Lifting of the Coulomb blockade of one-electron tunneling by quantum fluctuations

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(Submitted 24 May 1990)
Zh. Eksp. Teor. Fiz. **98**, 1834–1846 (November 1990)

An investigation is reported of the linear conductance of a system of two massive electrodes connected by point tunnel junctions to a small metallic granule. Quantum fluctuations of the granule charge lift the Coulomb blockade and give rise to a nonzero linear conductance G at any temperature T. A periodic dependence of G on the granule potential φ is determined at $T = 0$. Coherent transmission of electrons across the granule has the effect that at $T = 0$ the value of the conductance at the maxima of the $G(\varphi)$ dependence exceeds greatly its high-temperature value. Near the corresponding values of φ the amplitude of quantum fluctuations of the granule charge rises to a value of the order of the electron charge e.

1. INTRODUCTION

The first experiments[1] on tunnel junctions containing a layer of metallic granules in the barrier revealed nonlinearities of the current-voltage characteristics. The observed suppression of the current at low voltages was attributed in Ref. 1 to the fact that a tunneling electron alters the charge of a granule by an amount equal to the elementary charge e and increases its electrostatic energy by $E_a \sim e^2/C$ (where C is the granule capacitance). An electron can tunnel from a junction bank to a granule if its energy exceeds E_a, which can be ensured only by the application of sufficiently high voltages $V \sim e/C$. Recent years have seen the development of point-contact junctions in which the tunneling occurs mainly through one particular granule.[2-5] These experiments have revealed[5] a series of jumps in the current-voltage characteristic between which the granule charge remains constant and has the values e, $2e$, $3e$, ... The granule charge is then governed by the voltage applied to the junction. In the case of the more complex devices the granule charge can be altered independently by applying a voltage V_0 to an additional gate electrode,[6] as shown in Fig. 1. The linear conductance dependent on V_0 manifested in these experiments exhibits periodic oscillation maxima (see also Ref. 7). The origin of these maxima (and of the jumps in the current-voltage characteristic) is related to the degeneracy of the electrostatic energy of a granule

$$E(Q) = Q^2/2C + \varphi Q \qquad (1)$$

in respect of the magnitude of the charge. Equation (1) for the granule energy is derived in the Appendix and the parameter φ in Eq. (1) is proportional to V_0. We shall call φ (not quite correctly) the granule potential.[1)] The conditions for charged degeneracy $E((n-1)e) = E(ne)$ are satisfied if a potential assumes the values $\varphi = \varphi_n$, given by

$$\varphi_n = -(2n-1)e/2C. \qquad (2)$$

The existing experimental data can be explained using a semiclassical approach proposed in Refs. 8 and 9. In this approach the kinetic (transport) equation method is used to allow for tunnel transitions between real states at the banks and in the granule. The linear conductance of the junction calculated in this way[10] exhibits an activation-type temperature dependence in the limit $T \to 0$:

$$G(\varphi, T) \propto \frac{G_1 G_2}{G_1 + G_2} \exp\left(-\frac{E_a(\varphi)}{T}\right). \qquad (3)$$

Here, the activation energy $E_a(\varphi)$ is positive everywhere except at the points defined by Eq. (2) where it vanishes: $E_a(\varphi_n) = 0$; the conductances of the tunnel spacers separating the granule from the banks 1 and 2 (Fig. 1) are assumed to be always small: $G_1 + G_2 \ll e^2/\hbar$. The physical meaning of Eq. (3) is that, because of the law of conservation of energy, the real transitions between the granule and the banks involves only those electrons whose energy exceeds the energy needed to charge the granule E_a and the number of such electrons in the limit $T \to 0$ is exponentially small. Therefore, the semiclassical approach predicts a Coulomb blockade[11] of the tunneling process: at $T = 0$ the conductance $G(\varphi)$ vanishes for any value of φ, except at the charge degeneracy points described by Eq. (2).

We shall show that a self-consistent quantum-mechanical analysis of the problem of the tunneling across a granule shows that the Coulomb blockade [understood in the sense of vanishing of $G(\varphi)$ at $T \to 0$] does not appear. In fact, the law of conservation of energy does not forbid transitions between states close to the Fermi level in the banks involving virtual high-energy states in the granule. We shall show in

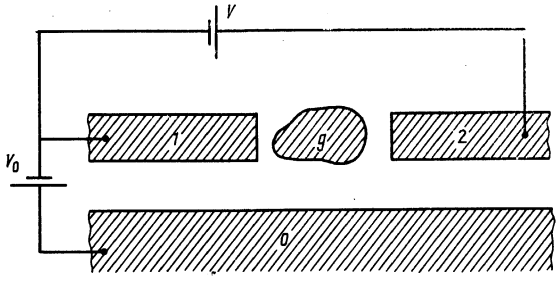

FIG. 1. Schematic representation of the investigated junction: g is a granule; *1* and *2* are massive electrodes tunnel-connected to the granule; *O* is the gate electrode; the present paper is concerned with the conductance between the electrodes *1* and *2* considered as a function of the gate voltage V_0 in the presence of a low longitudinal voltage $V \to 0$.

 0038-5646/90/111031-07$03.00

Sec. 2 that the conductance calculated in this way for $T=0$ differs from Eq. (3) by the replacement of $\exp(-E_a/T)$ with a small factor of the order of $(G_1+G_2)/(e^2/\hbar)$. Therefore, cooling reduces the conductance to $G\sim G_1G_2(e^2/\hbar)$, and not to zero. In this sense there is no complete Coulomb blockade.

