Excitation of nonlinear surface waves by electromagnetic beams

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An analysis is made of steady-state and dynamic transformations of Gaussian electromagnetic waves, propagating along an interface between linear and nonlinear media, into nonlinear surface waves (NSWs). A steady-state model (for a medium with a cubic nonlinearity) is used to demonstrate the feasibility of excitation of a stable NSW in two ways: direct excitation of an NSW by optimization of the beam parameters and excitation of an unstable NSW followed by its transformation into a stable wave. A relationship between the parameters of an NSW and the parameters of Gaussian beams is established. The excitation efficiency is determined numerically; it is close to 100% for optimal beams. A dynamic model is used to investigate the excitation of an NSW at an interface with a medium having a relaxation nonlinearity. It is shown that in the case of direct excitation we can reach steady-state conditions corresponding to the creation of an NSW. The characteristic excitation time is determined. It is shown that an attempt to excite an NSW via an unstable branch as a result of transformation creates a steady-state beam which is emitted by the interface and enters the linear medium.

1. INTRODUCTION

A classic example of an open system guiding electromagnetic radiation is a planar interface between two insulating media. In the case of linear and isotropic media the TE radiation is not confined to the interface. However, if the permittivity of the medium which is less dense increases in the presence of the field (i.e., in the case of a medium with a focusing nonlinearity), it is possible to use this property to generate a nonlinear surface wave (NSW) which represents a solution of the Maxwell equations localized at an interface between linear and nonlinear media (Refs. 1 and 2).[1)]

It would be desirable to determine the conditions for generation of NSWs because of the interest in the interaction of high-intensity beams incident on an interface between linear and nonlinear media at angles exceeding the total internal reflection angle[4–8] and also because of the current activity in the development of surface polariton spectroscopy (see Ref. 9 and the literature cited there).

An analysis of the dispersion relationship for NSWs shows that at an interface between linear and nonlinear media we can expect propagation of two waves with the same running power but with different propagation constants. Which of these two waves is obtained can be determined by investigating their stability and finding the efficiencies of the various excitation methods. A numerical investigation of the stability of NSWs in the presence of perturbations of their transverse structure has shown[8] that waves with propagation constant increasing as a function of the power are stable.

The ability of electromagnetic beams to excite NSWs has been analyzed theoretically on many occasions for the case of Gaussian beams incident on an interface at angles exceeding the total reflection angle;[6–8] however, contradictory conclusions on the possibility of exciting such waves were reached in Refs. 6 and 8, on the one hand, and in Ref. 7, on the other. This happened partly because the properties of NSWs have not been investigated sufficiently thoroughly and it has been found subsequently that beams with optimal parameters are needed for the excitation of NSWs. A method for determining the parameters of these beams by numerical calculation is proposed in Ref. 8. Deviation of the beam parameters from the optimal conditions leads to the appearance of reflected and transmitted beams, which usually split into separate filaments.[4–7]

Our aim was to investigate steady-state and dynamic transformations of a Gaussian beam propagating along an interface between two media into an NSW.

We formulate the problem in Sec. 2. A steady-state model is discussed in Sec. 3 in the approximation of a cubic nonlinearity which is used to describe media with Kerr, ponderomotive, thermal, relativistic, and other nonlinear mechanisms. In Sec. 4, we use the method of moments[10] to obtain analytic estimates of the parameters of optimal beams. In Sec. 5 we compare the analytic results with those obtained by numerical calculations. The dynamics of excitation of an NSW at an interface with a nonlinear medium with a relaxation-type nonlinearity, used to describe the Kerr nonlinearity (see, for example, Refs. 11 and 121) and also of the ponderomotive nonlinearity in an isothermal plasma,[13,14] is studied for the first time in Sec. 6. The conclusions deal with the applicability of the results to real physical systems.

This investigation may be useful in analyzing the excitation of nonlinear steady-state and dynamic structures and of more complex systems guiding electromagnetic radiation.[15–18]

2. FORMULATION OF THE PROBLEM

We assume that the half-space $x < 0$ is filled with a medium characterized by a permittivity $\varepsilon = \varepsilon_1$, whereas the half-space $x > 0$ is filled by a nonlinear medium $\varepsilon = \varepsilon_2 + \Delta\varepsilon$, where $\Delta\varepsilon$ is the wave-induced perturbation of the permittivity which can be described by

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$$\tau\frac{\partial\Delta\varepsilon}{\partial t}+\Delta\varepsilon=|E|^2/E_p^2. \quad (1)$$

Here E_p is a characteristic nonlinear field and τ is the relaxation time of a perturbation ($\tau=0$ corresponds to a nonlinearity with an instantaneous response). For $z=0$, the field distribution oscillates with time and, for example, it can be in the form of a Gaussian beam[2)]

$$\mathbf{E}=\mathbf{y}_0E_0(t)\exp[-(x-x_c)^2/2\sigma^2]\exp(-i\omega t). \quad (2)$$

We have to determine the conditions under which NSWs are excited when the transformation (2) takes place in a region defined by $z>0$ at times $t>0$.

