Classification of gasdynamical discontinuities and wave processes involved in the establishment of thermodynamic equilibrium

N. M. Kuznetsov

N. N. Semonov Institute of Chemical Physics, Academy of Sciences of the USSR, Moscow
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A classification is given of the gasdynamical discontinuities corresponding to a transition from a nonequilibrium state 1 to thermodynamic equilibrium under the condition that state 1 on the pressure-specific volume diagram is situated above the equilibrium adiabat, in contrast with what is called the detonation adiabat. The results are applied to the problem of ionization equilibrium wave propagation behind a shock wave compression discontinuity in a radiating gas, and to the question of possible shock-wave structural instability when radiation transfer from the equilibrium zone strongly enhances ionizational relaxation.

A substance with complicated physico-chemical properties (for example, one capable of changing chemical, ionic or phase composition) is found in a nonequilibrium state behind a shock wave compression discontinuity. The transition to the thermodynamic equilibrium state is realized, as is known, in the shock wave relaxation zone, which is broader than the compression discontinuity. Typically, this nonequilibrium state is not metastable: the "shaking" at the shock wave front is so strong that the subsequent complete relaxation is caused by the phenomena in the compression discontinuity, and the time at which the compression discontinuity passes across a mass element of the substance is the initial reference point for the relaxation. The transfer processes (transport phenomena) behind the compression discontinuity are unimportant in many cases. The problem of the relaxation and structure of the shock wave, mathematically speaking, is a Cauchy problem in this case. However, there are also cases of relaxation when the transfer processes play the principal role in forming the wave structure and, in particular, give rise to a strong contraction of the relaxation zone. As an example, one can take the qualitatively different role of radiation coming from the equilibrium zone behind the shock wave front, in forming an ionization structure[1] depending on the shock wave intensity.[1)] In optically active gases at low density, the width of the vibrational and chemical relaxation zone substantially depends on the equilibrium zone infrared-radiation.[3]

Calculations of the structure taking into account transport phenomena, involve, in the general case, solving a boundary value problem. The difference between the Cauchy and the boundary value problems are caused not only by different mathematical solution methods, but, what is far more important, the different physical nature of establishing the stationary structure in shock wave formation, for example, under the action of a piston. So if the source of any physical variable (energy, particles of a specific kind in a reacting gas mixture) transported then in the substance, is located in the equilibrium zone behind the shock wave front, then the establishment of stationary structure can depend strongly on how the shock wave forms. If the shock wave is formed by a piston, then it is very important whether the piston itself is the source of the physical variable under consideration, for example, the radiation. If the piston does not have such a property, then thermodynamic equilibrium can be established initially very far from the shock wave front in comparison with the stationary width of the relaxation zone. After accumulating a thermodynamically equilibrium layer of sufficient width in a neighborhood of the piston (in the case of radiation, we have in mind a layer of sufficient optical width: of the order of the photon mean free path at those frequencies at which electron excitation and ionization of molecules occur) and the inclusion thereby of supplementary relaxation mechanisms, the leading edge of the layer, advancing ahead of it with the shock wavefront velocity, starts to approach the front.

One can consider this stage of establishing the shock wave stationary structure as an independent wave process—the wave propagation transforming the substance from the nonequilibrium state which formed in the compression discontinuity, to the equilibrium state. For brevity, let us call it an equilibration wave. It can be subsonic in relation to the flow ahead of it, similar to a combustion wave, or supersonic, similar to a detonation wave. However, unlike the combustion or detonation cases, the state in front of the wave is not necessarily located under the adiabat of the equilibrium states on the P, V (pressure, specific volume) diagram. Let us agree to call the adiabat of the equilibrium states the equilibrium adiabat. It is determined by the same equation as the detonation adiabat, but its parameters (the sign of the "heat release," the equation of state) change over a range such that the equilibrium adiabat can be situated both above the initial point P_1, V_1 and below it.[4] The detonation adiabat is a particular case of an equilibrium adiabat.

