Thermoelectric mechanism of electromagnetic-acoustic transformation

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An electromagnetic wave of frequency ω incident on the surface of a crystalline conductor whose symmetry axis is parallel to the normal to the surface induces in the conductor (under conditions of normal skin effect), through the thermoelectric effect, inhomogeneous temperature oscillations of the same frequency ω. These oscillations excite a sound wave via the thermoelastic stresses (at the frequency ω). The amplitude u of this wave is calculated; it is linear in the amplitude of the electromagnetic wave, and is a function of the frequency ω and of the characteristics of the conductor (resistivity, thermal conductivity, Thomson coefficient, etc.).

An electromagnetic wave of frequency ω incident on a half-space ($z>0$) induces in the latter a sound wave of the same frequency ω. This phenomenon[1]—electromagnetic acoustic transformation (abbreviated EMAT), is driven by several mechanisms. In the absence of a constant magnetic field the EMAT is due to the Stewart–Tolman and strain transformation mechanisms.[2] No attention has seemingly been called so far to the fact that in a single crystal whose symmetry axis does not coincide with the normal to the surface there should exist one more EMAT mechansm due to the thermoelectric effect. In fact, according to Ref. 3 (see also Ref. 4, § 59), in a conductor whose symmetry axis is not parallel to the normal to the surface there are produced inhomogeneous temperature oscillations, $T=T(z,t)$ *having the frequency* ω *of the incident electromagnetic wave*; the temperature oscillations cause sound excitation via thermoelastic stresses.

Let $\theta(z,t)=T(z,t)-T_0$ (T_0 is the equilibrium temperature of the body, and the subscript "0" will henceforth be omitted). The thermoelastic stress tensor is then[5]

$$\sigma_{ik}^T=-B_{ik}\theta(z,t). \tag{1}$$

The tensor B_{ik} is proportional to the tensor of the thermal-expansion coefficient β_{ik}, and in order of magnitude we have $B_{ik}=\rho_m s^2\beta_{ik}$, where ρ_m is the density of the metal and s is the speed of sound.

The displacement vector (the sound amplitude) is calculated from the equations of motion of the elastic medium

$$\rho_m\ddot{u}_i-\frac{\partial\sigma_{ik}}{\partial x_k}=\frac{\partial\sigma_{ik}^T}{\partial x_k},\quad \frac{\partial\sigma_{ik}^T}{\partial x_k}=-B_{ik}\frac{\partial\theta}{\partial x_k},$$
$$\sigma_{ik}=\lambda_{iklm}u_{lm}. \tag{2}$$

We use the standard notation (see Ref. 5). The sound-excitation problem reduces thus in this case to calculation of the field of the temperature $\theta=\theta(z,t)$ and to solution of the acoustic problem (2) with boundary conditions corresponding to the free boundary:

$$\sigma_{iz}=0. \tag{2'}$$

To calculate $\theta(z,t)$ we must solve a related system of equations consisting of the Maxwell equations and the thermal-conductivity equation. We choose the crystal surface to be the xy plane; the z axis is the inward normal to the surface; the principal symmetry axis makes an angle φ with the z axis. The magnetic field of the wave on the crystal surface is directed along the y axis: $H_y=He^{-i\omega t}$. It has then the same direction everywhere inside the crystal. Recognizing that the quantities depend only on the coordinate z and on the time like $e^{-i\omega t}$, we readily write down the complete system of equations for the nonzero components of the fields and currents, and also for the temperature θ:

$$-H_y'=(4\pi/c)j_x,\ E_x'=(i\omega/c)H_y,\ -i\omega C\theta+q_z'=0.$$
$$E_x=\rho_{xx}j_x+\alpha_{xz}\theta',\ q_z=T\alpha_{zx}j_x-\varkappa_{zz}\theta',\ E_z=\rho_{zx}j_x+\alpha_{zz}\theta'. \tag{3}$$

A prime denotes differentiation with respect to z. We shall not need the last of these equations, which defines E_z. All the tensors in (3) (the resistivity ρ_{ik}, the thermal conductivity $\varkappa_{ik}$, the thermoelectric coefficients α_{ik}) are symmetric.[1)] We present the dependences of the quantities (3) on the angle φ using as an example the components of the tensor ρ_{ik}:

$$\rho_{xx}=\rho_\parallel\sin^2\varphi+\rho_\perp\cos^2\varphi,\quad \rho_{zx}=(\rho_\parallel-\rho_\perp)\sin\varphi\cos\varphi, \tag{4}$$

$\rho_\parallel$ and $\rho_\perp$ are the principal values of the tensor ρ_{ik}. A uniaxial crystal is assumed.

