

# Absorption of sound in metals with quantum defects

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(Submitted 3 May 1990; resubmitted 7 June 1990)

Zh. Eksp. Teor. Fiz. **98**, 1454–1464 (October 1990)

An analysis is made of the absorption of sound in metals containing mobile quantum defects. In a normal metal the main contribution to the absorption is due to an electron mechanism. At low temperatures electrons relax due to their scattering by quantum defects, which gives rise to a nontrivial temperature dependence of the absorption coefficient of sound. It is shown that the relaxation absorption mechanism, associated with redistribution of defects between interstices, can play an important role when two types of interstices separated by a small energy interval are present in a crystal and also in the superconducting phase of a metal if the trapping of mobile and immobile defects results in the formation of two-level defects.

In studies of the impurity scattering of electrons and metals it is usual to assume that impurities are distributed at random and “frozen” at fixed positions. The impurities then play the role of an external field which acts on electrons. The physical characteristics of metals with frozen defects are frequently calculated by the well-known “cross” technique (see, for example, Ref. 1).

However, some light interstitial impurities retain their mobility even at low temperatures because of quantum tunneling.<sup>2</sup> Such mobile defects (“impuritons”) should be regarded as internal degrees of freedom of a crystal. Theoretical estimates<sup>3</sup> and experimental results<sup>4,5</sup> show that the characteristic width of an energy band  $\varepsilon_0$  of hydrogen in a metal is 0.1–10 K.

It is undoubtedly of interest to investigate the transport characteristics of metals containing mobile defects (hydrogen isotopes and helium). The most promising objects for the investigation of these characteristics are single crystals of transition metals containing oxygen in a small and controlled amount. The scattering of electrons by mobile defects gives rise to a fundamentally new temperature dependence of the relaxation time of the electron subsystem and, consequently, of the transport coefficients of a metal.

The interaction of impuritons with one another causes them to cluster when a metal is cooled, leading to a fundamental modification of the impurity subsystem kinetics.<sup>6</sup>

The present paper reports an investigation of the absorption of sound in metals with mobile defects at low temperatures including the clustering temperature.

## 1. INFRARED RENORMALIZATIONS

The Hamiltonian describing the interaction of electrons with a system of impuritons not interacting with one another is<sup>7</sup>

$$H = \sum_{\mathbf{k}} \varepsilon(\mathbf{k}) a^+(\mathbf{k}) a(\mathbf{k}) + \sum_{\mathbf{k}} \omega(\mathbf{k}) c^+(\mathbf{k}) c(\mathbf{k}) \quad (1)$$

$$+ \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} V_0(\mathbf{q}) a^+(\mathbf{k}' - \mathbf{q}) c^+(\mathbf{k} + \mathbf{q}) c(\mathbf{k}) a(\mathbf{k}'),$$

where  $\varepsilon(\mathbf{k})$  and  $\omega(\mathbf{k})$  are the dispersion laws,  $a^+(\mathbf{k}), a(\mathbf{k})$  and  $c^+(\mathbf{k}), c(\mathbf{k})$  are the operators representing creation and annihilation of electrons and impuritons, respectively, and  $V_0(\mathbf{q})$  is the unrenormalized (“bare”) vertex of the electron–impuriton interaction. The summation with respect to

$\mathbf{k}$  to  $\mathbf{k}'$  is carried out over the Brillouin zone.

Kondo showed in a series of papers (see, for example, Ref. 7) that the electron–impuriton interaction results in significant infrared renormalizations of both  $V_0(\mathbf{q})$  and of the Green impuriton function  $\Psi_0(\mathbf{k}, \varepsilon_n)$ ,

$$\Psi_0(\mathbf{k}, \varepsilon_n) = [i\varepsilon_n - \omega(\mathbf{k}) + \zeta]^{-1}, \quad (2)$$

where  $\varepsilon_n$  is the Matsubara frequency (we assume specifically that impuritons are fermions) and  $\zeta$  is the chemical potential of impuritons. Calculations of the renormalizations in the parquet approximation, similar to that developed earlier in Ref. 8, yield the following result:

$$\Psi(\mathbf{k}, \varepsilon_n) = [i\varepsilon_n - \tilde{\omega}(\mathbf{k}) + \zeta]^{-1} \left[ \frac{\max(T, \varepsilon_0)}{E_0} \right]^g, \quad (3)$$

$$V(\mathbf{q}) = V_0(\mathbf{q}) [E_0 / \max(T, \varepsilon_0)]^g. \quad (4)$$

Here  $T$  is the absolute temperature,  $E_0$  is the width of the conduction electron and energy band, and  $\tilde{\omega}(\mathbf{k})$  is the renormalized impuriton dispersion law.