Thus, the process of establishing the stationary structure of a shock wave can include an equilibrium wave propagation stage. Moreover, as will be shown later, the disposition of the point P_1, V_1 above the equilibrium adiabat is typical.

Depending upon the intensity of the initial shock wave $S1$ and other parameters, the properties of the equilibrium wave (subsonic or supersonic velocity, the pressure jump sign, etc.) and its interaction with $S1$ can be qualitatively distinct. Therefore, it is useful to conduct a qualitative analysis of "nondetonation" equilibrium waves, that is with P_1, V_1 situated above the equilibrium adiabat.

A qualitative analysis of waves of such a type is also of

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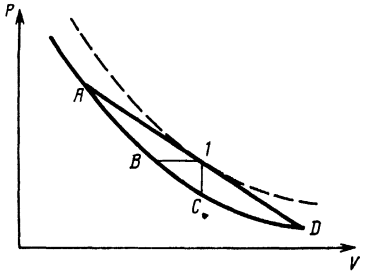

FIG. 1. Equilibrium adiabat (solid curve) and the isentrope of the metastable states (dashed line).

interest in connection with wave (front) transformations of metastable states to stable ones and in connection with general questions of classifying gasdynamic discontinuities.

Further, in Sec. 1, a classification is given of "nondetonation" waves (discontinuities) (see Fig. 1). This disposition of the equilibrium adiabat with respect to point 1 is possible for phase transitions, polymerization[4,5] and for the method of "preparation" of the initial state noted above in the shock wave compression discontinuity. In the last case, as has already been noted, just such an equilibrium adiabat disposition is typical, at least, in gases. Actually, the relaxation of the postulated "superheated" motions of a molecule in the compression discontinuity, is accompanied in an isochoric process by the lowering of the temperature T of the subsystem of the translational degrees of freedom in the gas. For a constant molecular weight μ this leads to a lowering of the pressure as shown in Fig. 1. The decrease of μ caused by dissociation or ionization for $D/kT \gg 1$ or $I/kT \gg 1$, where D and I are, respectively, the dissociation energy and the ionization potential of the molecule, is relatively small:

$$\frac{|\Delta\mu|}{\mu} \ll \frac{|\Delta T|}{T}$$

(let us recall that the actual dissociation and ionization of a gas of a normal or lower density occur, respectively, for $D/kT \gtrsim 10$ and $I/kT \gtrsim 10$). Consequently, in the conditions under consideration, the pressure in the isochoric process also decreases for $\mu \neq \text{const}$.

In Sec. 2, the qualitative nature is considered of the interaction of an equilibrium wave with the flow in front of it and behind it, as a function of its velocity, and the results are applied to the propagation of an ionization equilibrium wave behind a shock-wave compression discontinuity in a radiating gas, and to the question of possible shock-wave structural instability when the radiation transport from the equilibrium zone strongly enhances the ionization relaxation.

In Sec. 3, an estimate is given of the ionization equilibrium wave velocity in xenon at the temperature of the translational motion of the atoms of $3 \cdot 10^4$ K (without excitation of the electron levels and ionization) in front of it, and it is shown that the wave front, in this case, is a supersonic rarefaction discontinuity.

1. CLASSIFICATION OF "NONDETONATION" DISCONTINUITIES

Let us consider the equilibrium adiabat mentioned above. Our classification will be based on distinguishing portions of the adiabat, differing in how the discontinuity moves relative to the incoming and outgoing fluxes, that is, differing in the direction of the inequalities between the Mach numbers M_1, M_2 and unity.

A classification of the hydrodynamic discontinuities when the equilibrium adiabat is "below" point 1 (Fig. 1) has been previously given in connection with phase transformation[4] or polymerization.[5] However, the analysis of the nature of the discontinuities was very brief and incomplete. In particular, the question of the stability (evolution) of the jumps was almost not considered. Therefore, it is necessary to turn to classifying the jumps once more.

