# Spectral distributions of the intensity and polarization in single-photon quantum transitions in a strong electromagnetic field

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The spectral profiles of the intensity and polarization of the radiation in spontaneous quantum single-photon transitions in a strong magnetic field are obtained by computer calculations. Effects of the radiation depolarization are discussed. The results can be used, on the basis of available observation data, for astrophysical estimates of the parameters of cosmic gamma-ray bursts.

## 1. INTRODUCTION

Study of photon emission and absorption in a strong magnetic field $B \lesssim B_c$ ($B_c = m^2c^3/e\hbar = 4.4 \cdot 10^{14}$ G) is of interest in view of the presence of such fields in neutron stars such as pulsars and gamma-burst sources (see, e.g., Refs. 1 and 2). In such fields, quantum transitions between Landau levels are the main processes of photon emission and absorption. Analysis and interpretation of the observation data calls for theoretical models of the intensity and polarization spectra of the radiation from an ensemble of particles (electrons and positrons) for which the elementary acts of interaction with photons are due to the transitions in question.

If the distribution function of the electrons (and positrons) is maintained constant by some excitation mechanism, one can speak of stationary synchrotron radiation. On the contrary, if the excitation time is much longer than the time of the particle transition from the excited to the ground state, synchrotron-cooling radiation[3–5] must be considered. Synchrotron radiation spectra are obtained by folding the distribution function of an ensemble of electrons over the excited levels with the spectra corresponding to individual single-photon acts transitions (in the case of stationary synchrotron radiation) or with the spectra of multiphoton transitions via intermediate states to the ground level (in the case of synchrotron-cooling radiation). Construction of theoretical models of the ensemble radiation calls therefore for a detailed study of the intensity and polarization spectra of the harmonics of the synchrotron radiation generated in individual quantum transitions. Investigations of the profiles of synchrotron-radiation harmonics are of independent interest in connection with the probable observation of the corresponding lines in gamma-burst spectra.[6]

The radiation line shape for a transition between the first-excited and the ground state was investigated for the relativistic case in Refs. 7 and 8. The influence of the form of the electron distribution function in the longitudinal momenta on the line shape was investigated in Ref. 9. In a strong magnetic field, an important role is played by quantum effects such as quantum recoil and transitions with spin flip. A strong influence on the formation of the spectra is exerted also by low population of the excited Landau levels. Under these conditions the form of the profile and of the line polarization in a quantum transition can differ significantly from the classical case $B \ll B_c$.

The object of the present study is the influence of quantum and relativistic effects on the profile and polarization of radiation in a strong ($B \lesssim B_c$) magnetic field for spontaneous single-photon quantum transitions from one arbitrary state to another (with account taken of the spins). Such a study is possible only by using relativistic covariant wave functions of an electron in a magnetic field, and the corresponding expressions for the probabilities of such transitions.[5,9,10]

## 2. EMISSIVITY OF RELATIVISTIC ELECTRONS (POSITRONS) IN A MAGNETIC FIELD

In a magnetic field, the energy of an electron (positron)

$$E_n = (1 + 2nB + p^2)^{1/2} \tag{1}$$

is determined by its longitudinal momentum $p$ and by the principal quantum number $n = 0, 1, 2...$ (here and below $\hbar = 1$, $mc^2 = 1$ and $B_c = 1$). Beside the quantum numbers $n$ and $p$, a pure quantum state $(n, s, p)$ is determined by the projection of the spin $s$ along the magnetic field. The energy $\omega$ and the photon emission direction $\mathbf{n} = \mathbf{k}/|\mathbf{k}|$ are characterized by a wave vector $\mathbf{k}$. Two polarization states of the photon can be considered in a strong magnetic field: with polarization vector $\mathbf{e}_\parallel$ in the $\mathbf{kB}$ plane and with a vector $\mathbf{e}_\perp$ perpendicular to this plane.