According to Ref. 9, the dimensionless electron–impurity interaction constant is

$$g = 2 \int \frac{d\mathbf{k}_1}{(2\pi)^3} \frac{d\mathbf{k}_2}{(2\pi)^3} \frac{|V_0(\mathbf{k}_1 - \mathbf{k}_2)|^2}{|\nabla \varepsilon(\mathbf{k}_1)| |\nabla \varepsilon(\mathbf{k}_2)|}, \quad (5)$$

where integration is carried out over the Fermi surface. The order of magnitude is  $g \approx [N(0) V_0]^2$ , where  $N(0)$  is the density of the electron states on the Fermi surface. If  $V_0 \approx 1$  eV and the values of  $N(0)$  are typical of a metal, we find  $g \approx 0.1$ –1.

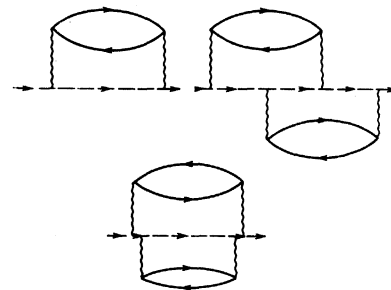


FIG. 1. Plots of the impuriton Green's function. The dashed line corresponds to  $\Psi_0(\mathbf{k}, \varepsilon_n)$ , the continuous curve represents the electron Green's function, and the wavy line is  $V_0(\mathbf{q})$ .

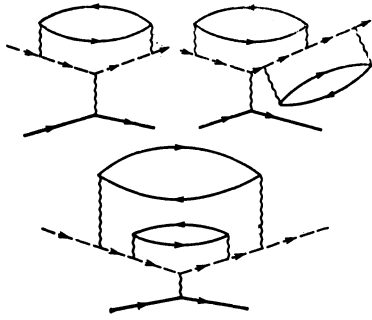


FIG. 2. Plots of the vertex of the electron-impuriton interaction. The notation is the same as in Fig. 1.

Figures 1 and 2 show the first terms of a series based on the parquet variant of perturbation theory applied to  $\Psi(\mathbf{k}, \epsilon_n)$  and  $V(\mathbf{q})$ , respectively. The range of validity of the parquet approximation is governed by the inequality<sup>9</sup>

$$N^2(0) V_0^3 \ln [E_0 / \max(T, \epsilon_0)] \ll 1. \quad (6)$$

We now formulate briefly the main results of our analysis, carried out in the approximation adopted above, of the scattering of electrons by impuritons.<sup>9</sup> In the range of low impuriton concentrations characterized by  $x \ll 1$  their contribution to the damping of electrons is described by the plot shown in Fig. 3.

At temperatures  $T \gg \epsilon_0$  the electron relaxation time  $\tau_{e,imp}$ , governed by the interaction with impuritons, is

$$\tau_{e,imp}^{-1} = \tau_0^{-1} \left( \frac{T}{E_0} \right)^g = xN(0) V_0^2 \left( \frac{T}{E_0} \right)^g; \quad (7)$$

here,  $\tau_0$  is the relaxation time of electrons interacting with randomly distributed frozen impurities of the same chemical nature. The dependence  $\tau_{e,imp}(T)$  implies that in a wide range of temperatures the temperature dependence of the resistance obeys  $R \propto T^g$ .

We can see that Eq. (7) does not include the tunnel matrix element  $\epsilon_0$  representing the quantum properties of a defect. Therefore, the same temperature dependence of the resistance should be observed also in a metal with heavy thermalized defects. In other words, the results of our analysis do not agree in the limit  $\epsilon_0 \rightarrow 0$  with those obtained using the cross technique.<sup>1</sup> We consider the scattering of electrons by an equilibrium system of defects. However, the relaxation time of the defect subsystem is governed by the value of  $t_0$  and increases exponentially with the mass of a defect. Measurements must however be carried out after a time  $t \gg t_0$

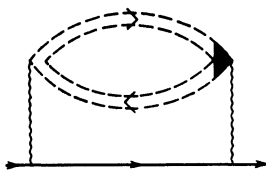


FIG. 3. Graph showing the contribution of impuritons to the self-energy part of the electron Green's function. The double dashed curve corresponds to  $\Psi(\mathbf{k}, \epsilon_n)$  and the black triangle with the wavy line corresponds to  $V(\mathbf{q})$ ; the rest of the notation is the same as in Fig. 1.

after the change in temperature, which realistically requires a very long "expectation" time. Therefore, at the usual rates of measurements a system of heavy defects does not reach equilibrium and can be regarded as frozen. The presence of the frozen defects in a real sample gives rise to a temperature-independent contribution  $\tau_{e,st}^{-1}$  to electron relaxation ( $\tau_e^{-1} = \tau_{e,imp}^{-1} + \tau_{e,st}^{-1}$ ), which in turn gives rise to a residual resistance, superposed on which we can easily see the temperature-dependent contribution of impuritons when the concentration of frozen defects is  $c \lesssim x$ .

