# Creation, annihilation and solitonlike traversal of clusters of vertical Bloch lines in ferromagnet domain walls

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Discussed below are the results of a numerical solution of a system of abbreviated Slonczewski equations which describe the dynamics of interacting clusters of vertical Bloch lines in an isolated domain wall in a film of a uniaxial ferromagnet of high perpendicular anisotropy. It is shown that, depending on the relationship between the parameters, when the clusters collide, they can annihilate each other or create new ones. The clusters are also observed to pass through each other without distorting their shape (solitonlike traversal).

## INTRODUCTION

Relatively simple mathematical models have always attracted the attention of theoreticians, since they permit a detailed understanding of the basic mechanisms of complicated phenomena. At the present time, active use of numerical experiments makes it possible to consider as basic (very simple) models even some that until recently were not included among traditional ones.

This paper gives a discussion of the results of a numerical analysis of a system of truncated Slonczewski equations,[1,2] a mathematical model describing the dynamics of an isolated domain wall (DW) in films of a uniaxial ferromagnet of high transverse anisotropy (so-called CMD materials). It was found that this model, on the one hand, leads to conclusions consistent (not only qualitatively, but often quantitatively as well) with the results of physical experiments,[3-6] and on the other hand, is a generator of new solitonlike solutions whose behavior depends appreciably on the relationship of the parameters of the model.

Interest in the dynamics of vertical Bloch lines (VBL)—a 1-*D* boundary between DW subdomains having opposite polarization—began to increase after Konishi's[7] constructive proposal to use a pair of VBL instead of cylindrical magnetic walls (CMD) as the carrier of information on magnetic memory systems.

As is well known, monopolar VBL have a tendency to collect into clusters.[1] According to theory, a static cluster produces a very slight distortion of the DW,[2] and it is only during motion along the boundary that the cluster can produce a visually observable distortion of the DW, whose displacement can be used to follow the displacement of the cluster. The velocity of the cluster and the magnitude of the accompanying deflection of the DW depend on the number of VBL it contains.[3,4] Methods of high-velocity photography make it possible to observe the interaction of different clusters when they counterstream or move together.[5,6,8]

## TRUNCATED SLONCZEWSKI EQUATIONS ALLOWING FOR A NONLOCAL MAGNETOSTATIC INTERACTION OF VBL

A theoretical discussion of VBL in a CMD film is based on the truncated Slonczewski equations, derived by averaging the complete equations in film thickness $h$, assuming a slight twisting of the DW (a small change in azimuthal angle with film thickness). A correct allowance for the magnetostatic interaction of VBL within a cluster and between the clusters makes it necessary to keep in the equations the nonlocal term $H_x^{cf}$ characterizing the magnetic field component along the DW, due to a nonuniform distribution in azimuthal angle $\varphi$. In dimensionless variables, the equations are[2]

$$\alpha\frac{\partial q}{\partial t}+\varepsilon\frac{\partial\varphi}{\partial t}=\frac{1}{2}\frac{\partial^2 q}{\partial x^2}+H_z-H'q-\varepsilon\frac{\partial\varphi}{\partial x}\cos 2\varphi,$$

$$-\frac{\partial q}{\partial t}+\alpha\varepsilon\frac{\partial\varphi}{\partial t}=\frac{\varepsilon}{2}\frac{\partial^2\varphi}{\partial x^2}-\frac{1}{2}\sin 2\varphi-\frac{\pi}{2}(H_x+H_x^{cf})\sin\varphi+\frac{\pi}{2}H_y\cos\varphi+\frac{\partial q}{\partial x}\cos 2\varphi. \quad (1)$$

