The nature of nonlinear interaction of waves in the plasma-maser effect

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The properties of nonlinear interaction of nonresonant and resonant waves in a plasma are investigated by quantum electrodynamical methods. The nature of the interaction mechanism of interaction between the particles and the waves, on which the effect under consideration is based, is elucidated.

1. INTRODUCTION

The "plasma-maser effect" is the name given in the literature[1] to the nonlinear interaction of resonant waves (ω, $\mathbf{q}$), for which Cherenkov resonance

$$\omega - \mathbf{q}\cdot\mathbf{v} = 0 \qquad (1)$$

with certain particles of the plasma is possible (the case is usually considered in which some low-frequency plasma oscillations correspond to the resonant waves, for example, ion sound), and to the nonlinear interaction of nonresonant waves (Ω,$\mathbf{k}$), for which conditions such as Cherenkov resonance are forbidden:

$$\Omega - \mathbf{k}\cdot\mathbf{v} \neq 0, \qquad (2)$$

and also the condition of resonant scattering with a resonant mode:

$$\Omega - \omega - (\mathbf{k} - \mathbf{q})\cdot\mathbf{v} \neq 0 \qquad (3)$$

(nonresonant waves usually include high-frequency waves—Langmuir or electromagnetic). A number of researches have been devoted to different aspects of such nonlinear interaction (see, for example, Refs. 2–11).

In the "canonical" statement of the problem of the plasma maser, a plasma is considered in which there is some (essentially epithermal) level of resonant oscillations: the object of study has been the evolution of a nonresonant test wave under such conditions. The prospect, based on the considered phenomenon of amplification of high-frequency nonresonant waves at the expense of the energy of low-frequency resonant oscillations, i.e., the effective conversion of the energy of the plasma oscillations upward in frequency, is very attractive.

It was shown in Ref 12. that the growth rate of the amplification of the nonresonant waves γ (or the damping decrement) under the considered conditions is determined not only by the imaginary part of the corresponding nonlinear dielectric constant $\varepsilon_{\Omega\mathbf{k}}^{NL}$ [the imaginary part of the linear dielectric constant is $\mathrm{Im}_{\Omega\mathbf{k}}^{L} = 0$ by virtue of the condition (2)], but also by an existing additional contribution, of the same order, of the generated quasilinear interaction of the plasma particles with the resonant oscillations; this contribution is proportional to $\partial\varepsilon_{\Omega\mathbf{k}}^{L}/\partial t$ and is connected with the time dependence of the quantity $\varepsilon_{\Omega\mathbf{k}}^{L}$. Detailed investigation[12] shows that the combined account of the two phenomena under these conditions—the direct nonlinear interaction and the effect of a nonstationary medium—leads in the final analysis to conservation of the number of quanta of nonresonant waves as an adiabatic invariant.

In the more general statement of the problem, we consider this same nonlinear interaction in the presence of any sort of other effects (in the terminology of Refs. 12, 13—an open system with external sources). Under these conditions, the number of quanta of nonresonant waves $N_\mathbf{k}$ cannot be conserved,[13] and the growth rate

$$\gamma = N_\mathbf{k}^{-1} dN_\mathbf{k}/dt$$

turns out to be independent of the energy density of the resonant waves and proportional to the density of such sources,[13] which include the Coulomb collision of particles in the plasma, the presence of a certain "effective frictional force" in the kinetic equation, and so on (see Refs. 12, 13). Here the amplification (damping) of the nonresonant waves is produced only by an external source and does not depend on the characteristics of the resonant turbulence.

The quantity $N_\mathbf{k}$ also ceases to be conserved upon application of an external magnetic field $\mathbf{H}_{\mathrm{ext}}$ to the plasma.[14]

An interesting feature of the given nonlinear interaction is also the fact[15] that the inverse effect—the simultaneous effect of the nonresonant and resonant waves on the distribution $\Phi_\mathbf{p}$ of the plasma particles—differs from zero (under the same conditions for which $N_\mathbf{k}$ is conserved).