In Fig. 1, the isentrope of the metastable state is given by the dashed line or, in the case of a "substance" prepared by compression in the shock wave discontinuity—the isentrope for the "switched-off" slow relaxation processes behind the compression jump. The derivative $-V^2(\partial P/\partial V)$ along such an "isentrope" equals the square of the velocity of sound at frequencies ω large in comparison with the characteristic inverse time τ for establishing equilibrium. Let us emphasize that τ pertains to the spontaneous bulk process of establishing equilibrium in spatially uniform conditions. The tangent to the "isentrope" at point 1 along with the isochore $1C$ and the isobar $1B$ divide the equilibrium adiabat into five parts, each of which determines specific properties of the corresponding discontinuity peculiar to itself.

The adiabat segment above point A corresponds to a compression wave supersonic with respect to the substance ahead of the front and subsonic with respect to the flow behind the front. This is the analogue of an ordinary shock wave. A self-sustaining method of propagating it without a pushing piston, or an infinite domain of constant flow behind the front, is impossible. For the existence of a wave, it is still required that it significantly accelerate the relaxation process, at least by an order of magnitude.

The segment AB corresponds to a compression wave subsonic on both sides of the discontinuity (Fig. 1). The number of parameters in terms of which one can express an arbitrary small one-dimensional perturbation of such a wave, equals four—the magnitude of the jump, acoustic perturbations propagating from the discontinuity into the regions in front of it and behind it, and an entropic perturbation. This is more than the number of boundary conditions (three conservation laws). If the transition to equilibrium is induced by a compression jump, the magnitude of which is given by the boundary conditions behind the jump independent of the initial state P_1, V_1 and of the transition kinetics, then such a wave is unstable (nonevolutionary).[6] Its front "smears out," generating practically undamped[2)] perturbations—compression waves of smaller amplitude ahead.

But if the transition to equilibrium is caused not by a gasdynamic discontinuity, but for some other reason, then besides the three boundary conditions (conservation laws), there is one more condition determining the equilibrium wave velocity. This velocity can, for example, be determined by the physical relaxation processes—the transport phenomena in combination with the kinetics of the chemical reactions, ionization, phase transformations, etc., or the time of origin of the initial state P_1, V_1 with the simultaneous inclusion of local relaxation mechanisms. In this case, the number of parameters equals the number of boundary conditions. But for an independently given state P_1, V_1 ahead of

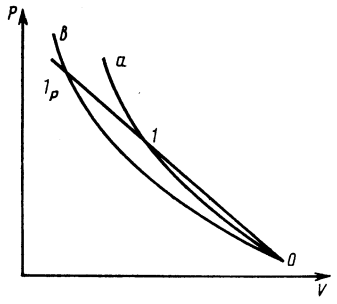

FIG. 2. Shock adiabats: a) of the metastable states, b) the equilibrium adiabat; $1 - 1_p$ is a subsonic compression shock.

the wave, in order that the equations have a solution and that the regime under consideration exist in practice, it is necessary to satisfy specific conditions behind the jump, for example, by introducing a piston moving with a velocity which is connected in a one-to-one manner with the state $P_1 V_1$. This one-to-one correspondence with $P_1 V_1$ is satisfied by the well-known transition from a nonequilibrium state to the equilibrium one $1 \to 1_p$ in the stationary relaxation zone of the initial shock wave $S1$ (see Fig. 2). One can interpret this transition as a subsonic (compression) equilibrium wave.

The segment of the adiabat BC does not correspond to any real waves (the values of the flow J are imaginary).

The segment CD of the adiabat corresponds to a rarefaction shock, supersonic with respect to the flow on both sides of it. The number of free parameters equals four: the size of the jump and three perturbations stemming from it downstream—two acoustic and one entropic. The number of boundary conditions also equals four—three conservation laws and an equation determining the spontaneous propagation velocity of an equilibrium wave. As in the case of AB, for a given initial state the problem may not have any solution at all. (A solution does not exist for all initial states $P_1 V_1$.)

