Tangential molecular forces caused between moving bodies by a fluctuating electromagnetic field

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One aspect of the molecular interaction between two semi-infinite media separated by a plane vacuum gap is analyzed. Specifically, the tangential forces, i.e., the forces acting along the boundaries, are analyzed. One medium is at rest, and the other is in uniform motion with respect to it at a velocity **v** which is parallel to the boundaries of the media. In general, the media are at different temperatures. A general expression is derived for that tangential interaction force between the media which results from a fluctuating electromagnetic field. The force exerted on the moving medium may either slow it or accelerate it. In the latter case the moving medium may perform work on external bodies. There is the interesting possibility of an "anomalous" situation in which the cold moving medium cools and the hot medium at rest heats up while this work is performed. It is shown that this case does not contradict the second law of thermodynamics. A detailed study is made of the particular case of highly conducting metals separated by a small gap. If the velocities involved are low, the interaction of the metals can be described with a "hydrodynamic" viscosity coefficient.

Fluctuations of the polarizability and magnetization of bodies give rise to a fluctuating electric field both inside and outside these bodies. The interaction between bodies mediated by this field is manifested in several effects, among which we will mention the following. First, there are the well-known molecular attraction forces (van der Waals forces).[1–3] Second, there is the heat exchange through a fluctuating electromagnetic field between bodies heated differently (see, for example, Ref. 4 and the bibliography there). Third, tangential molecular forces may act between gyrotropic media which have different temperatures and which are at rest in an external magnetizing field. The existence of these forces was pointed out in Ref. 5.

In the present paper we examine the interaction between moving media mediated by a fluctuating electromagnetic field. We restrict the discussion to the following formulation of the problem. Media 1 and 2, which fill half-spaces separated by a plane vacuum gap of width a (Fig. 1), are assumed to be homogeneous and isotropic with permittivities and permeabilities ε_1, μ_1 and ε_2, μ_2, respectively. These are dispersive media; i.e., their permittivities and permeabilities are complex functions of the frequency[1)] ω. Medium 1 is assumed to be at rest, while medium 2 is in motion with respect to it at a constant velocity **v** which is directed along the boundaries of the media. The temperatures of the media are held constant at T_1 and T_2, respectively. We assume that the temperature of each medium is measured by a thermometer connected to it. The interaction of the media through the fluctuating electromagnetic field has the consequence that tangential forces act on each medium.

1. GENERAL EXPRESSION FOR THE TANGENTIAL FORCE

The Cartesian laboratory coordinate system fixed in medium 1 is chosen so that the axis $z \equiv x_3$ is orthogonal to the boundaries of the media. We denote the x and y axes by x_1 and x_2, respectively. We denote by $\mathbf{F}_1$ the tangential force acting on a unit surface area of medium 1. This force has two components $F_{1\alpha}$ (here and below, the Greek-letter indices take on the values 1, 2). The components of this force are obviously determined by the average values of $T_{\alpha 3}$, the average components of the energy-momentum tensor of the electromagnetic field, in the vacuum gap. In other words, $F_{1\alpha} = \overline{T}_{\alpha 3}$. The superior bar here means a statistical average over the fluctuations of the electromagnetic field.

We assume that random electric and magnetic sources of electromagnetic fields are distributed in a statistically independent way in media 1 and 2. We can then write the force $\mathbf{F}_1$ as the sum of two terms, $\mathbf{F}_1^{(1)}$ and $\mathbf{F}_1^{(2)}$, the first of which is the contribution from sources distributed in medium 1, and the second from those in medium 2. The expression for the correlation functions of sources distributed in medium 1, at rest, is well known.[6] By solving the corresponding electrodynamic problem in the presence of a moving medium 2, one can find the fields generated by these sources in the vacuum gap. It is then a simple matter to calculate the average $\alpha 3$ components of the energy-momentum tensor and to thus determine the part of the force $\mathbf{F}_1^{(1)}$. The part $\mathbf{F}_1^{(2)}$, on the other hand, can be found in the following way. We transform to a coordinate system which is fixed in the moving medium, 2. In this coordinate system, the part of the force $\mathbf{F}_1^{(2)}$ can be found from the expression for $\mathbf{F}_1^{(1)}$ through appropriate changes in notation, as is easily understood. We then transform to the laboratory coordinate system and find the expression for $\mathbf{F}_1^{(2)}$ which we want, thereby completing our

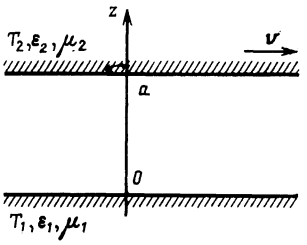

FIG. 1.