Let $dR(n,s \to m,s',\mathbf{k},\mathbf{e})/d\Omega$ be the rate of a spontaneous transition of an electron from a state $(n,s,p)$ into a state $(m,s',q)$, with emission of a photon of energy $\omega$ into a solid angle $d\Omega$ in the direction $\theta$, and let $f_{n,s}(p)$ be the distribution function of electrons in a state $(n,s)$ with longitudinal momenta $p$ [it is assumed that $\int f_{n,s}(p)dp = 1$]. The spectral density of the electron emissivity in the $\theta$ direction for a quantum transition $(n,s) \to (m,s')$, per particle in the state $(n,s)$, is then

$$\varepsilon(n, s \to m, s'; \omega, \theta, \mathbf{e}) = \int \frac{dR(n, s \to m, s'; \mathbf{k}, \mathbf{e})}{d\Omega} \delta(E_n - E_m - \omega) f_{n,s}(p)\, dp. \tag{2}$$

If $dR(n,s \to m,s';\mathbf{k},\mathbf{e})/d\Omega$ is given by Eq. (7) of Ref. 5, the right-hand side of (2) must be multiplied by $(1 - q\cos\theta/E_n)$, since integration over the emitted photon has been carried out in the cited equation with allowance for a $\delta$ function.

Photon emission by an electron is subject to the energy and momentum conservation laws:

$$E_n = E_m + \omega, \tag{3a}$$

$$p = \omega\cos\theta + q. \tag{3b}$$

Simultaneous solution of Eqs. (3) yields for the momentum

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$p$ two possible values that satisfy the condition of emission of a photon of energy $\omega$ in the direction $\theta$:

$$p_{1,2}=\frac{1}{2\omega\sin^2\theta}\{(E_{n0}{}^2-E_{m0}{}^2+\omega^2\sin^2\theta)\cos\theta$$
$$\mp[((E_{n0}+E_{m0})^2-\omega^2\sin^2\theta)((E_{n0}-E_{m0})^2-\omega^2\sin^2\theta)]^{1/2}\}. \quad (4)$$

These two solutions (4) exist only if $\omega<\omega_d$, where

$$\omega_d=(E_{n0}-E_{m0})/\sin\theta, \quad (5)$$

where $E_{l0}=(1+2lB)^{1/2}$ and $l=n,m$. If $\omega>\omega_d$, there are no real roots of $p$: for a transition $(n,s)\to(m,s')$ an electron with arbitrary momentum cannot emit in a specified direction a photon of energy higher than $\omega_d$. At $\omega=\omega_d$ the roots $p_1$ and $p_2$ are equal:

$$p_1=p_2=p_d=E_{n0}\operatorname{ctg}\theta. \quad (6)$$

Integration of (2) with allowance for the $\delta$ function leads to an emissivity

$$\varepsilon(n,s\to m,s';\omega,\theta,\mathbf{e})$$
$$=\sum_{k=1,2}\frac{dR(n,s\to m,s';\mathbf{k},\mathbf{e})}{d\Omega}\frac{E_nE_m}{|pE_m-qE_n|}f_{n,s}(p)\,|_{p=p_k}, \quad (7)$$

where summation over $k$ corresponds to the two possible momenta $p_{1,2}$. For $\omega<\omega_d$ the $\varepsilon(\omega,\theta)$ spectrum has a form determined by the distribution function in the longitudinal momenta. For $\omega=\omega_d$ the denominator in (7) vanishes and the emissivity becomes formally infinite, while at $\omega>\omega_d$ no photons are emitted (see Ref. 8).

It is known[11] that to take into account the finite lifetime of the excited state it is necessary to replace, in the first approximation, the $\delta$ function in (2) by the Lorentzian

$$L=\frac{1}{2\pi}\frac{R}{(E_n-E_m-\omega)^2+1/4R^2}. \quad (8)$$

Here $R$ is the natural width of the emission line

$$R=R(n,s,p=0)\frac{E_{n0}}{E_n}+R(m,s',q=0)\frac{E_{m0}}{E_m}, \quad (9)$$

and is expressed in term of the sum of the corresponding level widths $R(n,s,p=0)$ and $R(m,s',q=0)$ of the initial and final states. The level widths are defined in the rest system of the electron: the factors $E_{n0}/E_n$ and $E_{m0}/E_m$ in (9) are the result of a Lorentz transformation from the electron rest system into the laboratory system. The width of a level, in the units chosen above, is equal to the total rate of spontaneous departure from the corresponding state. This total rate is in turn a sum of the rates of transitions into all possible final states of both the electron (principal quantum number, spin) and photon (polarization, radiation direction):

$$R(n,s)=\int d\Omega\sum\frac{dR(n,s\to m,s';\omega,\theta,\mathbf{e})}{d\Omega}. \quad (10)$$

The summation is over $s'=\pm 1$, $m<n$, $\mathbf{e}=\mathbf{e}_\parallel,\mathbf{e}_\perp$. If the frequency of electron collisions with other particles or photons exceeds $R$, the Lorentzian profile width (8) is determined by this frequency.

