Scaling under conditions of the integral quantum Hall effect

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The scaling behavior of the conductivity tensor components of a two-dimensional electron gas in silicon MOS structures and in AlGaAs/GaAs heterostructures was investigated in the temperature range from 20 mK to 1.5 K and in magnetic fields up to 16 T. The existence of a separatrix in the flow line pattern in the $(\sigma_{xx},\sigma_{xy})$ plane was confirmed experimentally for the first time and a saddle point was detected. The flow line pattern was found to be asymmetric relative to the half-integral value $(h/e^2)\sigma_{xy}$ and nonperiodic along the σ_{xy} axis. The saddle point position varied not only from sample to sample, but even for different quantum levels in the same sample. The saddle point ordinate was much smaller than predicted theoretically. The existence of single-parameter scaling in the vicinity of the separatrix was confirmed. In the case of silicon MOS structures at the lowest Landau level and minimal temperatures the contributions of a single quantum state to the dissipative and nondissipative conductivities were proportional to one another.

INTRODUCTION

There have been several experimental investigations of the problem of scaling, i.e., of the dependences of the components of the conductivity tensor of a two-dimensional electron system on the size of a sample under the conditions of the integral quantum Hall effect.[1-9] The interest in this topic was triggered by the theoretical work of Pruisken *et al.*,[10,11] Khmel'nitskiĭ,[12] and Ando.[13] It was predicted in Refs. 10 and 12 that the number of the parameters that describe changes in the components of the conductivity tensors σ_{xx} and σ_{xy} with change in the dimensions of a sample is two. These parameters can be the conductivities σ_{xx} and σ_{xy} themselves. In this case a change in the dimensions of the system should result in motion of the point which represents the state of the electron system, in the $(\sigma_{xx},\sigma_{xy})$ plane in accordance with the equations

$$\partial\sigma_{xx}/\partial \ln L=\beta_{xx}(\sigma_{xx},\ \sigma_{xy}),$$
$$\partial\sigma_{xy}/\partial \ln L=\beta_{xy}(\sigma_{xx},\ \sigma_{xy}). \qquad (1)$$

Here, L is the system size, whereas β_{xx} and β_{xy} are functions which have to be found theoretically.

It follows from the system (1) that the lines describing the motion of a point in the $(\sigma_{xx},\sigma_{yy})$ plane and corresponding to different values of the occupancy factor cannot intersect. Figures 1a and 1b show two possible diagrams of such a system, characterized by stable points corresponding to the Hall plateau. They are distinguished by the fact that Fig. 1b has a saddle point and a separatrix passing through it so that the flow lines starting from points above the separatrix cannot drop below it. It follows from Ref. 14 that near the separatrix the motion in the vicinity of the saddle point of single-parameter nature.

Single-parameter motion along a curve with one fixed point at $\sigma_{xy}=e^2/h(n+1/2)$, where n is the number of completely filled quantum levels, is predicted also by numerical calculations of Ando and Aoki.[13,15-17]

We can represent the expected experimental results if we know not only the flow lines, but possible positions of the start points for relatively small (comparable with the magnetic length) dimensions of a sample. The positions of the start points relative to the separatrix are quite different in different theories. In the work of Pruisken and Khmel'nitskiĭ the start points lie above the separatrix, whereas according to Ando the start curve and the separatrix are identical for the zeroth and first Landau levels.

It therefore follows that the following predictions and problems have to be investigated experimentally:

1) changes in the values of σ_{xx} and σ_{xy} due to changes in the dimensions of a sample;