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task of finding the total force $\mathbf{F}_1$. The force density $\mathbf{F}_2$, acting on medium 2, differs from $\mathbf{F}_1$ only in sign: $\mathbf{F}_2 = -\mathbf{F}_1$. Since media 1 and 2 are isotropic, the only direction which is distinguished in this problem is that of the velocity at which medium 2 is moving. It follows that the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written

$$\mathbf{F}_2=-\mathbf{F}_1=\frac{\mathbf{v}}{v}F. \tag{1}$$

Calculations from (1) yield the following expression for F:

$$F=\frac{1}{v}(P_1+P_2/\gamma), \tag{2}$$

where

$$\gamma=1/(1-u^2)^{1/2}, \quad u=v/c,$$

and c is the velocity of light in vacuum. The quantities P_1 and P_2 in (2) have the following meaning. The quantity P_1 is the heat given up by medium 1 from a unit surface area per unit time in the coordinate system fixed in medium 1, i.e., in the laboratory coordinate system. The quantity P_1 is obviously equal to the average projection of the Poynting vector of the fluctuating electromagnetic field onto the outward normal to medium 1: $P_1 = \overline{S}_3$. In turn, the component S_3 is expressed in terms of the component T_{34} of the energy-momentum tensor of the electromagnetic field: $P_1 = c\overline{T}_{34}$. The quantity P_2 is the heat given up by medium 2 per unit time from a unit surface area, but in the coordinate system fixed in medium 2. We obviously have $P_2 = -c\overline{\tilde{T}}_{34}$. The tilde ($\sim$) means that the component $\overline{T}_{34}$ is taken in the moving coordinate system, fixed in medium 2. The minus sign means that the z axis is directed into medium 2. The quantities P_1 and P_2 are thus determined in the same way. Actually, P_1 and P_2 are the same quantity, looked at from different coordinate systems. We note in this connection that relation (2) is actually the law of transformation of this quantity under the transformation from one coordinate system to the other.

We now write expressions for P_1 and P_2 (since the calculations are fairly lengthy, we will write only the final results):

$$P_1=\frac{\hbar}{8\pi^3}\int_{-\infty}^{\infty}d\omega\int d^2\varkappa\left(\frac{\omega}{|\omega|}-\frac{\tilde\omega}{|\tilde\omega|}\right)\omega M(\omega,\varkappa;\mathbf{u})$$

$$+\frac{1}{4\pi^3}\int_{-\infty}^{\infty}d\omega\int d^2\varkappa\left\{\frac{\Pi(T_1,\omega)}{\omega}-\frac{\Pi(T_2,\tilde\omega)}{\tilde\omega}\right\}\omega M(\omega,\varkappa;\mathbf{u}), \tag{3}$$

$$P_2=-\frac{\hbar}{8\pi^3}\int_{-\infty}^{\infty}d\omega\int d^2\varkappa\left(\frac{\omega}{|\omega|}-\frac{\tilde\omega}{|\tilde\omega|}\right)\tilde\omega M(\omega,\varkappa;\mathbf{u})$$

$$-\frac{1}{4\pi^3}\int_{-\infty}^{\infty}d\omega\int d^2\varkappa\left\{\frac{\Pi(T_1,\omega)}{\omega}-\frac{\Pi(T_2,\tilde\omega)}{\tilde\omega}\right\}\tilde\omega M(\omega,\varkappa;\mathbf{u}), \tag{4}$$

where $\hbar$ is Planck's constant, $\varkappa = (\varkappa_1,\varkappa_2)$ is a two-dimensional wave vector, and