The main part of the present paper is Sec. 3, where the conductance $G(\varphi)$ is calculated for the most interesting case of φ near the points of charge degeneracy given by Eq. (2). It is shown that if the value $|\varphi-\varphi_n|$ is sufficiently small, then at $T=0$ the conductance is higher than at $T\sim e^2/C$. Therefore, near the charge degeneracy points at a low temperature the Coulomb interaction does not block the tunneling, but conversely stimulates it. The average charge of a granule $Q(\varphi)$ is calculated in Sec. 4 in the simplest case of zero temperature and zero longitudinal voltage. The prediction obtained using the semiclassical approach that the charge should be discrete, $Q=ne$, is approximate. A correction $\delta Q\sim e(G_1+G_2)/(t^2/\hbar)$, due to quantum fluctuations, is small to the extent that the conductances of the spacers are low compared with the quantum unit $e^2/\hbar$. In the immediate vicinity of the charge degeneracy points defined by Eq. (2) the correction reaches the value $\delta Q\sim e$. The conditions for experimental observation of the predicted dependences are discussed in Sec. 5.

2. TUNNEL JUNCTION HAMILTONIAN. CALCULATION OF THE CONDUCTANCE USING PERTURBATION THEORY

We shall consider a system of two massive electrodes connected by tunnel spacers to a granule. We shall describe a junction of this type under the Coulomb blockade conditions by allowing, inside the Hamiltonian, for tunnel transitions between the granule and the banks of the junction and also for the electrostatic energy of a charged granule [Eq. (1)]. The simplest Hamiltonian satisfying these requirements is

$$H=H_0+H_T, \tag{4}$$

$$H_0=\sum_k \varepsilon_k A_k^+A_k+\sum_k \varepsilon_k B_k^+B_k+\sum_p \varepsilon_p c_p^+c_p+\frac{\hat{Q}^2}{2C}+\varphi\hat{Q}, \tag{5}$$

$$H_T=\sum_{kp} t_1(A_k^+c_p+c_p^+A_k)+\sum_{kp} t_2(B_k^+c_p+c_p^+B_k). \tag{6}$$

Here, A_k, B_k, and c_p are the electron annihilation operators at the left- and right-hand banks of the junction and at the granule, respectively; their energies ε_k and ε_p, measured from the Fermi level, fill uniformly a band $-\mu<\varepsilon_k,\varepsilon_p<\mu$ with the densities of the states are ν and ν_g, respectively;[2)] t_1 and t_2 are the matrix elements describing the tunnel transitions between the banks and the granule. The granule charge operator is of the form

$$\hat{Q}=e\sum_p [c_p^+c_p-\theta(-\varepsilon_p)]. \tag{7}$$

The second term in Eq. (7), containing the Heaviside function $\theta(x)$, is selected so that the ground state of the granule corresponds to the charge $Q=0$ when $\varphi=0$. It follows directly from the nature of the Hamiltonian described by Eqs. (4)–(6) that the dependence of the junction conductance $G(\varphi)$ on the potential φ is periodic. In fact, a shift of the potential $\varphi\to\varphi+e/C$ is equivalent to a redefinition of the charge operator $\hat{Q}\to\hat{Q}+e$, which in turn is equivalent to a shift of all the levels ε_p in the granule by an amount ν_g^{-1} representing the separation between the levels (so that the number of states under the Fermi level decreases by unity). In the case of a granule whose dimensions are greater than atomic the separation between the levels is small compared with the width of the energy band 2μ and compared with the characteristic Coulomb energy of the granule e^2/C, so that such a shift of the levels does not alter the conductance. Since the period of the dependence of G on the potential φ is e/C, we shall consider only the interval

$$-\frac{e}{2C}<\varphi<\frac{e}{2C}. \tag{8}$$

The conductance of each of the tunnel spacers separating the granule from the banks is proportional to the square of the corresponding matrix element of the Hamiltonian H_T:

$$G_{1,2}=G_q\nu\nu_g t_{1,2}^2. \tag{9}$$

As pointed out in the Introduction, the values of the conductances $G_{1,2}$ are assumed to be always small compared with the fundamental quantity $G_q=2\pi e^2/\hbar$. This circumstance makes it possible to consider the tunnel Hamiltonian H_T as a small perturbation when calculating the conductance $G(\varphi)$ of the whole junction. In this approach the linear conductance $G(\varphi)$ is governed by the probability of the scattering of a Fermi electron from the left bank to the right because of transitions described by Eq. (6) and it can be expressed in terms of a matrix element of the $\hat{T}$ operator

$$\hat{T}(z)=H_T+H_T\frac{1}{z-H_0}H_T+H_T\frac{1}{z-H_0}H_T\frac{1}{z-H_0}H_T+\ldots \tag{10}$$

using the standard relationship

$$G=G_q\sum_{kk'}|\langle\Phi|B_{k'}\hat{T}(z=0)A_k^+|\Phi\rangle|^2\delta(\varepsilon_k)\delta(\varepsilon_{k'}). \tag{11}$$

Here, $|\Phi\rangle$ is the ground state of the Hamiltonian H_0 where all the levels below the Fermi energy are filled.