The present investigation has the aim of clarifying the nature of the mechanism of interaction of particles and waves that lies at the base of the studied effect. It is necessary to give answers to the following questions: 1) why do other influences (for example, the Coulomb collisions and so on) guarantee the differing from zero of the quantity γ, while the interaction with resonant waves (i.e., actually, the consideration of the quasilinear interaction as external relative to the nonresonant waves of the source in Refs. 12 and 13) does not lead to a change in $N_\mathbf{k}$, and also why the growth rate γ differs from zero in the presence of a regular external magnetic field and is equal to zero in its absence; 2) why, $dN_\mathbf{k}/dt = 0$ but $d\Phi_\mathbf{p}/dt \neq 0$ under the conditions of one and the same nonlinear interaction. The most natural answer to these questions involves investigating the given effect with the help of the methods of quantum electrodynamics.

It should be noted that the study of the plasma maser effect in Refs. 4–15 was based on classical kinetic theory. In the present work it was judged appropriate to consider the plasma maser from the point of view of quantum electrodynamics, which allows a better understanding of the physical nature of the investigated phenomenon.

2. GENERAL RELATIONS AND STATEMENT OF THE PROBLEM

We shall consider a system consisting of electrons and of photons interacting with them. The operator of the field

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function of the electron-positron field ψ_a is given by the expression

$$\psi_a(t,\mathbf{r})=\sum_{\mathbf{p}\sigma}\frac{1}{(2\varepsilon_{\mathbf{p}})^{1/2}}(a_{\mathbf{p}\sigma}u^{(+)}_{\mathbf{p},\sigma,a}e^{i\mathbf{pr}-i\varepsilon_{\mathbf{p}}t}+b^{+}_{\mathbf{p}\sigma}u^{(-)}_{-\mathbf{p},-\sigma,a}e^{i\varepsilon_{\mathbf{p}}t-i\mathbf{pr}}), \qquad (4)$$

where a is the spinor index running through the values 1, 2, 3, 4 (this is often omitted below); σ enumerates the polarization states and takes on the values $\pm 1/2$; $\varepsilon_p=(p^2+m^2)^{1/2}$; a is the annihilation operator of the electron; b^+ is the positron creation operator; the amplitudes $u_{p,\sigma}$ are normalized by the condition $\bar{u}^{(\pm)}_{p,\sigma}u^{(\pm)}_{p,\sigma}=\pm 2m$; we set the normalized volume V equal to unity; in what follows, $\hbar=c=1$.

The operator

$$A^0=0,$$
$$\mathbf{A}(t,\mathbf{r})=\sum_{\mathbf{k}\alpha}\left(\frac{4\pi}{2\Omega_{\mathbf{k}}}\right)^{1/2}(c_{\mathbf{k}\alpha}\mathbf{e}_{\mathbf{k}\alpha}e^{i\mathbf{kr}-i\Omega_{\mathbf{k}}t}+c^{+}_{\mathbf{k}\alpha}\mathbf{e}^{*}_{\mathbf{k}\alpha}e^{i\Omega_{\mathbf{k}}t-i\mathbf{kr}}) \qquad (5)$$

corresponds to the quantized field A^μ, interaction with which relates to the possibility of emission and absorption of a photon by an electron. (In this equation, a three-dimensional transverse gauge is used; the index α relates to the different polarization states of the photon and takes on the values ± 1; c and c^+ are the photon annihilation and creation operators; the unit polarization vector $\mathbf{e}_{k\alpha}$ satisfies the conditions $\mathbf{k}\cdot\mathbf{e}_{k\alpha}=0$ and $|\mathbf{e}_{k\alpha}|=1$.)