$$\tilde\omega=\gamma(\omega-\varkappa\mathbf{v}).$$

The integration in (3) and (4) is over the entire wave-vector space. The function $\Pi(T,\omega)$ is given by

$$\Pi(T,\omega)=\hbar|\omega|/(e^{|\omega|/\omega_T}-1), \quad \omega_T=T/\hbar,$$

where T is the temperature in energy units. The function $M(\omega,\varkappa;\mathbf{u})$ in (3) and (4) depends in a complicated way on the permittivities and permeabilities of the media, the width of the gap between them, and the velocity $\mathbf{v}$. It is given by

$$M=\frac{4|q|^2}{|Q|^2}\left\{\left(\frac{q_1}{\varepsilon_1}\right)''\left(\frac{\tilde q_2}{\tilde\varepsilon_2}\right)''(1+\beta)|Q_\mu|^2\right.$$
$$+\left(\frac{q_1}{\mu_1}\right)''\left(\frac{\tilde q_2}{\tilde\mu_2}\right)''(1+\beta)|Q_\varepsilon|^2$$
$$\left.+\left(\frac{q_1}{\varepsilon_1}\right)''\left(\frac{\tilde q_2}{\tilde\mu_2}\right)''|\beta|\,|Q_{\mu\varepsilon}|^2+\left(\frac{q_1}{\mu_1}\right)''\left(\frac{\tilde q_2}{\tilde\varepsilon_2}\right)''|\beta|\,|Q_{\varepsilon\mu}|^2\right\}, \tag{5}$$

where

$$q_1=(\varkappa^2-k^2\varepsilon_1\mu_1)^{1/2}, \quad q_2=(\varkappa^2-k^2\varepsilon_2\mu_2)^{1/2},$$
$$q=(\varkappa^2-k^2)^{1/2}, \quad k=\omega/c.$$

The branches of the square roots are chosen to satisfy

$$q'_{1,2}>0. \tag{6}$$

A single prime and a double prime specify the real and imaginary parts, respectively, of a complex quantity. The choice of the sign of the square root in the expression for q is arbitrary, as we will see below. In addition, we have

$$\beta=\frac{\gamma^2u^2q^2\varkappa_\perp^2}{\varkappa^2\tilde\varkappa^2},$$

where $\varkappa_\perp$ is the component of the wave vector $\varkappa$ which is perpendicular with respect to the velocity, i.e.,

$$\varkappa_\perp^2=\left[\varkappa-\frac{\mathbf{u}(\varkappa\mathbf{u})}{u^2}\right]^2,$$

and $\tilde\varkappa$ is the magnitude of the wave vector in the moving coordinate system,

$$\tilde\varkappa=\varkappa+(\gamma-1)\frac{\mathbf{u}(\varkappa\mathbf{u})}{u^2}-\gamma k\mathbf{u}.$$

The tilde here means that the corresponding quantities which depend on ω and $\varkappa$ are taken at $\omega=\tilde\omega$, $\varkappa=\tilde\varkappa$. The quantities Q_ε, Q_μ, $Q_{\varepsilon\mu}$, $Q_{\mu\varepsilon}$, and Q are given by

$$Q_\varepsilon=\left(q+\frac{q_1}{\varepsilon_1}\right)\left(q+\frac{\tilde q_2}{\tilde\varepsilon_2}\right)e^{qa}-\left(q-\frac{q_1}{\varepsilon_1}\right)\left(q-\frac{\tilde q_2}{\tilde\varepsilon_2}\right)e^{-qa}, \tag{7}$$

$$Q_\mu=\left(q+\frac{q_1}{\mu_1}\right)\left(q+\frac{\tilde q_2}{\tilde\mu_2}\right)e^{qa}-\left(q-\frac{q_1}{\mu_1}\right)\left(q-\frac{\tilde q_2}{\tilde\mu_2}\right)e^{-qa}, \tag{8}$$

$$Q_{\varepsilon\mu}=\left(q+\frac{q_1}{\varepsilon_1}\right)\left(q+\frac{\tilde q_2}{\tilde\mu_2}\right)e^{qa}+\left(q-\frac{q_1}{\varepsilon_1}\right)\left(q-\frac{\tilde q_2}{\tilde\mu_2}\right)e^{-qa}, \tag{9}$$