As the first step, we shall substitute in Eq. (11) a matrix element of the $\hat{T}$ operator calculated in the second order of perturbation theory, because all the odd terms of the expansion vanish. It follows from Eq. (10) that

$$\langle\Phi|B_{k'}\hat{T}(z=0)A_k^+|\Phi\rangle\approx-\langle\Phi|B_{k'}H_T\frac{1}{H_0}H_TA_k^+|\Phi\rangle$$
$$=-\sum_p\frac{t_1t_2}{\varepsilon_p+(e^2/2C+e\varphi)}\theta(\varepsilon_p)$$
$$+\sum_p\frac{t_2t_1}{-\varepsilon_p+(e^2/2C-e\varphi)}\theta(-\varepsilon_p). \tag{12}$$

Two terms in Eq. (12) allow for the scattering processes in which virtual states have an excess electron or an excess hole in the granule, respectively. In view of the anticommutative nature of the Fermi operators A_k and B_k^+, these two terms have opposite signs, i.e., the contributions made to the scattering by the electron and hole channels partly compensate one another. Substituting in Eq. (11) an approximate expression for a matrix element from Eq. (12), calculated on the assumption that the width of the energy band is large ($\mu\gg e^2/C$), we find that

$$G(\varphi)\approx\frac{G_1G_2}{G_q}\ln^2\frac{e/2C+\varphi}{e/2C-\varphi}. \quad (13)$$

The relationship (13) describes correctly the dependence of the conductance on the potential φ in a large part of the interval defined by Eq. (8), but at the ends of this interval, where $\varphi=\pm e/2C$, the charge degeneracy of the electrostatic energy of Eq. (1) gives rise to a singularity in Eq. (13). This means that in calculation of $G(\varphi)$ near the points of charge degeneracy we have to allow for higher orders of perturbation theory in terms of the tunnel Hamiltonian of Eq. (6).

3. CALCULATION OF THE CONDUCTANCE IN THE MAIN LOGARITHMIC APPROXIMATION

In studies involving higher orders of a perturbation series it is convenient to transform the problem of the tunneling of an electron from the left to the right bank into the scattering problem. We shall do this by a unitary transformation of the Fermi operators describing the electrons in the banks:

$$a_k=(t_1A_k+t_2B_k)/t,\quad b_k=(t_2A_k-t_1B_k)/t,\quad t=(t_1^2+t_2^2)^{1/2}. \quad (14)$$

In terms of the new variables the Hamiltonian of Eqs. (4)–(6) becomes

$$H=\sum_k\varepsilon_ka_k^+a_k+\sum_k\varepsilon_kb_k^+b_k+\sum_p\varepsilon_pc_p^+c_p+\frac{\hat{Q}^2}{2C}+\varphi\hat{Q}+\sum t(a_k^+c_p+c_p^+a_k). \quad (15)$$

The purpose of the transformation described by Eq. (14) is to separate those electron states (described by the operators b_k and b_k^+) which, in accordance with Eq. (15), do not interact with the granule. Using the fact that a state of the $b_k^+|\Phi\rangle$ type is not scattered, we shall find the law describing the transformation of a matrix element of the $\hat{T}$ operator related to Eq. (14):

$$\langle\Phi|B_{k'}\bar{T}(z=0)A_k^+|\Phi\rangle=\frac{t_1t_2}{t^2}\langle\Phi|a_{k'}\bar{T}(z=0)a_k^+|\Phi\rangle.$$

The new notation of Eq. (11) for the conductance becomes

$$G=G_q\frac{G_1G_2}{(G_1+G_2)^2}\sum_{kk'}|T_{k\to k'}|^2\delta(\varepsilon_k)\delta(\varepsilon_{k'}), \quad (16)$$

$$T_{k\to k'}=\langle\Phi|a_{k'}\bar{T}(z=0)a_k^+|\Phi\rangle. \quad (17)$$

In this section we shall consider the case when φ is close to one of the charge degeneracy points. We shall consider the specific potential $\varphi=-e/2C+U$, and assume that $U\ll e/C$. It follows from the results of the preceding section that the matrix element described by Eq. (17), found in the second order of perturbation theory, contains a large logarithmic factor $T_{k\to k'}\sim\nu_gt^2\ln(e/CU)$. The appearance of this factor is due to the fact that in summation over intermediate states in Eq. (12) some of the electron contributions corresponding to energies $\varepsilon_p\lesssim e^2/C$ are not compensated by the hole contributions. A similar situation occurs also in the case of higher orders of perturbation theory. The maximum degree of the logarithm which can be obtained in nth order of perturbation theory using a factor of the t^n type in front of the terms is equal to the number of summations over intermediate states, i.e., it is $n-1$. Since it is not possible to sum the whole series for the matrix element $T_{k\to k'}$, we shall use the fact that $\ln(e/CU)$ is a large parameter and confine ourselves to the main logarithmic approximation. This means that we shall calculate the sum of only those terms of the expansion in which the smallness of the parameter t is compensated to the greatest degree by the large logarithm:

$$T_{k\to k'}\approx\alpha_1t+\alpha_2t^2\ln\frac{e}{CU}+\alpha_3t^3\ln^2\frac{e}{CU}+\ldots+\alpha_nt^n\ln^{n-1}\frac{e}{CU}+\ldots=tf\left(t\ln\frac{e}{CU}\right). \quad (18)$$