Depending on the relationship between  $c$  and  $x$ , cooling may either result in localization of impuritons by the potential of the frozen defects or in clustering of these defects. The latter process either immediately establishes stratification into high- and low-concentration impuriton phases or creates clusters containing a finite number of particles and then stratification into phases with high and low cluster concentrations occurs at a lower temperature.<sup>6</sup>

The case  $c \gg x$  corresponds to dominance of the process of localization in a random potential, and was considered by us earlier.<sup>10</sup> Cooling causes impuritons to freeze in, so that they become localized in the deepest minima of the random potential and their contribution to the resistance made in the limit  $T \rightarrow 0$  becomes equal to the result obtained by the cross technique.

If cooling creates several phases in the system, then a comparison with the results of the cross technique obtained for a single-phase system is meaningless. It is the existence of an ultrapure phase and of a phase with a high concentration of impurities (distributed in an ordered manner) that corresponds to a thermodynamic equilibrium of a crystal with impurities in the limit  $T \rightarrow 0$ . However, geological times are required for the establishment of such an equilibrium in the case of heavy impurities.

## 2. ABSORPTION OF SOUND BY ELECTRONS

It is well known that at low acoustic frequencies, when the sound wavelength  $\lambda$  is much greater than the electron mean free path  $l_e$ , the absorption coefficient of  $\alpha_e$  of sound by electrons is directly proportional to their relaxation time  $\tau_e$  (see, for example, Refs. 11-13):

$$\alpha_e = \frac{n e_F \omega^2 \tau_e}{\rho s^3}, \quad (8)$$

where  $n$  is the electron density,  $\epsilon_F$  is the Fermi energy of electrons,  $\omega$  and  $s$  are the frequency and velocity of sound, and  $\rho$  is the density of the investigated metal.

At temperatures in the range  $T < \theta$  (where  $\theta$  is the Debye temperature) the contribution of defects to  $\tau_e$  dominates when their concentration is  $x \sim 10^{-3} - 10^{-2}$ , because  $\tau_e = \tau_{e,imp}$ .

A more thorough analysis shows<sup>14</sup> that Eq. (8) includes the time  $\tau_{2,e}$  which differs from the usual relaxation time by the factor  $[1 - P_2(\cos \varphi)]$  when integration is carried out with respect to the scattering angle  $\varphi$ ;  $P_2(y)$  is a Legendre polynomial.

When the scattering is by static defects, we have  $\tau_{2,e} = \text{const}$  and the temperature dependence of the absorption is entirely due to the small contribution made to the relaxation by the electron-phonon and electron-electron interactions.<sup>15</sup>

However, if the scattering is by mobile defects, the temperature dependence of  $\alpha_e$  is due to the temperature dependence of  $\tau_{e,imp}$ . However, in the range  $T \gg \varepsilon_0$  the main contribution to  $\tau_{2,e,imp}^{-1}$  comes from collisions involving scattering by large angles and the time  $\tau_{2,e,imp}$  differs from  $\tau_{e,imp}$  simply by a numerical factor of order unity.

Hence the absorption coefficient of sound is described by the following temperature dependence:

$$\alpha_e = \alpha_0 \left( \frac{E_0}{T} \right)^g, \quad (9)$$

where  $\alpha_0$  is the absorption coefficient in the case of frozen defects.

In the range  $T \ll \bar{\varepsilon}_0$ , where  $\bar{\varepsilon}_0$  is the renormalized width of an impuriton energy band, the main role is played by low-angle scattering. We then have  $\tau_{2,e}^{-1} = 3\tau_{tr,e}^{-1}$ , where  $\tau_{tr,e}$  is the transport scattering time of electrons. It is shown in Ref. 9 that in the case of an open Fermi surface the relaxation time  $\tau_{tr,e}$  is given by the following order-of-magnitude relationship:

$$\tau_{tr,e} = \tau_0 \left( \frac{E_0}{\varepsilon_0} \right)^g \left( \frac{\bar{\varepsilon}_0}{T} \right)^2. \quad (10)$$

Consequently, we have

$$\alpha_e = \alpha_0 \left( \frac{E_0}{\varepsilon_0} \right)^g \left( \frac{\bar{\varepsilon}_0}{T} \right)^2 T^{-2}. \quad (11)$$

Such a dependence is observed up to a temperature  $T^*$  at which the condition  $\lambda \gg l_e$  is no longer obeyed. When we have  $\lambda \ll l_e$  the quantity  $\alpha_e$  is completely independent of  $\tau_{2,e}$  (Refs. 11–13). Therefore, if  $T < T^*$  holds, the quantity  $\alpha_e$  reaches a constant value. Moreover, for  $T > T^*$ , the inequality  $\tau_{e,imp}^{-1} \gg \tau_{e,st}^{-1}$  may no longer be obeyed, which again causes  $\alpha_e$  to assume a constant value.

However, experimental observation of the dependence (Ref. 11) is hindered by the process of defect clustering<sup>6</sup> or the process of their stratification into phases with high and low defect concentrations. The characteristic temperature of these processes  $T_0$  is

$$T_0 = W_0 / |\ln x|, \quad (12)$$

where  $W_0$  is the specific binding energy of a defect in a cluster or in a highly concentrated impurity phase.