Here $x$ and $q(x,t)$ are the coordinate along the DW and the amount by which the DW deviates from the equilibrium position $y=0$, both normalized to the characteristic length of the magnetic material; $\varphi(x,t)$ is the azimuthal angle of magnetization at the DW center; $\alpha$ is a dimensionless damping parameter; $\varepsilon=(2Q)^{-1}$ is a dimensionless small parameter ($Q$ being the quality factor); $t$ is the time normalized to the quantity $T_0=(4\pi M\gamma)^{-1}2Q$, where $\gamma$ is the gyromagnetic ratio and $M$ is the saturation magnetization; $H_x$, $H_y$, $H_z$ are the components of the external field, and $H'$ is the displacement field gradient. All the fields are normalized to the quantity $4\pi M$. The expression for $H_x^{cf}$ is in the form of an improper integral:

$$H_x^{cf}(x,t)=-\frac{\varepsilon}{2h}\int_{-\infty}^{\infty}d\xi\frac{\partial}{\partial\xi}\cos\varphi(\xi,t)\frac{[(x-\xi)^2+h^2]^{1/2}-|x-\xi|}{x-\xi}. \quad (2)$$

It is easy to show[2] that in the absence of external fields, i.e., for $H_x=H_y=H_z=0$, the functions

$$q(x)=0,\quad \varphi(x)=2\ \text{arc. tan}\exp[p(2/\varepsilon)^{1/2}x]+\pi n$$

($p=\pm 1$ being the polarity of the VBL, and $n\in Z$) are the leading term of the solution, asymptotic in $\varepsilon$, of the system of equations (1). The field (2) corresponding to this solution for $h\gtrsim 1$ is

$$H_x^{cf}=\pi\left(\frac{\varepsilon}{2}\right)^{1/2}ps\sinh\left[\left(\frac{2}{\varepsilon}\right)^{1/2}x\right]\cosh^{-2}\left[\left(\frac{2}{\varepsilon}\right)^{1/2}x\right],$$

where $s=\text{sign}\cos\varphi(0)$.

This field offsets the repulsion force of exchange origin arising from two VBL of the same polarity coming closer together to a distance $a\sim\sqrt{\varepsilon}$, so that the expression $\Sigma_{i=1}^{n}\varphi(x-a_i)$, with $a_{i+1}-a_i\sim\sqrt{\varepsilon}$, as shown by numeri-

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cal modelling, is also an asymptotic solution of the initial system (1) and according to the physical terminology, corresponds to a cluster of $n$ VBL.

In this work, the numerical solution of the system (1) was performed by means of an implicit difference scheme using the matrix dispersion method. In view of the presence of an integral term in the second equation, a numerical solution of this system requires a significantly longer time than does the solution of an ordinary system of Slonczewski equations. In the calculation using Eq. (2), the integration near the singular point involved the use of difference formulas derived by expanding a part of the integrand as a Taylor series at this point, and Simpson's formula was used on the other portions. It was assumed that at the boundaries of the calculation interval in $x$, the derivatives $\partial q/\partial x$ and $\partial \varphi/\partial x$ were zero. The results, obtained in the calculations by this method for a statistical pair of VBL, are in good agreement with the result of Ref. 9, where a different calculation method was used to study static VBL clusters. All the calculations in this work were performed for the parameter values $\varepsilon = 0.1, \alpha = 0.2, h = 4, H' = 0.1$ for zero $H_x$ and $H_y$. Initially all the VBL in the cluster were located at the same distance from each other, equal to the distance between the VBL in the pair in the static case for the given parameter values. Allowing for the magnetostatic interaction of the clusters through the field $H_x^{cf}$ makes it possible (in contrast to the results given in Ref. 4) to follow the cluster interaction process in detail.

## RESULTS OF NUMERICAL EXPERIMENTS

When a DW moves under the action of an $H_z$ field, the clusters begin to move along the boundary owing to the gyrotropic force [the term $\partial q/\partial t$ in the second equation of the system (1)] acting on them. Since the presence of the DW-stabilizing $H'$ gradient causes rapid deceleration of the motion of the DW in a constant displacement field, the numerical experiment involved the consideration of a field $H_z$ that changed linearly with time and provided for nearly uniform motion of the DW outside the region containing the VBL. Note that all the results given in this work were obtained for DW velocities from 0.05 to 0.27; this is lower than the Walker velocity, which in dimensionless variables is 0.5. Obviously, the VBL velocity increases with the velocity of the DW, and below we discuss the velocities of these clusters.