This allows us to simplify greatly the Hamiltonian used in the calculation of $T_{k\to k'}$. It follows from Eq. (12) that a large logarithmic factor appears as a result of summation over intermediate states of energies $E\lesssim e^2/C$. Therefore, we shall assume that the half-width of the energy band μ is e^2/C and exclude from our discussion the states with the granule charge different from $Q=0$ or $Q=e$.[3)] Therefore, in calculations carried out in the main logarithmic approximation the Hamiltonian of Eq. (15) can be replaced by

$$H'=\left(\sum_k\varepsilon_ka_k^+a_k+\sum_p\varepsilon_pc_p^+c_p\right)(\hat{P}_0+\hat{P}_1)+eU\hat{P}_1+\sum t(c_p^+a_k\hat{P}_0+a_k^+c_p\hat{P}_1). \quad (19)$$

Here, $\hat{P}_0$ and $\hat{P}_1$ are the projectors (on the subspaces of the eigenstates) of the granule charge operator $\hat{Q}$, which correspond to the eigenvalues $Q=0$ and $Q=e$; the energies ε_k and ε_p lie within the interval

$$-D_0<\varepsilon_k,\ \varepsilon_p<D_0,\quad D_0\equiv e^2/C. \quad (20)$$

We note that the problem of the scattering by a granule is similar to the Kondo problem of the scattering by a localized spin. In both cases the logarithmic singularity of the scattering amplitude observed at low energies is related to the absence of complete compensation of the electron channel by the hole channel and to the degeneracy of the ground state of the scatterer. The only difference is that in the Kondo problem the scatterer has two states, whereas the granule in Eq. (19) has a wide spectrum. Nevertheless, in calculation of the scattering amplitude we can use the renormalization group method, which is fully analogous to that used by Anderson[12] in an analysis of the Kondo problem.

We shall define the renormalization-group transformations using a class of Hamiltonians of the type

$$H=\tilde{H}_0+\tilde{V}, \quad (21)$$

$$\tilde{H}_0=\left(\sum_k\varepsilon_ka_k^+a_k+\sum_p\varepsilon_pc_p^+c_p\right)(\hat{P}_0+\hat{P}_1)+eU\hat{P}_1,$$

$$\tilde{V}=\sum_{kk'}a_{k'}^+a_k(v_{kk'}^{(0)}\hat{P}_0+v_{kk'}^{(1)}\hat{P}_1)+\sum_{pp'}c_{p'}^+c_p(v_{pp'}^{(0)}\hat{P}_0+v_{pp'}^{(1)}\hat{P}_1)+\sum_{kp}(v_{kp}c_p^+a_k\hat{P}_0+v_{pk}a_k^+c_p\hat{P}_1). \quad (22)$$

The energies ε_k and ε_p occurring in $\tilde{H}_0$ fill the interval of Eq. (20). It is also assumed that the transitions described by Eq. (22) occur only between the states with the energies from the interval

$$-D<\varepsilon_k,\ \varepsilon_p<D, \quad (23)$$

which can be narrower than the interval of Eq. (20): $D\leqslant D_0$. The initial Hamiltonian in the renormalization-group ap-

proximation should be the Hamiltonian of Eqs. (19) and (20), which gives the following initial values of the parameters:

$$D=D_0, \quad v_{kk'}^{(0)}=v_{kk'}^{(1)}=v_{pp'}^{(0)}=v_{pp'}^{(1)}=0, \quad v_{kp}=v_{pk}=t. \qquad (24)$$

Our aim is to calculate the matrix element $T_{k\to k'}$ of Eq. (17). We shall therefore introduce an infinitesimally small renormalization group transformation for the class of Hamiltonians of Eqs. (21)–(23) when the width of the band D decreases by δD and the parameters of the perturbation described by Eq. (22) are renormalized due to ignored transitions involving high-energy states in such a way that the amplitude $T_{k\to k'}$ is not affected in the main logarithmic approximation. We shall first determine an operator $\hat{M}$, which projects any state of the system (21)–(23) to the subspace of states which do not contain quasiparticle excitations with energies $|\varepsilon_k|$ and $|\varepsilon_p|$ in the interval from $D-\delta D$ to D. Obviously, if ε_k is close to zero, then $\hat{M}a_k^+|\Phi\rangle = a_k^+|\Phi\rangle$. We can therefore modify the definition of Eq. (17) by the replacement $\hat{T}\to\hat{T}'=\hat{M}\hat{T}\hat{M}$. We shall show that $\hat{T}'$ is also a $\hat{T}$ operator, but it applies to a different renormalized Hamiltonian. In fact, using the definition

$$\hat{T}(z)=\hat{V}+\hat{V}\frac{1}{z-\hat{H}_0}\hat{T}(z) \qquad (25)$$

and writing down the second term in the form of a sum of two operators, we find that

$$\hat{T}(z)=\hat{V}+\hat{V}\hat{M}\frac{1}{z-\hat{H}_0}\hat{T}(z)+\hat{V}(1-\hat{M})\frac{1}{z-\hat{H}_0}\hat{T}(z).$$

