Dynamic interaction and collisions of Bloch lines in a ferromagnet

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An analysis is made of the interaction of Bloch lines moving under the action of a gyroscopic force in a domain wall inside a uniaxial ferromagnet characterized by a strong anisotropy. A numerical analysis of the Slonczewski equations shows that in the case of head-on collisions of Bloch lines there is a critical velocity above which the annihilation of these lines creates a soliton-like transmission effect. The dependences of the critical velocity on the magnitude of magnetic relaxation and on the flexural rigidity of a domain wall are found. It is shown that several Bloch lines moving along one direction form a stable cluster which is stabilized by gyroscopic pressure forces. A study is made of the collision processes between clusters containing several Bloch lines. It is shown that in addition to the range of velocities where annihilation takes place and the critical velocity above which a soliton-like transmission effect is observed, there is an intermediate range of velocities at which partial reconstruction of clusters after a collision takes place.

INTRODUCTION

Studies of Bloch lines separating regions of a Bloch domain wall with different directions of spin rotation are of fundamental importance for the understanding of dynamic processes in a magnetic material.[1,2] Bloch lines are also of interest in microelectronics in connection with the development of very-large-capacity memories.[3]

From the topological point of view a Bloch line represents a nonlinear defect of the vector magnetization field, i.e., a magnetic vortex. The vortex properties of a Bloch line determine its gyrotropic interaction with a domain wall within which the line is moving. The structure of a moving Bloch line is quite complex. It consists of a compact core where the spin orientation changes rapidly and an extended "shell" representing bending of a domain wall at the point where the line is located.[4–11] The core of a moving Bloch line is a carrier of its topological charge, governing the degree of rotation of the spins in the wall. Bending of a domain wall occurs only due to the motion of a Bloch line under the influence of a gyrotropic force acting on the wall along the normal to its plane. Under static conditions there is no such bending and we can expect only a slight change in the domain wall thickness at the points of location of a Bloch line.[2] Bending is particularly important from the experimental point of view in observations of moving Bloch lines, although other ways of observing Bloch lines are currently under development. An amplitude of such a bending or sag due to a one moving Bloch line is slight and it is experimentally easier to investigate clusters containing N Bloch lines, where N is the topological charge of the cluster equal to the total angle of rotation of the spins divided by π.

In the case of a uniaxial strongly anisotropic ferromagnet both Bloch lines and their clusters can be described using the Slonczewski equations,[2] which represent essentially suitably reduced Landau–Litshitz equations in which a single moving Bloch line can be described by a soliton-type solution.[9] It is natural to assume that Bloch line clusters can be described by multisoliton mathematical structures. Although the Slonczewski equations do not belong to the familiar classes of integrable systems, this link is quite significant, especially as the Slonczewski equations have an important asymptote in which they go over to the generalized sine–Gordon equation.[8] If we allow for the damping and external pumping, this asymptote is completely isomorphous to equations of a distributed Josephson junction for which the soliton solutions have been thoroughly investigated both theoretically and experimentally. It follows that we can expect manifestation of the soliton properties in the dynamics of Bloch lines and their clusters, particularly in collision processes.

Chetkin *et al.*[13,14] reported the results of experimental investigations of the dynamics of Bloch line clusters confirming the above ideas. In particular, they showed that fairly fast Bloch line clusters pass through each other in the case of head-on collisions and they still retained the individual topological charges, whereas slow clusters become annihilated. A classical theory of solitons gives only guiding ideas for the design and interpretation of relevant experiments. It is necessary to study the real situation allowing for pumping, dissipation, deformability of domain walls, etc. The results of such an investigation are presented below.

One should mention that an interesting attempt to investigate numerically the dynamics of topological and nontopological soliton-like excitations in a domain wall was recently made on the basis of the Slonczewski equations for a number of specific conditions where the stability of a domain wall would not be obvious because of the occurrence of bending perturbations (for details see later). This makes it difficult to use the undoubtedly interesting results directly in discussing the available experimental data.