For  $T_0 > \bar{\varepsilon}_0$ , which holds in the case of realistic defect concentrations, we can no longer use Eq. (11) obtained in the approximation of noninteracting impuritons. For clustering in the range  $T < T_0$  there is a steep increase in the time taken by the impurity system to reach equilibrium with the

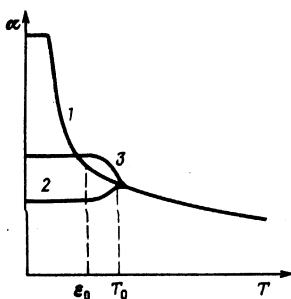


FIG. 4. Temperature dependence of the absorption coefficient of sound: 1) without allowance for clustering, 2), 3) after allowing for clustering of defects (the curves represent defects of two different types).

crystal after a change in temperature. Therefore, when measurements are carried out, the impurity system is no longer in equilibrium and if  $T \ll T_0$ , defects can be regarded as frozen. Consequently, we must show that  $\alpha_e(T)$  reaches a constant value  $\alpha'$  in the range  $T \ll T_0$ . The value of  $\alpha'$  depends on the nature of clusters which are then formed. The dependence  $\alpha_e(T)$  is plotted in Fig. 4.

In the case of stratification into phases the value of  $\alpha_e$  depends strongly on the size of nuclei of the highly concentrated phase.

### 3. RELAXATION ABSORPTION OF SOUND

In addition to the above-mentioned mechanism of absorption in crystals with quantum defects, there is also a relaxation mechanism of the absorption of sound. The greatest contribution to this mechanism comes in the case when there are two kinds of inequivalent interstices and the energy of an impuriton at these interstices differs by an amount  $\Delta E \lesssim T$ . In the case of *bcc*, *fcc*, and *hcp* metals the impurity band formed from tetrahedral interstitial states is degenerate. A static elastic deformation lifts this degeneracy partially or completely and the situation described above may be established.

The deformation created by an acoustic wave alters the value of  $\Delta E$ , which results in redistribution of impuritons between inequivalent interstices. In this case the mechanism is fully analogous to the mechanism of the absorption of sound by two-level system in amorphous metals.<sup>16,17</sup>

Let us assume that

$$\Delta E(t) = \Delta E + \gamma u(t), \quad (13)$$

where  $u(t)$  is the deformation created by an acoustic wave. The quantity  $\gamma$  is of the same order as the atomic scale of energies, because when the deformation (strain) is of order unity, the energy changes by an atomic value. We assume that the condition  $\lambda \gg l_{imp}$  is satisfied, where  $l_{imp}$  is the mean free path of impuritons. Therefore, in considering this contribution to the damping, we can assume that the deformation created by an acoustic wave has a homogeneous distribution.

For simplicity, we assume that the impuriton bands formed from interstitial states of one kind are not degenerate and we ignore their width compared with  $\Delta E$ .

When we consider the contribution of the change in the populations of interstices of different kind to the elastic stress in a crystal in accordance with Ref. 17, we find that the coefficient representing the relaxation absorption of sound is

$$\alpha_r = \frac{x\omega^2\tau_{imp}\gamma^2}{4T\Omega\rho s^3(1+\omega^2\tau_{imp}^2)\text{ch}^2(\Delta E/2T)}, \quad (14)$$

where  $\Omega$  is the volume of a unit cell and  $\tau_{imp}$  is the relaxation time of the impuriton subsystem. Following Ref. 18, we find that  $\tau_{imp}$  is given by

$$\tau_{imp}^{-1} = 4\pi \int \frac{d\mathbf{p} d\mathbf{q}}{(2\pi)^6} |V_{1,2}(\mathbf{q})|^2 n(\mathbf{p}-\mathbf{q})(1-n(\mathbf{p})) \times \delta[\varepsilon(\mathbf{p}-\mathbf{q}) - \varepsilon(\mathbf{p}) - \Delta E], \quad (15)$$

where the integration is carried out over the Brillouin zone and  $n(\mathbf{p})$  is the Fermi distribution function of electrons. The matrix element describing an interband transition of impuritons  $V_{1,2}(\mathbf{q})$  contains a small parameter  $\bar{\varepsilon}_0/\Delta E$ , where  $\bar{\varepsilon}_0$  is

the tunnel matrix element of an impuriton transition between neighboring interstices of different kinds.

The order of magnitude of the relaxation time of the impuriton subsystem is given by

$$\tau_{imp}^{-1} = \frac{g\tilde{\epsilon}_0^2}{\Delta E^2} \begin{cases} T, & \Delta E < T, \\ \Delta E \exp(-\Delta E/T), & T < \Delta E. \end{cases} \quad (16)$$

The quantity  $\alpha_r$  reaches its maximum at  $\omega\tau_{imp} = 1$ .