2) the scaling hypothesis including a check of the ab-

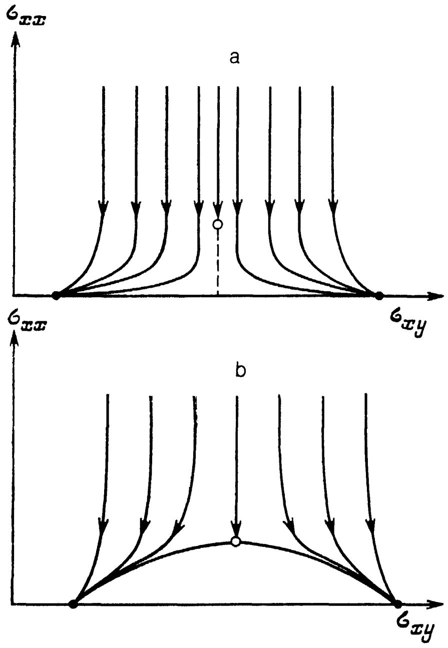

FIG. 1. Flow line pattern in the $(\sigma_{xx},\sigma_{xy})$ plane expected on the basis of the work of Pruisken for $T=0$. The arrows show the directions of motion of a mapping point, corresponding to a fixed occupancy factor, on increase in the size of a sample L. The black dots on the abscissa correspond to the neighboring plateau σ_{xy}. Figure 1b shows a separatrix identifying, in the $(\sigma_{xx},\sigma_{xy})$ plane, a region which cannot be reached by starting from higher values of σ_{xx}. The upper point of the separatrix is a saddle. The pattern is symmetric relative to the half-integral value of $(h/e^2)\sigma_{xy}$ and periodic along the σ_{xy} axis.

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sence of intersections of the flow lines in the $(\sigma_{xx},\sigma_{xy})$ plane and determination of the number of parameters describing the phase pattern;

3) the presence or absence of a separatrix and, in its presence, determination of the number of the scaling parameters in its vicinity;

4) check of periodicity of the flow-line pattern when σ_{xy} is altered by an amount which is a multiple of e^2/h and a check, carried out for different samples and materials, of the degree of universality of the flow pattern.

Partial answers to these questions can be found in Refs. 1–9. In real experiments the changes in σ_{xx} and σ_{xy} due to cooling rather than due to an increase in the dimensions of a sample are investigated. It is assumed that at finite temperatures an increase in the dimensions of a sample alters the conductivity only as long as the dimension in question is less than the characteristic length of dephasing induced by inelastic scattering. A further increase in the dimensions does not alter the conductivities and, therefore, it is possible to increase the dephasing length by cooling and thus increase the effective size of a sample.

However, it should be pointed out that not every displacement of a point in the $(\sigma_{xx},\sigma_{xy})$ plane is due to a change of temperature, but reflects a change in the dephasing line.[1] At sufficiently high temperatures, which for real samples can amount to several tenths of a kelvin, the main reason for the displacements of points is a change in the thermal smearing of the Fermi function. It was this effect that was investigated by Babkin *et al.*,[18] who observed a fall of the value of σ_{xx} at the maxima due to an increase in temperature in accordance with the law

$$\sigma_{xx}^{max}=\sigma_0^{max}(1-AT^2). \qquad (2)$$

In our opinion the published experimental results demonstrate reliably the following:

1) the changes in the conductivities σ_{xx} and σ_{xy} due to cooling obey the scaling transformation;[1,4,9]

2) there are no intersections of the flow lines in the $(\sigma_{xx},\sigma_{xy})$ plane;[1-9]

3) single-parameter scaling occurs in the vicinity of the points $\sigma_{xy}=e^2/h(n+1/2)$.[7]

The present paper represents a further attempt to answer the questions and solve the problems formulated above.

SAMPLES AND METHOD

Our measurements were carried out on four (100) silicon MOS transistors and two AlGaAs/GaAs heterostructures. The transistors had a gate of 250×250 μm dimensions. The thickness of the oxide layer in the first two transistors was 2450 Å, whereas for the other two it was 1810 Å (Ref. 19). The electron mobility in the oxide layer was a function of the electron density (Fig. 2). The temperature dependence of the conductivity given in Ref. 20 was used to determine the parameters of sample No. 1. The density of charged impurities in the layer was $N_i=2.6\times10^{10}$ cm^{-2}, the characteristic height of the irregularities of the Si/SiO_2 interface was $\Delta=3.14$ Å, and the period of these irregularities was $\Lambda=70$ Å. Hence, such irregularities could be regarded as short-period in magnetic fields up to 16 T.

The n-type channel in the heterostructures had dimensions of $1000\times4600\,\mu$m and $40\times500\,\mu$m, whereas the thickness of the spacer was 150 Å. In the first of these heterostructures (labeled MP0) the electron density was increased to $N_S=3.6\times10^{11}$ cm^{-2} by illumination with red light and the mobility was $\mu=37$ m$^2\cdot$V$^{-1}\cdot$s^{-1}. In the second heterostructure (MP2) the dark density and the mobility of electrons were 4.4×10^{11} cm^{-2} and 27 m$^2\cdot$V$^{-1}\cdot$s^{-1}, respectively.