Figure 1 shows the curves $q(x)$, $\varphi(x)$, as well as the form of $H_x^{cf}$ for a cluster consisting of six VBL and moving steadily from right to left at a velocity $v = 0.29$. The velocity of the DW is $u = 0.05$. It is evident that the field $H_x^{cf}$ has a fairly complicated form, but the number of points where this field is zero is equal to the number of VBL in the cluster. As the velocity increases, the average distance between the VBL and hence the total extent of the cluster decrease somewhat. Note that in this case, as well as those discussed below, the asymmetry of the DW deflection accompanying the cluster is not very distinct. This is due to the fairly large value of the gradient $H'$, which, together with $\alpha$ and $v$, is shown by theoretical analysis to exert an appreciable influence on the asymmetry. Thus, for $H' \gg \alpha^2 v^2$, which corresponds to the case under consideration, the shape of the deflection is nearly symmetric. For the same values of $\alpha$ and $v$ and $H' = 0.01$, the lengths of the leading and trailing edges will differ severalfold. It is the smallness of the gradient $H'$ that accounts for the marked asymmetry of the DW deflection, observed in experiments performed by Chetkin's group.[3–6]

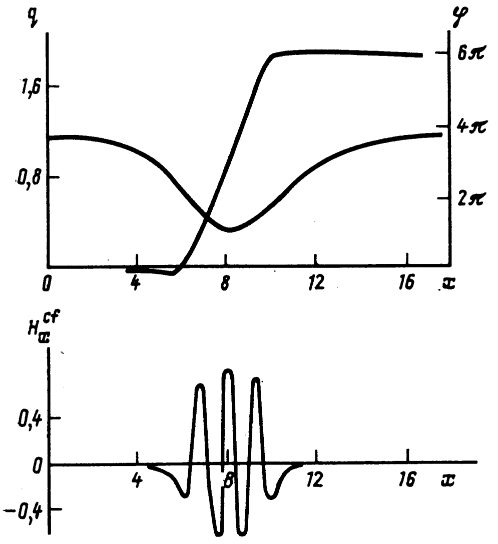


FIG. 1.

Let us consider the interaction of two clusters of opposite polarity, consisting of two and four VBL and initially located at a distance of ~10 (the position of the cluster taken to be the average value of the coordinates entering into its VBL). During the motion of the DW in the negative direction, the clusters begin to come closer together (Fig. 2). The smaller cluster moves faster, and prior to collision, the veloc-

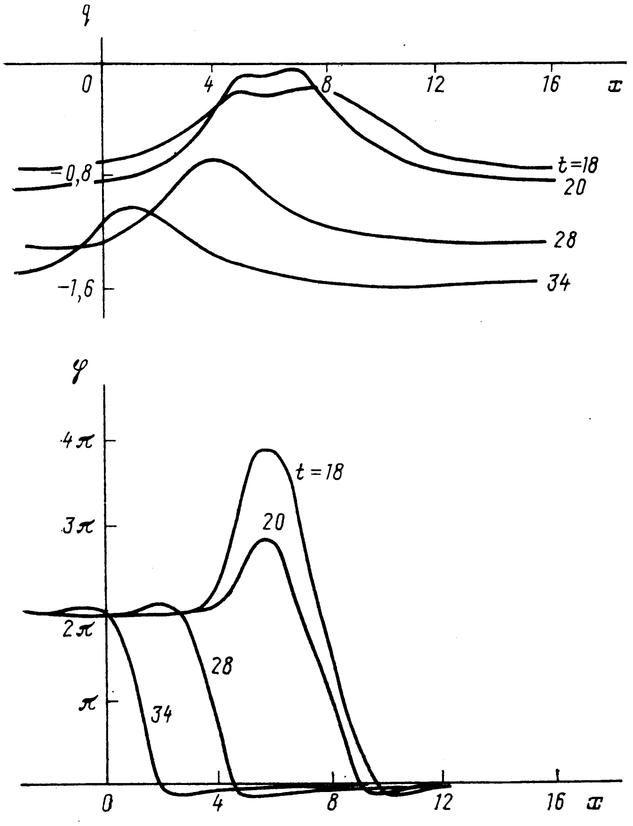


FIG. 2.

