Kinematic analysis of cosmological models with rotation

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We study observational manifestations of rotation in the class of spatially homogeneous shearless cosmological models. We show that pure rotation does not lead to causal anomalies and to parallax effects, nor to anisotropy of the microwave relict radiation. We obtain exact solutions for isotropic geodesics for a nonstationary model of the Gödel type. We find that cosmological rotation causes rotation of the photon-polarization plane.

1. INTRODUCTION

The natural physical idea that the universe (metagalaxy) is subject to global rotation in addition to expansion has repeatedly attracted researchers' interest.[1-7] Principal attention has been paid to the dynamic aspect of the construction of models with rotation, i.e., to a search of exact or approximate solutions of the gravitational equations. Much less studied is the kinematic aspect of such cosmologies, i.e., possible observational manifestations of cosmic rotation. Here the research was confined mainly to analysis of the only (but of fundamental importance) problem of the influence of the rotation on the anisotropy on the microwave relict radiation (MRR).[4,5,8,9] So far, however, only rotating models with shear were always considered, so that observational effects of "pure" rotation could not be separated.

We present here a detailed analysis of the kinematics of shear-free cosmological models with rotation. This task, which is also of independent interest, is a necessary preliminary step in the construction of viable rotating cosmologies that do not contradict the basic observational data. The dynamic realization of such models as solutions of gravitational equations with realistic material sources will be considered separately.

2. GEOMETRIC PROPERTIES AND METRIC OF SPACE-TIME

In accord with contemporary observation data, we considered the nearest expansions of standard Friedmann models, with preservation of the main features—spatial homogeneity and complete causality. We choose the space-time matrix in the form

$$dS^2=dt^2-2Rn_i dx^i dt-R^2\gamma_{ij}dx^i dx^j, \tag{2.1}$$

where t is the cosmological time, $R = R(t)$ is a scale factor, x^i are 3-space coordinates,

$$n_i=\mu_a e_i{}^{(a)}, \tag{2.2}$$

$$\gamma_{ij}=\lambda_{ab}e_i{}^{(a)}e_j{}^{(b)}. \tag{2.3}$$

Here μ_a, λ_{ab}, $a, b = 1,2,3$ are constants, ($\det \lambda_{ab} \neq 0$).

We assume that the space-time (2.1) admits of a three-parameter group of motions that acts simply transitively on hypersurfaces $t = \text{const}$; $e^{(a)} = e_i^{(a)}dx^i$ are the corresponding invariant 1-forms, and their Lie derivatives with respect to Killing fields $\xi^{\mu}_{(a)}$ are equal to zero; $L_{\xi(a)}e^{(b)} = 0$.

It is known that such manifolds are Bianchi-classified in accordance with the values of the structural constants in the commutator $[\xi_{(a)}\xi_{(b)}] = C^{c}_{\cdot ab}\xi_{(c)}$.

The class of models of (2.1) is quite extensive, contains both "open" and "closed" models, and admits of various topological structures of the universe.

Let us find, as usual, the main kinematic parameters of the model (2.1), determined for co-moving matter characterized by a 4-velocity $u^\mu = \delta^\mu_0$. For the rotation tensor we obtain

$$\omega_{\mu\nu}=\begin{cases}\omega_{0\mu}=0,\\ \omega_{ij}=-\dfrac{R}{2}\hat{C}^{k}_{\cdot ij}n_k.\end{cases} \tag{2.4}$$

the shear tensor is trivial $\sigma_{\mu\nu} = 0$; the volume expansion is $\theta = 3\dot{R}/R$. Here

$$\hat{C}^{k}_{\cdot ij}=e^{k}_{(a)}(\partial_i e_j{}^{(a)}-\partial_j e_i{}^{(a)}).$$

The explicit form of the structure constants C^{a}_{bc}, of the non-holonomy objects $\hat{C}^{k}_{ij}$, and also of $\xi_{(a)}$ and $e^{(a)}$, are given in Ref. 10.