It is natural to begin an analysis of the possibility of excitation of NSWs by Gaussian beams with the steady-states case.

3. MAIN EQUATIONS IN THE STEADY-STATE APPROXIMATION

In the steady-state case the relationship between a nonlinear perturbation of the permittivity and the field causing the perturbation is described, as in the case of a medium with an instantaneous-response nonlinearity, by an expression derived from Eq. (1) subject to $\partial\Delta\varepsilon/\partial t=0$:

$$\Delta\varepsilon=|E|^2/E_p^2. \quad (3)$$

We seek the solution of the wave equation for the field in the form

$$\mathbf{E}=\mathbf{y}_0E(x,z)\exp[-i(\omega t-hz)]. \quad (4)$$

Then, the amplitude $E(x,z)$, which varies smoothly on a scale of $l_z=2\pi/h$, is described by the following average equations:

$$2ih\frac{\partial E}{\partial z}+\frac{\partial^2E}{\partial x^2}+(k_0^2\varepsilon_1-h^2)E=0,\qquad x<0, \quad (5)$$

$$2ih\frac{\partial E}{\partial z}+\frac{\partial^2E}{\partial x^2}+(k_0^2\varepsilon_2-h^2+k_0^2|E|^2/E_p^2)E=0,\qquad x>0, \quad (6)$$

where $k_0=\omega/c$ is the wave number and c is the velocity of light in vacuum.

We supplement Eqs. (5) and (6) by boundary conditions which specify continuity of the tangential components of the electric field E_y and of the magnetic field H_z of the wave and reduce, in the selected geometry, to the continuity of the function E and its derivative $\partial E/\partial x$ at $x=0$. The solutions localized in x can be selected by imposing the condition that the field vanishes in the limit $|x|\to\infty$.

We consider the case when $h^2>k_0^2\varepsilon_1$ and $\varepsilon_1>\varepsilon_2$, and we go over to dimensionless variables in Eqs. (5) and (6) by means of the following expressions:

$$x=x(h^2-k_0^2\varepsilon_1)^{1/2},\quad z=z(h^2-k_0^2\varepsilon_1)/2h,$$
$$E=k_0E/E_p(h^2-k_0^2\varepsilon_1)^{1/2},\quad \alpha^2=(h^2-k_0^2\varepsilon_2)/(h^2-k_0^2\varepsilon_1). \quad (7)$$

Then Eqs. (5) and (6) become

$$i\frac{\partial E}{\partial z}+\frac{\partial^2E}{\partial x^2}-E=0,\qquad x<0, \quad (8)$$

$$i\frac{\partial E}{\partial z}+\frac{\partial^2E}{\partial x^2}-\alpha^2E+|E|^2E=0,\qquad x>0. \quad (9)$$

The boundary conditions remain the same when expressed in terms of dimensionless variables.

Equations (8) and (9) with these boundary conditions have a solution homogeneous in z ($\partial E/\partial z=0$):

$$E=\begin{cases}[2(\alpha^2-1)]^{1/2}\exp(x), & x<0,\\ 2^{1/2}\alpha\,\mathrm{ch}^{-1}[\alpha(x-x_0)], & x>0,\end{cases} \quad (10)$$

which is called a nonlinear surface wave.[1,2] When the propagation constant differs from H by an amount such that $\Delta h\ll h$, we can seek the solution of Eqs. (8) and (9) in the form

$$E=A(x)\exp(i\gamma z),$$

where the precision of the average description is ensured by imposing the following conditions on the dimensionless propagation constant γ:

$$|\gamma|\ll 2h/(h^2-k_0^2\varepsilon_1)^{1/2}. \quad (11)$$

Then, we can find $A(x)$ from

$$\frac{\partial^2A}{\partial x^2}-(1+\gamma)A=0,\qquad x<0, \quad (12)$$

$$\frac{\partial^2A}{\partial x^2}-(\alpha^2+\gamma)A+A^3=0;\qquad x>0, \quad (13)$$

which have the following solution that satisfies the boundary conditions:

$$A(x)=\begin{cases}[2(\alpha^2-1)]^{1/2}\exp[(1+\gamma)^{1/2}x], & x<0,\\ 2^{1/2}(\alpha^2+\gamma)^{1/2}\mathrm{ch}^{-1}[(\alpha^2+\gamma)^{1/2}(x-x_0)], & x>0\end{cases} \quad (14)$$

where

$$x_0=2^{-1}(\alpha^2+\gamma)^{-1/2}\ln\frac{(\alpha^2+\gamma)^{1/2}+(1+\gamma)^{1/2}}{(\alpha^2+\gamma)^{1/2}-(1+\gamma)^{1/2}}.$$

Equation (14) describes an NSW with a propagation constant different from that used in averaging the wave equation. Naturally, Eq. (14) reduces to Eq. (10) if $\gamma=0$.