1. MAIN EQUATIONS. SINE–GORDON ASYMPTOTICS

The present paper develops further an earlier treatment[9] and the interested reader can find details of formulation of the problem of the motion of a Bloch line in a domain wall in that earlier paper. In accordance with the theory of

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domain walls in uniaxial magnetic films,[1,2] we shall consider a wall as a surface, i.e., we shall ignore its thickness. The state of a wall is described by two coordinates: $q(\mathbf{r}, t)$ and $\varphi(\mathbf{r}, t)$, where $q(\mathbf{r}, t)$ is the shift of the center away from the equilibrium position; $\varphi(\mathbf{r}, t)$ is the azimuthal angle governing the spin orientation at the center of a domain wall in a plane normal to the wall; $\varphi = 0$ corresponds to the case when the spins lie within the domain wall plane.

The main equations of the dynamics of a domain wall with a Bloch line can be represented as follows:

$$\begin{aligned} &q_t+\varphi_{xx}-0{,}5\sin 2\varphi=-V-\alpha\varphi_t, \\ &\varphi_t-q_{xx}+b^2q=\alpha q_t, \end{aligned} \qquad (1)$$

where q is measured in units of Δ; Δ is the domain wall thickness; x is the coordinate along the domain wall measured in units of the Bloch line thickness $\Delta_L = (A/2\pi M^2)^{1/2}$; M is the saturation magnetization; A is the exchange stiffness constant; time is normalized to $(4\pi M\gamma)^{-1}$; γ is the gyromagnetic ratio; α is the dimensionless magnetic relaxation parameter; A is the velocity of translational motion of a domain wall measured in units of $4\pi\gamma M\Delta$. The term b^2q in Eq. (1) describes a restoring force acting on a domain wall and ensuring the existence of a stable equilibrium position $q = 0$ of a plane domain wall in the absence of pumping ($V=0$). Under real experimental conditions the restoring force is created by, for example, a gradient field H_z' in the case of an isolated domain wall (when $b^2 = H_z'\Delta/4\pi M$) or it is governed by the demagnetization fields in a stripe domain structure.

The left-hand side of the system (1) describes free motion of Bloch lines and their clusters among a domain wall. The right-hand side of the same system of equations describes the action of the gyroscopic force (pumping) exerted by a moving domain wall and effects of dissipation. In the case of steady-state motion of Bloch lines and clusters the pumping and dissipation balance each other out.

The equations in the system (1) differ somewhat in form from the corresponding system of equations used in Ref. 9, since pumping is expressed directly via a gyrotropic force exerted by a moving domain wall, whereas in Ref. 9 it is allowed for by a bias field h_z. We can easily go over from this field to the gyrotropic force. We can do this by substituting simply $h_z(t) = \dot{h}_z t$, where $\dot{h}_z = \text{const}$, so that after the substitution of $q \to q + Vt + \alpha V^2/b^2$, where $V = \dot{h}_z/b^2$, we obtain the system (1) from the equations given in Ref. 9.

It is assumed in Ref. 15 that $b = 0$. It is known that in films with the transverse (perpendicular) uniaxial anisotropy and an open domain structure a plane unpinned domain wall is unstable when flexural perturbations are present.[16] The development of such an instability may be suppressed by periodic boundary conditions used in Ref. 15 provided the period of the structure is sufficiently small. However, it is not clear what specific experimental situation corresponds to the undoubtedly interesting results of the mathematical modeling obtained in this formulation, i.e., it is not clear how such boundary conditions can be realized experimentally.

Inclusion of the restoring force in the system of equations (1) is also important from the mathematical point of view. This is because if $b = 0$, then the system of equations does not have any soliton-type solutions describing free motion of a Bloch line, i.e., the motion of such a line in the absence of pumping and dissipation when $\alpha = 0$ and $V = 0$, and satisfying the natural boundary conditions (for details see Ref. 9).

We can easily see how the asymptotic behavior of the system of equations (1) is obtained when $b \gg 1$. If we assume that $\|b^2q\| \gg \|q_{xx}\|$ and $\|\alpha q_t\|$, we obtain

$$b^{-2}\varphi_{tt}-\varphi_{xx}+0{,}5\sin 2\varphi=V+\alpha\varphi_t. \qquad (2)$$

This equation is isomorphous with the equation for a phase discontinuity of a wave function of superconducting electrons at a distributed Josephson junction.[17-19] Other examples of physical systems with the same "mathematics" are a one-dimensional conductor with a charge density wave,[20,21] a domain wall in a weak ferromagnet,[22] etc.

