Method of collective coordinates for an anisotropic Heisenberg ferromagnet

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The method of collective coordinates is used to develop a theory of perturbations in the vicinity of the classical solution for an anisotropic Heisenberg ferromagnet. A detailed analysis is made of the first three orders of perturbation theory for a ferromagnet whose ground state is of the domain wall type.

Recent investigations of nonlinear field-theory models have been concerned particularly with the problem of quantization of the field close to the classical solution of the equations of motion.[1-9] This problem arises in the theory of magnetism when, for example, a study is made of entities such as domain walls and of their interactions with structure defects or with quasiparticles.[10-12]

There are several approaches which can be used to tackle this problem (for reviews, see Refs. 4 and 9), among which the most consistent and providing the fullest treatment is the method of collective coordinates, proposed by Bogolyubov in connection with the problem of the interaction of a particle with a boson field.[13] The method was developed subsequently to deal with strongly interacting systems[14] and also as the basis of a procedure for canonical quantization in models of extended objects.[1-7]

The fundamental side of the method has been discussed frequently.[1,14] We shall recall briefly its main features. The essence of the method is the selection, from the variables of a system, of those parameters of the symmetry group of the Hamiltonian of the problem which ensure that the laws of conservation are satisfied rigorously to all orders of perturbation theory and thus one of the main difficulties known as the problem of zeroth modes is eliminated.[1,9] It is important to stress that, in contrast to other approaches,[6,10,11] the method of collective variables provides means for treating the problem at the quantum level right from the beginning without recourse to the classical analog of the investigated system, and makes it possible to avoid the operator ordering problem.

We use the method of collective coordinates to develop a perturbation theory close to the classical solution of an anisotropic Heisenberg ferromagnet whose ground state is of the domain wall type. The small parameter of the theory is $1/S^{1/2}$, where S is the spin.

1. We shall consider the model of a biaxial ferromagnet described by the Hamiltonian

$$H=\int\left[J_1\left(\frac{\partial S_\alpha}{\partial x}\right)^2-J_2S_2^2-J_3S_3^2\right]dx;\quad \alpha=1,\ 2,\ 3,$$
$$J_1>0;\quad J_2<J_3;\quad J_3>0, \tag{1}$$

where S_α are spin operators satisfying the following commutation relationships:

$$[S_\alpha(x),\ S_\beta(x')]=ie^{\alpha\beta\gamma}S_\gamma(x)\delta(x-x'). \tag{2}$$

Following the general idea of the method of collective coordinates, we introduce an additional variable ρ and the conjugate variable Π: $[\rho,\Pi]=i$, which is the generator of translations in the space of $\mathscr{H}_\rho$ (i.e., in the space of the functions of the variable ρ). We assume that $\mathscr{H}=\mathscr{H}_S\otimes\mathscr{H}_\rho$ is a tensor product of $\mathscr{H}_\rho$ and $\mathscr{H}_S$ in the space of the states H, while $|\lambda,\varphi(x)\rangle$ are the eigenvectors of the commuting operators ρ and $S_3(x)$. We assume that $R\{\varphi(x)\}$ is a functional whose actual form will be defined later, but which has the following property:

$$R\{\varphi(x-\varepsilon)\}=R\{\varphi(x)\}+\varepsilon. \tag{3}$$

We consider the space of the states H as a subspace of $\mathscr{H}$, described by the condition

$$\rho|\cdot,\cdot\rangle=0, \tag{4}$$

and define in $\mathscr{H}$ a unitary transformation

$$|\lambda,\varphi(x)\rangle\xrightarrow{U}|\tilde\lambda,\tilde\varphi(x)\rangle, \tag{5}$$

where

$$\tilde\lambda=R\{\varphi(x)\},\quad \tilde\varphi(x)=\varphi(x+\tilde\lambda-\lambda).$$

The corresponding operator is

$$U=\int d\xi\, e^{-iP\xi}e^{i\Pi\xi}\delta(\rho-R\{S_3(x)\}-\xi), \tag{6}$$

where $e^{-iP\xi}$, $e^{-i\Pi\xi}$ are the operators of the translations in $\mathscr{H}_S$ and $\mathscr{H}_\rho$, respectively:

$$e^{-iP\xi}S_\alpha(x)e^{iP\xi}=S_\alpha(x+\xi),\quad e^{-i\Pi\xi}\rho e^{i\Pi\xi}=\rho-\xi,\quad [P,\ \Pi]=0. \tag{7}$$