The value of  $\alpha_{r,max}$  is of the order of

$$\alpha_{r,max} = \frac{x\omega^2\gamma^2 \ln(g\tilde{\epsilon}_0^2/\omega\Delta E)}{\Omega\rho s^3 g\tilde{\epsilon}_0^2} \quad (17)$$

and can exceed  $\alpha_e$ .

Therefore, in the presence of two types of interstices with  $\Delta E < \theta$  the relaxation contribution to the absorption of sound at low temperatures may compete with the electron contribution and exceed the latter.

#### 4. INTERNAL FRICTION

In addition to the relaxation mechanism discussed above there is also a contribution to the absorption of sound from internal friction, i.e., due to a redistribution of impurities between the regions of compression and dilatation in an acoustic wave.

We show that this contribution is unimportant. Since  $\lambda \gg l_{imp}$ , the distribution of impurities occurs by diffusion and the characteristic time of the diffusion process is of order

$$\tau_j = \lambda^2/D, \quad (18)$$

where  $D$  is the diffusion coefficient of impurities. An estimate of the relevant contribution to the absorption of sound is readily obtained from Eq. (14) assuming  $\Delta E = 0$  and  $\tau_{imp} = \tau_f$ :

$$\alpha_f = \frac{x\gamma^2\omega^2\tau^*}{\Omega T\rho s^3 [1 + (\omega\tau^*)^2]}, \quad (19)$$

where

$$\tau^* = D/s^2. \quad (20)$$

The ratio of the two values is

$$\alpha_f/\alpha_e = x^2\tau^*E_0 \left(\frac{E_0}{T}\right)^{1-g}. \quad (21)$$

Even in the case of the lightest hydrogen isotope (protium) the contribution of the internal friction to the absorption of sound at temperatures  $T < \theta$  is much less than the electron contribution (the values of  $D$  are taken from Ref. 3). In the case of heavy impurities the values of  $D$  and  $\tau^*$  friction to the absorption of sound can be ignored throughout the investigated temperature range.

#### 5. TWO-LEVEL SYSTEMS

As pointed out already, at temperatures  $T < T_0$  a system of clustered defects becomes frozen. If the particles in a cluster are then practically immobile, the relaxation contribution to the absorption of sound below  $t_0$  disappears.

However, there may be a situation<sup>4,5</sup> when a mobile defect (hydrogen atom) is trapped by a heavy immobile impurity, but it still remains capable of tunneling between two (or more) interstices which are equivalent on the energy scale. This effect has been observed in niobium single crystals when hydrogen and deuterium are trapped by immobile

carbon, nitrogen, and oxygen impurities.<sup>4,5</sup> The resultant two-level systems (TLSs) naturally contribute to the absorption of sound.<sup>16,17</sup> An important role is played by an asymmetry of TLSs characterized by a difference between the energies of an impuriton at interstices before we allow for the tunneling represented by  $\Delta E_i$ . This asymmetry is due to the action of other clusters and point defects on the resultant cluster. It follows from Refs. 16 and 17 that the contribution of TLSs to the absorption is

$$\alpha_{TLS} = \sum_i \frac{\omega^2\tau_i}{4T\rho s^3(1+\omega^2\tau_i^2)} \left(\frac{\Delta E_i}{E_i}\right)^2 \frac{\gamma_i^2}{\text{ch}^2(E_i/2T)}, \quad (22)$$

where  $\tau_i$  is the relaxation time of a TLS  $\gamma_i = d(\Delta E_i)/du$ , and the splitting of the TLS levels is given by

$$E_i = [(\Delta E_i)^2 + \tilde{\epsilon}_0^2]^{1/2}, \quad (23)$$

and  $\tilde{\epsilon}_0$  is a matrix element representing tunneling between energy minima. In the range of frequencies of interest to us we have  $\tilde{\epsilon}_0 \gg \omega$ .

We now estimate the quantities occurring in Eq. (22) on the assumption that the TLS asymmetry is due to the action of frozen randomly distributed defects whose concentration is  $c$ . It is known<sup>6</sup> that the long-range part of the potential of the interaction between defects is due to the sum of the elastic interaction and the interaction via Friedel oscillations of the electron density (first and second terms, respectively):

$$W(\mathbf{R}) = [W_{el}(\mathbf{R}/R) + W_e \cos(2k_F R)]\Omega/R^3, \quad (24)$$

where  $k_F$  is the Fermi momentum of electrons. The value of  $W_{el}$  changes its sign depending on the orientation of the vector  $\mathbf{R}$  relative to the crystallographic axes whereas  $W_e \sim N(0)V_{1,0}V_{2,0}$ , where  $V_{1,0}$  and  $V_{2,0}$  are the potentials acting on electrons and contributed by the first and second interacting defects, respectively. The values of  $W_{el}$  and  $W_e$  are of the same order of magnitude (0.01–1 eV).