Naturally, in a more detailed theory we have to allow for the boundary conditions and for the dissipation field due to magnetic charges on the surface of a film. It is known that an allowance for such a magnetostatic field results in twisting of a domain wall across the film thickness.[2] It is shown in Ref. 23 that this effect is weak if the film is sufficiently thick so that its thickness is comparable with the Bloch line thickness. Moreover, in the case of films with the thickness not too large and governed by the characteristic sag of a domain wall, an allowance for the twisted shape of a domain wall is unimportant (apart from numerical renormalization of the parameters) in the case of dynamics of a single Bloch line.[24] We shall therefore assume that the adopted approximation of the local nature of the magnetostatic fields (Winter approximation[25]) can be applied also to problems of the dynamic interaction of Bloch lines.

The equilibrium positions in the phase space of the system of equations (1) are the points defined by

$$q_x=0, \quad \varphi_x=0, \quad q=0, \quad \varphi=n\pi+0{,}5\arcsin 2V,$$

where $n = 0, \pm 1, \pm 2, \ldots$. In the case of a domain wall at rest these equilibrium positions are joined by separatrices in the phase space describing isolated Bloch lines characterized by

$$q=0, \quad \varphi=n\pi\pm 2\,\mathrm{arctg}\,[\exp(x-x_0)]. \qquad (3)$$

There are no other solutions satisfying the boundary conditions $q_{x|x=\pm\infty} = \varphi_{x|x=\pm\infty} = 0$ in the static case. Therefore, in describing clusters of Bloch lines in the static case we have to allow for the long-range magnetostatic fields describing the attraction of adjacent Bloch lines.[3,26] In the case of a moving domain wall we can expect a gyroscopic pressure which compresses closely spaced Bloch lines and plays the same role as the magnetostatic attraction between these lines. This gives rise to separatrix solutions showing not only the adjacent equilibrium positions. If the Bloch line velocity is sufficiently high, the gyroscopic compression exceeds the magnetostatic interaction force of Bloch lines and the dynamics of clusters can be described satisfactorily by the local approximation.

In estimating the characteristic velocity we shall assume that the force of the magnetostatic interaction is approximately equal to the force of attraction between two charged lines separated by a distance equal to the Bloch line thickness and the gyroscopic pressure is governed by the normal component of the displacement of a domain wall in the region with the highest slope of its sag, which travels together with a cluster along the wall at a velocity u normalized to the value of S, where S is the maximum velocity of a Bloch line in an untwisted domain wall. It follows from Ref. 9 that $S = \gamma(8\pi A)^{1/2}$. In this case we can ignore the magnetostatic interaction if $u \gg (2\pi Q^{1/4})^{-1}$, where $Q = K/2\pi M^2$ and K is the uniaxial anisotropy constant. It should be pointed out that the maximum velocity of a Bloch line in thin films [$(d \leqslant 1)$, where d is the film-thickness in units of Δ_L] is unity.[8,9] Conditions of validity of the local approximation become more stringent in the case of thick films $(d > 1)$. In fact, because the domain wall is twisted, the maximum velocity of a Bloch line (like the maximum velocity of the domain wall[2]) is governed by the mechanism of generation of horizontal Bloch lines and we can show that under these conditions the maximum velocity of a Bloch line expressed in terms of the same dimensionless units is given by $u_{\max} \propto d^{-1/2}$ in the case of an isolated line and by $u_{\max} \propto (dN)^{-1/2}$ in the case of a cluster of Bloch lines with a topological charge N.

2. COLLISIONS OF BLOCH LINES: MUTUAL CROSSING OF THE LINES AND ANNIHILATION. CRITICAL VELOCITIES

We shall now consider head-on collisions of Bloch lines. Analytic solutions describing multisoliton Bloch lines can be obtained only in the asymptotic case when $b \gg 1$ when they are described by Eq. (2). These equations have been investigated thoroughly theoretically (see, for example, Ref. 27) in the case when there is no dissipation and no pumping $(\alpha = V = 0)$. Such equations have mult-soliton solutions which can be obtained by the Hirota method, by the method of the inverse scattering problem, or by other approaches (see Ref. 28). We shall give a solution which describes collisions of two kinks of Eq. (2), representing isolated Bloch lines moving at the same velocity u in a head-on manner:

$$\varphi(t,x) = 2\,\mathrm{arctg}\left\{\frac{b}{u}\,\mathrm{sh}\left[\frac{b(ut-x_0)}{(b^2-u^2)^{1/2}}\right]\Big/\left[\mathrm{sh}\left[\frac{b(x-x_1)}{(b^2-u^2)^{1/2}}\right]\right]\right\}, \tag{4}$$