In addition to the quantized field A^μ, which corresponds to nonresonant electromagnetic waves, we shall consider the classical (non-quantized) field φ^μ, which describes resonant longitudinal waves:

$$\boldsymbol{\varphi}=0, \quad \varphi^0\equiv\varphi(t,\mathbf{r})=\int d\omega\, d\mathbf{q}\, \varphi_{\omega\mathbf{q}}\, e^{i\mathbf{qr}-i\omega t}. \qquad (6)$$

The resonant field φ could also be regarded as quantized—this is not important for what follows. Our choice—nonresonant quantized transverse fields and resonant longitudinal nonquantized fields—is made for simplicity in the subsequent exposition, and also so as not to focus attention on the quantization of the longitudinal plasma waves for which, naturally, a detailed consideration of the electrodynamics of the medium (plasma) would be necessary, while investigation under the conditions described above could in practice lead to a much simpler basis for the electrodynamics of the vacuum (see below).

In previous works[1-15] on the problem of the plasma maser, the dielectric constant $\varepsilon_{\Omega\mathbf{k}}$ was first found with the help of the usual procedure of the solution of the kinetic equation, and then the growth rate γ was determined (for transverse electromagnetic waves) from the formula

$$\gamma=-\frac{\Omega^2\,\mathrm{Im}\,\varepsilon_{\Omega\mathbf{k}}}{\partial(\Omega^2\,\mathrm{Re}\,\varepsilon_{\Omega\mathbf{k}})/\partial\Omega}. \qquad (7)$$

Under the conditions of a nonstationary medium,[12] the contributions mentioned above, which are proportional to $\partial\varepsilon/\partial t$, also appear in the numerator of (7)—see Ref. 12.

In the present approach, we shall not calculate the function $\varepsilon_{\Omega\mathbf{k}}$, but shall find directly the change in the number of quanta of nonresonant waves $dN_{\mathbf{k}\alpha}(t)/dt$. The time dependence of the quantity $N_{\mathbf{k}\alpha}(t)$ (the operator of which is given by the expression $N_{\mathbf{k}\alpha}=c_{\mathbf{k}\alpha^+}c_{\mathbf{k}\alpha}$) is determined by the relation

$$N_{\mathbf{k}\alpha}(t)=\langle\ldots|S^+(t)c_{\mathbf{k}\alpha}{}^+c_{\mathbf{k}\alpha}S(t)|\ldots\rangle \qquad (8)$$

and should in principle take into account the two contributions to $dN_{\mathbf{k}\alpha}(t)/dt$ mentioned above—both that connected with the nonlinear interaction and that due to the nonstationarity of the medium. The averaging in (8) is carried out over the state $|\ldots\rangle$, which contains a certain quantity of electrons and photons in various states; the S matrix is a chronological exponential of the Lagrangian of interaction of the system:

$$S(t)=T\exp\left[i\int_{-\infty}^{t}dt_1\,\mathscr{L}(t_1)\right], \qquad (9)$$

$$\mathscr{L}(t)=\mathscr{L}_\varphi(t)+\mathscr{L}_A(t)=-\int d\mathbf{r}\, j_\mu(t,\mathbf{r})\,[\varphi^\mu(t,\mathbf{r})+A^\mu(t,\mathbf{r})], \qquad (10)$$

$$j^\mu=e{:}\bar{\psi}\gamma^\mu\psi{:}, \qquad (11)$$

where :...: is the symbol for normal ordering.

Before proceeding to the calculation of the law of evolution of the quantity (8), it is necessary to make a number of observations that provide the basis for the very possibility of such calculation procedures. First, we note that for the given interaction $\mathrm{Im}\,\varepsilon_{\Omega\mathbf{k}}$ in (7) turns out to be connected only with a nonlinear response of third order in the field (i.e., it is determined only by the direct interaction of the modes) in the standard iteration scheme of solution of the kinetic equation; so far as the additional "polarization" contribution is concerned, a contribution connected with products of responses of second order, it is identically equal to zero for the given type of interaction (as was already assumed in Ref. 1 and rigorously proved in Ref. 12).