Transferring the last term to the left-hand side of the above equation and using the commutative nature of the operators $\hat{M}$ and $\hat{H}_0$ we obtain an equation of the (25) type for the operator $\hat{T}'$:

$$\hat{T}'(z)=\hat{V}'+\hat{V}'\frac{1}{z-\hat{H}_0}\hat{T}'(z).$$

The renormalized operator

$$\hat{V}'=\hat{M}\frac{1}{1-\hat{V}(1-\hat{M})\frac{1}{z-\hat{H}_0}}\hat{V}\hat{M} \qquad (26)$$

describes transitions between the states with energies $|\varepsilon_k|$, $|\varepsilon_p|<D-\delta D$. Unfortunately, it is not possible to allow rigorously for the renormalization of Eq. (26) of the Hamiltonian of Eq. (22). We shall therefore expand the exact equation (26) up to the second order in terms of a small perturbation $\hat{V}$:

$$\hat{V}'=\hat{M}\hat{V}\hat{M}+\hat{M}\hat{V}(1-\hat{M})\frac{1}{z-\hat{H}_0}\hat{V}\hat{M}. \qquad (27)$$

We shall show that this is equivalent to calculation of $T_{k\to k'}$ in the main logarithmic approximation. We can easily show that if we allow for the renormalization of Eq. (27), the operator $\hat{V}'$ remains within the class described by Eq. (22). The difference between $\hat{V}'$ and $\hat{V}$ is that transitions now occur between states in a narrower band $|\varepsilon_k|$, $|\varepsilon_p|<D-\delta D$, and the values of the parameters v change. For example, we can calculate the renormalization of the coefficient $v_{kk'}^{(0)}$. It follows from Eq. (27) that

$$\delta v_{kk'}^{(0)}=\sum_p\frac{v_{kp}v_{pk'}}{z-\varepsilon_p-eU}\theta(\varepsilon_p-(D-\delta D))\theta(D-\varepsilon_p)$$
$$+\sum_{k_1}\Big[\frac{v_{kk_1}^{(0)}v_{k_1k'}^{(0)}}{z-\varepsilon_{k_1}}\theta(\varepsilon_{k_1}-(D-\delta D))\theta(D-\varepsilon_{k_1})$$
$$-\frac{v_{kk_1}^{(0)}v_{k_1k}^{(0)}}{z+\varepsilon_{k_1}}\theta(-\varepsilon_{k_1}-(D-\delta D))\theta(D+\varepsilon_{k_1})\Big]. \qquad (28)$$

If $z=0$, the last term vanishes because of the mutual compensation of the electron and hole contributions mentioned above. The first term corresponding to the scattering via a granule in the electron channel is not compensated by the corresponding hole contribution because the latter is related to transitions to the states with $Q=-e$, which are forbidden in Eq. (22). Replacing the sum remaining in Eq. (28) with the integral, we find that

$$v_{kk'}^{(0)}\to v_{kk'}^{(0)}-\nu_g\frac{\delta D}{D+eU}v_{kp}v_{pk'}.$$

Assuming that the scale or gauge D is large, $D\gg eU$, we obtain the first renormalization group equation:

$$\frac{dv_{kk'}^{(0)}}{d\xi}=-\nu_g v_{kp}v_{pk'}, \qquad \xi=\ln\frac{D_0}{D}. \qquad (29)$$

Similarly, we can obtain equations for the remaining five coefficients in Eq. (22):

$$\frac{dv_{kk'}^{(1)}}{d\xi}=\nu_g v_{pk}v_{kp}, \quad \frac{dv_{pp'}^{(0)}}{d\xi}=\nu v_{kp}v_{pk}, \quad \frac{dv_{pp'}^{(1)}}{d\xi}=-\nu v_{pk}v_{kp},$$

$$\frac{dv_{kp}}{d\xi}=-\nu v_{kk'}^{(0)}v_{k'p}+\nu v_{k'p}v_{kk'}^{(1)}-\nu_g v_{kp'}v_{p'p}^{(1)}+\nu_g v_{p'p}^{(0)}v_{kp'},$$

$$\frac{dv_{pk}}{d\xi}=-\nu v_{pk'}v_{k'k}^{(0)}+\nu v_{k'k}^{(1)}v_{pk'}-\nu_g v_{pp'}^{(1)}v_{p'k}+\nu_g v_{p'k}^{(0)}v_{pp'}. \qquad (30)$$

Substitution of the variables

$$\lambda=-\nu v_{kk'}^{(0)}=\nu v_{kk'}^{(1)}=\nu_g v_{pp'}^{(0)}=-\nu_g v_{pp'}^{(1)},$$
$$\eta=(\nu\nu_g)^{1/2}v_{kp}=(\nu\nu_g)^{1/2}v_{pk} \qquad (31)$$

reduces the system of differential equations (29)–(30) with the initial conditions of Eq. (24) to a pair of dimensionless renormalization-group equations:

$$\frac{d\lambda}{d\xi}=\eta^2, \quad \frac{d\eta}{d\xi}=4\lambda\eta, \qquad (32)$$

where

$$\lambda(\xi=0)=0, \quad \eta(\xi=0)=(\nu\nu_g)^{1/2}t.$$

The solution of the system (30) is of the form

$$\lambda={}^1/_2(\nu\nu_g)^{1/2}t\,\mathrm{tg}\,[2(\nu\nu_g)^{1/2}t\xi],$$
$$\eta=\frac{(\nu\nu_g)^{1/2}t}{\cos[2(\nu\nu_g)^{1/2}t\xi]}. \qquad (33)$$