where x_0 and x_1 are arbitrary constants. We can use perturbation theory of solitons to obtain equations describing the evolution of the parameters u and x_0 with time in the presence of dissipation and pumping $(\alpha \neq 0, V \neq 0)$, as was done in Ref. 18. A reduction in the velocity u to zero as a result of such a head-on collision of solitons is accompanied by the transition of kinks to a bound state resembling a breather, which subsequently relaxes to a vacuum state corresponding to $\varphi = \text{const}$. This gives the critical value of the velocity at which annihilation takes place.

The limiting value of the velocity of a domain wall at which the process of restoration of a Bloch line (after annihilation) can begin is on the other hand governed by the balance of the initial and final soliton energies

$$E(u) = 8/(1-u^2/b^2)^{1/2}$$

and the dissipation energy

$$E_d(\tau) = \int_{-\infty}^{0} dt \int_{-\infty}^{+\infty} dx\,(V\varphi_t + \alpha\varphi_t^2),$$

which is not compensated by external pumping during the soliton collision time $\tau \approx b/u$. Therefore, we have

$$2E(u_{cr}) = 2E(0) + E_d(\tau). \tag{5}$$

It is not possible to calculate E_d analytically. Therefore, we shall use the results of a numerical calculation reported in Ref. 18, where it is shown that the critical pumping rate V_{cr} is proportional to the relaxation parameter α. Bearing in mind that we can go over from Eq. (2) to an equation employed in Ref. 18 by the substitution of $t \to bt$ and $\alpha \to \alpha b$, we obtain the following expression for the $b \gg 1$ case under discussion:

$$V_{cr} \propto \alpha b. \tag{6}$$

In reality we usually have $b \ll 1$. In this case there are no solutions of the system (1) which would describe even isolated Bloch lines. There are only approximate solutions representing Bloch lines in the limit of low velocities $u \ll 1$ (Refs. 8 and 9). Therefore, the subsequent calculations must be carried out numerically.

Our calculations were made using an implicit difference scheme with an iteration calculation of nonlinear terms at each time step. The spatial step in the integration procedure was $\delta x = 0.1$ and the temporal was $\delta t = 0.1$. The total number of points splitting a domain wall along the coordinate x was $N_{\delta x} = 4000$. The boundary conditions were selected in the form $\varphi_x = p_\pm \varphi$, $q_x = p_\pm q$, where $p_\pm$ is the characteristic parameter representing the fall of the solution at the periphery of a Bloch line on approach to an equilibrium point. This parameter is obtained from a linearized system (1) near the equilibrium position characterized by $q = 0$ and $\varphi = n\pi + 0.5$ allowing for the self-similarity of the solution $q = q(x - ut)$ and $\varphi = \varphi(x - ut)$. At moderately high velocities $u \ll 2b/\alpha$, we have $p_\pm = \pm b$. The selected boundary conditions allow us to minimize the influence of reflections and pinning of domain walls at the limits of the numerical calculation "window" on the dynamics of a Bloch line at the center of this window.

The results of our calculations are presented in Figs. 1–6. At low pumping velocities $V < V_1$ a collision between Bloch lines with different directions of the "twistedness" of the magnetization in a domain wall (with different topological charges, of a kink and an antikink) results in their annihilation (Fig. 1). In this case the energy of a moving Bloch line is insufficient to recreate a new pair of Bloch lines with an opposite topological charge, i.e.,

$$2E(u) < 2E(0) + E_d(\tau),\quad E(u) = E(0) + m_L u^2/2, \tag{7}$$

where $E(u)$ is the energy of a moving Bloch line, m_L is the mass of a Bloch line, $u(V)$ is the velocity of a Bloch line, and E_d is the dissipation energy during the collision time τ.