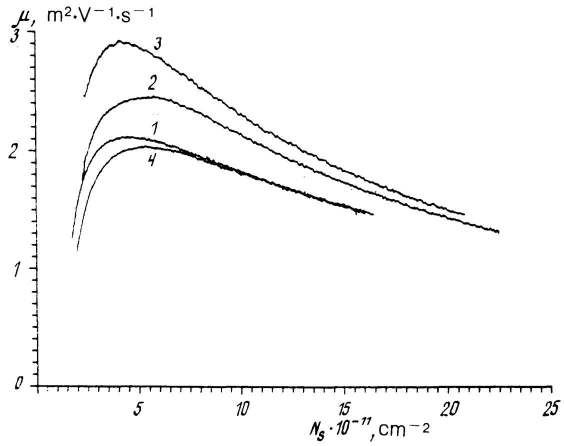

FIG. 2. Electron mobility in silicon MOS transistors plotted as a function of the electron density at $T=1.5$ K. The numbers alongside the curves are the serial numbers of samples.

All the samples had the Hall configuration, which made it possible to determine the components of the magnetoresistance tensor ρ_{xx} and ρ_{xy} that were then converted to the conductivities σ_{xx} and σ_{xy}. In the case of the silicon transistors the dependences of ρ_{xx} and ρ_{xy} on the electron density were determined in a specific magnetic field, whereas in the case of the heterostructures they were obtained as a function of the applied magnetic field.

The measurements were carried out using an alternating current of frequency 10 Hz under linear conditions in the temperature range from 20 mK to 1.5 K and in magnetic fields up to 16 T. The measuring currents were in the range 0.3–10 nA.

Typical experimental dependences are shown in Figs. 3 and 4. It is clear from these figures that the quantum levels corresponding to low occupancy factors were well resolved. The maxima of σ_{xx} or ρ_{xx} did not always correspond to the half-integral occupancy factor, indicating an overlap of the quantum levels in spite of the disappearance of the dissipative conduction process in a wide range of the occupancy factors. The maxima corresponding to valleys of lower energy in the silicon transistors were shifted toward higher occupancy factors. A similar effect was observed also for the heterostructures: the spin sublevels corresponding to the lower energy were shifted by a larger amount.

RESULTS OF MEASUREMENTS ON SILICON MOS TRANSISTORS

As already mentioned, the components ρ_{xx} and ρ_{xy} were determined in a fixed magnetic field and at a fixed temperature as a function of the electron density in a two-dimensional channel. The measurements were made at seven or eight temperatures in the range 20 mK–1.5 K. Qualitatively the results obtained for all four transistors were identical.

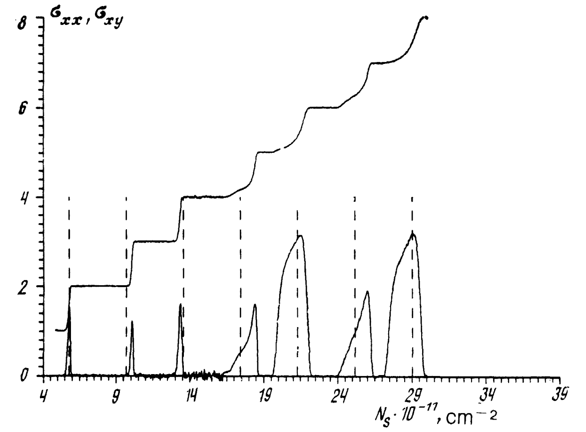

FIG. 3. Dependences of σ_{xx} and σ_{xy} on the electron density in sample No. 1. The ordinate is in units of e^2/h and the values of σ_{xx} are increased by a factor of 10. The dashed vertical lines represent the position of the half-integral occupancy factors. $H = 16$ T, $T \approx 20$ mK.