It should be noted that in terms of the variables

$$x_n=x(1+\gamma)^{1/2},\quad A_n=A(1+\gamma)^{-1/2}$$

we can reduce Eqs. (12) and (13) to Eqs. (8) and (9) where $\partial E/\partial z=0$ if we introduce α_n such that

$$\alpha_n=[(\alpha^2+\gamma)/(1+\gamma)]^{1/2}. \quad (15)$$

We can therefore say that the properties of NSWs are governed by the only parameter α_n which can be expressed in terms of linear parts of the permittivities of the media and its propagation constant.

In the case of the field distributions (8) and (9) localized in x we have

$$J=\int_{-\infty}^{\infty}|E(x)|^2dx=\text{const}, \quad (16)$$

which is proportional to the integrated power in the wave. In the case of an NSW with $\gamma\neq0$, this integral is

$$J=\frac{\alpha^2-1}{(1+\gamma)^{1/2}}+2[(1+\gamma)^{1/2}+(\alpha^2+\gamma)^{1/2}] \quad (17)$$

and one value of J and α corresponds to two possible values of γ. The results of an investigation of the stability of NSWs carried out in Ref. 8 shows that the waves characterized by $\alpha_n<2$ are stable, whereas those with $\alpha_n>2$ are unstable. Since, as is easily shown, we have $\partial J/\partial\gamma>0$ if $\alpha_n<2$ and $\partial J/\partial\gamma<0$ if $\alpha_n>2$, this NSW stability criterion agrees, as shown in Ref. 8, with the Kolokolov–Vakhitov criterion for self-focused solutions in an infinite medium.[19]

4. ANALYSIS OF STEADY-STATE EXCITATION OF NONLINEAR SURFACE WAVES

We can estimate analytically the parameters of the Gaussian beams which are optimal for the excitation of NSWs on the basis of the method of moments used in Ref. 10 to find the field distribution. For example, the equation for the first moment of the field distribution (center-of-mass position)

$$\bar{x} = \int_{-\infty}^{\infty} x|E(x)|^2\,dx \tag{18}$$

follows from Eqs. (8) and (9) and can be written in the form

$$\frac{d^2x}{dz^2} = E_b^2(E_b^2 - 2(\alpha^2 - 1)), \tag{19}$$

where $E_b = |E(x=0)|$. It follows from Eq. (19) that a beam moves as a whole along an interface if the field E_b excited at this interface is equal to the field of an NSW at the same interface, i.e., if

$$E_b = E_c = [2(\alpha^2-1)]^{1/2}. \tag{20}$$

We can easily show that of the two equilibrium positions of a beam with a Gaussian profile the stable position is that which corresponds to a positive amplitude gradient, i.e., to a beam with the field maximum in a nonlinear medium. We note that if $E_b > E_c$, the beam is "attracted" to the nonlinear medium.

The definition of the path of a Gaussian beam of a given width within the framework of Eq. (19) reduces to an analysis of the potential

$$U(x_c) = J_b[2(\alpha^2-1)\Phi(2^{1/2}x_c/\sigma) - 2^{-1/2}E_0^2\Phi(2x_c/\sigma)] \tag{21}$$

as a function of the beam amplitude (Figs. 1a–1d). Here,

$$J_b = E_0^2\pi^{1/2}\sigma = \mathrm{const} \tag{22}$$

is the integrated beam power and

$$\Phi(x) = (2\pi)^{-1/2}\int_{-\infty}^{x} \exp(-y^2/2)\,dy$$

is the normal distribution function, whereas the beam parameters are assumed to be dimensionless here and later. The main results of our analysis are as follows.

The potential well exists if $x > 0$, provided $E_0 > E_c$ (Figs. 1b, 1c, and 1d). The condition under which such a well traps beams described by Eq. (2) and characterized by $|x_c| \ll \sigma$ is (Fig. 1c)

$$E_c < E_0 < 2^{1/4}E_c \approx 1{,}19E_c. \tag{23}$$

This condition can be refined for beams with the field maximum in a nonlinear medium ($x_c < 0$), as shown in Fig. 1b:

$$x_c \geqslant -0{,}5\sigma, \qquad E_0 < 1{,}13E_c. \tag{24}$$

In the case of beams propagating in a nonlinear medium ($x_c > 0$) there are no restrictions on the amplitude (Fig. 1d). In the case of beam localization it is necessary to ensure that it is close to the bottom of a potential well of width $\sim\sigma$, while the position of the maximum is

$$x_c = 2^{1/2}\sigma(-\ln(E_c/E_0))^{1/2}.$$

However, if $x_c \geqslant \sigma$ ($E_0 \gg E_c$) we cannot ignore the change in $\sigma(z)$ in an analysis of $U(x_c)$ and we have to provide a self-consistent description of the evolution of the quantities $x_c(z)$ and $\sigma(z)$.