Second, the calculation procedure just mentioned is in fact the calculation of the numerator of (7) (with account also of the effects produced by the nonstationarity), expanded in a series in the constant e^2 of electromagnetic interaction. Generally speaking, the denominator of (8) is also represented by some series in e^2; but for the study of the effect at hand (and not the corrections to it), it suffices to seek only the expansion of the numerator of (7) [with the necessary degree of accuracy—to terms $\propto e^4$, since the term $\propto e^2$ in $\mathrm{Im}\,\varepsilon_{\Omega k}$ is absent according to Ref. 1, assuming the denominator of (7) to be equal to the expression $2\Omega+O(e^2)$].

Finally, we note that the use of the formulas of the quantum electrodynamics of a vacuum causes us in actuality to set $\Omega_{\mathbf{k}}=|\mathbf{k}|$ in (5), and not $\Omega_{\mathbf{k}}=(\mathbf{k}^2+\omega_p^2)^{1/2}$ as in a plasma. For the present calculation this is not very significant in principle; the technique developed below for consideration of the evolution of the nonresonant waves does not permit us to find the difference of the dependence of the energy density of the waves $W(t)$ on the law of change of $N_{\mathbf{k}\alpha}(t)$, since this difference is connected with the dependence $\Omega_{\mathbf{k}}=\Omega_{\mathbf{k}}(t)$, which we have ignored.

3. NATURE OF THE CONSERVATION OF THE NUMBER OF QUANTA

Thus, further investigation of the function $N_{\mathbf{k}\alpha}(t)$ [Eq. (8)] be based on the relations (8)–(11). [The correctness of such a method is not difficult to verify by the example of the calculation of simple quantities—Landau damping and the quasilinear integral; it can be established that the calculation of these quantities by means of the S matrix (9) leads to correct general relativistic quantum expressions.]

Differentiating (8), we find the following, with the necessary accuracy:

$$\frac{dN_{\mathbf{k}\alpha}(t)}{dt}=\langle\ldots|\int_{-\infty}^{t}dt_1\int_{-\infty}^{t_1}dt_2\int_{-\infty}^{t_2}dt_3\{\mathscr{L}(t_3)\mathscr{L}(t_2)\mathscr{L}(t_1)$$

$$\times[\mathscr{L}(t),N_{\mathbf{k}\alpha}]-[\mathscr{L}(t),N_{\mathbf{k}\alpha}]\mathscr{L}(t_1)\mathscr{L}(t_2)\mathscr{L}(t_3)\}$$

$$-\int_{-\infty}^{t}dt_1\int_{-\infty}^{t_1}dt_2\int_{-\infty}^{t}dt_3\{\mathscr{L}(t_2)\mathscr{L}(t_1)[\mathscr{L}(t),N_{\mathbf{k}\alpha}]\mathscr{L}(t_3)$$

$$-\mathscr{L}(t_3)[\mathscr{L}(t),N_{\mathbf{k}\alpha}]\mathscr{L}(t_1)\mathscr{L}(t_2)\}|\ldots\rangle. \qquad (12)$$

Of the four Lagrangians entering into each component of (12), two should correspond to $\mathscr{L}_A$ and two to $-\mathscr{L}_A$. It is not difficult to see that $\mathscr{L}_\varphi$ commutes with the operator of the number of quanta, $N_{\mathbf{k}\alpha}$, as a consequence of which we must set $\mathscr{L}(t)=\mathscr{L}_A(t)$; the contributions with $\mathscr{L}(t)=\mathscr{L}_\varphi$ are automatically cancelled.