ities are approximately 0.25 and 0.35. The interaction then involves annihilation of the smaller cluster with part of the VBL of the large cluster, and as a result, there remains a cluster consisting of two VBL moving in the same direction in which the large cluster was moving prior to collision. A similar result was observed experimentally in Ref. 6. However, at cluster velocities approximately twice as high (0.57 and 0.78), the clusters interpenetrate, and after the collision, we again have clusters consisting of two and four VBL, but now they are moving away from each other.

As was shown by the calculations, if two identical clusters, each consisting of four VBL, are oppositely directed, then at low velocities they annihilate completely, and at high velocities they also pass through each other. At the location of the cluster interaction, a particularly substantial deviation of the DW [$\sim O(1)$] takes place. We note that such behavior of the solutions apparently is not directly related to the presence of the integral term in the system (1). Thus, Ref. 10 discussed the Slonczewski equations in the absence of a DW-stabilizing gradient, ignoring the magnetostatic interaction, and at the same time, owing to a special selection of the initial conditions, solitary-wave type structures arose in the DW that, depending on the velocity, either annihilated or traversed each other like solitones.

However, the examples discussed above still do not describe all the possibilities. Let us consider in more detail the opposite motion of two pairs of VBL. A characteristic feature of the solutions of the system (1) is the fact that in view of the presence in the equations of nonlinear terms containing the first derivatives with respect to $x$, especially the term containing $\partial\varphi/\partial x$, the velocities of clusters of opposite polarity containing the same number of VBL will be somewhat different. For Fig. 3, corresponding to the opposite motion of two pairs of VBL, the velocities of the left-hand and right-hand clusters prior to collision are 0.67 and 0.56, respectively. It is evident that the deviation of the DW for the left-hand cluster slightly exceeds the corresponding deviation for the right-hand cluster, while on the contrary, the range of variation of angle $\varphi$ for the left-hand cluster is somewhat smaller than for the right-hand cluster. During the interaction, the clusters traverse each other only partially, and as a result, by $t = 19$ two single VBL of opposite polarity have formed which move away from each other. At the same time, repolarization of the boundary takes place. It is evident that if the motion of the DW ceases, these VBL will eventually annihilate each other. This type of pair interaction is characteristic only of a certain range of velocities.

Studies have shown that if the pairs approach at a relative velocity $v_{rel} \leqslant 0.90$, complete annihilation of the clusters takes place, while at $v_{rel} = 1.35$, they already pass completely through each other. The dynamics in the latter case resembles the pattern, known from soliton theory, of the two-soliton solution which corresponds to noninteracting solitons at $t \to \pm\infty$. As the velocities increase further, not only soliton-like interaction of the clusters, but also the formation of new VBL becomes possible. Thus, for a relative velocity $v_{rel} = 1.53$, after the collision three VBL of opposite polarity that move away from each other are formed in each case. The maximum deviation of the DW during the collision will be greater than when the pairs simply pass through each other.