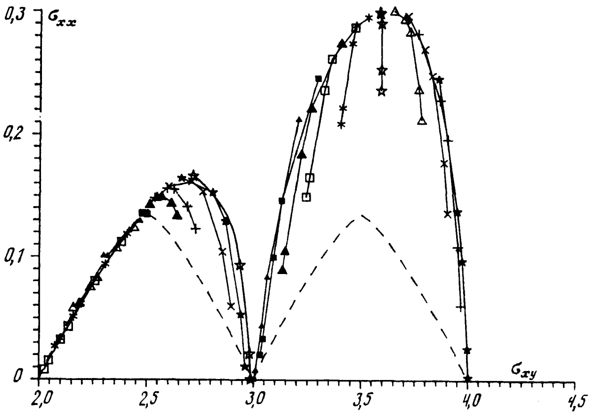

FIG. 5. Flow line pattern for sample No. 3. The dashed curve represents the results of a symmetric reflection and translation of a part of a separatrix in a segment $2 \leqslant (h/e^2)\sigma_{xy} \leqslant 2.5$. The axes are plotted in units of e^2/h.

A nonmonotonic variation of σ_{xx} at the maxima was observed at the lowest Landau level: right down to ≈ 0.7 K. The conductivity increased as a result of cooling, but at lower temperatures the values of σ_{xx} decreased. This indicated an increase in the dissipative conductivity at the maxima as a result of cooling, which could be explained only by thermal smearing of the Fermi distribution [see Eq. (2)], so that the temperature interval of interest to us was below 0.7 K. As in Ref. 4, we found that the value of σ_{xx} at the maxima decreased as a result of cooling at the first Landau level and this was true throughout the investigated temperature range. An increase in the magnetic field from 10 to 16 T had practically no effect on the values of σ_{xx} at the maxima.

Typical experimental results are presented in Figs. 5–7. It is clear from Fig. 5 that in the case of the maximum corresponding to the occupancy factor of 2.5 in sample No. 3 there was clear evidence of a separatrix. Moreover, the experiments demonstrated the existence of a saddle point at $\sigma_{xy} = 2.5e^2/h$. All the points starting from lower values of σ_{xy} moved to the left along the separatrix. On the other hand, all the points starting from higher values of σ_{xy} shifted to the right. The point corresponding to $\sigma_{xy} = 2.5e^2/h$ remained immobile (within the experimental error) when temperature was lowered and this was true throughout the investigated temperature range. We used dashed curves in Figs. 5 and 6 to plot the expected position of the separatrix for occupancy factors from 2 to 6. The dashed curves were plotted by symmetric reflection of a part of a separatrix in a segment $2e^2/h < \sigma_{xy} < 2.5e^2/h$ and translation along the σ_{xy} axis by a number which was a multiple of e^2/h. The separatrix plotted in this way coincided (Fig. 6) with the left-hand wing of the experimental curve for the maximum close to the occupancy factor 4.5. All the flow lines in the interval $2e^2/h < \sigma_{xy} < 6e^2/h$ were not in conflict with the hypothesis of the existence of a universal symmetric separatrix. We found that none of the start curves of all the investigated maxima coincided with the separatrix. Under the experimental conditions the right-hand wing of the separatrix was not attained.

In the case of sample No. 1 the flow line pattern was similar. The only difference was that at low temperatures all the maxima for the lowest Landau level were of approximately the same amplitude amounting to $\sigma_{xx} \approx 0.15e^2/h$. The flow lines of this sample could be described using a separatrix obtained by an analysis of the data of sample No. 3.

The data obtained for sample No. 2 (Fig. 7) were very different from those found for the other three samples. Once again the experimental pattern indicated the presence of a separatrix. However, this separatrix was located well below the corresponding curves for the other samples. This could be checked both for an occupancy factor of 2.5 and for an occupancy factor of 4.5. It is clear from Fig. 7 that there was a considerable asymmetry: the position of the saddle point was shifted to the left from the half-integral value of σ_{xy}.