The classical correlation function of the resonant fields $\varphi_{\omega\mathbf{q}}$ must now be replaced by the procedure of averaging over the random phases of the field $\varphi_{\omega\mathbf{q}}$ (cf. Refs. 1–15):

$$\varphi_{\omega\mathbf{q}}\varphi_{\omega'\mathbf{q}'}\to\langle\varphi_{\omega\mathbf{q}}\varphi_{\omega'\mathbf{q}'}\rangle=|\varphi_{\omega\mathbf{q}}|^2\delta(\omega+\omega')\delta(\mathbf{q}+\mathbf{q}'). \qquad (13)$$

In the method that we have used for the calculation of the nonivariant quantity $dN_{\mathbf{k}\alpha}(t)/dt$, the integration of the Lagrangian over the time in (12), after explicit substitution of the expressions (4)–(6), produces in the denominator expressions of the type $\pm\Omega_{\mathbf{k}}\pm\varepsilon_{\mathbf{p}}\pm\varepsilon/_{p-\mathbf{k}'}$, and so on. The classical conditions (1)–(3) are replaced by the general

$$\varepsilon_{\mathbf{p}-\mathbf{q}}-\varepsilon_{\mathbf{p}}+\omega=0,\qquad \varepsilon_{\mathbf{p}-\mathbf{k}}-\varepsilon_{\mathbf{p}}+\Omega_{\mathbf{k}}\neq0,\qquad \varepsilon_{\mathbf{p}-\mathbf{q}-\mathbf{k}}-\varepsilon_{\mathbf{p}}+\omega+\Omega_{\mathbf{k}}\neq0$$

[which in what follows we shall also designate as (1)–(3)]. To find the (real) quantity $dN_{\mathbf{k}\alpha}(t)/dt$ we must take the imaginary part of at least one of such factors in the denominator; this imaginary part is

$$\operatorname{Im}(\varepsilon_{\mathbf{p}-\mathbf{q}}-\varepsilon_{\mathbf{p}}+\omega\pm i0)^{-1}=\mp\pi\delta(\varepsilon_{\mathbf{p}-\mathbf{q}}-\varepsilon_{\mathbf{p}}+\omega),$$

where the sign of the infinitely small contribution $\pm i0$ in the exponential of the corresponding Lagrangian is determined by the causality principle—integration over the time from $t=-\infty$ in (9), (12) must have meaning. [We always consider the Cherenkov conditions (1)–(3); in (12), generally speaking, in addition to Cherenkov resonances of the type $\varepsilon_{\mathbf{p}-\mathbf{q}}-\varepsilon_{\mathbf{p}}+\omega=0$, pair-creation resonances can also arise, $\varepsilon_{\mathbf{p}-\mathbf{q}}+\varepsilon_{\mathbf{p}}-\omega=0$ and so on, which are not of interest to us at the present time.]

Explicit substitution of the expressions (4) and (5) in (10) can establish the fact that the commutator $[\mathscr{L}_A,N_{\mathbf{k}\alpha}]$ is very similar in its structure to the Lagrangian $\mathscr{L}_A$; the only important difference is that in the Lagrangian $\mathscr{L}_A$ the terms containing the photon annihilation operator c and the creation operator c^+ appear with the same signs, while in the expression $[\mathscr{L}_A,N_{\mathbf{k}\alpha}]$ they appear with opposite signs. This fact leads to the result that [upon satisfaction of the conditions (1)–(3)!], any two contributions to (12) of the type $\ldots\mathscr{L}_A\ldots[\mathscr{L}_A,N_{\mathbf{k}\alpha}]\ldots$ and $\ldots[\mathscr{L}_A,N_{\mathbf{k}\alpha}]\ldots\mathscr{L}_A,\ldots$ which are obtained from one another by the substitution $\mathscr{L}_A\leftrightarrow[\mathscr{L}_A,N_{\mathbf{k}\alpha}]$, accurately cancel one another, i.e, their sum is equal to zero. Roughly speaking, this takes place because the (resonant) Lagrangians which stand to the left of the first $\mathscr{L}_A$ or to the right of the second $\mathscr{L}_A$ enter into both components in one and the same of the two matrices (8) and consequently have the same sign of the imaginary contribution $i0$ to each denominator arising in the integration over time; the Lagrangians between the two $\mathscr{L}_A$ are nonresonant by virtue of the conditions (2) and (3), although they belong in the first component to S^+ and in the second, to S, and therefore they should contain denominators with infinitely small contributions of opposite sign. [These conditions are based on the form of the Lagrangian of electromagnetic interaction (10)].