3. FUNDAMENTAL PHYSICAL PROPERTIES OF SHEAR-FREE MODELS WITH ROTATION

It is known that in many models with rotation (for example, the classical Gödel solution) there exist closed time-like curves that can lead to causality violation. Let us show that in the considered class of metrics (2.1) the condition that ensures a correct causal structure of space-time is that the matrix λ_{ab} be positive-definite.

In fact, let the closed curve $x^\mu(s)$, $0\leqslant s\leqslant 1$, $x^\mu(0) = x^\mu(1)$ be everywhere timelike, i.e., for arbitrary s,

$$g_{\mu\nu}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}>0.$$

We choose a value of the parameter s_0 for which[11] $dt/ds = 0$ (such a point exists by virtue of the assumption that the curve is closed). We now have for (2.1)

$$g_{\mu\nu}\left(\frac{dx^\mu}{ds}\frac{dx^\nu}{ds}\right)\Bigg|_{s=s_0}=-R^2\lambda_{ab}e_i^{(a)}e_j^{(b)}\frac{dx^i}{ds}\frac{dx^j}{ds}.$$

This expression is always negative for $\lambda_{ab} > 0$, without contradicting the initial condition that $x^\mu(s)$ is time-like.

In particular, for the classical Gödel metric

$$ds^2=dt^2-2e^{x/2}dtdy-(dx^2-{}^1/{}_2e^xdy^2+dz^2) \tag{3.1}$$

we have

$$\lambda_{ab}=\mathrm{diag}(1,-{}^1/{}_2,1),$$

and it is this which leads to the existence of time-like curves. It is important to emphasize that the presence or absence of

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causality is not connected in any way with the presence or absence of cosmic rotation.

The most important test in relativistic cosmology is the reliably established experimental fact that the MRR is isotropic. We shall consider this problem traditionally in the geometric-optics approximation, when the light rays are null-geodesic

$$k^\mu \nabla_\mu k^\nu = 0, \quad k^\mu k_\mu = 0, \tag{3.2}$$

where $k^\mu = dx^\mu(\lambda)/d\lambda$, λ is an affine parameter.

The dependence of the radiation frequency on the relative motion of the source and receiver is determined by the rad shift Z (Ref. 12)

$$1+Z = \frac{(k_\mu u^\mu)_e}{(k_\mu u^\mu)_0}, \tag{3.3}$$

where the subscripts "e" and "0" denote respectively the space-time points of the source and observer. Assuming, as usual that the MRR has an absolute-blackbody spectrum,[13] we obtain for the radiation temperature the equation

$$T_0 = \frac{T_e}{1+Z} = T_e \frac{(k_\mu u^\mu)_0}{(k_\mu u^\mu)_e}.$$

In the general case, the recorded temperature T_0 depends on the observation direction via Z [Eq. (3.3)], as noted indeed in Refs. 4, 5, 8, 9, and 12. In our present case, however, this is not so. Indeed, it is easily seen that the metric (2.1) has a nontrivial conformal Killing vector

$$L_{\xi_{\text{conf}}} g_{\mu\nu} = \varphi g_{\mu\nu}, \qquad \xi^\mu_{\text{conf}} = R u^\mu = R\delta_0{}^\mu, \tag{3.4}$$

where the conformal factor is $\varphi = 2\dot{R}$.

Since, obviously

$$k_\mu \xi^\mu_{\text{conf}} = \text{const}$$

is the first integral of Eqs. (3.2), we obtain immediately

$$R(t_e)(u^\mu k_\mu)_e = R(t_0)(u^\mu k_\mu)_0.$$

Consequently,

$$T_0 = T_e \frac{R(t_e)}{R(t_0)}. \tag{3.5}$$

Thus, the MRR is fully isotropic in the considered class of models with rotation, as also in the Friedmann cosmology.