Of greatest interest from the point of view of excitation of NSWs by beams incident on an interface is an analysis of the first two cases when a considerable proportion of the beam power is concentrated initially in the linear medium.

In this case an analysis of the equation for the second moment of the field distribution (square of the effective beam width)

$$\frac{d^2a_{eff}^2}{dx^2} = 8\int_{-\infty}^{\infty}\left|\frac{\partial E}{\partial x}\right|^2 dx - 2\int_{0}^{\infty}|E|^4\,dx, \tag{25}$$

$$a_{eff}^2 = \int_{-\infty}^{\infty} x^2|E|^2\,dx \tag{26}$$

makes it possible to write down the condition that the beam confined to the interface does not experience spreading:

$$J > 2^{3/2}\pi^{1/2}/\sigma\Phi(2x_c/\sigma), \tag{27}$$

which can be rewritten in the form

$$\sigma > 2^{3/4}/\Phi^{1/2}(2x_c/\sigma)E_0. \tag{28}$$

It should be noted that the integrated power in a beam deduced from Eqs. (22), (23), (27), and (28) exceeds the NSW excitation threshold:

$$J > 3^{3/2}(\alpha^2-1)^{1/2}. \tag{29}$$

On the other hand, there is naturally a limit to σ from above. The maximum value of σ can be estimated, for example, from the condition that an initial distribution (2) "creates" in a homogeneous nonlinear medium only one steady-state channel (soliton):[20]

$$\sigma < 2\pi^{1/2}/[\Phi(x_c/\sigma)E_0]. \tag{30}$$

If $|x_c| \ll \sigma$, the inequalities of Eqs. (28) and (30) can be written in the form

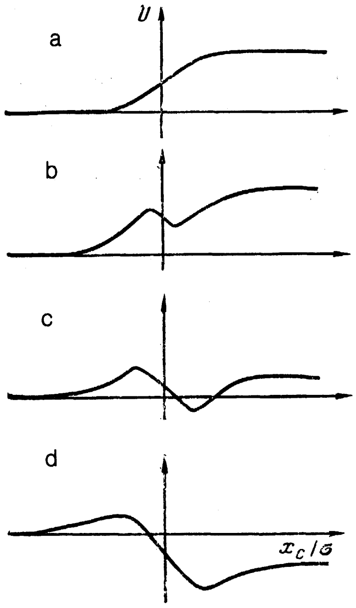

FIG. 1. Profiles of the potential $U(x_c)$ plotted as a function of the field amplitude in a Gaussian beam: 1) $E_0 < E_c$; b) $E_c < E_0 < 1.13\,E_c$; c) $E_c < E_0 < 2^{1/4}E_0$; d) $E_0 > E_c$.

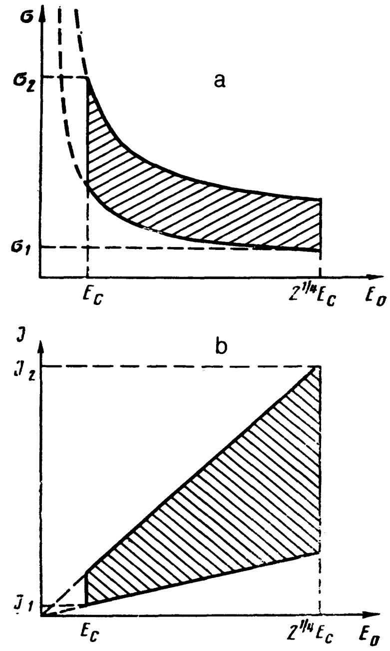

FIG. 2. Ranges of the parameters of the optimal beams. Here, $\sigma_1 = 2^{1/2}(\alpha^2-1)^{-1/2}$, $\sigma_2 = 2^{3/4}(\alpha^2-1)^{1/2}$, $J_1 = 2^{7/4}(\alpha^2-1)^{1/2}$, $J_2 = 8\pi(\alpha^2-1)^{1/2}$.

$$\sigma > 8^{1/4}(\alpha^2-1)^{-1/2} \approx 1{,}7(\alpha^2-1)^{-1/2}, \quad (31)$$

$$\sigma < 2^{3/2}\pi^{1/2}(\alpha^2-1)^{-1/2} \approx 5{,}0(\alpha^2-1)^{-1/2}. \quad (32)$$

Failure to obey the conditions for the excitation of NSWs results in the emission of the beams from the interface. Their subsequent evolution is described by the familiar solutions of the homogeneous linear and nonlinear problems.

Figures 2a and 2b give the ranges of the amplitudes and widths and the amplitudes and powers of the beams which are optimal for the excitation of nonlinear surface waves.[3)]

The above analysis is supported by numerical calculations reported below.