$$Q_{\mu\varepsilon}=\left(q+\frac{q_1}{\mu_1}\right)\left(q+\frac{\tilde q_2}{\tilde\varepsilon_2}\right)e^{qa}+\left(q-\frac{q_1}{\mu_1}\right)\left(q-\frac{\tilde q_2}{\tilde\varepsilon_2}\right)e^{-qa}, \tag{10}$$

$$Q=Q_\varepsilon Q_\mu-4\beta\varkappa^2\tilde\varkappa^2\left(1-\frac{1}{\varepsilon_1\mu_1}\right)\left(1-\frac{1}{\tilde\varepsilon_2\tilde\mu_2}\right). \tag{11}$$

Let us examine some properties of the function $M(\omega,\varkappa;\mathbf{u})$. Since the media which we are discussing here are dissipative, for $\omega>0$ we have

$$\varepsilon_1''>0, \quad \mu_1''>0, \quad \varepsilon_2''>0, \quad \mu_2''>0. \tag{12}$$

From (6) and (12) we find

$$\left(\frac{q_1}{\varepsilon_1}\right)''<0, \quad \left(\frac{q_1}{\mu_1}\right)''<0, \quad \left(\frac{q_2}{\varepsilon_2}\right)''<0, \quad \left(\frac{q_2}{\mu_2}\right)''<0.$$

These quantities are odd functions of the frequency. It then follows from (5) that we have

$$M(\omega, \varkappa; \mathbf{u})>0, \quad \omega\tilde{\omega}>0, \tag{13}$$
$$M(\omega, \varkappa; \mathbf{u})<0, \quad \omega\tilde{\omega}<0. \tag{14}$$

It is also simple to verify that M has the additional properties

$$M(\omega, \varkappa; \mathbf{u})=M(-\omega, -\varkappa; \mathbf{u})=M(\omega, -\varkappa; -\mathbf{u}). \tag{15}$$

Using (15), we can rewrite (3) and (4) in a form which contains an integration only over positive frequencies:

$$P_1=\frac{\hbar}{4\pi^3}\int_0^\infty d\omega \int d^2\varkappa\left(1-\frac{\tilde{\omega}}{|\tilde{\omega}|}\right)\omega M$$
$$+\frac{1}{2\pi^3}\int_0^\infty d\omega\int d^2\varkappa\left\{\frac{\Pi(T_1,\omega)}{\omega}-\frac{\Pi(T_2,\tilde{\omega})}{\tilde{\omega}}\right\}\omega M, \tag{16}$$

$$P_2=-\frac{\hbar}{4\pi^3}\int_0^\infty d\omega \int d^2\varkappa\left(1-\frac{\tilde{\omega}}{|\tilde{\omega}|}\right)\tilde{\omega} M$$
$$-\frac{1}{2\pi^3}\int_0^\infty d\omega\int d^2\varkappa\left\{\frac{\Pi(T_1,\omega)}{\omega}-\frac{\Pi(T_2,\tilde{\omega})}{\tilde{\omega}}\right\}\tilde{\omega} M. \tag{17}$$

In accordance with the original formulation of the problem, it is assumed that medium 2 is in uniform motion. The fluctuating electromagnetic field exerts on it a tangential force with a surface density $\mathbf{F}_2$. Consequently, if the uniform motion of medium 2 is to be sustained, an external force must be applied to it. We denote the surface density of this external force by $\mathbf{f}$. We assume that the axis $x = x_1$ is directed along the velocity of medium 2. It is then obvious that $\mathbf{F}_2$ and $\mathbf{f}$ have projections onto the x_1 axis alone. Medium 2 gives up heat for $P_2 > 0$ or absorbs it for $P_2 < 0$. Consequently, its rest mass is not a constant, and in turn we have $f \neq F$. The force f can be determined easily from the equation of motion of medium 2. Under the condition $v =$ const we have