We thus found that in the case of the lowest Landau level our measurements were carried out in the vicinity of a separatrix and this was true of all the samples. According to Refs. 7, 14, and 21 this was precisely the condition needed for the realization of single-parameter scaling. Following Refs. 7, 14, and 21, we plotted in Fig. 8 the temperature dependence of the derivative $(\partial\sigma_{xy}/\partial N_s)^{-1}$ at a maximum of σ_{xx}. It is clear from this figure that there was a range of temperatures where the derivative was proportional to

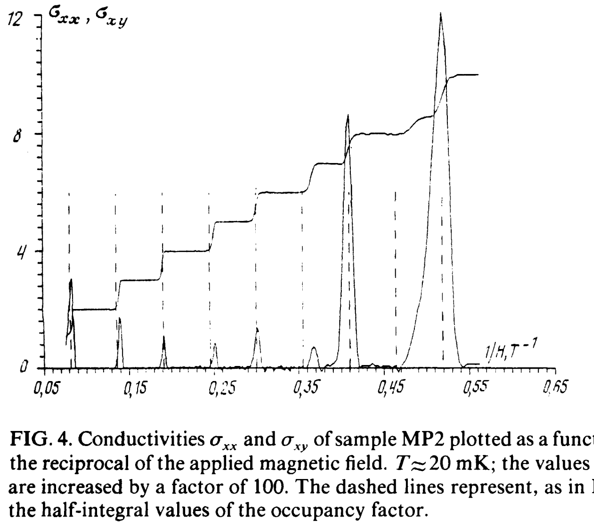

FIG. 4. Conductivities σ_{xx} and σ_{xy} of sample MP2 plotted as a function of the reciprocal of the applied magnetic field. $T \approx 20$ mK; the values of σ_{xx} are increased by a factor of 100. The dashed lines represent, as in Fig. 3, the half-integral values of the occupancy factor.

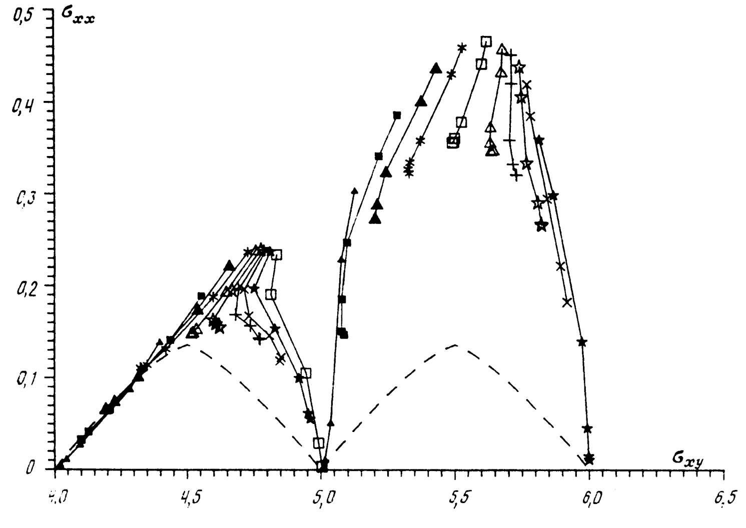

FIG. 6. Same as in Fig. 5, but for higher values of σ_{xy}. The dashed curve represents a periodic continuation, along the σ_{xy} axis, of the curve plotted in Fig. 5.

$T^{0.9 \pm 0.1}$. Figure 9 shows the temperature dependence of the second derivative $(-\partial^2\sigma_{xx}/\partial i^2)^{-1}$ at the same point. In this temperature range the latter derivative was proportional to $T^{1.7 \pm 0.2}$. Unfortunately, plotting of higher-order derivatives of our samples reduced strongly the precision with which we could determine the power exponent of the temperature dependence. However, the relationship between the temperature dependences of the first and second derivatives was a clear demonstration of the existence of single-parameter scaling.[14]

The data presented in Fig. 8 can be interpreted in a different manner. The off-diagonal conductivity can be written in the form

$$\sigma_{xy}=2\pi l^2(e^2/h)\int_0^{\varepsilon_F}\alpha(\varepsilon)\nu(\varepsilon)\,d\varepsilon, \tag{3}$$

where l is the magnetic length, $\nu(\varepsilon)$ is the density of states, and $\alpha(\varepsilon)$ is the effectiveness of the states corresponding to an energy ε. In estimates we can assume that in the case of delocalized states we have $\alpha = \alpha_0 = \text{const}$, whereas localized states are characterized by $\alpha = 0$.