This result does not contradict in any way the conclusions of Refs. 4, 5, and 8, where cosmologic models with shear were considered. The result (3.5) above demonstrates clearly the qualitative difference between effects of "pure" rotation and shear: whereas the latter always leads to anisotropy of the MRR, arbitrary rotation (not generated by shear) has no effect whatever on the angular distribution of the MRR temperature.

Note that the conformal Killing vector ensures also the absence of parallax effects that could impose, as shown in Refs. 14 and 15, serious restrictions on the size of the rotation. A recently proved[16] theorem states that the necessary and sufficient condition for the absence of parallax effects in cosmology is the existence of a conformal Killing vector proportional to the average 4-velocity u^μ of matter. This condition is satisfied for the metric (2.1) [see (3.4)].

4. GÖDEL COSMOLOGICAL MODEL

We have established in the preceding sections that the general geometric and physical properties of the class of metrics (2.1) agrees well with the principal data of observational cosmology. Namely, the universes described by (2.1) are spatially homogeneous and fully causal (for positive λ_{ab}), they contain no parallaxes, and the MRR is fully isotropic. The class (2.1) is quite extensive: it includes the Bianchi models II–IX, so that one can hope to construct a viable cosmology with rotation within the context of the considered approach.

To discern other specific rotation effects in cosmology it is more illustrative to make (2.1) more specific and proceed to a detailed consideration of some particular case. In our opinion, one of the most interesting models is the Gödel-type metric

$$ds^2 = dt^2 - 2\sigma^{1/2} R(t) e^{mx} dt dy - R^2(t)(dx^2 + k e^{2mx} dy^2 + dz^2), \tag{4.1}$$

where m, σ, $k \geqslant 0$ are constant parameters. This metric is a natural nonstationary generalization of the standard Gödel model,[3] and the condition $k \geqslant 0$ guarantees (see Sec. 3) the absence of closed timelike curves.

The absolute magnitude of the rotation

$$\omega = \left(\frac{1}{2}\omega_{\mu\nu}\omega^{\mu\nu}\right)^{1/2} = \frac{m}{2R}\left(\frac{\sigma}{k+\sigma}\right)^{1/2} \tag{4.2}$$

decreases as the universe expands.

The three Killing vectors for (4.1) are

$$\xi_{(1)} = \frac{1}{m}\frac{\partial}{\partial x} - y\frac{\partial}{\partial y}, \quad \xi_{(2)} = \frac{\partial}{\partial y}, \quad \xi_{(3)} = \frac{\partial}{\partial z} \tag{4.3}$$

and satisfy the commutation relations

$$[\xi_{(1)}, \xi_{(2)}] = \xi_{(2)}, \quad [\xi_{(1)}, \xi_{(3)}] = [\xi_{(2)}, \xi_{(3)}] = 0.$$

The metric (4.1) is therefore of type III in the Bianchi classification.

It is interesting to note that if m is small the 3-geometry of the hypersurfaces $t = \text{const}$ is close to the geometry of the Friedmann flat model (they coincide at $m = 0$). In the general case it is easily seen that the hypersurface $t = \text{const}$ is the direct product of a two-dimensional (xy) surface of constant negative curvature $-m^2$ and a straight line (the z axis) [here $R(t)$ introduces the scale for this product].

We choose at each point of the space-time (4.1) a local orthonormalized (Lorentzian) reference (tetrad) h^{a}_{μ} with components

$$h^{\hat 0}_0 = 1, \quad h^{\hat 2}_0 = -\sigma^{1/2} aR, \quad h^{\hat 1}_1 = h^{\hat 3}_3 = R, \qquad h^{\hat 2}_2 = aR(k+\sigma)^{1/2}, \quad a = e^{mx}. \tag{4.4}$$

As usual, $g_{\mu\nu} = h^{a}_{\mu} h^{b}_{\nu} \eta_{ab}$, where $\eta_{ab} = \text{diag}(+1, -1, -1, -1)$ is a Minkowski matrix. Here and hereafter a caret designates tetrad indices; $a, b \ldots = 0,1,2,3$.