$$\gamma v\frac{dm_2}{dt}=F+f. \tag{18}$$

We wish to stress that all the quantities involved here refer to a unit surface area of medium 2 in the laboratory coordinate system. In particular, m_2 is the rest mass of a cylindrical column whose axis runs parallel to the z axis and which has a unit cross-sectional area. It is not difficult to see that for an arbitrary rest mass m_2 we have

$$\frac{dm_2}{dt}=-P_2/c^2.$$

From (18) we then find

$$f=-F-\frac{\gamma v}{c^2}P_2. \tag{19}$$

Substituting in F from (2), we find an expression for f:

$$f=-\frac{1}{V}(P_1+\gamma P_2). \tag{20}$$

The external force f performs work with a surface density

$$vf=-(P_1+\gamma P_2)=-U \tag{21}$$

on medium 2 per unit time. The quantity U is obviously the work performed by medium 2 on the source of the external force. If $U<0$, this source of the external force performs work on the medium; if $U>0$, on the other hand, medium 2 performs work on the external bodies. Let us analyze the various possible cases here.

It can be seen from (16) and (17) that P_1 and P_2 are complex functionals of the permittivities and permeabilities of the media and can vary over wide ranges. This result does not, however, mean that P_1 and P_2 can independently take on any prespecified values. Analysis of expressions (16) and (17) shows (we omit the proof) that the permissible values of P_1 and P_2 are determined by the inequality

$$\frac{P_1}{T_1}+\frac{P_2}{T_2}<0, \tag{22}$$

which is essentially the Clausius inequality from thermodynamics.

Under what conditions can useful work be obtained from this system? In other words, we wish to determine the conditions under which the following inequality holds:

$$U=(P_1+\gamma P_2)>0. \tag{23}$$

Figure 2 shows the regions determined by inequalities (22) and (23) in the plane of the variables P_1 and P_2. The region of P_1 and P_2 values which are allowed is the region below the line $P_2 = -(T_2/T_1)P_1$. The region of P_1 and P_2 values which correspond to a positive work U, on the other hand, is the region above the line $P_2 = -P_1/\gamma$. The intersection of these regions determines that set of permissible values of P_1 and P_2 for which positive work is performed on external bodies. The following cases are possible here: (1) $T_2 > T_1$. In this case, positive work is performed in the region with

$$P_2>0, \quad P_1<0,$$

as can be seen from Fig. 2. In other words, the cold body (medium 1) obtains heat ($P_1 <0$), and the hot one gives it up. Such a situation should be regarded as normal.

(2) $T_1 > \gamma T_2 > T_2$. In this case, positive work is performed in the region

$$P_1>0, \quad P_2<0.$$

Again, the situation is normal; i.e., the hot body (medium 1) gives up heat ($P_1 >0$), and the cold one acquires it.

(3) $T_2 < T_1 < \gamma T_2$. In this case, positive work corresponds to $P_2 >0$, $P_1 <0$; i.e., positive work is performed as the cold body (medium 2) cools down and as the hot one

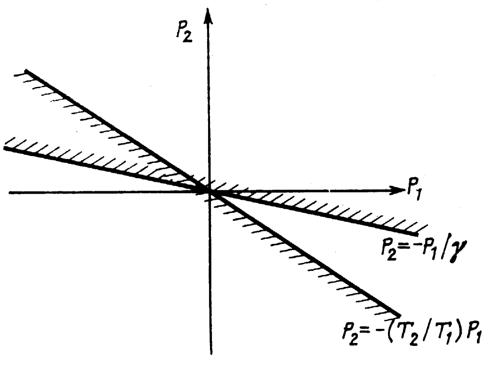

FIG. 2.

(medium 1) warms up. We call this situation "anomalous," but we do not mean to imply that the second law of thermodynamics is being violated. Essentially, work is being performed by virtue of a loss of mass from medium 2 as a result of thermal radiation ($P_2 > 0$). Losing mass, medium 2 tends to increase in velocity in order to compensate for the loss of momentum. If the uniform motion of medium 2 is to be sustained, this medium must be "held back" by an external force f. This is necessary if the force F due to the fluctuating electromagnetic field is inadequate for the purpose. As a result, medium 2 performs work on the source of the external force.