In observation of the renormalization-group equations the parameter eU/D is assumed to be small, so that the solution of Eq. (33) is meaningful only up to the scales $D\sim eU$. In calculation of the conductance this is sufficient provided $U\gg U^*$, where

$$eU^*=D_0\exp\Big\{-\frac{\pi}{4(\nu\nu_g)^{1/2}t}\Big\}\equiv\frac{e^2}{C}\exp\Big\{-\frac{\pi}{4}\Big(\frac{G_q}{G_1+G_2}\Big)^{1/2}\Big\}. \qquad (34)$$

In fact, in this case the estimate $\eta^2 \ll \lambda \ll 1$ applies and this means that we can calculate the matrix element of the $\hat{T}$ operator by limiting ourselves to the first order of perturbation theory: $T_{k\to k'} = v^{0}_{(kk')}$. This is due to the fact that the factor $\ln(D/eU)$ which appears in the higher orders is not a large parameter on the scale of $D \sim eU$. Therefore, it follows from Eqs. (31) and (33) that

$$T_{k\to k'} = -\frac{1}{2}\left(\frac{\nu_g}{\nu}\right)^{1/2} t \operatorname{tg}\left[2(\nu\nu_g)^{1/2} t \ln\frac{e}{CU}\right]. \qquad (35)$$

Substituting Eq. (35) into the expression for the conductance given by Eq. (16), we obtain

$$G(\varphi) = \frac{1}{4}\frac{G_1 G_2}{G_1+G_2}\operatorname{tg}^2\left\{2\left(\frac{G_1+G_2}{G_q}\right)^{1/2}\ln\frac{e/2C+\varphi}{e/2C-\varphi}\right\}. \qquad (36)$$

The dependence (36) is the main result of our investigation.[4)] The argument of the logarithm is written in such a form that the function $G(\varphi)$ describes the dependence of the potential throughout the range defined by Eq. (8) with the exception of narrow intervals $|\varphi \pm e/2C| \lesssim U^*$ near the ends of the range. If we allow for the periodicity of the conductance, $G(\varphi + e/C) = G(\varphi)$, we find that Eq. (36) covers all the values of φ with the exception of the vicinities of the charge degeneracy points of Eq. (2). On approach to these points the solution of Eq. (33) increases and at the point $U = U^*$ it becomes infinite. This singularity has no physical meaning, because it appears as a result of the approximation (27) for the renormalization group of Eq. (26). An allowance for the next terms in the expansion of Eq. (26) in powers of $\hat{V}$ would have given rise to terms of the third and higher orders of λ and η on the right-hand side of Eq. (32). Therefore, the range of validity of the main logarithmic approximation is limited by the condition $\lambda, \eta \ll 1$. At the limit of this range where $\lambda \sim 1$ and the conductance reaches a resonance value $G \sim G_q G_1 G_2/(G_1+G_2)^2$ (in particular, if $G_1 = G_2$, the conductance becomes $G \sim e^2/\hbar$).

We shall conclude this section by noting that our result [Eq. (35)] is of the form $T_{k\to k'} = tf[t\ln(e/CU)]$, which corresponds to the main logarithmic approximation of Eq. (18). The correction terms on the right-hand sides in Eq. (32) are polynomials of a degree $n \geqslant 3$, which are functions of λ and η and—according to Eq. (33)—are of the form $t^n f(t\xi)$. Their inclusion leads to corrections of the $t^{n-1} f(t\ln(e/CU))$ type to the matrix element $T_{k\to k'}$. The right-hand sides of the system (32) originate from the simplified form of Eq. (27) of the renormalization transformation of Eq. (26). Therefore, as pointed out already, the approximation (27) is equivalent to a calculation of $T_{k\to k'}$ in the main logarithmic approximation.

4. GRANULE CHARGE

In a semiclassical analysis the granule charge considered in the limit $T = 0$ can assume only discrete values:

$$Q = ne. \qquad (37)$$

In this limit we have $Q = 0$ for the values of φ in the interval of Eq. (8) and outside this interval the charge changes abruptly by $\pm e$. However, it is clear that because of the possibility of tunnel transitions of electrons between the granule and the banks, the value of Q exhibits quantum fluctuations. These fluctuations should grow on approach of the potential φ to the points defined by Eq. (2), where the classical value $Q(\varphi)$ exhibits jumps. In this section we shall calculate the granule charge $\bar{Q}(\varphi)$ in the limit of zero longitudinal voltage, when $\bar{Q}(\varphi)$ is governed by the average value of the operator $\hat{Q}$ of the ground state of the system. We shall begin with the Hamiltonian described by Eqs. (4)–(6) in the transformed modification of Eq. (15). It follows from Eq. (15) that $\hat{Q} = \partial H/\partial\varphi$, so that the average charge $\bar{Q}$ can be calculated as a derivative of the ground-state energy $E_0(\varphi)$:

$$\bar{Q}(\varphi) = \frac{\partial E_0}{\partial\varphi}. \qquad (38)$$

Equation (38) reproduces the result of Eq. (37) if we substitute the energy $E_0^{(0)}$ found ignoring tunnel transitions ($t = 0$). The quantum correction to Eq. (37) can be estimated by substituting in Eq. (38) the energy E_0 calculated in the second order of perturbation theory in terms of the small parameter t:

$$E_0 = E_0^{(0)} - t^2 \sum_{kp}\left(\frac{\theta(-\varepsilon_k)\theta(\varepsilon_p)}{\varepsilon_p - \varepsilon_k + e^2/2C + e\varphi} + \frac{\theta(\varepsilon_k)\theta(-\varepsilon_p)}{\varepsilon_k - \varepsilon_p + e^2/2C - e\varphi}\right). \qquad (39)$$