5. ISOTROPIC GEODESICS IN GÖDEL-TYPE WORLD

Practically all the information we received in observational cosmology consists of various forms of electromagnetic energy (in the optical, radiofrequency, x-ray, and other bands). Therefore the most important task is an investigation of the structure of the isotropic (null) geodesics. Integrating (3.2), we thus obtain in fact exhaustive information on all the observable manifestations of global rotation.

Integration of the equations of null-geodesics is made much simpler by the existence of the three Killing vectors (4.3): we obtain directly three first integrals:

$$q_i = -\xi_{(i)}{}^{\nu} k_\nu, \quad i=1,2,3. \tag{5.1}$$

Hence, adding the isotropy condition, we have

$$k^1 = \frac{m}{R^2}(q_1 + y q_2), \tag{5.2}$$

$$k^2 = \frac{1}{kaR^2}\left(\frac{q_2}{a} - \sigma^{1/2} R k^0\right), \tag{5.3}$$

$$k^3 = \frac{q_3}{R^2}, \tag{5.4}$$

$$(Rk^0)^2 = \frac{k}{k+\sigma}\left[m^2(q_1+yq_2)^2 + \frac{q_2{}^2}{ka^2} + q_3{}^2\right]. \tag{5.5}$$

Using the fact that $k^0 = dt/d\lambda$, we eliminate from Eqs. (5.2)–(5.5) the affine parameter λ and change over to the variable t:

$$R\frac{dx}{dt} = \frac{m}{Rk^0}(q_1+yq_2), \tag{5.6}$$

$$R\frac{dy}{dt} = \frac{1}{ka^2}\left(\frac{q_2}{Rk^0} - \sigma^{1/2} a\right), \tag{5.7}$$

$$R\frac{dz}{dt} = \frac{q_3}{Rk^0}. \tag{5.8}$$

To solve this system we must know the function Rk^0. Note that it has already been defined by the isotropy condition (5.5), and consequently a general solution of the system (5.6)–(5.8) can be obtained without integrating the equations of the gravitational field [i.e., for arbitrary $R(t)$]. Indeed, for any realistic law of universe evolution one can in fact change to a new temporal variable

$$t \to \tau(t), \quad \tau = \int_{t_0}^{t} dt'/R(t')$$

such that

$$R\frac{d}{dt} = \frac{d}{d\tau}$$

and consequently R is completely eliminated from (5.6)–(5.8).

Before writing the exact solution of the system (5.6)–(5.8), we determine the initial conditions and introduce thus parametrization of null-geodesics passing through the observation point. As usual, taking spatial homogeneity into account, we assume that the observer coordinates are $P = (t = t_0, x = 0, y = 0, z = 0)$. It is now convenient to "number" the geodesics passing through the point P with the aid of the spherical angles θ and φ, which determine the light-ray direction in the locally Lorentzian basis of the observer at the point P. Let $k^a = h^a_\mu k^\mu$. Without loss of generality we put

$$k^{\hat 0} = 1, \ k^{\hat 1} = \sin\theta\cos\varphi, \ k^{\hat 2} = \sin\theta\sin\varphi, \ k^{\hat 3} = \cos\theta, \tag{5.9}$$

for $t = t_0$ and $x = y = z = 0$. The constants q_i can now be expressed in terms of the introduced angles. Using (5.1) and (5.9) we get

$$q_1 = \frac{R_0}{m}\sin\theta\cos\varphi, \tag{5.10}$$

$$q_2 = R_0(\sigma^{1/2} + (\sigma+k)^{1/2}\sin\theta\sin\varphi), \tag{5.11}$$

$$q_3 = R_0\cos\theta, \tag{5.12}$$

where $R_0 = R(t_0)$ is the value of the scale factor at the observation instant t_0.

Differentiating (5.5) and using (5.6)–(5.8), we obtain

$$Rk^0 = \frac{k}{k+\sigma}\left(\frac{\sigma^{1/2} q_2}{ka} + R_0\right). \tag{5.13}$$

Substituting next (5.13) in (5.5) we obtain the first integral of the system (5.6)–(5.8). With the aid of the last relation it is now easy to write the general solution.