5. NUMERICAL CALCULATIONS OF PARAMETERS OF STEADY-STATE EXCITATION OF NONLINEAR SURFACE WAVES

Our numerical calculations were carried out using the difference approximation for Eqs. (8) and (9) using with the Crank–Nicholson scheme. The passage of the beams to infinity was modeled by smooth introduction of decay of the field [addition of a term $i\beta(x)E$ to Eqs. (8) and (9) at the limits of the interval $-L < x < L$ where the calculation was carried out]. The length of the interval L and the decay parameters $\beta(x)$ were selected from the condition that the electromagnetic wave not reflect and that it decay sufficiently at the limits of the interval, so that zero boundary conditions can be used in Eqs. (8) and (9). The boundary between the media was placed in the middle of the calculated interval. The selected scheme ensured the continuity of E and $\partial E/\partial x$ everywhere, including at the interface between the media, so that the boundary conditions were satisfied "automatically" at $x = 0$. In each medium the same number of sites was taken. The initial ($z = 0$) field distribution was selected in the form of either a Gaussian beam field when investigation was made of the transformation of the beam into NSWs or an NSW field whose stability was investigated.

The parameters of a Gaussian beam were selected by numerical calculations based on the above scheme using analytic estimates given by Eqs. (31) and (32). Figure 3a shows a variant demonstrating the trapping of a beam in an NSW corresponding to a stable branch.

When use was made of a beam with the parameters satisfying the conditions for the excitation of NSWs belonging to the unstable branch ($\alpha > 2$), it was found that NSWs were generated again for $z \gg 1$. It is obvious that these waves are stable and the values of γ for them satisfy $\alpha_n < 2$ (see Fig. 3b). The relationship between the parameters of an NSW and the parameters of a beam is then obtained by assuming that this NSW is a result of decay of an unstable NSW created by the beam. This assumption was confirmed also in an investigation (by the numerical scheme outlined above) of the nonlinear stage of the instability of an NSW. It was found that typical evolution of an unstable NSW is its transformation into a stable NSW retaining, to within a few percent, the integrated power J, which agrees with the results of Ref. 8. The condition that J be conserved in the course of such a transformation readily yields the following expression for the propagation constant γ of a stable NSW:

$$\gamma = (\alpha+1)^2[\alpha+3+((\alpha+3)^2-16)^{1/2}]^2/64-1. \quad (33)$$

The expression (33) can be generalized to the case when the value of γ for the unstable branch differs from zero. It should

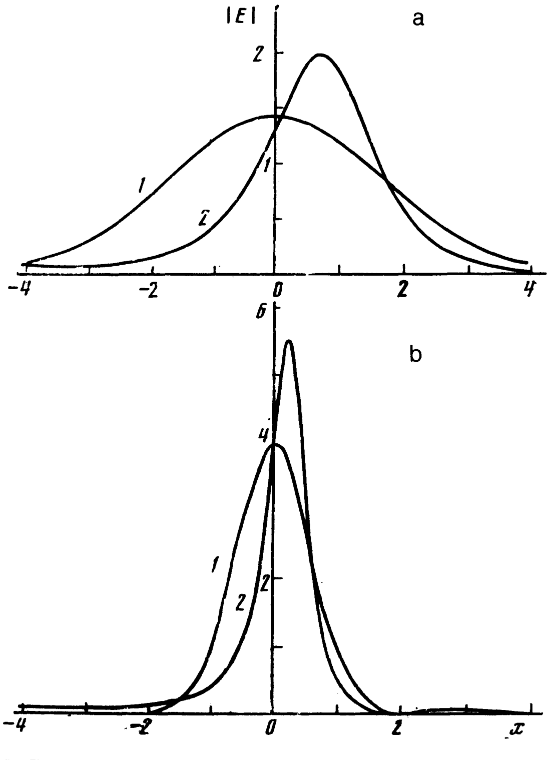

FIG. 3. Initial ($z = 0$, curve 1) and "final" ($z \gg 1$, curve 2) distributions of the field amplitude in the case of direct excitation of a stable NSW ($\alpha = 2^{1/2}$, $\gamma = 0.15$, $\alpha_n \leqslant \alpha$) by a beam with the parameters $E_0 = 2^{1/2}$, $\sigma = 1.7$, $x_c = 0$ (a) and excitation via an unstable branch ($\alpha = 3$, $\gamma = 4.8$, $\alpha_n = 1.54$, $E_0 = 4$, $\sigma = 0.57$, $x_c = 0$) (b).