We also mention a relationship, which will be used later and which follows from Eqs. (3) and (7):

$$e^{-iP\xi}R\{S_3(x)\}e^{iP\xi}=R\{S_3(x)\}-\xi. \tag{8}$$

The significance of the transformation given by Eq. (5) is that in the new representation the spin variables are invariant under translations in $\mathscr{H}_S$, as is easily demonstrated. This makes it possible to identify the classical component corresponding to a domain wall without disturbing the translation symmetry of the Hamiltonian of the system. The zeroth (translation) mode is excluded by the condition (4), which after the transformation of Eq. (5) becomes

$$R\{S_3(x)\}|\cdot,\cdot\rangle=0. \tag{9}$$

Unless stated otherwise, we always assume that S_α represents the following operators: S_3, $S_\pm=S_1\pm iS_2$.

Using Eqs. (6)–(8), we find that

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$$U\prod_{k=1}^{N}S_{\alpha_k}(x_k)U^+$$

$$=\int d\eta\, d\xi \exp[i\xi(\Pi-P)]\prod_{k=1}^{N}S_{\alpha_k}(x_k-\eta)\delta\Big(R\{S_3(x)\}$$

$$-\tilde{R}\Big\{\prod_{n=1}^{N}S_{\alpha_n}(x_n-\eta)\Big\}-\xi\Big)\delta(\rho-R\{S_3(x)\}-\eta),\quad (10)$$

where $\tilde{R}\{\Pi_k S_{\alpha_k}(x_k)\}$ is given by the equality

$$R\{S_3(x)\}\prod_{k=1}^{N}S_{\alpha_k}(x_k)=\prod_{k=1}^{N}S_{\alpha_k}(x_k)\tilde{R}\Big\{\prod_{n=1}^{N}S_{\alpha_n}(x_n)\Big\}.$$

We must mention that in view of the condition (9), the term $R\{S_3(x)\}$ in the argument of the last δ function can be omitted and then, since $[\rho,S_\alpha]=0$, there is no need for ordering of the operators when integrating with respect to η.

Bearing in mind that $[S_\pm(x),S_3(x')]=\pm\delta(x-x')S_\pm(x)$, we have

$$\tilde{R}\Big\{\prod_{k=1}^{N}S_{\alpha_k}(x_k)\Big\}=R\Big\{S_3(x)+\sum_{k=1}^{N}\varepsilon(\alpha_k)\delta(x-x_k)\Big\},\quad (11)$$

where $\varepsilon(3)=0$, $\varepsilon(+)=1$, $\varepsilon(-)=-1$.

We define $R\{\varphi(x)\}$ as the solution of the equation

$$\int_{-\infty}^{\infty}f(x-R)\varphi(x)\,dx,\quad (12)$$

where $f(x)$ is a certain given function, which falls sufficiently rapidly at infinity. The functional $R\{\varphi\}$ introduced in this way clearly has the necessary property described by Eq. (3). In our subsequent analysis it is sufficient to expand $R\{\varphi\}$ in the range of low values of $\delta R=R\{\varphi_0+\delta\varphi\}-R\{\varphi_0\}$. Varying the expression (12) and using the condition $|\delta R|\ll 1$, we find that the method of successive approximations yields

$$\delta R=\delta R^{(1)}+\delta R^{(2)}+O[(\delta R)^3],$$

$$\delta R^{(1)}=c\int f(x-R_0)\delta\varphi(x)\,dx,\quad c^{-1}=\int f'(x-R_0)\varphi_0(x)\,dx,$$

$$\delta R^{(2)}=c\delta R^{(1)}\Big[\frac{1}{2}\delta R^{(1)}\int f''(x-R_0)\varphi_0(x)\,dx$$

$$-\int f'(x-R_0)\delta\varphi(x)\,dx\Big],\quad (13)$$

where $R_0=R\{\varphi_0(x)\}$.

We carry out one further unitary transformation corresponding to rotation of the coordinate axes:

$$G=\exp\Big[-i\int\beta(x)S_2(x)\,dx\Big]\exp\Big[-i\int\alpha(x)S_3(x)\,dx\Big],\quad (14)$$

where α and β are the Euler angles, and we use the Holstein–Primakoff representation for spin operators:[15,16]

$$S_+=(2S-a^+a)^{1/2}a,\quad S_-=a^+(2S-a^+a)^{1/2},\quad S_3=S-a^+a,$$

$$[a(x),a^+(x')]=\delta(x-x'),\quad (15)$$

$$[a(x),a(x')]=[a^+(x),a^+(x')]=0.$$

Since in the representation obtained by the transformation of Eq. (6) the spin variables are invariant under translations in $\mathscr{H}_S$ and the classical solution corresponding to a domain wall is not degenerate, on the basis of Eq. (9), there are angles $\alpha(x)$ and $\beta(x)$ such that as a result of the transformation given by Eq. (14) the classical solution becomes $(S_1(x),S_2(x),S_3(x))=(0,0,S)$. Consequently, following Refs. 15 and 16, this makes it possible to consider weakly excited states using the quantity $1/S^{1/2}$ as the formal parameter of the expansion of the expressions given in the system (15).