The derivative is then given by

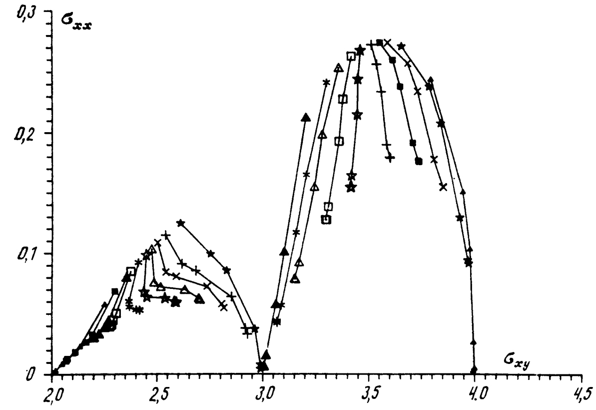

FIG. 7. Flow line pattern for sample No. 2. The saddle point is plotted for the peak σ_{xx} corresponding to $i = 2.5$ and it is shifted to the left relative to the half-integral value of $(h/e^2)\sigma_{xy}$. The ordinate of the saddle point is considerably less than for the other silicon MOS transistors.

$$(h/e^2)\,\partial\sigma_{xy}/\partial N_S=2\pi l^2\alpha_0. \tag{4}$$

A comparison of Eq. (3) for a completely filled quantum level with Eq. (4) shows that the derivative $\partial N_s/\partial\sigma_{xy}$ at a maximum of σ_{xx} is proportional to the fraction of delocalized states α_0^{-1} at the quantum level. The line plotted in Fig. 8 is the temperature dependence of the number of delocalized states at a quantum level. It is worth noting two circumstances. Firstly, according to Fig. 8, the number of delocalized states at a given level depended weakly on the applied magnetic field, at least between 10 and 16 T. Secondly, the number of delocalized states in our samples ceased to vary completely at temperatures below 60 mK.

This disappearance of the temperature dependence below 60 mK could be explained in a natural manner by assuming that the electron gas temperature was not equal to the temperature of the helium bath in a solution cryostat. However, the temperature of a standard resistance thermometer measured at the same rate of power dissipation showed no significant deviations from the helium bath temperature. Moreover, in experiments on single GaAs/AlGaAs heterojunctions, mounted on the same substrates as the silicon MOS structures and provided with the same numbers of current and potential contacts, we found that the conductivity was a function of temperature right down to 20 mK.

One should point out one other feature of the results plotted in Fig. 8. The fraction of delocalized states at the lowest quantum levels ($i < 4$) was 5–8% of the total number of electrons at a level. This was the fraction of delocalized states in all the investigated samples when they were subjected to a field of 16 T at the lowest temperature. This number could not be increased to any significant effect by employing any other method of estimating the number of delocalized states. Such a low number was in conflict with the results of Ref. 22. The reason for this discrepancy between the two sets of results was that the conclusions were reached in Ref. 22 by extrapolation of the "high-temperature" data from 1.5 K to absolute zero.

At the lowest temperatures we found that the silicon MOS structures exhibited a direct proportionality between the values of σ_{xx} and the derivative $\partial\sigma_{xy}/\partial i$ in the vicinity of all the investigated maxima of σ_{xx} when $i < 4$ in fields from

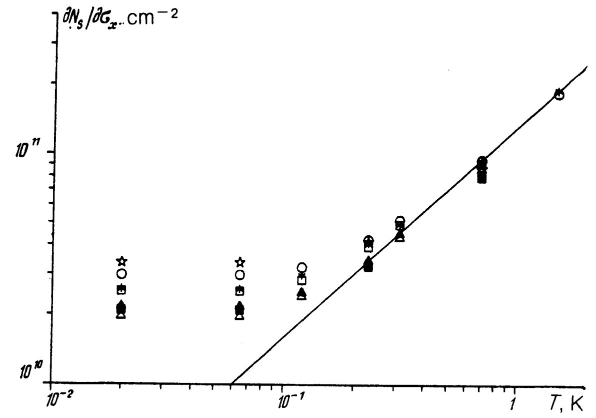

FIG. 8. Temperature dependences of the reciprocal of the maximum value of the derivative $\partial\sigma_{xy}/\partial N_S$: ○) $i\approx 3.5$, *) $i\approx 2.5$, sample No. 3, $H = 16$ T; □) $i\approx 3.5$, △) $i\approx 2.5$, sample No. 2, $H = 16$ T; ▲) $i\approx 3.5$, sample No. 2, $H = 10$ T; ☆) $i\approx 3.5$, ■) $i\approx 2.5$, sample No. 1, $H = 14$ T.