With the finite lifetime of the state taken into account, the energy conservation law (3a) (which is explicitly expressed in (2) by a $\delta$ function) no longer holds exactly: the photon energy $\omega$ and the electron initial momentum $p$ are no longer connected by the solution (4). For $\omega<\omega_d$ the values of $p_1$ and $p_2$ correspond to the maximum of the Lorentzian (8): electrons with such momenta make the principal contribution to the emission of a photon of specified energy in a given direction.

## 3. EFFECT OF MAXWELLIAN DISTRIBUTION OF THE EMITTED PARTICLES ON THE LINE SHAPE

To investigate the line shape, we consider transitions between two pure states $(n,s)$ and $(m,s')$. It follows from (7) that the line shape is substantially influenced by the form of the electron distribution function in the longitudinal momenta, namely by the fractions of electrons with momenta $p_1$ and $p_2$ in the electron ensemble.

Let the electrons in the initial state $(n,s)$ be described by the thermal distribution function

$$f_n(p)=C_n(T)\exp[-E_n(p)/T], \quad (11)$$

where $C_n(T)$ is determined by the normalization condition. Without allowance for the natural level width, the line shape is specified by convolution of $f(p)$ with a $\delta$ function [Eq. (2)]. In the cases of the thermal function (11) this leads to a Gaussian profile with Doppler broadening. The characteristic electron-momentum scatter, $\Delta p=(2T)^{1/2}$, corresponds to a Doppler line width $G=\omega(\Delta p)-\omega(-\Delta p)$, where $\omega(p)$ can be obtained from the solution of Eqs. (3). For $nB\ll 1$ we have $G=(n-m)B\Delta p\cos\theta$. When the natural width is allowed for, the line profile acquires Lorentz wings (Fig. 1). The form of its central part depends on the ratio of the natural (or collisional) level width [e.g., (9)] to the Doppler broadening. In the relativistic limit this profile is asymmetric and has an abrupt break at $\omega>\omega_d$ (Fig. 1).

In the classical nonrelativistic limit the line has a Lorentz profile if $R>G$ or a Voigt profile with a Doppler center and Lorentz wings at $R<G$. In contrast to the classical case, in the relativistic treatment the line profile can be substantially influenced by the singularity at $\omega=\omega_d$. For a thermal distribution of the radiating electrons, the line profile in the Doppler center reaches its maximum at $p=0$, in which case

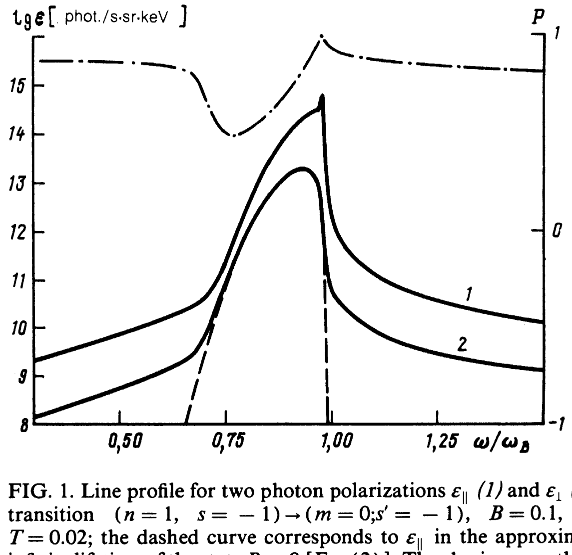


FIG. 1. Line profile for two photon polarizations $\varepsilon_\parallel$ *(1)* and $\varepsilon_\perp$ *(2)* in the transition $(n=1,\ s=-1)\to(m=0;s'=-1)$, $B=0.1$, $\theta=75°$, $T=0.02$; the dashed curve corresponds to $\varepsilon_\parallel$ in the approximation of infinite lifetime of the state $R=0$ [Eq. (2)]. The abscissas are the photon energies in units of $\omega_B=eB/mc$. The degree of linear polarization $P$ is shown by the dash–dot line.