When the critical velocity is exceeded, i.e., when $V > V_1$, we have

$$2E(u) \geqslant 2E(0) + E_d(\tau), \tag{8}$$

which results in restoration of a pair of Bloch lines after a collision, in other words, "mutual transmission" of Bloch lines takes place. Since the leading and trailing edges of moving waves representing a sag of a domain wall differ in respect of the slope because of the presence of dissipation,[8,9] the profiles of a dynamic sag of a domain wall accompanied by a moving Bloch line demonstrate clearly (see Fig. 2) that this is precisely the effect of soliton-like transmission of

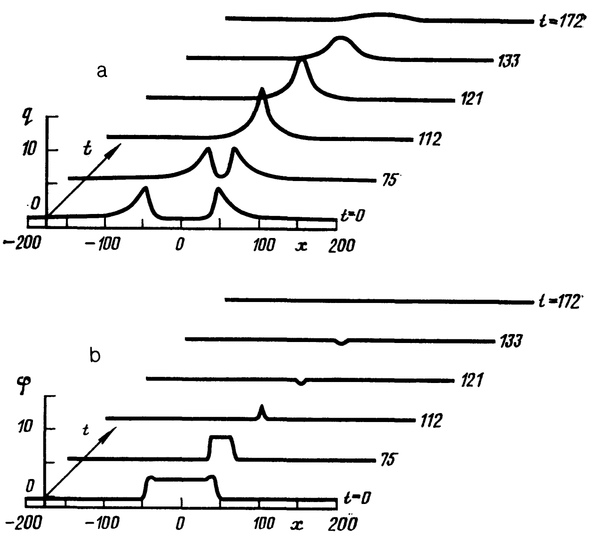

FIG. 1. Annihilation of Bloch lines due to collisions: $\alpha = 0.4$, $b = 0.1$, $V = 0.26$.

Bloch lines. In contrast to Ref. 15, there is no inversion of the sag on collision and the restored profile of a domain wall sag demonstrates the soliton-like transmission of Bloch lines conserving the individual topological charges, as found experimentally.[13,14]

The critical velocity V_1 is governed by the equality in Eq. (8) and depends on the damping parameters α and on the domain wall stiffness b. Figure 3a gives the dependence $V_1(\alpha)$ if $b = 0.1$, showing that an increase in the damping increases $V_1(\alpha)$ and this is due to an increase in the dissipation energy E_d during a collision. Figure 3b demonstrates the dependence of the critical velocity on the domain wall stiffness.

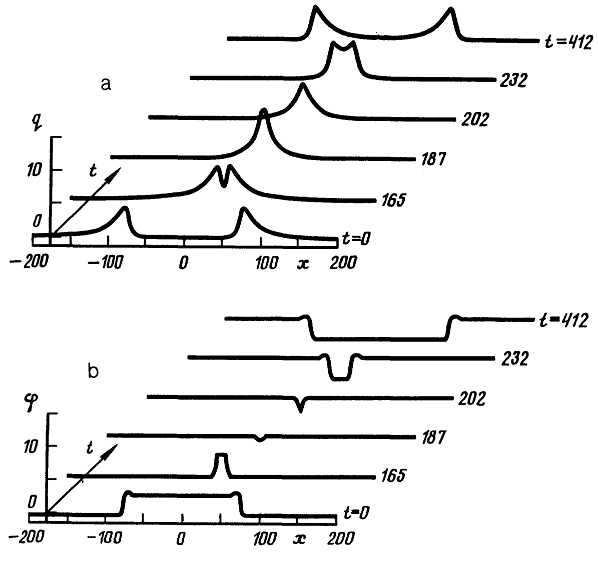

FIG. 2. Mutual transmission of Bloch lines in collisions: $\alpha = 0.4$, $b = 0.1$, $V = 0.28$.

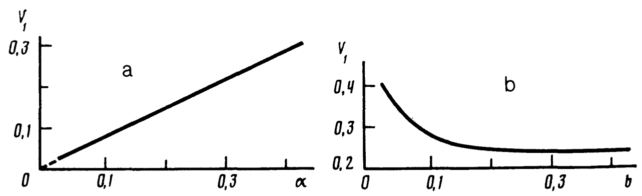

FIG. 3. Dependences of the critical velocity in collisions of two Bloch walls on the parameter α if $b = 0.1$ (a) and on the parameter b if $\alpha = 0.4$ (b).

3. COLLISIONS OF BLOCH LINE CLUSTERS. PHASE DIAGRAMS

In addition to isolated Bloch lines a domain wall may contain also clusters with several lines. In particular, a pair of Bloch lines is used as a data unit (bit) In magnetic memory devices.[3] Such clusters are stable because of the magnetostatic attraction and in the absence of long-range magnetostatic fields they dissociate into isolated Bloch lines because of the exchange stiffness. Nevertheless, as pointed out already, in the course of their motion they become stabilized because of the formation of a general sag of a domain wall and the appearance of a gyroscopic pressure of a moving sag which compresses such a cluster. Therefore, at high velocities of Bloch line clusters we can ignore the magnetostatic attraction and investigate the dynamics of these lines ignoring long-range magnetostatic fields.