The segments AB and CD have analogs on a typical detonation adiabat corresponding, respectively, to slow and fast (supersonic) combustion.[6,8] (An analogy in the number of parameters and boundary conditions, but not in the direction of change of the density and other variables in the jump.)

The last adiabat segment below D, corresponds to a discontinuity subsonic with respect to the flow in front of it and supersonic with respect to the stream behind it. It does not satisfy the necessary stability condition in the relation between the numbers of parameters (five) and boundary conditions (four). Such a jump would quickly "erode" as a consequence of the isentropic rarefaction waves escaping ahead.

2. A QUALITATIVE ANALYSIS OF THE WAVE PROCESS OF TRANSITION TO THE EQUILIBRIUM STATE

Let us now consider the qualitative picture of the development of the wave relaxation process of a monatomic (inert) gas converted to a nonequilibrium state by an initial shock wave (we have in mind that state which arises as a result of an impulsive compression of the gas without exciting the electron subsystem).

The spontaneous transition to equilibrium in any part of an originally homogeneous substance (point 1) and constant flow (rest) when point 1 is situated over the equilibrium adiabat (Fig. 1) gives rise to a lowering of the pressure in this part, and a rarefaction wave with sound velocity c_1 goes from it to the nonequilibrium region of the substance. If equilibrium is established not "three-dimensionally," but by shifting the boundary dividing the nonequilibrium and equilibrium states (with some velocity v_p depending on the state of the substance at point 1), the subsequent development of the process will depend critically on the magnitude of v_p/c_1 (the Mach number M_1) in comparison with unity.

For the case $M_1 < 1$ (subsonic wave), we have the adiabat segment to the right of D. As was noted in the previous section, the rarefaction shock corresponding to this adiabat segment is unstable. The rarefaction waves generated by it change the position of point 1 and decrease the pressure difference ΔP on the boundary between the equilibrium and nonequilibrium states. When point 1 is "prepared" in a compression discontinuity, the resulting rarefaction waves reflecting from the compression discontinuity are transformed into compression waves of smaller amplitude.[3)]

Without specific calculations, it is hard to see the details of the electron-ion relaxation process behind the initial shock wave front. But a decrease of the pressure jump ΔP and the amplitude of the perturbations reflected by the shock wave front, speak in favor of the process being convergent: a weakening of the "smeared-out" rarefaction shock causes it to disappear completely and causes the stationary ionization structure of the shock wave to form, characterized, as is known, not by a rarefaction but by a compression of the substance in it. For a given initial state P_1, V_1 (point 1) and the asymptote corresponding to it—the equilibrium flow behind the shock front, there exists a unique point on AB corresponding to a stationary discontinuity. In other words, a stationary shock structure corresponds to a subsonic "compression jump" (a specific point on AB), connecting the two extreme points of the structure, the initial point 1 and the equilibrium state. This is the unique evolutionary subsonic compression shock on the entire interval AB of the adiabat, the rest corresponding to nonevolutionary jumps (see Sec. 1).

The relaxation process establishing ionization equilibrium develops qualitatively differently in the case $M_1 > 1$: the supersonic rarefaction shock corresponding to CD. The propagation velocity of such an equilibrium wave, as was noted in Sec. 1, is not arbitrary and is uniquely determined by the state in front of the wave (point 1) and the front relaxation kinetics.

In the coordinate system of the rarefaction discontinuity, the velocity of the substance increases in passing across the discontinuity. If state 1 is "prepared" by a shock wave (by a sustained piston), then, in this case, a difference of velocities arises between the substance directly behind the rarefaction discontinuity and the piston, the velocity of which equals u_1. The substance accumulates on the piston with a velocity $\Delta u = u_1 (V_2/V_1 - 1)$. This causes another shock wave to appear, which advances in front of the piston in the equilibrium substance (see Fig. 3).