Let us look at some simple consequences and particular cases of the general results. We assume that the media 1 and 2 are absolutely cold, i.e., $T_1 = T_2 = 0$ [in this case we have $\Pi(T_1,\omega) = \Pi(T_2,\omega) = 0$]. Using (13) and (14), we then find from (16) and (17)

$$P_1<0, \quad P_2<0.$$

The meaning of this result is that both media warm up as a result of the relative motion. External objects must perform work if this is to happen.

We now consider the case in which media 1 and 2 are absolutely black: All the radiation incident on them is absorbed. Media 1 and 2 may be regarded as blackbodies only if they have unit permittivities and permeabilities: $\varepsilon_1 = \varepsilon_2 = 1$ and $\mu_1 = \mu_2 = 1$. In this case the function $M(\omega, \varkappa; \mathbf{u})$ takes the very simple form [as can be seen easily from (5) and (7)-(11)]

$$M(\omega, \varkappa; \mathbf{u}) = \begin{cases} \dfrac{1}{2}, & \varkappa<k, \\ 0, & \varkappa>k. \end{cases}$$

In other words, it depends on neither the width of the gap nor the relative velocity of the media. In this case the integrals in (16) and (17) can be evaluated easily; we find

$$P_1=P_v(T_1)-\gamma P_v(T_2), \qquad P_2=P_v(T_2)-\gamma P_v(T_1),$$

where

$$P_v(T)=\frac{\pi^2 T^4}{60c^2\hbar^3}$$

is the energy flux into vacuum from an isolated blackbody heated to the temperature T. For the work performed by the external force f we then find from (21)

$$U=-\frac{u^2}{1-u^2}P_v(T_1).$$

Using this result, we find the following expressions for the forces F and f from (2) and (19):

$$F=-\frac{\gamma u}{c}P_v(T_2), \qquad f=\frac{\gamma^2 u}{c}P_v(T_1).$$

Interestingly, the force F depends only on the temperature of the second medium, and the force f depends only on the temperature of medium 1. In this case we have $U<0$; i.e., external objects perform work on medium 2. Clearly, the work performed by the external force is an insignificant part of $P_v(T_1)$ at any realistic velocity of medium 2.

2. METALS WHICH ARE GOOD CONDUCTORS

In the general case of arbitrary permeabilities of the media, the heat fluxes P_1 and P_2 are given by very complex, opaque expressions. In this section of the paper we consider the case of metals which are good conductors, in which case the general expressions simplify substantially.

The electrodynamic properties of metals which are good conductors can be described by their surface impedances[7]

$$\zeta_1(\omega)=(\mu_1/\varepsilon_1)^{1/2}, \qquad \zeta_2(\omega)=(\mu_2/\varepsilon_2)^{1/2},$$

which lead to Leontovich boundary conditions. The surface impedances are small: $|\zeta_{1,2}|\ll 1$. The calculation of the fluxes P_1 and P_2 can thus be limited to the first nonvanishing order in the impedances of the metals. We make the further assumption that the width of the gap is small (much smaller than the Wien wavelength of the thermal radiation corresponding to the temperature of the hotter body). The calculations in this case are similar to those which were carried out in Ref. 4, in a study of heat exchange between metals which are good conductors. Omitting the calculations, we write the final expressions for the heat fluxes P_1 and P_2 in the case of thin gaps:

$$P_1=\frac{1}{2\pi^2 ac}\int_0^\infty d\omega \int_0^\pi d\theta \left\{\frac{\Pi(T_1,\omega)}{\omega}-\frac{\Pi(T_2,\tilde\omega)}{\tilde\omega}\right\}\omega Z(\omega,\tilde\omega), \tag{24}$$

$$P_2=-\frac{1}{2\pi^2 ac}\int_0^\infty d\omega \int_0^\pi d\theta \left\{\frac{\Pi(T_1,\omega)}{\omega}-\frac{\Pi(T_2,\tilde\omega)}{\tilde\omega}\right\}\tilde\omega Z(\omega,\tilde\omega). \tag{25}$$