When $q_2 = 0$, i.e., the initial directions are specified by angles that satisfy the condition

$$\sin\theta\sin\varphi = -\left(\frac{\sigma}{k+\sigma}\right)^{1/2},$$

the light-ray trajectories are of the form

$$x = \tau\left(\frac{k+\sigma}{k}\right)\sin\theta\cos\varphi, \tag{5.14}$$

$$y = \frac{\sigma^{1/2}}{m(k+\sigma)}\left\{\exp\left[-m\tau\left(\frac{k+\sigma}{k}\right)\sin\theta\cos\varphi\right] - 1\right\}, \tag{5.15}$$

$$z = \tau\left(\frac{k+\sigma}{k}\right)\cos\theta. \tag{5.16}$$

In the general case, for $q_2 \neq 0$, it is easy to introduce a function $\Phi(\tau)$ with an initial condition $\Phi(0) = \varphi$ that satisfies the differential equation

$$\frac{d\Phi}{d\tau} = -m\left(\frac{\left(\frac{\sigma}{k+\sigma}\right)^{1/2} + \sin\theta\sin\Phi}{1 + \left(\frac{\sigma}{k+\sigma}\right)^{1/2}\sin\theta\sin\Phi}\right). \tag{5.17}$$

The solution of the system (5.6)–(5.8) can then be written in the form

$$e^{-mx} = \frac{\sigma^{1/2} + (k+\sigma)^{1/2}\sin\theta\sin\Phi}{\sigma^{1/2} + (k+\sigma)^{1/2}\sin\theta\sin\varphi} \tag{5.18}$$

$$y = \frac{\sin\theta(\cos\Phi - \cos\varphi)}{m(\sigma^{1/2} + (k+\sigma)^{1/2}\sin\theta\sin\varphi)}, \tag{5.19}$$

$$z = \left(\frac{k+\sigma}{k}\right)\cos\theta\left\{\tau + \left(\frac{\sigma}{k+\sigma}\right)^{1/2}\left(\frac{\Phi-\varphi}{m}\right)\right\}. \tag{5.20}$$

Equation (5.17) can be integrated in terms of elementary functions and Φ is implicitly determined from the relation

$$\left(\frac{\sigma}{k+\sigma}\right)^{1/2}(\Phi-\varphi) - \frac{k}{k+\sigma}\int_{\varphi}^{\Phi}\frac{d\Phi'}{(\sigma/(k+\sigma))^{1/2} + \sin\theta\sin\Phi'}$$
$$= -m\tau, \tag{5.21}$$

where the form of the integral depends on the relative values of $\sin\theta$ and $[\sigma/(k+\sigma)]^{1/2}$.

6. ROTATION OF POLARIZATION OF ELECTROMAGNETIC RADIATION

In Sec. 5 we obtained the general structure of light rays in a Gödel-type universe (4.1). We shall obtain important additional information on electromagnetic waves in a rotating world by investigating the behavior of the polarization vector along an isotropic geodesic.

Polarization-rotation effects were previously investigated in detail for the case of ray propagation in the gravitational field of a rotating compact object[17-19] (see also the literature cited in these references). To our knowledge, an analogous problem has so far not been considered in cosmology with rotation.