Differentiation of Eq. (39) with respect to φ gives

$$\bar{Q}^{(2)}(\varphi) = eg\ln\frac{e/2C-\varphi}{e/2C+\varphi}, \qquad (40)$$

where the dimensionless quantity $g = \nu\nu_g t^2 = (G_1+G_2)/G_q$ is a small parameter, $g \ll 1$. Over the large part of the interval of Eq. (8) the correction (40) to the classical value of the charge $Q = 0$ is small: $\bar{Q}^{(2)} \ll e$, but on approach to the charge degeneracy points $\varphi = \pm e/2C$ it diverges logarithmically. As in the case of Eq. (13) for the conductance $G(\varphi)$, it is desirable to refine Eq. (40) in the range of values of φ close to $\pm e/2C$ by calculating the charge $\bar{Q}(\varphi)$ in the main logarithmic approximation. We shall do this by applying again the renormalization group method described in Sec. 3. We shall consider only the interval of values of φ near the left-hand end of the range defined by Eq. (8), assume that $\varphi = -e/2C + U$, and calculate the charge $\bar{Q}(U)$ as a derivative of the ground-state energy $E_0(U)$ of the Hamiltonian of Eq. (19). We note the renormalization described by Eq. (27) of the Hamiltonian of the type given by Eqs. (21)–(23) narrows the space where the perturbation of Eq. (22) acts and, therefore, it makes some of the tunnel transitions forbidden. This alters the ground-state energy by $\delta E_0(U)$. The change $\delta E_0(U)$ can be estimated in the second order of perturbation theory:

$$\delta E_0(U) \approx \sum_{0<\varepsilon_p<D} + \sum_{\substack{-D<\varepsilon_k<0 \\ D-\delta D<\varepsilon_p<D}} \frac{v_{kp}v_{pk}}{-(|\varepsilon_k|+|\varepsilon_p|+eU)} + \text{const} \qquad (41)$$

(here, const denotes the sum of terms independent of U due to transitions without a change in the granule charge). We shall repeat the renormalization group procedure described in Sec. 3 by reducing gradually the truncation parameter D to the scale eU (as before, we shall assume that $U \gg U^*$). The ground-state energy of the Hamiltonian obtained in this way is $E_0^{(0)}$, apart from a small correction due to the residual processes of tunneling in the narrowed band of width $D \sim eU$. Therefore, the ground-state energy E_0 of the initial

Hamiltonian of Eq. (19) is described by the sum of all the corrections of the type given by Eq. (41), which appear as a result of consecutive stages of reduction in the width of the band $D_0 \to D_0 - \delta D$, $D_0 - \delta D \to D_0 - 2\delta D$, etc.:

$$E_0(U) = E_0^{(0)} + \sum \delta E_0(U).$$

Differentiation of this relationship with respect to U gives the required granule charge:

$$\overline{Q}(U) = \sum \frac{\partial}{\partial U} \delta E_0(U) = \sum e\nu\nu_g v_{kp} v_{pk} \frac{\delta D}{D}$$

$$= e \int_0^{\ln(D/eU)} \eta^2(\xi)\, d\xi. \qquad (42)$$

Comparing Eq. (42) with the first equations in the system (32), we find that

$$\overline{Q}(U) = e\lambda\left[\xi = \ln(D/eU)\right].$$

Therefore, the solution of the problem of calculation of the average charge $\overline{Q}$ in the main logarithmic approximation is given already by the function $\lambda(\xi)$ of Eq. (33). We shall write down the answer in the form convenient for comparison with the results of perturbation theory [Eq. (40)], which leads to

$$\overline{Q}(\varphi) = \frac{1}{2} e g^{1/2} \operatorname{tg}\left(2g^{1/2} \ln \frac{e/2C - \varphi}{e/2C + \varphi} \right). \qquad (43)$$

Like Eq. (36) for the conductance $G(\varphi)$, Eq. (43) is valid throughout the range (8) with the exception of exponentially narrow intervals near its ends where $|\varphi \pm e/2C| \lesssim U^*$. At the end of this region $\lambda[\xi = \ln(D/eU)]$ reaches $\lambda \sim 1$, i.e., $\overline{Q} \sim e$.

It therefore follows that the semiclassical theoretical prediction of the existence of a Coulomb blockade which forbids tunneling of electrons to a granule at $T = 0$ is only approximate. Quantum theory lifts this forbiddenness. Quantum fluctuations of the charge, which are small far from the points of degeneracy of Eq. (2), grow and reach $\delta Q \sim e$ on approach to these points.