the photon energy is

$$\omega_0=\omega(p=0)=\{E_{n0}-[(E_{n0}\cos\theta)^2+(E_{m0}\sin\theta)^2]^{1/2}\}/\sin^2\theta, \tag{12}$$

which corresponds at $nB \ll 1$ to $\omega_0=(n-m)B$. Since $\omega_d>\omega_0$ always [cf. (5) and (12)], at $D=\omega_d-\omega_0 \lesssim G$ the relativistic singularity influences the right-hand wing of the line (Fig. 1). The line becomes asymmetric, and the maximum of the spectral distribution of the photons diverges in general between $\omega_0$ and $\omega_d$. The relativistic singularity drops out as $\theta\to 90°$ or $\theta\to 0$: in the first case ($\omega_0\to\omega_d$ and $G\to 0$) the line has the Lorentz profile (8), and in the second ($p_d\to\infty$ and $\omega_d\to\infty$) this singularity cannot be seen because of the small contribution to the radiation from particles with momentum $p_d$; the line has therefore a Voigt profile with a Doppler center and Lorentz wings.

In the general case the line shape is determined by the relations between the Doppler broadening $G$, the Lorentz broadening $R$, and the distance $D=\omega_d-\omega_0$ between the maximum of the Doppler center and the position of the relativistic singularity. If the temperature of the particles [see (11)] is low ($R\gg G$) and the observation conditions are such that $R\gg D$, the line has a Lorentz profile. In Fig. 2, these values of $T$ and $B$ correspond to the region $A$. For $G>R\gg D$ (region $B$) the position of the relativistic singularity coincides with the maximum of the Doppler center—the line has a distinctive "toothlike" form (Fig. 1). For $G>D\gg R$ (region $C$ in Fig. 2) the line has a Doppler center whose right-hand wing is bounded by a relativistic singularity. The regions $B$ and $C$ correspond thus to the presence of a relativistic singularity on the line profile (see Figs. 1 and 3). For $G<D$ (region $D$) the relativistic singularity is located in the far right-hand wing and is therefore not seen, and the line has a Voigt profile. It should be noted that the onset of a relativistic singularity depends strongly on the observation conditions (on the angle $\theta$ between **k** and **B**)– the regions $B$ and $C$ shift towards higher temperature with decrease of the angle (Fig. 2).

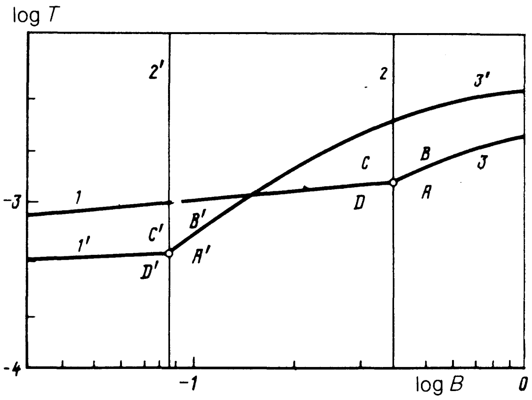


FIG. 2. Lines of equal values of the quantities $G(\theta,T,B)$, $D(\theta,B)$ and $R(B)$ (see the text) for the transition $(n=1, s=-1)\to(m=0, s'=-1)$. Abscissas—magnetic field in units of $B_c$, ordinates—temperature in units of $mc^2$. Curves: 1—$G=D$, 2—$R=D$, 3—$R=G$, angle $\theta=87°$ (1',2',3')— the same for the angle $\theta=88°$). The circles mark the equalities $R=G=D$. Region $A$—$R>D>G$, Lorentz line profile; $B$—$G>R>D$, Doppler line center, Lorentz wings—the position of the relativistic singularity coincides with the maximum of the Doppler center; $C$—$G>D>R$, a relativistic singularity is seen on the Gaussian profile; $D$—$D>R>G$; Gaussian line shape, the relativistic singularity is located in the far right-hand wing of the Doppler center and is not shown. Regions $A'$, $B'$, $C'$, $D'$ correspond to the regions $A$, $B$, $C$, $D$, but for $\theta=88°$.

