universal and do not depend on the properties of the contact, if $a \ll L_l$, L_H, and L_v, where a is the size of the contact. In particular, for $\lambda < a < l$ the quantity B in Eq. (9) acquires the extra factor $(\lambda/a)^2$ and for $l < a \ll L_z$, L_y, and L_x the extra factor is $(Da\langle\nu\rangle\hbar)^{-1}$.

Moreover, the formulas (10)–(12) are also valid for the case of a point contact when there is no tunneling contact. To check this assertion we studied the geometry of the sample shown in Fig. 2, when all dimensions, including the size of the contact, are greater than l. In this case the situation can be described by Langevin equations with a special form of the correlation function of the external currents.[16] Solving Langevin's equation for L_T, $L_H > a$ we obtain the expression (9) with $B \approx D\hbar a\langle\nu\rangle$. The expression (12) can be derived analogously. In the case $L_T > a \gg l$ the expressions obtained above for $l < \{L_H, L_V\} < a$ transform into the corresponding expressions for the amplitude of the fluctuations of G as a function of $\mathbf{H}$ and V.[4,15]

It is important to note that the ergodicity hypothesis,[4] according to which averaging over the magnetic field is equivalent to averaging over a random distribution of impurities, is not applicable to the case studied above and to formulas (9) and (10). This is evident at least from the fact that Eq. (9) depends explicitly on $\mathbf{H}$, so that the amplitude $\delta G(\mathbf{H})$ of the oscillations and the characteristic period ΔH increase as H increases. The ergodicity hypothesis is first satisfied only when $l < L_H < a$.

3. Conclusion. In Ref. 6 mesoscopic fluctuations of $G(\mathbf{H})$ were already observed experimentally for a single-mode contact. However these experiments were performed under conditions when L_H is of the order of the size of the contacts and the oscillations were determined by the inter-

ference of waves inside the channel, while the formula (9) is valid in much weaker fields $L_H > l > a$. In addition, it should be kept in mind that all results obtained above pertain to the case when the scattering potential has the form of white noise. This means that in real semiconductor systems with high mobility, when the electrons are scattered by the random potential at small angles, l must be interpreted as the transport mean free path.

We note that in applying the results to metal-insulator-metal systems with resonance tunneling (see Ref. 9) it should be kept in mind that there exists another source of fluctuations of $G(H)$—mesoscopic fluctuations of the capacitance of the resonance state in the insulator. Such fluctuations arise as a result of the screening of this charge state by the metallic borders.

Finally we note that the expression for the correlation function of the electron densities (6) was derived neglecting the electron-electron interaction. Including this interaction results in effective screening of the density fluctuations. When this is done all other results of this work remain unchanged, but there arise fluctuations of the electric field $\delta\mathbf{E}(\mathbf{r})$ in the sample. The correlation function of these fluctuations has the form ($|\mathbf{r}-\mathbf{r}'| > l > \varkappa$, where $\varkappa$ is the Debye radius)

$$\langle \delta E_i(\mathbf{r})\, \delta E_j(\mathbf{r}') \rangle = (4\pi\varkappa^2 e)^2 \frac{\partial^2}{\partial r_i\, \partial r_j'} \langle \delta n(\mathbf{r})\, \delta n(\mathbf{r}') \rangle .$$

These fluctuation electric fields can be measured with the help of a scanning tunneling microscope of atomic forces.

Another phenomenon associated with the existence of such fields is adsorption of inert gases on the surface of a metal. In this case the correction to the adsorption energy is proportional to $\alpha(\delta\mathbf{E})^2$, where α is the polarizability of an atom of the adsorbed gas.

The attraction between macroscopic bodies at sufficiently low temperatures can also be determined by the fluctuation mechanism noted above and can compete with the Van-der-Waals mechanism.[17,18]

Analogously to Refs. 2, 4, 11, and 14 switching on a magnetic field results in fluctuations of $\delta\mathbf{E}(\mathbf{r})$ as a function of $\mathbf{H}$ and in a reduction of the quantity $\langle(\delta\mathbf{E}(\mathbf{r}))^2\rangle$, i.e., it causes the attractive force acting between the bodies and the adsorption energy to depend on $\mathbf{H}$.

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