Since at the same DW velocity, clusters containing a smaller number of VBL move faster, a study can be made of what will occur in a collision of two clusters during unidirectional motion. Consider clusters of the same polarity, consisting of two and four VBL, moving in the same direction at velocities of 0.46 and 0.34. In this case, the DW velocity is 0.05. Figure 4a shows the positions of the VBL included in

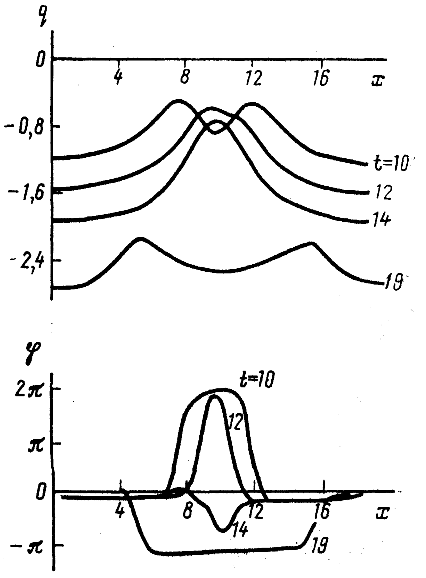


FIG. 3.

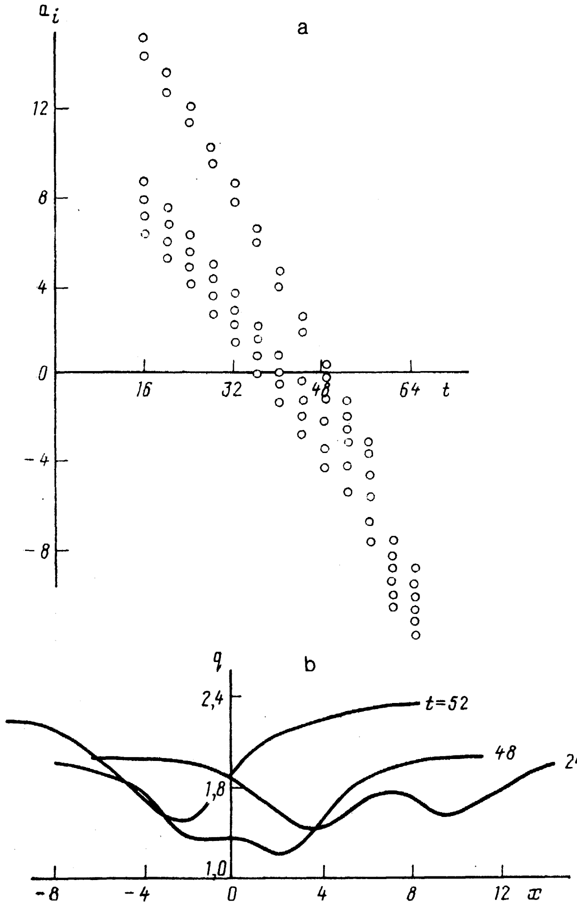


FIG. 4.

the clusters, at different times, starting with $t = 16$. When the clusters come closer together after some vibrations and displacements of the VBL relative to each other, a chain of six VBL is formed that will move in the previous direction. Figure 4b shows how the deflection of the DB changes in this case. The result of the interaction does not change qualitatively as the velocities approximately double. A similar interaction was observed in Ref. 8. In Ref. 5, a result was obtained in the case where a small cluster, catching up with a large one, seems to traverse it and then is found to be in front, gradually moving away. As is evident from Fig. 4, at the instant of interaction of the clusters there is a definite tendency in this direction, but the difference between the velocities is insufficient to permit the detachment of a tiny cluster in front.

## CONCLUSIONS

1. This system of Slonczewski equations is not in the class of completely integrable systems, but has solitonlike solutions for a certain relationship of the parameters. For other parameters, there exist solutions corresponding to the creation and annihilation of solitons.

2. The described effects of VBL cluster interaction are weakly dependent on the terms which are present in the Slonczewski equations and which contain the first spatial derivatives. However, these terms cause a certain difference between the absolute values of the velocities of clusters of opposite polarity, moving along the DW at the same DW velocity. This also leads to a slight drift of a VBL pair under action of a periodically changing displacement field.

3. The results of the numerical experiment performed are in good agreement with the observations made in physical experiments of Refs. 3–6.

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