We thus find that $dN_{\mathbf{k}\alpha}(t)/dt=0$. However, our discussions are still not completely valid, since both S matrices in (8) should transform the initial state of the system into another one that is also allowable; however, the general expression (8) also contains this same number of such contributions as $\mathscr{L}_\varphi\mathscr{L}_A\mathscr{L}_\varphi\cdot N_{\mathbf{k}\alpha}\cdot\mathscr{L}_A$, for example, when the second S matrix describes the forbidden [by the law of conservation of 4-momentum and by the nonresonance condition (2)] process of radiation of a photon by a free electron (or also the forbidden process of transformation of the photon into an electron-positron pair, etc.). Such contributions, obviously need to be excluded from (12). However, we can show that the general sum of such exclusions of "nonphysical" contributions is also equal to zero. Actually, for any method of pairing of the creation and annihilation operators in the Lagrangians entering into (12), the sum of the expressions $\pm\omega\pm\varepsilon_{\mathbf{p}}\pm\varepsilon_{\mathbf{p}-\mathbf{q}}$ for all four $\mathscr{L}$ is equal to zero; this leads to the result that the sum of the two excluded expressions, for example,

$$\int_{-\infty}^{t}dt_1\int_{-\infty}^{t_1}dt_2\int_{-\infty}^{t_2}dt_3\,\mathscr{L}_\varphi(t_3)\mathscr{L}_A(t_2)\mathscr{L}_\varphi(t_1)\cdot N_{\mathbf{k}\alpha}\cdot\mathscr{L}_A(t)$$

and

$$\int_{-\infty}^{t}dt_1\int_{-\infty}^{t_1}dt_2\,\mathscr{L}_\varphi(t_2)\mathscr{L}_A(t_1)\mathscr{L}_\varphi(t)\cdot N_{\mathbf{k}\alpha}\cdot\int_{-\infty}^{t}\mathscr{L}_A(\tau_1)d\tau_1,$$

is exactly equal to zero. These expressions differ in the choice of the factor in which there is no integration over time. Here, it is again important that the expression $\Omega_{\mathbf{k}}+\varepsilon_{\mathbf{p}-\mathbf{k}}-\varepsilon_{\mathbf{p}}$, which arises upon integration of $\mathscr{L}_A$ over time is nonresonant.

We have thus obtained a result that generalizes the conclusion of Ref. 12: in the interaction of nonresonant waves with resonant fields and particles, the number of quanta of nonresonant waves is conserved: $dN_{\mathbf{k}\alpha}(t)/dt=0$. For this conclusion, two circumstances used above are essential: I. $[\mathscr{L}_\varphi,N_{\mathbf{k}\alpha}]=0$; II. "absolute" nonresonant character of $\mathscr{L}_A$, i.e., non-fulfillment of all resonance conditions that can contain the quantities $(\Omega_{\mathbf{k}},\mathbf{k})$ corresponding to $\mathscr{L}_A$ [the hindrances (2) and (3)].