We are interested in the difference between the values of  $W(\mathbf{R})$  in two adjacent interstices. We can easily see that because of a rapid change in  $\cos(2k_F R)$  the main contribution to  $\Delta E_i$  comes from the interaction via Friedel oscillations of the electron density. In the case of TLSs with the coordinates of the minima  $\mathbf{R} \pm \mathbf{a}/2$ , we obtain

$$\Delta E_i = 2 \sum_j W_e \frac{\Omega}{|\mathbf{R}_i - \mathbf{R}_j|^3} \sin \left[ \frac{k_F(\mathbf{a}, \mathbf{R}_i - \mathbf{R}_j)}{|\mathbf{R}_i - \mathbf{R}_j|} \right] \times \sin(2k_F |\mathbf{R}_i - \mathbf{R}_j|). \quad (25)$$

The summation occurs between static defects with the coordinates  $\mathbf{R}_j$ .

Since  $\cos(2k_F R)$  oscillates rapidly over atomic distances, we can consider a simplified model on the assumption that the quantity

$$\sin \left[ \frac{k_F(\mathbf{a}, \mathbf{R}_i - \mathbf{R}_j)}{|\mathbf{R}_i - \mathbf{R}_j|} \right] \sin(2k_F |\mathbf{R}_i - \mathbf{R}_j|)$$

assumes values in the interval  $(-b, b)$ , where  $b$  is of the order of unity, with a probability independent of  $|\mathbf{R}_i - \mathbf{R}_j|$ .

In this case the problem of the distribution of the quantity  $\Delta E_i$  is analogous to the problem of the dipole width of a

magnetic resonance line.<sup>19</sup> In the range of concentrations  $c \ll 1$  the distribution of the value  $\Delta E_i$  is Lorentzian with a characteristic width

$$\delta = cW_e, \quad (26)$$

where a typical value  $\gamma_i = d(\Delta E_i)/du$  is again equal to  $\delta$ .

The value of  $\tau_i$  is given by Eq. (16), where  $\Delta E$  should be replaced with  $E_i$ . Since there is a distribution of  $\Delta E_i$  (and, consequently, of  $E_i$ ), we are dealing with an exponentially wide distribution of times  $\tau_i$ . The difference between single crystals and metallic glasses is the absence, at low concentrations  $c$ , of a broad distribution of  $\tilde{\epsilon}_0$ . For simplicity, we assume that  $\tilde{\epsilon}_0$  is the same for all TLSs of a given kind.

Substituting the expression for  $\tau_i$  into Eq. (22), we find that the absorption coefficient is

$$\alpha_{\text{TLS}} = \begin{cases} \sum_i \frac{\omega^2 \delta^2 (\Delta E_i)^2}{4T \rho s^3 \max(T, E_i) g \tilde{\epsilon}_0^2}, & \omega \tau_i < 1, \\ \sum_i \frac{\delta^2 g \tilde{\epsilon}_0^2 \max(T, E_i) (\Delta E_i)^2}{4T \rho s^3 E_i^4 \text{ch}^2(E_i/2T) \exp(E_i/T)}, & \omega \tau_i > 1. \end{cases} \quad (27)$$

We introduce a quantity  $E^*$  representing the splitting of the TLS levels such that  $\omega \tau_i = 1$ . Then, to logarithmic accuracy we have

$$E^* = T \ln \left( \frac{g \tilde{\epsilon}_0^2}{\omega T} \right). \quad (28)$$

Since in reality we find  $E_i \gg \tilde{\epsilon}_0$ , it follows that at low temperatures we have  $\omega \tau_i \gg 1$ , and this applies to all TLSs.

If  $E^* \gg \epsilon_0$ , then the main contribution to the absorption comes from TLSs with  $\Delta E_i$  located in the middle of a band whose width is of the order of  $E^*$  and is located near  $E^*$ . In the case when  $E^* \lesssim \tilde{\epsilon}_0$ , the main contribution to the absorption comes from TLSs with  $\Delta E_i$  of the order of  $(\tilde{\epsilon}_0 T)^{1/2}$ .

Averaging over the distribution of  $\Delta E_i$  and assuming  $\ln(g \tilde{\epsilon}_0^2 / \omega T) \sim 1$ , we obtain two forms of the expression for  $\alpha_{\text{TLS}}$ .

a)  $\tilde{\epsilon}_0 \ll \delta$ .