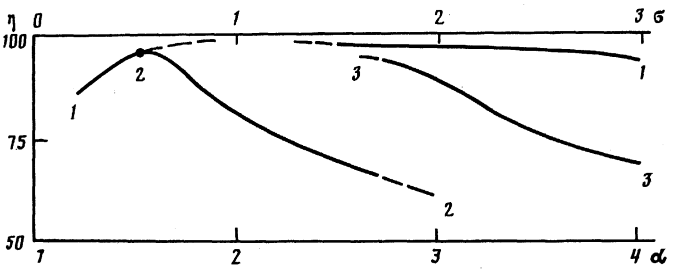

FIG. 4. Results of calculations of the excitation efficiency $\eta(\alpha)$ for beams described by Eq. (35) (curve 1) and the dependence $\eta(a)$ for $E_0 = E_c$, $x_0 = x_c$, and $\alpha = 2^{1/2}$ (curve 2), and $\alpha = 3$ (curve 3).

be noted that variation of the spectrum of initial perturbations imposed on an unstable NSW can yield the regime of Ref. 8 with emission of an NSW into a linear medium, which however is not observed when a wave is excited by a beam described by Eq. (2).

The excitation efficiency can be represented in a natural manner by the ratio of the integrated power J in the wave to the integrated power in an exciting beam J_b:

$$\frac{J}{J_b} = \eta(E_0, \sigma, x_c, \alpha). \tag{34}$$

Curve 1 in Fig. 4 shows the dependence $\eta(\alpha)$ for beams with the following parameters:

$$E_0 = E_c, \quad x_c = x_0, \quad \sigma = \frac{(\alpha+1)^2}{\pi^{1/2} E_0^2} \tag{35}$$

(beam width is deduced from the condition that the integrated power in a beam equal to that of an NSW with the parameters $\alpha = 0$ and $\gamma = 0$). The discontinuity in the dependence $\eta(\alpha)$ is due to the fact that we are in the vicinity of the limit of trapping of a beam by a wave defined by Eq. (27) and the beam width is only a few percent higher than the maximum permissible value [see the inequality of Eq. (28)]. For example, if $E^0 = E_c$, then σ is given by Eq. (33) and $x_c = 0$ represents the limiting points of a discontinuity on α which assume the following values: $\alpha_1 = 1.37$, $\alpha_2 = 3.41$. In general, the excitation efficiency is a function of given parameters and can be improved by optimization. By way of example, Fig. 4 gives the $\eta(\sigma)$ for $E_0 = E_c$ and $c_c = 0$ when $\alpha = 2^{1/2}$ (curve 2) and $\alpha = 3$ (curve 3), demonstrating that the maximum values of $\eta(\sigma)$ for beams with the critical amplitude are observed near the limits of the trapping interval.

6. DYNAMIC EXCITATION OF NONLINEAR SURFACE WAVES

We investigate the dynamics of excitation of NSWs by Gaussian beams on the assumption that the characteristic relaxation time of the nonlinear medium is $\tau \gg \tau_e$, which is the time taken to establish an electromagnetic field (or, in the adopted dimensionless units it amounts to $\tau \gg 1$). This condition makes it possible to describe the field by steady-state equations which, in the normalization described by Eq. (7) are

$$i\frac{\partial E}{\partial z} + \frac{\partial^2 E}{\partial x^2} - E = 0, \qquad x<0,$$

$$i\frac{\partial E}{\partial z} + \frac{\partial^2 E}{\partial x^2} - \alpha^2 E + \Delta\varepsilon E = 0, \qquad x>0, \tag{36}$$

and we can retain the time derivative only in the equation for $\Delta\varepsilon$:

$$\frac{\partial \Delta\varepsilon}{\partial t} + \Delta\varepsilon = |E|^2, \tag{37}$$

where we shall introduce a new time $t_n = t/\tau$ in the nonlinear perturbation

$$\Delta\varepsilon = \frac{k_0^2 \Delta\varepsilon}{(h^2 - k_0^2 \varepsilon_1)}.$$

The boundary conditions for the field equations are the same in the dynamic formulation of the problem as in the steady-state case.

To integrate Eqs. (26) and (37) we used an implicit scheme based on the use of the discrete Fourier transformation. Since the spectral method imposes artificial boundary conditions of periodicity, the limits of the calculation interval L ($-L \leqslant x \leqslant L$) were selected so that the fields at the limits were small and had practically no influence on the solution near the interface between two media. The reflection effects can be suppressed, as in the calculations carried out using the steady-state model, by introducing artificial decay which is weak near the interface and rises smoothly to the edges of the calculation interval. To smooth the spectra of the solution the discontinuity of the permittivity is "smeared out" by means of a transition function [$\tan^{-1}(\delta x), \delta \gg 1$]. The field of a Gaussian beam described by Eq. (2) grows smoothly with time at the front of the layer ($x = 0$). There are no initial perturbations of the permittivity.