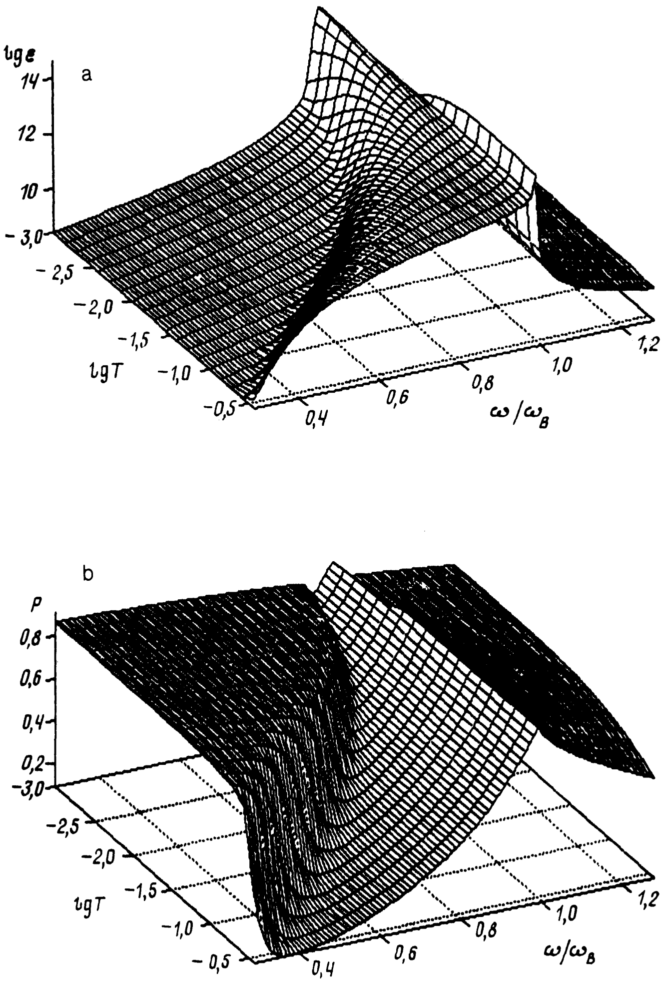


FIG. 3. Line profile for unpolarized radiation for the transition $(n=1, s=-1)\to(m=0, s'=-1)$, $B=0.1$, $\theta=75°$ (a) and profile of the degree of linear polarization for the same transition (b) as functions of the temperature.

Figure 3a shows the dependence of the emission line profile for the transition $(n=1,\ s=-1)\to(m=0,\ s'=-1)$ on the electron temperature at a fixed magnetic field $B$. At low temperature the line has a Doppler center and corresponds to region $D$ of Fig. 2. As the temperature rises, a relativistic singularity appears in the high-frequency wing, and the Doppler broadening increases in the low-frequency part of the profile (region $C$ of Fig. 2).

At fixed values of $B$ and $T$ corresponding to the condition $G>D\gg R$ (region $C$ in Fig. 2), the position $\omega=\omega_d$ of the relativistic singularity on the line profile depends on the initial and final numbers of the Landau levels. As already noted, for a singularity to set in it is necessary that the ensemble contain a sufficient number of electrons with momentum $p\sim p_d$, as follows from (6), an increase of the number $n$ leads to an increase of $p_d$ and $D$, and the condition $G>D$ may be violated. The best conditions for the onset of this singularity are therefore realized, for a given harmonic $\nu=n-m$, in transitions from the level $n=\nu$ into the ground state ($m=0$, $s'=-1$).

The spectral width of the relativistic singularity depends only on the widths of the initial and final levels [see (9)]. It is known that these widths increase with increase of

$n$ and $m$, therefore the "contrast" of the singularity on the line profile is decreased. The relativistic singularity in a given magnetic field will thus have maximum contrast on the line profile corresponding to the transition $(n=1, s=1) \rightarrow (m=0, s'=-1)$.

Of greatest interest is the study of an emission line profile that is made asymmetric by a relativistic singularity. Comparison of the observable spectral-line parameters with theoretical estimates of $G$ and $D$ makes it possible to determine the magnetic field and the temperature in the generation region. In addition, as will be shown below, it is precisely in this case that a characteristic singularity should also be present in the polarization spectrum.