Here

$$\tilde\omega=\gamma\omega(1-u\cos\theta),$$
$$Z(\omega,\tilde\omega)=\frac{\omega\zeta_1'(\omega)\tilde\omega\zeta_2'(\tilde\omega)}{\omega\zeta_1'(\omega)+\tilde\omega\zeta_2'(\tilde\omega)}. \tag{26}$$

There is a useful mathematical result which makes it possible to rewrite the results in (24) and (25) in various equivalent forms. We denote by $\Phi(\omega,\tilde\omega)$ an arbitrary function of the variables ω and $\tilde\omega$. We then have the identity

$$\int_0^\infty d\omega \int_0^\pi d\theta\, \Phi(\omega,\tilde\omega)=\int_0^\infty d\omega \int_0^\pi d\theta\, \Phi(\tilde\omega,\omega),$$

which is easy to prove by transforming from the integration variables ω and θ to the new variables $\tilde\omega$ and $\tilde\theta$:

$$\tilde\omega=\gamma\omega(1-u\cos\theta), \qquad \cos\tilde\theta=\frac{\cos\theta-u}{1-u\cos\theta}.$$

We now consider the case in which the velocity of medium 2 is small ($u\ll 1$). In this case it is easy to find the following expressions for the forces f and F from the general expressions (24) and (25), with the help of (2) and (20) (these expressions hold to first order in u):

$$f=-\frac{v}{4\pi ac^3}\int_0^\infty d\omega\left\{\Pi(T_1)\left(\frac{\omega}{\beta^2}\frac{\partial\beta_2}{\partial\omega}-\frac{2}{\beta}\right)+\Pi(T_2)\frac{\omega}{\beta^2}\frac{\partial\beta_1}{\partial\omega}\right\},$$

$$F=\frac{v}{4\pi ac^3}\int_0^\infty d\omega\left\{\Pi(T_1)\frac{\omega}{\beta^2}\frac{\partial\beta_2}{\partial\omega}+\Pi(T_2)\left(\frac{\omega}{\beta^2}\frac{\partial\beta_1}{\partial\omega}-\frac{2}{\beta}\right)\right\}. \tag{27}$$

Here

$$\beta_1=1/\omega\zeta_1', \quad \beta_2=1/\omega\zeta_2', \quad \beta=\beta_1+\beta_2.$$

We rewrite the force F, exerted on medium 2 by the fluctuating electromagnetic field, in the form

$$F=-\lambda v/a, \tag{28}$$

where, according to (27),

$$\lambda=-\frac{1}{4\pi c^3}\int_0^\infty d\omega\left\{\Pi(T_1)\frac{\omega}{\beta^2}\frac{\partial\beta_2}{\partial\omega}+\Pi(T_2)\left(\frac{\omega}{\beta^2}\frac{\partial\beta_1}{\partial\omega}-\frac{2}{\beta}\right)\right\}. \tag{29}$$

In this connection we recall a problem from hydrodynamics: the problem of the relative motion of two parallel planes which are separated by a gap of width a and between which there is an incompressible fluid with a viscosity coefficient η (see, for example, §17 in Ref. 8). We assume that one of these planes is at rest, while the other is moving at a velocity v parallel to the planes. We then have the following expression for the density of the tangential force F exerted on the moving plane:

$$F=-\eta v/a.$$

This expression has the same form as (28). The quantity λ is thus playing the role of a viscosity coefficient representing a viscosity caused by the fluctuating electromagnetic field. In contrast with hydrodynamics, however, the coefficient λ can in general be either positive or negative. It is simple to show that in the particular case in which the metals are at the same temperature this coefficient is always positive. To show this, we assume that the temperatures are equal ($T_1=T_2\equiv T$), and we find from (29)

$$\lambda=\frac{1}{4\pi c^3}\int_0^\infty d\omega\frac{\Pi(T)}{\omega}\frac{\partial}{\partial\omega}\frac{\omega^2}{\beta}.$$

Integrating by parts here, and noting that the terms which have been integrated vanish, we find the following expression for λ:

$$\lambda=-\frac{1}{4\pi c^3}\int_0^\infty d\omega\frac{\omega^2}{\beta}\frac{\partial\Pi(T)}{\partial\omega}$$

$$=\frac{\hbar^2}{4\pi c^3T}\int_0^\infty d\omega\frac{\zeta_1'\zeta_2'}{\zeta_1'+\zeta_2'}\frac{\omega^3e^{\omega/\omega_T}}{(e^{\omega/\omega_T}-1)^2}. \tag{30}$$

This quantity is evidently positive.