Using Eqs. (9), (10), (13), and (15), as well as the familiar relationship

$$\exp(-i\varepsilon S_\alpha)S_\beta\exp(i\varepsilon S_\alpha)=A_{\beta\gamma}{}^{\alpha}(\varepsilon)S_\gamma,\quad (16)$$

where A^α are matrices of the rotations in $\mathscr{R}^3$, we find

$$GU\Big(R\{S_3(x)\}-\tilde{R}\Big\{\prod_k S_{\alpha_k}(x_k)\Big\}\Big)U^+G^+$$

$$=S^{-1}(q_1+S^{-1/2}q_2+S^{-1}q_3)+O(S^{-5/2}),$$

$$q_1\Big\{\prod_k S_{\alpha_k}(x_k)\Big\}=-g^{-1}\sum_k\varepsilon(\alpha_k)f(x_k),\quad g=\int_{-\infty}^{\infty}f'\cos\beta\,dx,$$

$$(17)$$

$$q_2\Big\{\prod_k S_{\alpha_k}(x_k)\Big\}=\frac{1}{2^{1/2}g^2}\sum_k\varepsilon(\alpha_k)f(x_k)\int_{-\infty}^{\infty}f'(a^++a)\sin\beta\,dx.$$

As far as $q_3\{\Pi_k S_{\alpha_k}(x_k)\}$ is concerned, in subsequent calculations we need to know that x_k occurs in this expression only in the combination $\Sigma_k\varepsilon(\alpha_k)f(x_k)$.

We now consider Eq. (10). The transformation of Eq. (14), subject to the comment made immediately after Eq. (10), yields

$$GU\prod_k S_{\alpha_k}(x_k)U^+G^+$$

$$=\int F(\xi)\prod_k[A^3(\alpha(x_k-\rho))A^2(\beta(x_k-\rho))]_{\alpha_k\beta_k}$$

$$\cdot S_{\beta_k}(x_k-\rho)\delta\Big(\xi-\delta R\Big\{\prod_n S_{\alpha_n}(x_n-\rho)\Big\}\Big)d\xi,\quad (18a)$$

where the following notation is introduced:

$$F(\xi)=G\exp(-iP\xi)G^+\exp(i\Pi\xi),$$

$$\delta R\Big\{\prod_k S_{\alpha_k}(x_k)\Big\}=GU\Big(R\{S_3(x)\}-\tilde{R}\Big\{\prod_k S_{\alpha_k}(x_k)\Big\}\Big)U^+G^+.$$

Since the expansion of $F(\xi)$ in powers of $S^{-1/2}$ is not regular, we use the fact that $\delta R=O(S^{-1})$, which can be deduced from Eq. (17), and we represent F in the form

$$F(\nu/S)=\sum_{k=0}^{\infty}F_k\nu^k.$$

We show below that the coefficients F_k of this series now have a regular expansion in the powers of $S^{-1/2}$ and they obey $F_k=O(S^{-k/2})$. Then, after integration with respect to ξ, Eq. (18a) becomes

$$\sum_{m=0}^{\infty} F_m \prod_k [\ldots]_{\alpha_k \beta_k} S_{\beta_k}(x_k-\rho)\left[S\delta R\left\{ \prod_n S_{\alpha_n}(x_n-\rho)\right\}\right]^m , \tag{18b}$$

so that the problem reduces to finding expansions for F_k. Using Eqs. (7) and (16), we can transform F to an expression more convenient in expansions:

$$G\{S_\alpha(x)\}e^{-iP\xi}G^+\{S_\alpha(x)\}e^{i\Pi\xi}$$

$$=G\{S_\alpha(x)\}G^+\{S_\alpha(x+\xi)\}e^{i\xi(\Pi-P)}$$

$$=\exp\Big\{i\int\alpha(x)\,[A_{3\gamma}^2(\beta(x+\xi))S_\gamma(x+\xi)$$

$$-A_{3\gamma}^2(\beta(x))S_\gamma(x)]dx\Big\}\cdot$$

$$\cdot\exp\Big\{i\int\beta(x)\,[S_2(x+\xi)-S_2(x)]dx\Big\}\exp\{i\xi(\Pi-P)\}. \tag{19}$$