## 4. POLARIZATION IN A LINE

The degree of linear polarization is defined as

$$P=(\varepsilon_{\parallel}-\varepsilon_{\perp})/(\varepsilon_{\parallel}+\varepsilon_{\perp}), \tag{13}$$

where $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ are the electron emissivities (2) for photons with respective polarizations $e_{\parallel}$ and $e_{\perp}$. Polarization in a line is influenced by several factors: 1) dependences of $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ on the type of transition (without or with spin flip); 2) dependences of $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ on the radiation direction $\theta$, and 3) dependences of $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ on the temperature of the radiating ensemble of electrons (positrons).

If $R \gg G, D$ (region $A$ in Fig. 2), the last effect is absent, and the polarization is constant along the line profile. Its degree is a maximum for the angle $\theta = 90°$ and decreases with decrease of this angle (nonmonotonically in some cases according to numerical calculations[5]). It is known that for transitions $(n,s) \rightarrow (m,s')$ between pure quantum states the line polarization has opposite signs for transitions without and with spin flip (electric and magnetic types of radiation, respectively).

If the line has for the $\theta$ direction a Doppler wing with spectral width $G$ (the condition $R \ll G$ is met), the main contribution to the radiation is made in this region by particles with momenta $p_1$ and $p_2$. For $D < G$ (region $B$ of Fig. 2) the high-frequency boundary of the Doppler center is a relativistic singularity at a frequency $\omega = \omega_d$, corresponding to a momentum $p_d(\theta) = p_1 = p_2$. In a reference frame $F'$ in which $p'_d = 0$, the radiation direction $\theta$ corresponds to the radiation direction $\theta' = 90°$. In this frame, the particles making the main contribution to the radiation at the relativistic-singularity frequency $\omega_d$ are at rest ($p'_1 = p'_2 = 0$).

It is known that for transitions between pure quantum states with equal spins $(n,s) \rightarrow (m,s)$, there is no radiation with longitudinal polarization $\varepsilon_{\parallel}$ in the 90° direction. On the contrary, for transitions with spin flip $(n,s) \rightarrow (m,-s)$, in the case $\theta = 90°$ there is no radiation with transverse polarization $\varepsilon_{\perp}$. It follows hence that there should be no relativistic singularity on the line profile for the corresponding radiation components ($\varepsilon_{\parallel}$ for transitions without spin flip, $\varepsilon_{\perp}$ with spin flip). Thus, the degree of linear polarization in the relativistic singularity is a maximum (Fig. 1) at 100% in the limit of infinitesimally small natural line width ($R = 0$).

In a Doppler line center, $\omega < \omega_d$, radiation in the direction $\theta$ is emitted by particles with momenta $p_1 < p_d$ and $p_2 > p_d$. In the reference frame $F'$ ($p'_d = 0$ and $\theta' = 90°$) introduced above these momenta correspond to $p'_1 < 0$ and $p'_2 > 0$. To determine the polarization of radiation from particles moving along the field in the direction $\theta'$ with momenta $p'_1$ and $p'_2$ we must transform to systems $F''_1$ and $F''_2$ in which these particles are at rest ($p''_1 = 0$ and $p''_2 = 0$ respectively), and determine the polarization of the radiation in those directions $\theta''_1$ and $\theta''_2$ to which $\theta'$ corresponds on the system $F'$. (The degree of linear polarization is not altered by a Lorentz transformation of the reference frame[12]). At $\theta' = 90°$ we have $90° < \theta''_1 < 180°$ and $90° > \theta''_2 > 0°$, and when the frequency is lowered, $\omega < \omega_d$, the absolute values of $p'_{1,2}$ increase and the angles $\theta''_{1,2}$ approach 180° and 0°, respectively. It is known (see, e.g., Ref. 5) that the difference between the intensities of radiation with different polarizations $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ decreases in this case.

The degree of polarization is thus decreased with decrease of frequency in the spectral region of the Doppler center. If the width $G$ of the Doppler center is large enough, the polarization can decrease to zero. With further decrease, the frequency goes outside the region of the Doppler center when $|p_1, p_2| > \Delta p$. A transition to a Lorentz-wing asymptote takes place then on the line profile, and the degree of polarization approaches the value corresponding to this asymptote.