In this case, we have

$$\alpha_{\text{TLS}} = \begin{cases} \frac{x_{\text{TLS}} \omega^2 \delta^3}{\Omega \rho s^3 T g \tilde{\epsilon}_0^2}, & T > \delta, \\ \frac{x_{\text{TLS}} \omega^2 T \delta}{\Omega \rho s^3 g \tilde{\epsilon}_0^2}, & \tilde{\epsilon}_0 < T < \delta, \\ \frac{x_{\text{TLS}} g \delta T^{1/2} \tilde{\epsilon}_0^{1/2}}{\Omega \rho s^3 \exp(2\tilde{\epsilon}_0/T)}, & T < \tilde{\epsilon}_0. \end{cases} \quad (29)$$

The quantity  $\tilde{\epsilon}_0$  depends on  $T$  because of infrared renormalizations caused by the electron-impuriton interaction:<sup>7</sup>

$$\tilde{\epsilon}_0 = \epsilon_0 \left( \frac{\max(T, \epsilon_0)}{E_0} \right)^\beta, \quad (30)$$

where  $\beta \sim g$ ;  $\epsilon_0 = \text{const}$  is the unrenormalized tunnel matrix element. Therefore, for  $T = \delta$  there is a change in the temperature dependence from  $\alpha_{\text{TLS}} \propto T^{-1-2\beta}$  if  $T > \delta$  to  $\alpha_{\text{TLS}} \propto T^{1-2\beta}$  if  $\tilde{\epsilon}_0 < T < \delta$  ( $\beta \ll 1$ ). The dependence  $\alpha_{\text{TLS}}(T)$  is shown in Fig. 5.

Comparing the maximum of the function  $\alpha_{\text{TLS}}(T)$  corresponding to  $T = \delta$  with the electron contribution to the absorption, we find that

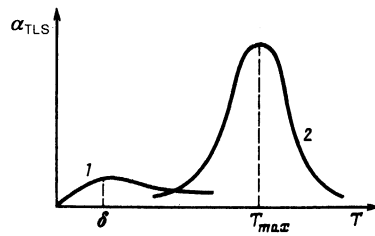


FIG. 5. Temperature dependence of the contribution of two-level systems to the absorption of sound: 1) in a normal metal; 2) in a superconductor.

$$\frac{\alpha_{\text{TLS}}^{\text{max}}}{\alpha_e} = \frac{c x_{\text{TLS}} \delta^2}{g \tilde{\epsilon}_0^2}. \quad (31)$$

For  $x_{\text{TLS}} \sim c \sim 10^{-3} - 10^{-2}$  the electron contribution to the absorption dominates the TLS contribution, as supported by the experimental results.<sup>5</sup>

b)  $\delta \ll \tilde{\epsilon}_0$ .

In this case, we find that

$$\alpha_{\text{TLS}} = \begin{cases} \frac{x_{\text{TLS}} \omega^2 \delta^3}{\Omega \rho s^3 T g \tilde{\epsilon}_0^2}, & T > \tilde{\epsilon}_0, \\ \frac{x_{\text{TLS}} \delta^3 g}{\Omega \rho s^3 (T \tilde{\epsilon}_0)^{1/2} \exp(2\tilde{\epsilon}_0/T)}, & \delta^2 / \tilde{\epsilon}_0 < T < \tilde{\epsilon}_0, \\ \frac{x_{\text{TLS}} g \delta T^{1/2} \tilde{\epsilon}_0^{1/2}}{\Omega \rho s^3 \exp(2\tilde{\epsilon}_0/T)}, & T < \delta^2 / \tilde{\epsilon}_0. \end{cases} \quad (32)$$

We can easily see that the maximum of  $\alpha_{\text{TLS}}$  is reached at  $T = \tilde{\epsilon}_0$  and its amplitude is less than in the case  $\tilde{\epsilon}_0 < \delta$ .

## 6. ABSORPTION OF SOUND BY TWO-LEVEL SYSTEMS IN A SUPERCONDUCTOR

It is known<sup>20</sup> that at temperatures  $T \ll T_c$ , where  $T_c$  is the superconducting transition temperature, the electron contribution to the absorption of sound falls exponentially as a result of cooling. Consequently, the contribution of two-level systems to the absorption becomes dominant.<sup>5</sup>

The difference between the situation where the absorption is due to TLSs in a normal metal is associated with an exponential rise of  $\tau_i$  because of a reduction in the number of electron excitations.

If the TLS splitting is much less than the absolute temperature, then  $\tau_i$  can be described by the following estimate:<sup>21</sup>

$$\tau_i^{-1} = g \frac{\tilde{\epsilon}_0^2}{E_i^2} \frac{2T}{1 + \exp(\Delta/T)}, \quad (33)$$

where  $\Delta$  is the superconducting gap in the spectrum of electron excitations. Then it has to be described by Eq. (30), where instead of  $\max(T, \epsilon_0)$ , we now have  $\max(T, \epsilon_0, \Delta)$  (Ref. 9). Hence for  $\epsilon_0, T < \Delta$  the value of  $\tilde{\epsilon}_0$  is independent of temperature.