Hence, using Eq. (15), we find

$$F(\nu/S)$$

$$=\exp\Big\{i\nu\int_{-\infty}^{\infty}\alpha\frac{d}{dx}\cos\beta\,dx\Big\}[1+O(S^{-1/2})]\exp\Big[\frac{i\nu}{S}(\Pi-P)\Big].$$

The first exponential function in the above expression may be allowed for by the transformation[13]

$$\Pi\to\Pi-S\int\alpha\frac{d}{dx}\cos\beta\,dx, \tag{20}$$

which has a simple meaning: the second term in Eq. (20) is simply the classical momentum of a domain wall; on the other hand, in the representation obtained as a result of the transformation (6) the quantity Π is found to be the operator of the total momentum of the system, so that the transformation of Eq. (20) represents inclusion of the momentum in the classical solution. Finally, using Eq. (20) and the expansions of the expressions in the system (15), we obtain

$$F(\nu/S)=1+\frac{\nu}{(2S)^{1/2}}(\sigma+\mu)+\frac{1}{S}\Big[i\nu(\Pi-P-\eta)$$

$$+\frac{\nu^2}{4}(\sigma^2+2\mu\sigma+\mu^2+2i\chi)\Big]+O(S^{-3/2}), \tag{21}$$

where

$$\sigma=\int_{-\infty}^{\infty}\beta\frac{d}{dx}(a-a^+)dx,\qquad \eta=\int_{-\infty}^{\infty}\alpha\frac{d}{dx}(a^+a\cos\beta)dx,$$

$$\mu=i\int_{-\infty}^{\infty}\alpha\frac{d}{dx}[(a+a^+)\sin\beta]dx,\qquad \chi=\int_{-\infty}^{\infty}\alpha\frac{d^2}{dx^2}\cos\beta\,dx.$$

2. We now have all the expansions necessary to find the Hamiltonian (1). The terms containing the derivatives of the spin operators are calculated as follows:

$$[\partial^2/\partial x\,\partial x'\;GUS_\alpha(x)S_\beta(x')U^+G^+]_{x=x'}.$$

Using Eqs. (15), (17), (18), and (21), we obtain

$$GUHU^+G^+=E_0+S^2\sum_{n=1}^{\infty}S^{-n/2}H_n,$$

where E_0 is the classical energy of a domain wall,

$$H_1=2^{1/2}\int dx\Big\{J_1\beta''-J_2\sin\beta\cos\beta+\frac{1}{4}J_3\Big[2\sin2\beta\sin^2\alpha$$

$$+2i\sin2\alpha\sin\beta-i\beta'\sin2\alpha\int q_1(x')\sin^2\beta(x')dx'\Big]\Big\}a+\text{H.c.},$$

$$H_2=\int\Big\{2J_1\frac{\partial a^+}{\partial x}\frac{\partial a}{\partial x}-[J_1(\beta')^2+J_2(3\sin^2\beta-2)$$

$$-J_3(1-3\sin^2\alpha\sin^2\beta)]a^+a+\frac{1}{2}[(J_1(\beta')^2-J_2\sin^2\beta$$

$$+J_3(\cos2\alpha+\sin^2\beta\sin^2\alpha-i\cos\beta\sin2\alpha))aa+\text{H.c.}] \tag{22}$$

$$-\frac{1}{2}[(J_1(2q_1'\beta'\cos\beta+q_1''\sin\beta)+J_3q_1\sin\beta(\cos2\alpha$$

$$-i\cos\beta\sin2\alpha))\sigma a+\text{э. c.}]-\frac{1}{8}[J_1(q_1')^2\sin^2\beta$$

$$-J_3q_1^2\cos2\alpha\sin^2\beta]\sigma^2-\frac{i}{2^{3/2}}J_3\sigma q_2\sin2\alpha\sin^2\beta$$

$$-\frac{1}{2}J_3q_1\sin2\alpha\sin^2\beta(P-\Pi)\Big\}dx,\qquad q_i=q_i\{S_+^2(x)\}.$$

The above expression reflects the fact that in the case under consideration for $x\to\pm\infty$, the quantity $\beta(x)$ tends to constant values, which are multiples of π. Moreover, we also assume $\alpha=$ const, because this considerably simplifies the calculation. We confirm the validity of this assumption later.