5. DISCUSSION OF RESULTS

The characteristic temperature T_0 at which the Coulomb effects begin to manifest themselves is governed by the charge energy of the investigated granule, which is $T_0 = e^2/2C$. At high temperatures $T > T_0$ the resistances of two series-connected tunnel junctions are additive and the conductance is $G = G_1 G_2/(G_1 + G_2)$. Lowering of the temperature of the system to $T \lesssim T_0$ gives rise to a periodic dependence of the conductance on the granule potential φ and the period is e/C. At such temperatures the behavior of the conductance is described by a semiclassical theory based on the kinetic (transport) equation method.[10] According to this theory, cooling reduces the conductance at the maximum of the dependence $G(\varphi)$ to $G = 1/2 G_1 G_2/(G_1 + G_2)$, and for all the other values of φ the fall is exponential [see Eq. (3)]. However, as shown in Sec. 2, even at $T = 0$ there is a nonzero residual conductivity described by Eq. (13) and due to quantum transitions of electrons via virtual states of the granule. A comparison of Eqs. (3) and (13) shows that the exponentially observed fall of the conductance stops at temperatures

$$T \sim T_1 = \frac{T_0}{\ln[G_q/(G_1+G_2)]}.$$

Therefore, the semiclassical theory is valid if $T > T_1$. At temperatures $T \sim T_1$ the detailed nature of the dependence $G(T)$ is unknown and its analysis would require allowance for quantum inelastic tunneling processes, which do not occur at $T = 0$. (The contribution of such processes to the dependence of the conductance on the longitudinal voltage was investigated in Ref. 13.) The dependence $G(\varphi)$ found in Sec. 3 [Eq. (36)] shows that near the charge degeneracy points of Eq. (2) the transmission of electrons by the granule exhibits resonances and at these resonances the conductance is

$$G \sim \frac{e^2}{\hbar} \frac{G_1 G_2}{(G_1+G_2)^2}. \qquad (44)$$

The resonance value of Eq. (44) exceeds the conductance in the high-temperature limit $T > T_0$ because of the large parameter $e^2/\hbar(G_1 + G_2)$. This demonstrates an anomalous increase in the conductance at maxima of the dependence $G(\varphi)$ when temperature is lowered.

We considered the properties of a junction with a small granule but nevertheless we ignored the discrete nature of the electron excitations in the granule. This is justified if the typical separation ν_g^{-1} between the size-quantization levels is considerably less than the energy scale eU^* introduced in Sec. 3 [see Eq. (34)]. In the opposite limiting case when $\nu_g^{-1} \gg eU^*$ our results are valid only far from the charge degeneracy points, when $e|\varphi \pm e/2C| \gg \nu_g^{-1}$. Near these points, we have $e|\varphi \pm e/2C| \lesssim \nu_g^{-1}$, and in the main logarithmic approximation we have to replace the argument of the logarithm in Eqs. (36) and (43) with $e/C\nu_g^{-1}$. Moreover, if $eU^* < \nu_g^{-1}$, the discrete nature of the granule levels applies to a mesoscopic fine structure of Ref. 14 at each maximum of the dependence $G(\varphi)$ (Ref. 14): a narrow resonance peak appears and the width and amplitude of the peak may depend on the number of the maximum.

The tunnel matrix elements t_1 and t_2 used in the simplest model Hamiltonian of Eq. (4)–(6) are independent of the momenta k and p. This imposes certain limitations on the geometry of the system: the tunnel junctions connecting the granule to the massive electrodes should be point-like and the distance between them should not exceed one electron wavelength. Fluctuations of the charge discussed in Sec. 4 can be observed if the granule is connected to just one massive electrode. This simplifies the requirements in respect of the geometry of the system.

The authors are grateful to D. E. Khmel'nitskiĭ for valuable discussions.

APPENDIX

In this Appendix we shall derive Eq. (1) for the electrostatic energy of a granule and find the meaning of the parameter φ (see also Ref. 9).

The electrostatic energy of a system of charged conductors is

$$E = \frac{1}{2} \sum_{ij} C_{ij}^{-1} Q_i Q_j. \qquad (A1)$$

In the case of the system shown in Fig. 1, the independent variables are the granule charge Q and the potentials of the massive electrodes $\varphi_i = \partial E/\partial Q_i$, where $i = 0$, 1, or 2. Transforming conventionally Eq. (A1) to these variables, we obtained the following quadratic form for the energy:

$$E = \frac{Q^2}{2C} + \sum_{i=0}^{2} \alpha_i \varphi_i Q + \frac{1}{2} \sum_{i=0}^{2} C_i \varphi_i^2. \tag{A2}$$

In the cases $\varphi_1 = \varphi_2 = 0$ and $\varphi_0 = V_0$ which are of interest to us, it follows from Eq. (A2) that

$$E = \frac{Q^2}{2C} + \alpha_0 V_0 Q + \frac{C_0}{2} V_0^2. \tag{A3}$$

The part of the energy of Eq. (A3) dependent on the granule charge Q is identical with that given by Eq. (1) provided we assume that $\varphi = \alpha_0 V_0$. The dimensionless coefficient $\alpha_0 \lesssim 1$ is governed primarily by the distance between the granule and the massive electrodes.

[1)] The true potential of the granule is $\varphi + Q/C$.
[2)] For simplicity, we shall assume that the left- and right-hand banks of the junction are identical.
[3)] It follows from Eq. (2) that the energy of such states exceeds e^2/C.
[4)] In the initial Hamiltonian of Eqs. (4)–(6) we ignored the fact that electrons have spin degrees of freedom. Inclusion of these degrees of freedom would have doubled the values of the conductance given by Eq. (36) and of the charge given by Eq. (43) [because the densities of states ν and ν_g in Eq. (9) are given for just one direction of the spin].

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Translated by A. Tybulewicz