Now, having clarified the nature of the conservation of the number of quanta in the plasma-maser effect, we can answer the questions posed at the beginning of the paper. That is, the absence of the two circumstances I and II mentioned above (or even of one of them) leads to the result that in open systems (see Refs. 12, 13) external sources can lead to a value of $dN_{\mathbf{k}\alpha}(t)/dt$ that differs from zero. For such external sources, the high-frequency electromagnetic radiation is not "absolutely" nonresonant (in contrast to the case of the interaction with resonant fields considered above). Thus, for example, for collision damping (bremsstrahlung), the "basis" diagram of the process is absorption (emission) of a photon by an electron and exchange of a virtual quantum with any other particle (similar to the process of plasma maser, such a "basis" diagram is the scattering of resonant waves into nonresonant). But in this case, the photon frequency $\Omega_{\mathbf{k}}$ is not absolutely arbitrary but is connected by the relation $\Omega_{\mathbf{k}} + \varepsilon_f + E_f - \varepsilon_i - E_i = 0$, where $\varepsilon_{i(f)}$ and $E_{i(f)}$ are the initial (final) energies of the radiating electron and the particle with which it collides, while in the "canonical" statement of the plasma-maser problem the parameters of the nonresonant wave are not at all connected with the parameters of the resonant wave—see Eqs. (2) and (3). This is certainly a violation of condition I; furthermore, condition I is also violated in bremsstrahlung, since exchange of a virtual quantum of the same kind as that of the radiated quantum is possible between the colliding particles.

It is also clear why the growth rate of the plasma maser differs from zero in an external field—the "basis" is now not the scattering diagram but a diagram containing lines corresponding to the external field in addition to the lines of resonant and nonresonant waves. As a consequence, the appearance of conditions connecting $\Omega_{\mathbf{k}}$ with $\omega_{\mathbf{q}}$, and the parameters of the external field [which does not violate (2), (3)], i.e., the appearance of a resonant denominator in integration of $\mathscr{L}_A$ over time in (12). In addition, our discussions of the external field are only illustrative, since the particles are bound even in a weak external magnetic field, and perturbation theory must directly be constructed in a representation that takes into accurate account the external field and the change in the $\varepsilon_{\mathbf{p}}$ dependency produced by it.

Finally, we shall show why, in the inverse plasma-maser effect the change in the particle distribution function differs from zero: $d\Phi_{\mathbf{p}}(t)/dt \neq 0$, while in the direct process the number of quanta does not change $dN_{\mathbf{k}\alpha}(t)/dt = 0$; i.e., why the nonresonant waves in the plasma maser, without being themselves changed, play the role of an unusual catalyst of additional (relative to the quasilinear) exchange of energy between the particles of the plasma and the resonant waves. Condition II, of course, applies equally both to the direct and the inverse effects; however, condition I for the inverse effect is violated. Actually, in the calculation of $d\Phi_{\mathbf{p}\sigma}(t)/dt$, in place of $N_{\mathbf{k}\alpha} = c_{\mathbf{k}\alpha}{}^+ c_{\mathbf{k}\alpha}$ we should investigate the time evolution of the values of the operator of the number of electrons $\Phi_{\mathbf{p}\sigma} = a_{\mathbf{p}\sigma}{}^+ a_{\mathbf{p}\sigma}$, which does not commute either with $\mathscr{L}_A$ or with $\mathscr{L}_\varphi$ (in contrast to $N_{\mathbf{k}\alpha}$).

4. CONCLUSION

Thus, we have clarified above the nature of the nonlinear interaction of waves and particles in the plasma maser. The result of Ref. 12 that the number of quanta of nonresonant waves is adiabatically conserved in such an interaction has been generalized further (spontaneous processes such as the exchange effect, etc., were not taken into account in Ref. 12). It is shown why the amplification (damping) of the waves becomes possible, due to account of the interaction of external sources and under conditions of interaction in an external field. Also, the reason is given for the change in the distribution function of electrons in the case of no change in the number of quanta of the nonresonant waves. A rigorous consideration in the quantum case (and a comparison with the classical results of Refs. 12–15) of the simultaneous interaction of resonant and resonant turbulence on the particle distribution, and also the analysis of the conditions under which amplification of high-frequency radiation in the plasma maser in an external field is possible (and also in the presence of external sources, sources of energy, particles, momentum and so on) will be the object of a separate investigation.

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