An important problem in the dynamic formulation is the determination of the steady-state regimes established above. Earlier calculations suggest that the use of the relaxation equation for the description of the evolution of a nonlinear perturbation results typically in the attainment of the steady-state situation for $t \gg 1$, although initially the transient regime may be very complex.[13,14] If we allow for the possible attainment of a steady state, the parameters of the beams used to excite NSWs can be selected in a natural manner on the basis of the results of the steady-state problem. Figures 5a–5c give the data on the excitation of NSWs demonstrating the dynamics of the field and of the perturbation ε corresponding to the trapping of such a beam by an NSW. The characteristic excitation time T of an NSW, which we define as the time taken for the wave intensity to reach a certain fraction of the steady-state value, can be approximated satisfactorily for the intensity levels 0.4–0.9 by the expression $T = k_1 L_z$, where $k_1 \approx 1.5$ ($L_z \gg 1$). The excitation time can be defined in terms of the ratio of the integrated energy flux through a section with a coordinate L_z (which is transferred by the NSW) and the flux through a section characterized by $z = 0$ (transferred by the beam):

$$\eta_2 = \frac{\int_0^t J(z=L_z, t)\,dt}{\int_0^t J(z=0, t)\,dt}. \tag{38}$$

In this case the result gives a good fit to the straight line $T = k_2 L_z$, where $k_2 \approx 2.5$. It should be noted that estimates characterized by $k_{1,2} \approx 1$ can be obtained directly assuming

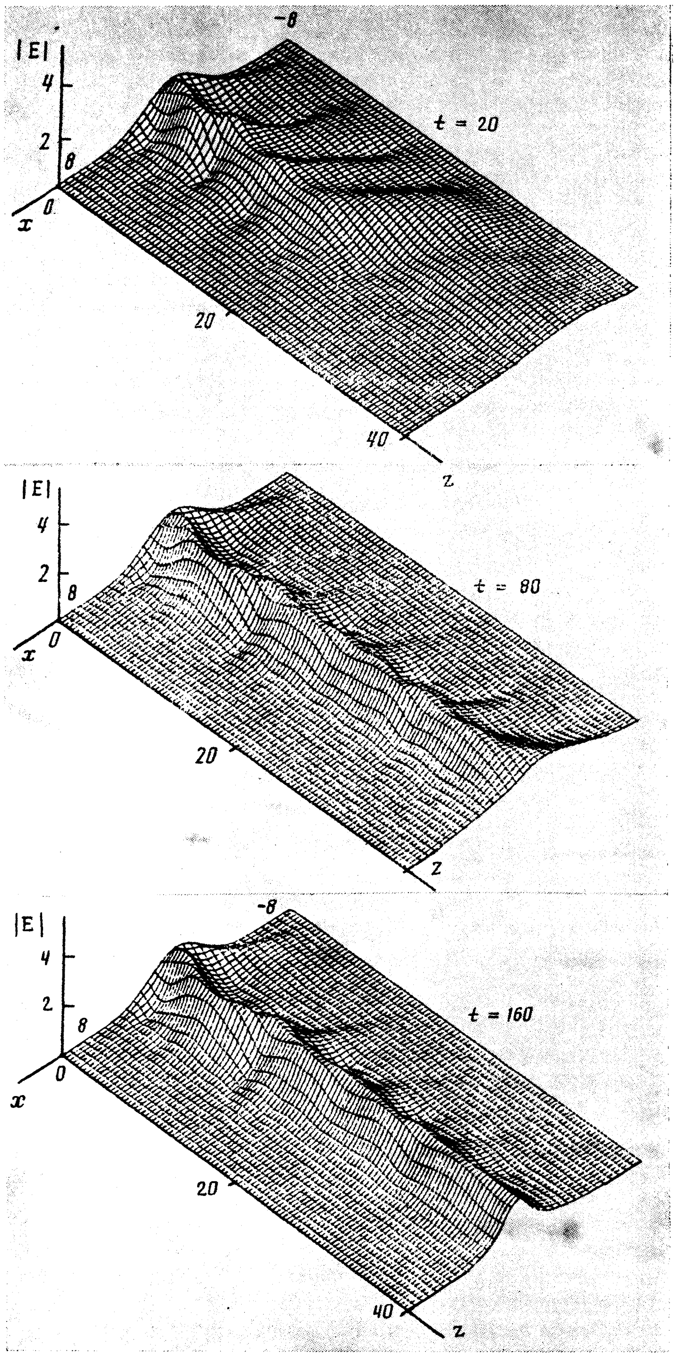

FIG. 5. Dynamics of excitation of the stable branch of a nonlinear surface wave ($\alpha = 2^{1/2}$, $E_0 = E_c$, $\sigma = 1.7$, $x_c = 0$).