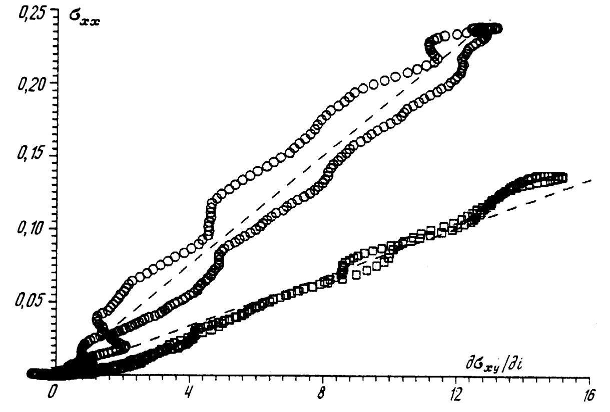

FIG. 10. Conductivity σ_{xx} plotted as a function of the derivative $\partial\sigma_{xy}/\partial i$ for two σ_{xx} peaks: ○) $i\approx 3.5$, □) $i\approx 2.5$, sample No. 3, $H = 16$ T, $T\approx 20$ mK.

10 to 16 T. The results obtained by plotting the dependence of σ_{xx} on $\partial\sigma_{xy}/\partial i$ are shown for two peaks of σ_{xx} in Fig. 10. Each of the maxima of σ_{xx} is represented in Fig. 10 by a point from which two branches emerge and these branches reach the origin of the coordinate system, and represent a rise and fall of σ_{xx} due to changes in the electron density. Subject to the error in the quantity $\partial\sigma_{xy}/\partial i$, we can regard these branches as coincident with one another and with the dashed line in Fig. 10.

RESULTS OF MEASUREMENTS ON GaAs/AlGaAs HETEROSTRUCTURES

In the case of the MP0 heterostructure, exactly as in the case of the silicon MOS transistors, the behavior of σ_{xx} at a maximum corresponding to $i\approx 1.5$ was nonmonotonic. The conductivity at the maximum fell as a result of cooling when the temperature of the sample was below 0.3 K. In the case of sample MP2 a detailed investigation of the maxima corresponding to $i\approx 1.5$ and $i\approx 3.5$ (Figs. 11 and 12) revealed a reduction in the conductivity σ_{xx} as a result of cooling at all temperatures below 1.5 K. In both cases the reduction in the conductivity was undoubtedly due to the scaling effects.

It is clear from Fig. 11 that the experimental results obtained for sample MP2 were evidence of the existence of a separatrix. In any case, the flow lines on the left-hand side of

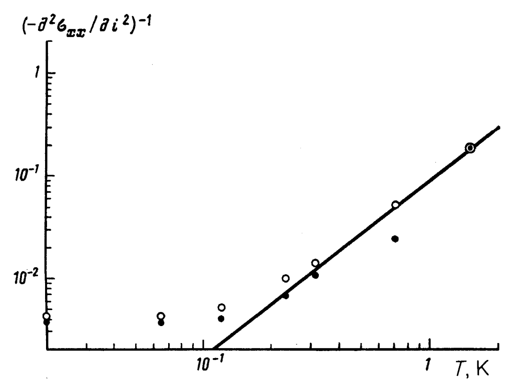

FIG. 9. Maximum absolute value of the second derivative $(-\partial^2\sigma_{xx}/\partial i^2)^{-1}$ plotted as a function of temperature: ○) $i\approx 2.5$; ●) $i\approx 3.5$; sample No. 3, $H = 16$ T.

this figure moved along the same curve at sufficiently low temperatures. It is worth noting two characteristic features of the results in this figure. Firstly, the ordinate of the saddle point lies below the positions obtained for the silicon MOS transistors. Secondly, the flow line pattern is strongly asymmetric.

The maximum value of σ_{xx} obtained for the same sample at the lowest temperature for the second peak of the conductivity ($i\approx 3.5$) was even less (Fig. 12). In this case the saddle point ordinate did not exceed $3\times 10^{-3}(e^2/h)$. The flow line pattern demonstrated an even stronger asymmetry than in Fig. 11 and in the vicinity of the left-hand saddle point $(3e^2/h, 0)$ there was motion along the separatrix.