The condition $H_1=0$ gives

$$J_1\beta''-(J_2-J_3\sin^2\alpha)\sin\beta\cos\beta=0,$$

$$\beta'\int_{-\infty}^{\infty}\sin^2\beta(x')q_1(x')dx'-2\sin\beta=0. \tag{23}$$

Using Eq. (13), we can readily show that for $\alpha=$ const, the system of equations (23) is internally self-consistent for any selection of the function f in Eq. (12), so that we obtain the familiar result[17]

$$\beta(x)=\pm2\,\text{arctg}\exp\Big(\pm\frac{x}{\Delta}\Big),\qquad \Delta=[J_1/(J_2-J_3\sin^2\alpha)]^{1/2}, \tag{24}$$

where the choice of the signs is governed by the boundary conditions imposed on $\beta(x)$ in the limit $x\to\pm\infty$.

This circumstance allows us to simplify greatly the expression for H_2. For this purpose we choose the function f in Eq. (12) as follows:

$$f(x)=\begin{cases}1, & |x|\le L,\\ 0, & |x|>L,\end{cases}$$

and we go to the limit $L\to\infty$ in Eq. (22). The result is then

$$H_2=\Big\{\int 2J_1\frac{\partial a^+}{\partial x}\frac{\partial a}{\partial x}+\frac{i}{2}J_3\Delta\sin2\alpha\Big(\frac{\partial a^+}{\partial x}a-a^+\frac{\partial a}{\partial x}\Big)$$

$$+[2J_2\cos2\beta+J_3(4\sin^2\alpha\sin^2\beta-1)]a^+a+\Big[\frac{1}{2}J_3(\cos2\alpha$$

$$-i\cos\beta\sin 2\alpha)aa+\text{э. c.}\Big]+\frac{1}{4\Delta}J_3\int dx'\sin\beta(x)\sin\beta(x')$$
$$\cdot[2(\cos 2\alpha-i\sin 2\alpha(\cos\beta(x')-\cos\beta(x)))a^+(x)a(x')$$
$$-((\cos 2\alpha-i\sin 2\alpha(\cos\beta(x)$$
$$+\cos\beta(x')))a(x)a(x')+\text{H.c.})]\Big\}dx$$
$$-J_3\Delta\Pi\sin 2\alpha. \tag{25}$$

We have allowed here for the fact that

$$P=\frac{i}{2}\int\left(\frac{\partial a^+}{\partial x}a-a^+\frac{\partial a}{\partial x}\right)dx.$$

3. In the diagonalization of H_2 we can use the method of canonical transformations of Ref. 15:

$$b_k=\int[u_k^*(x)a(x)-v_k^*(x)a^+(x)]dx,$$
$$b_k^+=\int[u_k(x)a^+(x)-v_k(x)a(x)]dx, \tag{26}$$

where u_k and v_k satisfy the conditions of unitarity

$$\int[u_k(x)u_p^*(x)-v_k(x)v_p^*(x)]dx=\delta_{kp},$$

$$\int[u_k(x)v_p(x)-u_p(x)v_k(x)]dx=0,$$

and in the case under consideration are given by the equations

$$\left(\frac{\partial^2}{\partial\xi^2}+\frac{i\varepsilon}{2}\sin 2\alpha\frac{\partial}{\partial\xi}+2\sin^2\beta+\frac{\varepsilon}{2}\cos 2\alpha-1+\frac{\Delta^2}{2J_1}\omega_k\right)u_k$$
$$-\frac{\varepsilon}{2}(\cos 2\alpha+i\sin 2\alpha\cos\beta)v_k-\frac{\varepsilon}{4}\int d\xi'\sin\beta(\xi)\sin\beta(\xi')\cdot$$
$$\cdot\{[\cos 2\alpha-i\sin 2\alpha(\cos\beta(\xi')-\cos\beta(\xi))]u_k(\xi')$$
$$-[\cos 2\alpha+i\sin 2\alpha(\cos\beta(\xi')+\cos\beta(\xi))]v_k(\xi')\}=0,$$

$$\tag{27}$$

$$\left(\frac{\partial^2}{\partial\xi^2}-\frac{i\varepsilon}{2}\sin 2\alpha\frac{\partial}{\partial\xi}+2\sin^2\beta+\frac{\varepsilon}{2}\cos 2\alpha-1-\frac{\Delta^2}{2J_1}\omega_k\right)v_k$$
$$-\frac{\varepsilon}{2}(\cos 2\alpha-i\sin 2\alpha\cos\beta)u_k-\frac{\varepsilon}{4}\int d\xi'\sin\beta(\xi)\sin\beta(\xi')$$
$$\cdot\{[\cos 2\alpha-i\sin 2\alpha(\cos\beta(\xi')-\cos\beta(\xi))]v_k(\xi')$$
$$-[\cos 2\alpha-i\sin 2\alpha(\cos\beta(\xi')+\cos\beta(\xi))]u_k(\xi')\}=0.$$