The wave $S2$ falls behind the supersonic rarefaction wave. This is apparent from the following estimates. The velocities of these waves (v_p and v_2) relative to the material between them are connected by the relation

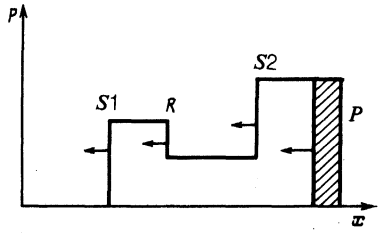

FIG. 3. Configuration of the waves: $S1$ is the initial shock wave, R is the supersonic equilibrium wave (rarefaction shock), $S2$ is the second shock wave, P is the piston.

$$\frac{v_p}{v_2}=\frac{1-V_3/V_2}{1-V_1/V_2}.$$

For a weak rarefaction shock (a neighborhood of the point V_1 of the adiabat) we have $v_p \gg c_1$, $v_2 \approx c_2$, where $c_2^2 = -V^2(\partial P/\partial V)_s$ is the velocity of sound of the gas 2 which is in thermodynamic equilibrium, where $c_2 < c_1$. In the case of a strong rarefaction shock—far from point C of the adiabat, we have $V_1/V_2 \approx 0.5$. Moreover, the wave $S2$ is strong and, consequently, $V_3/V_2 \ll 1$ holds.

The strength of $S2$ after the reflection from it of the rarefaction shock, behind which there is an extended region in which the motion is slowed by Δu, is greatly reduced. In such a weakened wave, the ionization rates drop sharply and again an extended region of the nonequilibrium substance arises. The subsequent enhancement of the transition to equilibrium occurs when $S2$ approaches the trailing edge of this domain. Depending on the strength of $S2$, the subsequent motion of the equilibrium boundary occurs in any of the wave processes considered in Sec. 1. It is conceivable that soon after this when $S2$ (or the waves formed during its collapse at the entrance to the nonequilibrium domain) overtakes $S1$, the process of establishing an equilibrium structure terminates. But on the whole, it proves to be highly drawn-out, especially as a consequence of the very slow initial accumulation of electrons and, correspondingly, large distance between the $S1$ front and the piston when equilibrium radiation appears near it. The time required to establish the equilibrium structure can prove to be at least as long as the characteristic time of the experiment in the shock tube. Under such conditions, the gas behind the shock wave front can be practically nonradiating (see Ref. 10).

This qualitative analysis of wave interaction suggests that one-dimensional motion in the wave relaxation zone may be unstable. So, if the "cloud" of equilibrium luminous gas forming initially is localized in a bounded part of a longitudinal tube section, then the equilibrium wave front (rarefaction shock) will not be planar, but expanding, for example, spherically. The collision of the oppositely directed fluxes of the substance behind such a wave generates a compression wave of the same nature as the wave $S2$ considered above, but far stronger and not one-dimensional. It is possible that the rarefaction and compression waves arising in the subsequent development of such a gasdynamic process, will not be damped, and the electron-ionic relaxation will occur in a shock wave "turbulated" structure. Judging by the nature of the emission from the gas, such structures exist in quite strong shock waves. The estimates given in the next section show that supersonic equilibrium waves (rarefaction shocks) can exist at about such intensities.

3. IONIZATION EQUILIBRIUM WAVE IN AN INERT GAS

As an example of a wave process establishing thermodynamic equilibrium, consider an ionization wave propagating in a monatomic (inert) gas, preliminarily compressed into a strong shock wave and thereby converted to nonequilibrium state 1 (on the nature of the departure of the states from equilibrium, see Sec. 2). We are interested in an estimate of the temperature T_1 of the atoms of the gas in state 1 (and the temperature T_p connected with it behind the equilibrium wave front), for which the equilibrium wave is a supersonic rarefaction shock that corresponds to a transition from point 1 to some point on the segment CD of the equilibrium adiabat (Fig. 1).