The solution of the system (3) can be written in the form

$$E_x(z)=A_1e^{ik_1z}+A_2e^{ik_2z},\quad z>0,\quad \operatorname{Im}k_{1,2}>0,$$
$$\theta'(z)=\frac{1}{\alpha_{xz}}\left[A_1\left(1-\frac{k_1^2}{k^2}\right)e^{ik_1z}+A_2\left(1-\frac{k_2^2}{k^2}\right)e^{ik_2z}\right]. \tag{5}$$

Here

$$k_{1,2}=k\left\{\frac{1+b\pm[(1-b)^2-4ab]^{1/2}}{2(1+a)}\right\}^{1/2},\quad k^2=\frac{4\pi i\omega}{c^2\rho_{xx}},$$
$$a=\frac{T\alpha_{xz}^2}{\rho_{xx}\varkappa_{zz}},\quad b=\frac{c^2C\rho_{xx}}{4\pi\varkappa_{zz}}. \tag{6}$$

The connection between the coefficients A_1 and A_2 should be determined from the boundary conditions for the temperature. The isothermal boundary corresponds to the condition $\theta(0)=0$, and the adiabatic one to $\theta'(0)=0$. The oscillation amplitudes $E_x(z)$ and $\theta'(z)$ should be connected with the value of the magnetic field on the boundary:

$$k_1A_1+k_2A_2=(\omega/c)H. \tag{7}$$

Thus:

1) the isothermal boundary ($\theta(0)=0$)

$$\begin{Bmatrix}A_1\\A_2\end{Bmatrix}=\frac{\omega}{c}\frac{H}{k_1^2-k_2^2}\begin{cases}k_1(1-k_2^2/k^2),\\-k_2(1-k_1^2/k^2);\end{cases} \tag{8}$$

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2) the adiabatic boundary ($\theta'(0)=0$)

$$\begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \frac{\omega}{c} \frac{H}{(k_1-k_2)(1+k_1k_2/k^2)} \begin{cases} 1-k_2^2/k^2, \\ -(1-k_1^2/k^2). \end{cases} \qquad (8')$$

For $\alpha_{xz}=0$ we have

$$k_1=k, \quad k_2=(i\omega C/\varkappa_{zz})^{1/2}\equiv k_T. \qquad (9)$$

Here and below the numbering of the wave vectors begins with the limiting transition (9).

The parameter a in good metals is as a rule small.[2)] It can therefore be assumed that $k_1\sim k$ and $k_2\sim k_T$. Using the smallness of a and the standard expressions for ρ_{xx} and $\varkappa_{zz}$, we readily obtain

$$|k_1/k_2|\sim|k/k_T|=b^{-1/2}\approx l/\delta_0,$$

where

$$\delta_0^2=c^2/\omega_0^2,\quad \omega_0^2=4\pi ne^2/m,\quad l=v_F\tau$$

(we use the standard notation, as above). The ratio $|k_1/k_2|$ can thus be either larger or smaller than unity, depending on the temperature and on the sample quality.

We turn now to a solution of the acoustic problem [see (2) and (2′)]. Since the z axis does not coincide with a principal symmetry axis, the sound waves propagating along it are generally speaking neither longitudinal nor transverse. There are always present, however, three independent waves with mutually perpendicular polarizations $\mathbf{e}_j$

$$\mathbf{e}_j\mathbf{e}_{j'}=\delta_{jj'};\quad j,\ j'=1,\ 2,\ 3$$

and sound velocities s_j. The amplitude u_j of the jth sound is determined by the component of the thermoelastic (in this case) force along the $\mathbf{e}_j$ direction:

$$u''+k_{\rm ac}^2u=(\rho_ms^2)^{-1}(\mathbf{eB})\theta',\quad k_{\rm ac}=\omega/s, \qquad (10)$$

where $\mathbf{B}$ is a vector with components B_{iz}. We have left out the subscript j. According to (2′), the boundary condition for (10) is