Far from  $T_c$ , for  $E^*, \delta \ll T \ll \Delta$ , we obtain [subject to Eq. (33)]:

$$E^* = \left\{ \frac{2g \tilde{\epsilon}_0^2 T}{\omega [1 + \exp(\Delta/T)]} \right\}^{1/2} \quad (34)$$

and

$$\alpha_{\text{TLS}}^s = \begin{cases} \sum_i \frac{\omega^2 \delta^2 (\Delta E_i)^2 \exp(\Delta/T)}{\rho s^3 g \bar{\epsilon}_0^2 T^2}, & \omega \tau_i < 1, \\ \sum_i \frac{\delta^2 g \bar{\epsilon}_0^2 (\Delta E_i)^2}{\rho s^3 E_i^4 \exp(\Delta/T)}, & \omega \tau_i > 1. \end{cases} \quad (35)$$

As in the case of a normal metal, two variants are possible.

a)  $\delta \gg \bar{\epsilon}_0$ .

For  $E^* > \bar{\epsilon}_0$ , the main contribution to the absorption is due to TLSs whose asymmetry is  $\Delta E_i \sim E^*$ ; in the opposite case the characteristic asymmetry of the TLSs is of order  $\bar{\epsilon}_0$ . After averaging over the distribution of  $\Delta E_i$ , we obtain

$$\alpha_{\text{TLS}}^s = \begin{cases} \frac{x_{\text{TLS}} \omega \delta^3}{\Omega \rho s^3 T E^*}, & E^* > \delta, \\ \frac{x_{\text{TLS}} \omega \delta E^*}{\Omega \rho s^3 T}, & \bar{\epsilon}_0 < E^* < \delta, \\ \frac{x_{\text{TLS}} \omega \delta (E^*)^2}{\Omega \rho s^3 T \bar{\epsilon}_0}, & E^* < \bar{\epsilon}_0. \end{cases} \quad (36)$$

The range of temperatures where the estimate (36) is valid is limited from above by the condition  $T > \bar{\epsilon}_0$ . For  $T < \bar{\epsilon}_0$ , we can no longer use Eq. (33) for  $\tau_i$ .

Since  $E^*$  depends exponentially on  $T$  [Eq. (34)], a sharp maximum of  $\alpha_{\text{TLS}}^s$  is observed at  $E^* = \delta$ . The temperature  $T_{\text{max}}$  corresponding to this maximum, to within logarithmic corrections, equals

$$T_{\text{max}} = \Delta / \ln \left( \frac{g \bar{\epsilon}_0^2 \Delta}{\omega \delta^2} \right) \quad (37)$$

and at low values of  $c$  it can exceed considerably the temperature  $T = \delta$  corresponding to a maximum of  $\alpha_{\text{TLS}}$  for a normal metal (Fig. 5).

The amplitude of the maximum  $\alpha_{\text{TLS}}^s$  for the superconducting phase is  $g \bar{\epsilon}_0^2 / \omega T_{\text{max}} \gg 1$  times greater than the corresponding value for the normal phase and is proportional to  $\omega$ , in good agreement with the experimental results.<sup>5</sup>

b)  $\bar{\epsilon}_0 \gg \delta$ .

We then have

$$\alpha_{\text{TLS}}^s = \begin{cases} \frac{x_{\text{TLS}} \omega \delta^3}{\Omega \rho s^3 T E^*}, & E^* > \bar{\epsilon}_0, \\ \frac{x_{\text{TLS}} \omega \delta^3 (E^*)^2}{\Omega \rho s^3 T \bar{\epsilon}_0^3}, & E^* < \bar{\epsilon}_0. \end{cases} \quad (38)$$

In this case a maximum of  $\alpha_{\text{TLS}}^s$  occurs at a temperature

$$T_{\text{max}} = \Delta / \ln \left( \frac{g \Delta}{\omega} \right), \quad (39)$$

at which we have  $\bar{\epsilon}_0 = E^*$  and the value  $(\alpha_{\text{TLS}}^s)^{\text{max}}$  contains, compared with the  $\delta \gg \bar{\epsilon}_0$  case, an additional factor  $\delta / \bar{\epsilon}_0$ .

In the range of values  $\delta \sim \Delta$  there is a gradual transition from Eq. (29) to Eq. (36). The corresponding expressions for  $\alpha_{\text{TLS}}^s$  are too cumbersome and we shall not give them here.

## CONCLUSIONS

We have thus shown that the electron contribution to the absorption of sound in a normal metal containing quantum defects dominates the relaxation contribution, and the temperature dependence of the absorption coefficient  $\alpha$  is due to the temperature dependence of the relaxation time of electrons related to their scattering by mobile defects. Below the clustering temperature, where the defects lose their mobility, the coefficient  $\alpha$  reaches a constant value.

The relaxation contribution to the absorption by a normal metal may dominate in the presence of two types of interstices separated by a small energy interval. The contribution of the internal friction to the absorption is small.

If as a result of capture of mobile and immobile defects we can expect formation of two-level systems, then these systems also contribute to the absorption of sound associated with the relaxation mechanism. The distribution of the asymmetry of these two-level systems is Lorentzian with a characteristic width proportional to the defect concentration. In the case of a superconductor the absorption of sound is related specifically to the presence of two-level systems and its magnitude is slightly greater than the contribution of these systems to the absorption in a normal metal.

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Translated by A. Tybulewicz