Let us assume, for example, that each metal can be described by the theory of a normal skin effect, i.e., that the impedances of the two metals are of the form

$$\zeta_1'=(\omega/8\pi\sigma_1)^{1/2}, \quad \zeta_2'=(\omega/8\pi\sigma_2)^{1/2},$$

where σ_1 and σ_2 are the conductivities of metals 1 and 2, respectively. For the coefficient λ we then find from (30)

$$\lambda=\frac{105}{128}\frac{1}{\sigma_1^{1/2}+\sigma_2^{1/2}}\frac{T^{7/2}}{2^{1/2}\pi c^3\hbar^{5/2}}\zeta_R(7/2), \tag{31}$$

where $\zeta_R(7/2)$ is the value of the Riemann zeta function $\zeta_R(x)$ at $x=7/2$. In particular, at room temperature, with conductivities $\sigma_1\sim\sigma_2\sim5\cdot10^{17}$ s^{-1}, we find $\lambda\sim3\cdot10^{-21}$ dyn·s/cm^2 from (31).

It was shown above that the limitations imposed on P_1 and P_2 allow the possibility of an "anomalous" situation in which useful work is extracted from the system while the cold moving body is cooling down, and the hot body at rest is warming up. This situation must be understood in the sense that it is possible to choose the permittivities and permeabilities of the media and the velocity of the relative motion in such a way that this anomalous situation would arise. Would this situation be attainable in a special class of media (good conductors)? If so, what restrictions would have to be imposed on the frequency dependence of the surface impedances? One can show, in particular, that if the impedances have a power-law frequency dependence, $\zeta_{1,2}'\sim\omega^s$ (with $s=1/2$, this dependence corresponds to the normal skin effect; with $s=2/3$, it corresponds to the anomalous skin effect), the work U is always negative. In other words, one cannot obtain useful work from the system.

Analysis of expressions (24) and (25) shows that the energy extracted from the system, $U=(P_1+\gamma P_2)$, will be positive if the function $Z(\omega,\tilde{\omega})$ lies primarily in the band

$$\theta_1<\theta<\theta_2, \tag{32}$$

where θ_1 and θ_2 are determined by

$$\cos\theta_1=u, \quad \cos\theta_2=\frac{1-T_2/\gamma T_1}{u},$$

and T_1 lies in the interval

$$T_2<T_1<\gamma T_2. \tag{33}$$

In particular, one can assume that $Z(\omega,\tilde{\omega})$ is always zero outside the band (32). We are then naturally led to ask whether it is possible to choose the impedances $\zeta_1'(\omega)$ and $\zeta_2'(\omega)$ in such a way that the function $Z(\omega,\tilde{\omega})$ has this property. We can achieve this result by choosing the impedances in the following way: $\zeta_1'(\omega)$ is nonzero in the band $\omega_{10}<\omega<\omega_{11}$, $\zeta_2'(\omega)$ is nonzero in the band $\omega_{20}<\omega<\omega_{21}$, and the boundaries of these bands satisfy the inequalities

$$\frac{\omega_{11}}{\gamma}<\omega_{20}<\omega_{21}<\frac{T_2}{T_1}\omega_{10}.$$

We see that the following condition must hold:

$$\omega_{11}<\frac{\gamma T_2}{T_1}\omega_{10}. \tag{34}$$

On the other hand, we have $\omega_{10}<\omega_{11}$, but by virtue of (33) inequality (34) can nevertheless be satisfied. This example shows that if this anomalous situation is to be realized the surface impedances of the metals must have some specially matched frequency dependences.

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[1)] In this paper, we are assuming an expansion in Fourier integrals with a time factor $\exp(-i\omega t)$ for all quantities.

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