$$u'(0)=0. \qquad (10')$$

It is easy to find from (10), (5), and (10′) that the sound-wave amplitude is equal to

$$u=\frac{\mathbf{eB}}{\rho_ms^2\alpha_{xz}k_{\rm ac}}\left[\frac{k_1(1-k_1^2/k^2)}{k_{\rm ac}^2-k_1^2}A_1+\frac{k_2(1-k_2^2/k^2)}{k_{\rm ac}^2-k_2^2}A_2\right],$$

or, substituting the values of A_1 and A_2 from (8) and (8′),

$$u=\frac{(\mathbf{eB})\,\omega H}{c\rho_ms^2\alpha_{xz}}\frac{(1-k_1^2/k^2)(1-k_2^2/k^2)}{(k_1^2-k_{\rm ac}^2)(k_2^2-k_{\rm ac}^2)}$$

$$\times\begin{cases} k_{\rm ac},\ \text{isothermal boundary}, & (11)\\ \dfrac{k_{\rm ac}^2+k_1k_2}{k_{\rm ac}},\ \text{adiabatic boundary}. & (12)\end{cases}$$

Equations (11) and (12) constitute the solution of the problem in general form. The equations can be simplified by using the smallness of the parameter a [see (6)]. Here

$$\left(1-\frac{k_1^2}{k^2}\right)\left(1-\frac{k_2^2}{k^2}\right)\approx a=\frac{T\alpha_{xz}^2}{\rho_{xx}\varkappa_{zz}}. \qquad (13)$$

Noting, furthermore, that the imaginary unity is present only in the expressions for the squared wave-vectors k^2 and k_T^2, we can recast the equations for $|u|$ in the form

$$|u|=\frac{(\mathbf{eB})HcT\alpha_{xz}}{4\pi\rho_ms^2C}(1+\beta_{\rm em}^2)^{-1/2}(1+\beta_T^2)^{-1/2}$$

$$\times\begin{cases}1, & \theta(0)=0,\\ |1-(\beta_{\rm em}\beta_T)^{-1/2}|, & \theta'(0)=0.\end{cases} \qquad (14)$$

Here

$$\beta_{\rm em}=\frac{\omega c^2\rho_{xx}}{4\pi s^2},\quad \beta_T=\frac{\omega\varkappa_{zz}}{s^2C}. \qquad (15)$$

To estimate $\beta_{\rm em}$ and β_T it is convenient to use their order-of-magnitude values

$$\beta_{\rm em}\sim\frac{\omega c^2}{\omega_0^2\tau s^2},\quad \beta_T\sim\omega\tau\left(\frac{v_F}{s}\right)^2.$$

We have used, just as throughout the article, the "traditional" expressions $\rho_{xx}\approx m/ne^2\tau$, $\varkappa_{zz}\sim Clv_F$, assuming that the heat is carried by the conductivity electrons. We point out that the parameters $\beta_{\rm em}$ and β_T behave differently as the temperature changes: when the temperature is lowered $\beta_{\rm em}$ decreases while β_T increases [the free-path time $\tau=\tau(T)$ depends on temperature].

To estimate the scale of the EMAT due to the thermoelectric effect, we compare the obtained expression (14) with $u_{\rm ST}$. The subscript "ST" indicates that we are dealing here with a transformation due to the Stewart–Tolman force:[2]

$$\mathbf{f}_{\rm ST}=\frac{im\omega}{e}\mathbf{j}. \qquad (16)$$

Substituting for $(\mathbf{e}\cdot\mathbf{B})\theta'/\rho_ms^2$ in the right-hand of Eq. (10)

$$\frac{im\omega He^{ikz}}{\rho_ms^2e(4\pi i\rho_{xx}/\omega)^{1/2}},\quad k=(4\pi i\omega/\rho_{xx}c^2)^{1/2},$$

we can readily show that

$$|u_{\rm ST}|\approx\frac{mcH}{4\pi\rho_Mse}\frac{1}{\beta_{\rm em}(\beta_{\rm em}^2+1)^{1/2}}.$$

Assuming that $\beta_{\rm em}$ and β_T do not differ excessively from unity, we get

$$\left|\frac{u}{u_{\rm ST}}\right|\approx\frac{(\mathbf{eB})T\alpha_{xz}e}{ms^2C}.$$