We shall now consider a collision between two clusters with opposite topological charges and assume that each of them contains two Bloch lines. At low domain wall velocities, lower than the first critical velocity $V_1^{(2)}$ (the upper index identifies here and later the number of Bloch lines in a cluster), the annihilation of clusters takes place as shown in Fig. 4. This demonstrates clearly the multistage nature of the pulses when the annihilation of a pair of Bloch lines gives rise to a time delay associated with oscillations of spin in the cluster interaction region.

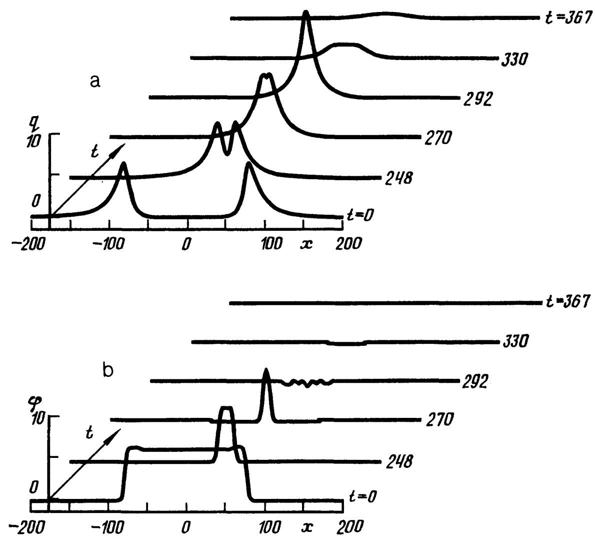

FIG. 4. Annihilation of clusters of two Bloch lines in head-on collisions: $\alpha = 0.4$, $b = 0.1$, $V = 0.17$.

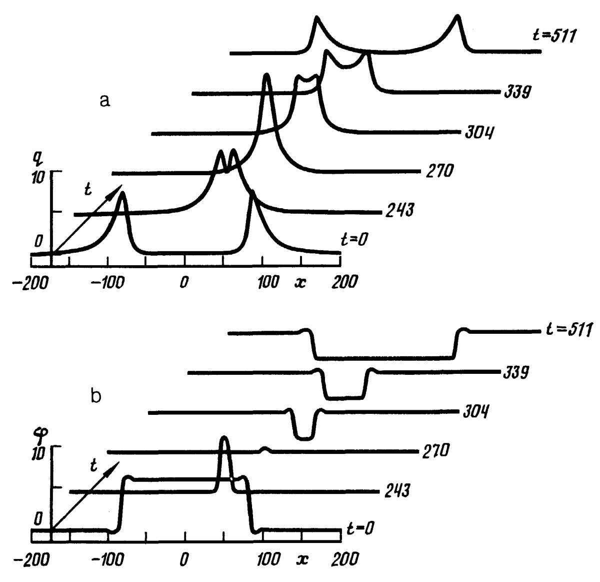

FIG. 5. Partial transmission of clusters of two lines: $\alpha = 0.4$, $b = 0.1$, $V = 0.22$.

If the velocity is higher than the first critical value, but less than the second critical velocity, partial restoration of the clusters exchanging topological charges takes place: an incomplete transmission effect occurs (Fig. 5). When the second critical velocity $V_2^{(2)}$ is exceeded, clusters are restored completely and a soliton-like transmission effect is observed. When a large number of lines collides, an increase in the velocity V results in a gradual restoration initially of one line in each cluster, then two lines, etc. until the recovery is complete. Obviously, the condition for restoration of m lines out of the initial number of n in a cluster after a collision can be described by:

$$nE(u)=mE(0)+E_d(m, n), \tag{9}$$

where $E_d(m, n)$ represents the dissipative losses in time $\tau(m, n)$ in the course of a collision.