For any nonequilibrium state P_1V_1, an equilibrium adiabat of the type depicted in Fig. 1 can be constructed. To each point on the segments AB, CD of the adiabat corresponds the propagation velocity of the discontinuity:

$$v_2=V_1[(P_2-P_1)/(V_1-V_2)]^{1/2}.$$

A necessary condition for the existence of an equilibrium wave is

$$v_2=v_p,$$

where v_p is the equilibrium wave propagation velocity caused by the front relaxation processes. In the case of ionization relaxation in an inert gas, the basic processes determining v_p are the excitation and ionization by radiation from the equilibrium zone with subsequent cascade ionization by the electrons.

The velocity v_p was evaluated previously,[1] but, in this case, the only photoprocess, considered was ionization

$$A+\hbar\omega \rightarrow A^+ + e. \quad (1)$$

The photoexcitation process

$$A+\hbar\omega \rightarrow A^* \quad (2)$$

with subsequent photoionization was not considered as a consequence of the narrowness of the effective band of frequencies in which the process (2) makes a significant contribution to the ionization of the gas behind the shock wave front when the absorption line has a Lorentzian profile. However, at high temperatures ($T_1 \gtrsim 1.5$ eV) the effective frequency band grows strongly as a consequence of the dimer absorption mechanism[11]

$$A+A+\hbar\omega \rightarrow A^*+A \quad (3)$$

and amounts to several eV. The lines corresponding to transitions to different excited states, moreover, overlap. As a result, the initial ionization comes chiefly not by the direct route (1), but in a sequence of processes (3) and

$$A^*+\hbar\omega \rightarrow A^+ + e, \quad (4)$$

$$A^*+A \rightarrow A^+ + e + A. \quad (5)$$

[The atomic collisional ionization (5) originates from those excited states which have an energetically accessible point where terms intersect with the states of ionization prod-

ucts.] As a result, the energetic boundary E of the radiation spectrum from the equilibrium zone contributing to the ionization kinetics drops out of the ionization potential around 1 eV. Under such conditions, at a radiation temperature $T_p > 1$ eV, almost all the photons participate in the excitation processes and the subsequent ionization (3), (4). (So, for example, more than 85% of all the radiation equilibrium photons with temperature $T_p = 1$ eV have an energy larger than 1 eV.) The temperature in the equilibrium wave $T_p \gtrsim (1\text{–}1.5)$ eV corresponds to a significantly higher atomic temperature ($T_1 \sim 3\cdot 10^4$ K) in front of the wave, that is behind the compression jump of the shock wave S 1. At such a temperature, the binary absorption coefficient $\varkappa$ depends weakly on the frequency and near the maximum of the Planck distribution amounts to[11]

$$\varkappa = \varkappa_0 \delta^2, \quad \varkappa_0 \sim (1\text{–}0{,}1)\ \text{cm}^{-1}, \tag{6}$$

where δ is the relative gas density, equal to unity in normal conditions. The absorption coefficient (6) satisfies[11] $\varkappa L > 1$, where L is the distance from the front of the shock wave S 1 to the point of origin of the equilibrium emitting region of the gas. When this inequality is satisfied and the frequency dependence of $\varkappa$ is weak in comparison with the Planck function, the equilibrium wave propagation velocity can be computed by the following simplified version of the methodology.[1] The initial ionization originates basically due to the photoprocesses (3) and (4). After the relative electron density grows to some magnitude $\alpha = \alpha^*$, the ionization basically follows the cascade mechanism of Petschek and Byron[12] through excitation of the atoms by an electron shock and heating of the electrons by elastic collisions with the ions. Hence

$$\frac{dN_e}{dt} \approx \frac{J^* \varkappa}{2}, \quad \alpha < \alpha^*, \tag{7}$$

$$\frac{dN_e}{dt} \approx \frac{N^2 A}{m\varepsilon} \alpha^2 \ln \frac{\alpha_p}{\alpha}, \quad \alpha > \alpha^* \tag{8}$$

$$J^* = J_p \exp(-\varkappa L^*), \quad J_p = \frac{(kT_p)^3}{4\pi^2\hbar^3 c^2} \int_{E/kT_p}^{I/kT_p} \frac{x^2\,dx}{\exp x - 1},$$