We shall assume in the estimates that the electron gas is degenerate ($\alpha_{xz}\sim T/e\varepsilon_F$); the heat capacity C and the thermal expansion coefficient β (which enters in B, see above) can vary in wide ranges, depending on the ratio of the temperature T to the Debye temperature T_D, but their ratio depends little on the temperature. If $B\sim Ms^2n\beta$, we have

$$\left|\frac{u}{u_{\rm ST}}\right|\sim\frac{M}{m}\frac{T}{\varepsilon_F}\frac{T\beta}{\tilde C}.$$

The dimensionless factor $T\beta/\tilde C$ where $\tilde C$ is the heat capacity per particle, is of the order of T/T_D if the Grüneisen relation holds (as is practically always in order of magnitude). Thus

$$\left|\frac{u}{u_{\rm ST}}\right|\sim\frac{M}{m}\frac{T}{\varepsilon_F}\frac{T}{T_D}. \qquad (17)$$

It is clear therefore that at not too low temperatures the thermoelectric transformation mechanism exceeds the Stewart–Tolman mechanism, which is the only macroscopic mechanism in the absence of a constant magnetic field. Another comparison method is to determine the constant magnetic field H_0 that ensures, on account of the Lorentz transformation mechanism, the same efficiency as the thermoelectric mechanism considered here. According to Ref. 1

$$|u_L| = \frac{HH_0}{4\pi\rho_m s\omega}(1+\beta_{em}{}^2)^{-1/2}.$$

From this and from (14) (assuming as before $\beta_{em}, \beta_T \sim 1$), we get $|u| \sim |u_D|$ at

$$\omega_c = \omega \frac{M}{m}\frac{T}{\varepsilon_F}\frac{T}{T_D}, \qquad \omega_c = eH_0/mc. \tag{18}$$

This is a reassuring estimate, although it must be remembered that the frequency ω of the electromagnetic field is bounded by the conditions $\omega\tau \ll 1$ and $|k_{1,2}|l \ll 1$, and when they are not met one cannot speak of temperature oscillations. Note that the estimates (17) and (18) must be approached with caution: they are approximate. On the other hand, Eqs. (11), (12), and (14) for the amplitude of an excited sound wave contain only macroscopic characteristics of the conductor and should therefore be regarded as exact.

Observation and identification of a new EMAT mechanism are apparently possible by using the nontrivial angular dependence, according to which the effect vanishes at $\varphi = 0$ (i.e., when the normal to the surface of the conductor coincides with the symmetry axis of the crystal), and also if the magnetic field in the electromagnetic wave incident on the crystal is polarized along the x axis (see the beginning of the article).

One more "strange" circumstance is noteworthy: at $\beta_{em}\beta_T = 1$, i.e., at

$$\omega = \frac{s^2}{c}\left(\frac{4\pi C}{\rho_{xx}\varkappa_{zz}}\right)^{1/2} \propto \frac{s^2}{v_F c}\omega_0, \qquad \omega_0{}^2 = \frac{4\pi n e^2}{m}$$

the transformation effect considered here vanishes in the case of an adiabatic boundary [$(\theta'(0) = 0)$; see the second equation of (14)]. A return to the exact expression (11) does not eliminate the vanishing of $|u|$, but only shifts somewhat the frequency at which this effect occurs.

The absence of temperature oscillations ($\theta \equiv 0$) at $\varphi = 0$ as well as in isotropic conductors is a consequence of the assumption that all the quantities depend only on one coordinate z. Under real conditions the x and y dimensions of the conductors are limited, all the field and current components depend not on the coordinate z alone, temperature oscillations set in, and all this should be manifest in the dependence of the electromagnetic characteristics of a sample on its thermal characteristics, particularly on the heat outflow conditions. When the thermoelectric coefficient is not too small, a distinction must be made between isothermal (ρ) and adiabatic (ρ_{ad}) resistivities

$$\rho_{ad} = \rho\left(1 + \frac{\alpha^2 T}{\rho\varkappa}\right).$$

We point out that $\rho_{ad} - \rho \propto \alpha^2$, and the effect predicted here is linear in α.

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[1)] The symmetry of the tensor α_{ik} is a consequence of the asumed relatively high symmetry of the crystal.

[2)] The smallness of a is a consequence of the dgeneracy of the conduction-electron gas.

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