The critical velocities $V_m^{(n)}$ depend on the damping and on the stiffness of a domain wall. Figure 6a shows the dependences of the first and second critical velocities of a domain wall (for two-line clusters) on α. In the shaded part of this figure the clusters are restored partly after a collision. Above the shaded region the restoration process is complete. Spontaneous generation of new pairs of Bloch lines during the cluster collision time begins near the Walker limit $V_w \approx 0.5$. At low velocities $V < 0.02$ an instability appears due to the exchange-induced pushing apart of Bloch lines in a cluster.

Obviously, similar effects should appear also on reflection of Bloch lines from an unpinned end of a domain wall at the boundary of a ferromagnetic crystal. In fact, in the latter case the boundary conditions are $q_x = \varphi_x = 0$. These conditions are satisfied at the center of a collision between two clusters because of the symmetry of the problem. However, the results of our calculations relating to collisions of Bloch lines with one another can be applied to collisions of Bloch lines with the boundary of a crystal.

CONCLUSIONS

We demonstrated that the dynamic properties of a Bloch line in a domain wall of a uniaxial ferromagnet resemble to some extent the properties of topological solitons, as manifested strikingly in head-on collisions of individual Bloch lines and clusters. It should be pointed out that the system of dynamic equations describing a Bloch line does not apply to the familiar classes of integrable equations even if we ignore pumping and dissipation. This behavior of Bloch lines is obviously associated with the presence of a compact Bloch-line core where the change in the direction of spins in space is faster (see the Introduction). A new effect of partial (multistage) annihilation of Bloch line clusters in the critical velocity range (shown shaded in the phase diagram in Fig. 6) is predicted.

The process of collision of clusters with different values of their absolute topological charge is considered in Ref. 29. A collision of this type forms one cluster moving in the same direction as the large cluster before a collision, i.e., partial annihilation of clusters takes place which to some extent is a process similar to that investigated by us.

There is a very close analogy between the behavior of Bloch lines and multisoliton excitations of a distributed Josephson junction. However, there are also important differences associated in particular with the fact that in the case of a Bloch line there is an additional factor (apart from pumping and dissipation) which destroys the integrability of the system and the soliton behavior, namely a sag of a domain wall due to the gyroscopic force in the course of motion of a Bloch line, i.e., a factor due to the vortex nature of a Bloch line. This factor opens up new opportunities for experimental investigation of the dynamic interaction of Bloch line clusters. We have here in mind collisions between clusters moving along the same direction "overtaking" (collisions), a phenomenon which is unknown if in the case of a distributed Josephson junction. In the case of Bloch lines this possibility appears because a sag of a domain wall makes the coefficient of the viscous friction acting on a cluster (i.e., its mobility) dependent on the velocity and on the topological charge. Consequently, in the case of a pumped domain wall it is possible to create simultaneously several clusters with

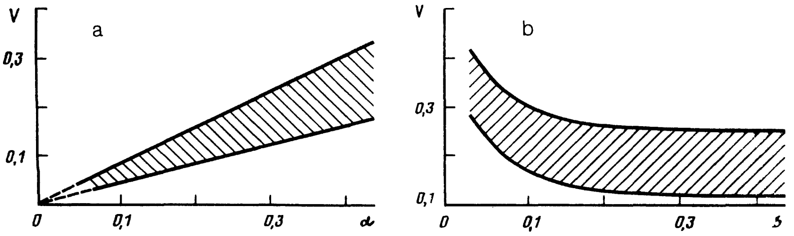

FIG. 6. Dependences of the first and second critical velocities of a domain wall for clusters of two lines on the parameter α when $b = 0.1$ (a) and on the parameter b when $\alpha = 0.4$ (b).

different topological charges and moving at different velocities,[30] i.e., the conditions may be established under which unidirectional collisions are possible.

These considerations and the results of numerical calculations dealing with the partial transmission of guiding Bloch line clusters, as well as the experimental data of Ref. 29 on partial annihilation of Bloch line clusters make the situation discussed above more general than would follow from the classical theory of solitons.

The results of the above theoretical analysis are in good agreement with the experimental data.[13,14,29] In a more detailed analysis of the dynamic interaction of Bloch lines and their clusters, particularly in the range of critical velocities where partial annihilation of clusters is possible, one has to allow more rigorously for the magnetic-dipole interaction (other than use the approximation adoped above). It would also be desirable to carry out a quantitative analysis of the role of the "twistedness" of a domain wall in collisions of Bloch lines, but dealing with this topic would increase the size of the problem.

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