The polarization vector f_μ is defined in the geometric-optics approximation as a unit spacelike vector orthogonal to the wave vector k^μ. It satisfies the equation of parallel shift along a geodesic

$$k^\mu \nabla_\mu f^\nu = 0, \quad k^\mu f_\mu = 0, \quad f^\mu f_\mu = -1. \tag{6.1}$$

It is not convenient to analyze the system (6.1) in the framework of the Newman–Penrose formalism.[18,20,21] We define an isotropic tetrad in respect of the local Lorentzian basis (4.4) in the form

$$\begin{aligned} l^a &= 2^{-1/2}(1, 0, 0, 1), \quad n^a = 2^{-1/2}(1, 0, 0, -1), \\ m^a &= 2^{-1/2}(0, i, 1, 0), \quad \bar{m}^a = 2^{-1/2}(0, -i, 1, 0). \end{aligned} \tag{6.2}$$

As usual, we have

$$l^a l_a = n^a n_a = m^a m_a = \bar{m}^a \bar{m}_a = 0,$$
$$l^a n_a = 1, \quad m^a \bar{m}_a = -1.$$

The nontrivial tetrad components of the Weyl tensor are

$$\begin{aligned} C_{\hat{0}\hat{1}\hat{0}\hat{1}} &= C_{\hat{0}\hat{2}\hat{0}\hat{2}} = -C_{\hat{2}\hat{3}\hat{2}\hat{3}} = -C_{\hat{3}\hat{1}\hat{3}\hat{1}} = {}^1/_2 C_{\hat{1}\hat{2}\hat{1}\hat{2}} \\ &= -{}^1/_2 C_{\hat{0}\hat{3}\hat{0}\hat{3}} = \frac{m^2}{6R^2}\left(\frac{k}{k+\sigma}\right). \end{aligned} \tag{6.3}$$

From this we obtain for the spinor components

$$\psi_0 = \psi_1 = \psi_3 = \psi_4 = 0, \quad \psi_2 = \frac{m^2}{6R^2}\left(\frac{k}{k+\sigma}\right). \tag{6.4}$$

This means that in accordance with the Petrov classification the metric (4.1) is of type D.

The spin coefficients for (4.1) in the gauge (6.2) are

$$\begin{aligned} &\bar{k} = \bar{\sigma} = \lambda = \nu = 0, \\ &\tau = -2^{-1/2}\frac{\dot{R}}{R}\left(\frac{\sigma}{k+\sigma}\right)^{1/2}, \quad \pi = 2^{-1/2}\frac{\dot{R}}{R}\left(\frac{\sigma}{k+\sigma}\right)^{1/2}, \\ &\rho = 2^{-1/2}\left(-\frac{\dot{R}}{R} + i\frac{m}{2R}\left(\frac{\sigma}{k+\sigma}\right)^{1/2}\right), \quad \mu = -\rho^*, \quad \varepsilon = -{}^1/_2\rho^*, \\ &\alpha = 2^{-1/2}\left(-\frac{\dot{R}}{R}\left(\frac{\sigma}{k+\sigma}\right)^{1/2} + i\frac{m}{R}\right), \quad \gamma = {}^1/_2\rho, \quad \beta = -\alpha^*, \end{aligned} \tag{6.5}$$

where the asterisk labels a complex conjugate.

It is interesting to note that in the gauge (6.2) the scalars (specified by ρ) that determine the expansion and rotation of zero congruence are directly connected (respectively) with the volume expansion θ and the space-time rotation ω(4.2).

Following Refs. 18 and 20, we note that at each point of a null geodesic there is defined a pair of vectors

$$a^\mu = (n^\nu k_\nu) l^\mu - (l^\nu k_\nu) n^\mu, \tag{6.6}$$

$$ib^\mu = (\bar{m}^\nu k_\nu) m^\mu - (m^\nu k_\nu)\bar{m}^\mu, \tag{6.7}$$

where $l^\mu = h^\mu_a l^a \ldots$. These vectors are spacelike in construction and are orthogonal to one another and to k^μ:

$$k^\mu a_\mu = k^\mu b_\mu = a^\mu b_\mu = 0,$$
$$a^2 = a^\mu a_\mu = -2(n^\nu k_\nu)(l^\mu k_\mu),$$
$$b^2 = b^\mu b_\mu = -2|(m^\mu k_\mu)|^2 = -2(n^\nu k_\nu)(l^\mu k_\mu).$$