The spectral dependence of the degree of the linear polarization of radiation of a quantum transition is illustrated by Fig. 3b. With rise of temperature, the maximum of the degree of linear polarization shifts from the frequency $\omega_0$ to the frequency $\omega_d$ of the relativistic singularity and approaches 100%. On the contrary, strong depolarization takes place in the low-frequency region of the Doppler center.

In the general case, no radiation is observed in spontaneous transitions $(n,s) \rightarrow (m,s')$ between pure quantum states. First, such a transition is simultaneously accompanied by a transition with spin flip, $(n,s) \rightarrow (m,-s')$, and it becomes necessary to sum over the final spin states. (No such problem arises for a transition into the ground state $(m=0)$, since the spin in this state can be only $s' = -1$.) Second, excitation of the initial pure state $(n,s)$ is usually accompanied by excitation of the state $(n,-s)$ and a spontaneous transition form the latter into the state $(m, s' = \pm 1)$ will also contribute to the radiation of frequency $\omega$ in the $\theta$ direction.

In the case of stationary synchrotron radiation, therefore, the intensity of the photon flux of the radiation line generated in the $n \rightarrow m$ transition can be determined from the equation

$$\varepsilon(n \rightarrow m; \omega, \theta) = \sum \eta(s)\, \varepsilon(n, s \rightarrow m, s'; \omega, \theta, \mathbf{e}), \tag{14}$$

where $\eta(s)$ is the probability of exciting the state $(n,s)$ with spin $s$ $(\eta(s) + \eta(-s) = 1)$, and the summation is over $s = \pm 1$, $s' = \pm 1$, $\mathbf{e} = \mathbf{e}_{\parallel}, \mathbf{e}_{\perp}$. The degree of linear polarization is described here by the expression

$$P(n \rightarrow m; \omega, \theta) = \frac{1}{\varepsilon(n \rightarrow m; \omega, \theta)} \sum_{\substack{s=\pm 1\\ s'=\pm 1}} \eta(s) P_{nm}(s, s')\, \varepsilon(n, s \rightarrow m, s'; \omega, \theta), \tag{15}$$

where

$$P_{nm}(s, s') = \frac{\varepsilon(n, s \rightarrow m, s'; \omega, \theta, \mathbf{e}_{\parallel}) - \varepsilon(n, s \rightarrow m, s'; \omega, \theta, \mathbf{e}_{\perp})}{\varepsilon(n, s \rightarrow m, s'; \omega, \theta, \mathbf{e}_{\parallel}) + \varepsilon(n, s \rightarrow m, s'; \omega, \theta, \mathbf{e}_{\perp})}.$$

If the spontaneous transitions are quite rapid and there is no stationary distribution in the excited levels (synchrotron-cooling radiation), the total number of radiation-line photons emitted over an infinite time is physically meaningful.[5] The spectral profile and the line polarization can be calculated in this case by using Eqs. (14) and (15), replacing in the definition (2) of $\varepsilon$ the transition probabilities per unit time $dR(n,s\rightarrow m,s';\omega,\theta)$ by the partial transition probabilities $dR(n,s\rightarrow m,s';\omega,\theta)/R(n,s)$, where $R(n,s)$ is the total rate (10) of deviation from the $(n,s)$ state. By $\eta$ is meant than the probability of various spin states at the initial instant of time $t=0$.

It follows from (15) that the linear-polarization spectrum of the harmonic $\nu=n-m$ of synchrotron radiation is expressed, in arbitrary cases of stationary synchrotron radiation or synchrotron-cooling radiation, in terms of functions $P_{nm}(s,s')$ that determine the polarization of radiation generated in transitions between pure states $(n,s)\rightarrow(m,s')$, and can be calculated numerically on the basis of the results of Ref. 5. These functions are crucial for the calculation of the polarization spectrum of synchrotron radiation with any distribution function of the radiating particles.

It is thus possible to calculate on the basis of the results, in the general relativistic case, the spectrum and polarization and synchrotron radiation of an ensemble of particles with arbitrary distribution function in both the momenta and the quantum levels. Radiation singularities observed for elementary transitions can be used, in the presence of observation data, for direct estimates of the temperature and magnetic field in the radiating region.


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