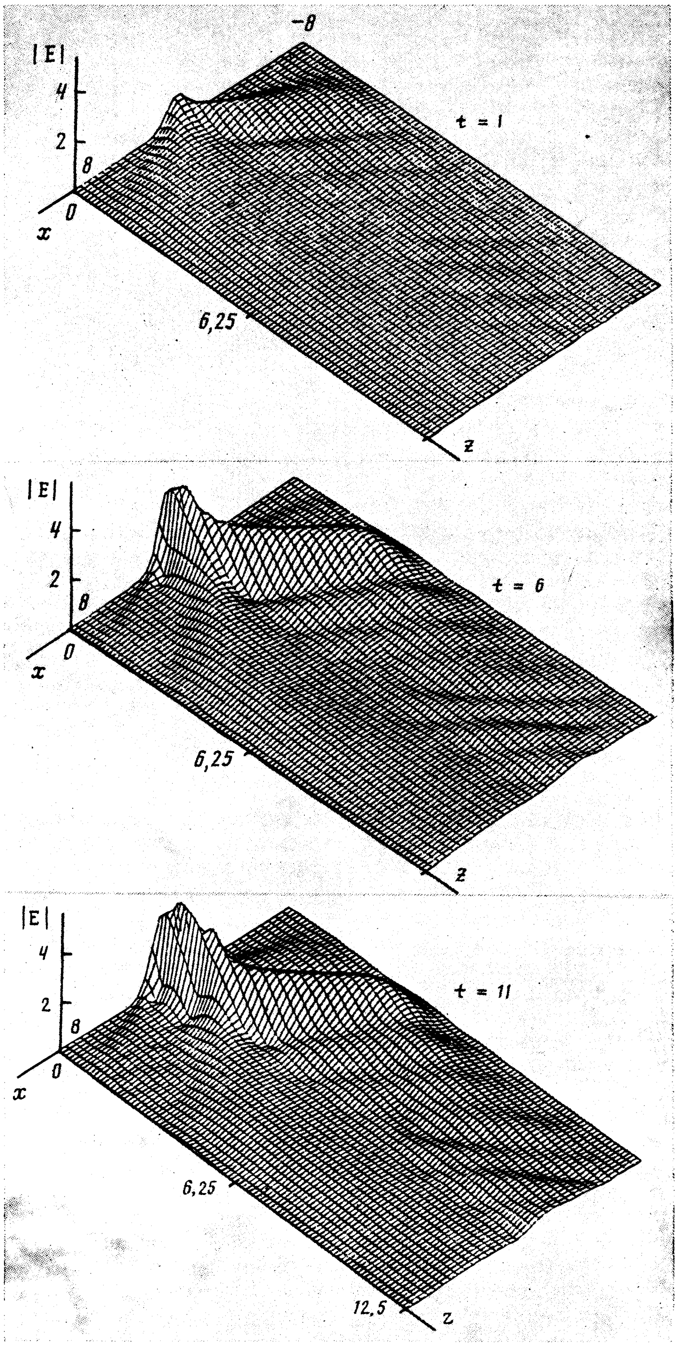

FIG. 6. Excitation of the unstable branch of a nonlinear surface wave ($\alpha = 2^{1/2}$, $E_0 = E_c$, $\sigma = 1.7$, $x_c = 0$) resulting in the emission of radiation into the linear medium.

that the medium–field system evolves simply both in space and time.

Attempts to excite an NSW by excitation of the unstable branch do not give the desired result. The steady-state regimes obtained in this case correspond to the emission of beams in a linear medium (Figs. 6a–6c, where $\alpha = 2.25$, $E_0 = 3.0$, $\sigma = 0.75$, and $x_c = 0.75$).[4)] Note that in all these variants of calculations of the dynamics of the unstable branch of NSWs, subject to various initial conditions and on the assumption of a nonlinear perturbation of the permittivity, an NSW is emitted into the linear medium, but further investigations are needed to check whether this is generally valid.

7. CONCLUSIONS

An important topic outside the framework of our models is the influence of the finite beam width in the transverse (y) direction on the stability and excitation of NSWs. It is known that in a homogeneous nonlinear medium an allowance for the two-dimensional localization of the beam causes it to collapse in a finite distance.[10] It is shown in Ref. 21 that a solution homogeneous in y is linearly unstable in the presence of perturbations in this direction and should split into two-dimensionally confined beams with a scale corresponding to the maximum unstable growth rate. Steady-state two-dimensional localized beams are obtained in Ref. 18 and these are confined to a layer of a linear insulator located within a nonlinear medium characterized by a cubic nonlinearity and saturation of this nonlinearity (these beams include also an asymmetric mode of this waveguide whose structure is close to that of a beam confined by the sole interface). The dependence of the power of this beam on the propagation constant in a medium with saturated nonlinear suggests that it should be stable.

These theoretical results may be subjected to an experimental check, for example, at the interface between crystalline quartz and nitrobenzene whose permittivities differ in the optical range anywhere between the third and fifth decimal point. The threshold integrated power for the excitation of NSWs is $\sim 10^6$–10^8 W/m.

[1)] In Ref. 3 it is suggested that these waves be called nonlinear surface polaritons.

[2)] The initial condition for the field is also the NSW field.

[3)] The parameters of the optimal beams incident on an interface[8] differ by less than a factor of 2 from the parameters of the optimal beams propagating along an interface between two media.

[4)] The parameters of NSWs excited by this beam under steady-state conditions are $\alpha = 2.25$ and $\gamma = 2.8$, which corresponds to $J = 11.6$ for $\eta = 97\%$.

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