They can therefore be used to specify (at each point) a polarization basis

$$E_1{}^\mu = \frac{a^\mu}{(-a^2)^{1/2}}, \quad E_2{}^\mu = \frac{b^\mu}{(-b^2)^{1/2}}. \tag{6.8}$$

In this basis the polarization vector is resolved in the form

$$f^\mu = (\cos\eta)E_1{}^\mu + (\sin\eta)E_2{}^\mu, \tag{6.9}$$

where the angle η depends on the position on the geodesic. Direct calculation shows that

$$\eta = \frac{m}{2}\left(\frac{\sigma}{k+\sigma}\right)^{1/2} z = \omega R z \tag{6.10}$$

i.e., the polarization-vector rotation angle [assuming $\eta = 0$ at the point P ($t = t_0$, $x = y = z = 0$)], on an arbitrary null geodesic at a point with a third coordinate z (and arbitrary x and y), is determined by the cosmic rotation ω.

7. CONCLUSION

Our analysis has shown that there exists an extensive class of cosmological models with rotation, whose properties do not contradict the basic observational data, viz., isotropy of the microwave background radiation, and absence of closed timelike curves and of parallax effects. It is important to note that these conclusions are independent of the magnitude of the cosmic rotation.

The results on the structure of isotropic geodesics contain complete information on the motion of light and permit calculation of all the corresponding observational effects of cosmology with rotation, for example standard cosmological tests: $m - Z$ (the visible quantity is the red shift) and $N - Z$ (calculation of the number of sources), etc. We emphasize especially the promise offered by observations of angular variations of the polarization properties of electromagnetic radiation. Equation (6.10) predicts the effect of rotation of the plane of polarization of light on the path between two points in space (with different z coordinates). Using the Kristian–Saks expansion technique,[12,13] we can rewrite (6.10) in the form

$$\eta = \omega r \cos\theta + O(Z^2) \tag{7.1}$$

where r is the distance to the source, θ the angle between the direction of the cosmic rotation and the position of the object on the celestial sphere, and Z is the red shift. It can be shown that in this form the effect of polarization rotation takes place not only for a model of the Gödel type (4.1), but also for the entire class of metric with rotation (2.1).

Such an effect can be observed by using data of polarization observations of extragalactic radio sources. It is known that, in accordance with universally accepted synchrotron theory of radio sources,[22,23] their radiation is partially linearly polarized, and the direction of the polarization vector is determined by the magnetic field of the object. Assuming as usual[24] that the magnetic field is as a rule parallel to the principal axis of an elongated source, the observation of polarization rotation can be reduced to measurement of the difference Δ between the position angle of the magnetic vector and the position angle of the principal axis of the radio source.

Birch[25,26] has recently analyzed a selection of 132 objects (from the catalogs 3*CR* and 4*C*) covering practically the entire celestial sphere, and established for Δ the presence of dipole anisotropy of type (7.1). The most probable explanation of the obtained anisotropy is assumed in Refs. 25 and 26 to be metagalaxy rotation, but there are no quantitative estimates. The theory developed above permits, for the first time, to determine on the basis of the observations of Refs. 25 and 26 the direction and magnitude of the cosmic radiation. We present a preliminary estimate: using least squares, Eq. (7.1) yields for the observations of Refs. 25 and 26, for the rotation direction in galactic coordinates

$$l=295°\pm25°, \quad b=24°\pm20°, \tag{7.2}$$

and

$$\frac{\omega}{H}=1{,}8\pm0{,}8, \tag{7.3}$$

where $H=(\dot{R}/R)_0$ is the present-day value of the Hubble constant. It is remarkable that even the rather rough estimate (7.2) agrees well with the direction obtained for the large-scale metagalaxy anisotropy obtained by other groups from independent (unconnected with polarization) observations.[27,28]

We have purposely confined ourselves to the kinematic aspect of the problem of constructing a cosmology with rotation, taking into account the fundamental character of the principal observational data. Note that several exact solutions for Gödel-type cosmological models are already known in general relativity theory.[29,30]

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