$$\frac{(kT_p)^3}{4\pi^2\hbar^3 c^2} = 1{,}53\cdot 10^{11} T_p^3\ [\text{cm}^{-1}\cdot\text{sec}^{-1}] \quad (T_p \text{ in K}),$$

$$\int_{E/kT_p}^{I/kT_p} \frac{x^2\,dx}{\exp x - 1} \approx \int_0^{\infty} \frac{x^2\,dx}{\exp x - 1} = 2{,}40, \ldots, \quad \frac{I}{kT_p} \gg 1,$$

$$\frac{E}{kT_p} \lesssim 1,$$

where N and N_e are the number of atoms and electrons in 1 cm^3, α_p is the equilibrium electron concentration at T_p, and J_p is the equilibrium density of the radiant flux of quanta participating in the photoexcitation (3) and in the subsequent photoionization (4). The factor $\frac{1}{2}$ in (7) is in consideration of the two-step nature of the photoionization process, and the sign of the approximation in (7) is connected with the fact that $\frac{1}{2}$ does not pertain to the entire photon spectrum: for $h\omega > I$ for the ionization of an unexcited atom, a single photon is sufficient, and for $h\omega \ll 1$ the photoionization cross section (4) is negligibly small. Therefore, in a more precise calculation, it would be necessary to replace the factor $\frac{1}{2}$ for these frequency ranges by, respectively, one and zero. In the above relations, A is a factor depending weakly on the atomic temperature[12] T and equal to $3.2\cdot 10^{-10}$ cm^3/sec^{-1} for $T = 2\cdot 10^4$ K, c is the velocity of light, m is the atomic weight, ε is the specific ionization potential, equal to one for a hydrogen atom, L^* is the distance from the point at which $\alpha = \alpha^*$ to the leading edge of the equilibrium emission region of the gas.

By integrating (7), (8), one can show that for $\delta \lesssim 1$ we have $\varkappa L^* \lesssim 1$. Therefore, we further let $J^* \approx (1\text{–}e^{-1})J_p$ in the numerical estimates. The concentration $\alpha = \alpha^*$ is determined by equating the right sides of (7) and (8):

TABLE I. Equilibrium adiabat in the variables V, T, the relative electron density on the adiabat, the Mach number and the velocities v_2, v_p of the equilibrium wave. The initial state: $T_1 = 3\cdot 10^4$ K, $\delta_1 = 1$.

V_1/V_2	T_p, 10^4 K	α_p	$M = v_2/c_1$	v_2, 10^3 m/sec	v_p, 10^3 m/sec
0,4	0,98	0,031	0,59	–	–
0,5	1,10	0,067	0,70	–	–
0,6	1,17	0,079	0,82	–	–
0,7	1,22	0,11	0,98	1,74	12,5
0,8	1,26	0,13	1,2	2,17	13,1
0,9	1,30	0,14	1,7	3,07	13,7
0,95	1,32	0,14	2,4	4,3	14,0
0,98	1,33	0,15	3,8	6,9	14,2
0,995	1,34	0,15	7,7	13,8	14,4
0,996	1,34	0,15	8,6	15,4	14,4
1,000	1,34	0,15	∞	∞	14,4
1,675	1,48	0,21	0,059	0,10	16,7
1,68	1,48	0,22	0,085	0,15	16,7
1,70	1,48	0,22	0,18	0,32	16,7
1,80	1,50	0,22	0,37	0,65	17,0
2,0	1,53	0,24	0,57	1,0	17,6
2,2	1,57	0,25	0,69	1,2	18,9
2,5	1,61	0,28	0,84	1,5	20,5
3,0	1,70	0,31	1,7	1,9	22,0

$$\alpha^* = \frac{1}{N_0}\left[\frac{J^*\varkappa_0 m\varepsilon}{2A\ln(\alpha_p/\alpha^*)}\right]^{1/2}, \quad (9)$$

$N_0 \approx 2.7\cdot 10^{19}$ cm^{-3} is the Loschmidt number.