Here, $\xi=x/\Delta$, ω_k is the corresponding eigenvalue, and $\varepsilon=J_3/(J_2-J_3\sin^2\alpha)$.

We now use the condition (9). We note that Eq. (12) implies that this condition is equivalent to

$$\int f(x)S_3(x)dx|\cdot,\cdot\rangle=0.$$

Using Eqs. (10), (15), (16), and (26) we obtain

$$GU\int f(x)S_3(x)dx\,U^+G^+\xrightarrow[L\to\infty]{}(S/2)^{1/2}\sum_k\int\sin\beta(x)$$
$$\cdot[(u_k(x)+v_k(x))b_k+\text{H.c.}]dx+o(S^{1/2}).$$

We subtract the equations of the system (27) from one another, multiply the difference by $\sin\beta$, and integrate with respect to x. Integrating by parts the terms containing the derivatives of u_k and v_k, and using Eqs. (23) and (24), we find that

$$\omega_k\int(u_k+v_k)\sin\beta\,dx=0.$$

We can see that the condition (9) reduces to elimination of the states with $\omega_k=0$ from the spectrum and, therefore, in the first three orders of perturbation theory the model is described by the Hamiltonian

$$H=E_0+S\sum_{\omega_k\neq 0}\omega_k b_k^+b_k-J_3\Delta\Pi\sin 2\alpha.$$

By way of example, we consider the case $\varepsilon\ll 1$. We seek u_k, v_k, and ω_k in the form of expansions in powers of ε. In the zeroth approximation, the solution of the problem described by the system (27) is well known:[16]

a) discrete spectrum:

$$u_0^{(0)}\sim\operatorname{sech}\xi,\quad v_0^{(0)}=0,\quad \omega_0=0;$$

b) continuous spectrum:

$$u^{k(0)}\sim(ik+\operatorname{th}\xi)e^{-ik\xi},\quad v_k^{(0)}=0,$$
$$\omega_k^{(0)}=2(J_2-J_3\sin^2\alpha)(1+k^2).$$

The solution a) should, as demonstrated above, be rejected because of the condition (9). Then, applying the standard methods, we find that to within terms of order ε, the result is

$$u_k=C\Big[(ik+\operatorname{th}\xi)e^{-ik\xi}$$
$$-\frac{i\varepsilon}{4}\sin 2\alpha\left(e^{-ik\xi}-\frac{\pi}{2}\operatorname{sech}\frac{\pi k}{2}\operatorname{sech}\xi\right)\Big],$$
$$v_k=-\frac{\varepsilon}{4}C\Big[\frac{\cos 2\alpha-k\sin 2\alpha}{1+k^2}(ik+\operatorname{th}\xi)e^{-ik\xi}$$
$$+i\sin 2\alpha\left(e^{-ik\xi}-\frac{\pi}{2}\operatorname{sech}\frac{\pi k}{2}\operatorname{sech}\xi\right)\Big],$$
$$C=[2\pi\Delta(1+k^2+\varepsilon k/2)]^{-1/2},$$

$$\omega_k=2(J_2-J_3\sin^2\alpha)\left[1+k^2-\frac{\varepsilon}{4}(2\cos 2\alpha+k\sin 2\alpha)\right].$$

It follows from the above solution that, in particular, in the case under consideration there is no scattering of domain walls by spin excitations, as one would expect on the basis of Ref. 17.

It should be stressed that although this model is fully integrable, a similar approach can be used also to deal with multidimensional models as well as with models that allow for the interactions that keep the equations from being completely integrable[5] and whose inclusion is important in studies of relaxation processes.[10,11] In particular, in the case of a domain wall on transition to the three-dimensional case the changes occur only in the stage of diagonalization of H_2 and they reduce to an additional Fourier transformation of the operators a^+ and a in the domain wall plane (however, in this case there is scattering of domain walls by spin excitations—see Ref. 11).

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