In this cascade ionization approximation (7), (8), the ionization wave velocity is connected by the following relation with the quanta flux J^* and α^*:

$$\alpha^* N v_p = J^*/2. \quad (10)$$

This ratio has a simple physical meaning: the quantity of free electrons generated per unit time by the photons ($\alpha^* N v_p$) equals the number of photons expended in ionization, that is $J^*/2$. It follows from (9) and (10) that

$$v_p = \frac{1}{\delta}\left[\frac{J^* A\ln(\alpha_p/\alpha^*)}{2\varkappa_0 m\varepsilon}\right]^{1/2}, \quad \ln\left(\frac{\alpha_p}{\alpha^*}\right)\approx 4,$$

and after substituting the numerical values of the coefficients, we obtain

$$v_p = \frac{10}{\delta} T_p^{3/2}\left(\frac{1}{m\varepsilon\varkappa_0}\right)^{1/2}\exp\left(-\frac{L^*\varkappa}{2}\right)[\text{cm}\cdot\text{sec}^{-1}],$$

where $\varkappa_0$ is expressed in cm^{-1}. Results of calculations for xenon ($m = 131$, $\varepsilon = 0.89$) for $T_1 = 3\cdot 10^4$ K, $\delta = 1$, $\exp(-L^*\varkappa/2)/\varkappa_0^{1/2} = 1$ are presented in Table I.

Consistent with these results, the values of v_p and v_2 coincide for $V_1/V_2 = 0.995$, that is practically on the isochore, and the Mach number M_1 at this point is larger than unity which corresponds to a supersonic equilibrium wave in the form of a rarefaction shock with a very weak density decrease. However, the pressure in it drops by a factor of two:

$$P_2/P_1 = T_p(1+\alpha_p)/T_1 \approx 0{,}5.$$

For a decrease of T_1, the method presented above of computing J_p, based on the smallness of E/kT, and assuming that $\varkappa$ independent of frequency become unsuitable and requires more laborious calculations. Based on qualitative considerations we anticipate that reducing the temperature rapidly reduces the equilibrium wave velocity and increases the amplitude (density change) of the rarefaction shock, and then the solution in the form of a rarefaction shock disappears. In place of it, there appears a solution corresponding to the segment of the adiabat above B (see Fig. 1). As a consequence of the strong dependence of v_p on T_1, the minimum temperature T_1 at which the solution in the form of a rarefaction shock disappears, is evidently bounded below by $\sim 2\cdot 10^4$ K.

As noted in Sec. 2, the experimentally observed variations in emission from the shock wave front in inert gases (xenon[10]) may be connected with the propagation of non-one-dimensional supersonic rarefaction shocks.

[1)] We have in mind wave intensities far less than what are called critical.[2] Therefore, the radiation contribution to the energy flux in the wave structure is insignificant.

[2)] As was noted above, the issue concerns perturbations with high frequencies ($\omega\tau \gg 1$). Such perturbations, strictly speaking, also damp, but so slowly that this can not give rise to a stationary structure of the Zel'dovich-type weak wave.[7] In the opposite case, the residence time of a particle of the substance in the wave structure would be comparable with the lifetime τ of the original nonequilibrium state, which contradicts the assumption that the transition to equilibrium occurs in a front (that is, by propagation of an equilibrium front, and not spontaneously and independently in each element of the volume).

[3)] The phase change with decreasing acoustic wave amplitude when it is reflected normally by the shock wave front occurs[9] when the inequality $J^2(\partial V/\partial P)_H < 1 - 2M_2$ is satisfied, where J is the density flux of the substance across the shock wave front, $(\partial V/\partial P)_H$ is the derivative of the specific volume with respect to the pressure along the shock adiabat, and M_2 is the Mach number behind the shock wave front. One can show that this inequality is satisfied for the initial shock wave compression jump under consideration (the adiabatic index is $\gamma = \frac{5}{3}$ for a monatomic gas with electron degrees of freedom "frozen").

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