The flow line pattern for sample MP0 ($i\approx 1.5$) was very similar to that observed for the silicon transistors. A characteristic value of the maximum at σ_{xx} at $T\approx 20$ mK was $0.1e^2/h$. The motion in the vicinity of the left-hand stable point $(e^2/h, 0)$ was along the separatrix. The pattern was extremely asymmetric.

In the case of this sample we found that in a wide range of temperatures (100 mK$\leqslant T \leqslant$1.5 K) the derivative $\partial\sigma_{xy}/\partial H$ at a maximum of σ_{xx} was proportional to $T^{-0.40\pm 0.05}$ and the derivative $\partial^2\sigma_{xx}/\partial H^2$ was proportional to $T^{-0.9\pm 0.1}$, in agreement with the behavior expected in the single-parameter scaling.

In the case of sample MP2 the vicinity of the separatrix was reached only at temperatures below 0.3 K. The temperature dependence of the quantity $(\partial\sigma_{xy}/\partial i)^{-1}$, governing the fraction of delocalized states at a quantum level, is plotted in Fig. 13. It is clear from this figure that the fractions of delocalized states were similar for the two investigated σ_{xx} peaks, corresponding to the occupancy factors 1.5 and 3.5, and depended on temperature as $T^{0.36\pm 0.07}$. The second derivative $\partial^2\sigma_{xx}/\partial H^2$ varied monotonically in the range 100 mK–1.5 K.

The quantities σ_{xx} and $\partial\sigma_{xy}/\partial i$ were not proportional in the case of our GaAs/AlGaAs samples.

DISCUSSION OF RESULTS

Before discussing the meaning of the experimental results, we must mention a methodological shortcoming which could distort the observed pattern. The final results of

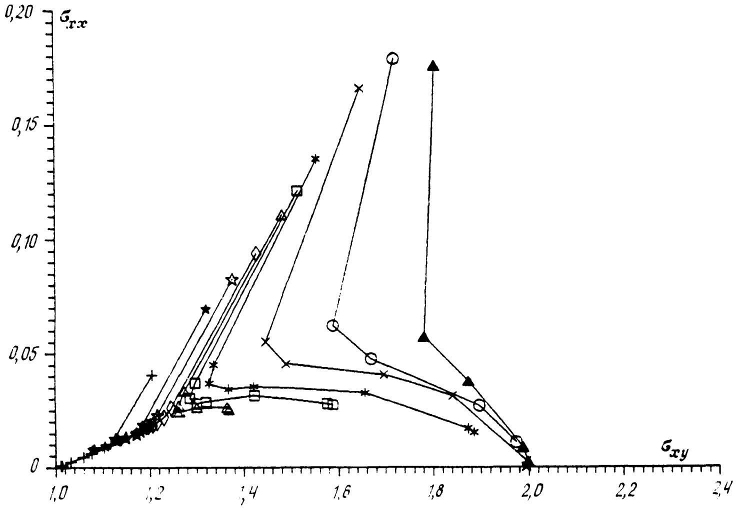

FIG. 11. Flow line pattern for sample MP2 in the case when $i \approx 1.5$.

our measurements were the values of σ_{xx} and σ_{xy}, but we in fact measured the resistivities ρ_{xx} and ρ_{xy}. Conversion of the tensor $\hat{\rho}$ to the tensor $\hat{\sigma}$ presumes, firstly, the absence of any significant contribution to the current from the edge states of the kind discussed on several occasions (see, for example, Refs. 23, 24, and 11). A justification for such a calculation is provided by the experiments reported by Kotthaus's group[24] on surface acoustic waves showing that conversion of this kind is permissible right up to values of $\sigma_{xx} \geqslant 10^{-7}$ Ω^{-1}. The same experiments demonstrated also a homogeneous distribution of the current in the plane of a sample when the dissipative conductivity was sufficiently high.

Secondly, conversion of the tensor $\hat{\rho}$ to the tensor $\hat{\sigma}$ presumes a macroscopic homogeneity of a sample, because we are assuming that the quantities ρ_{xx} and ρ_{xy} are measured at the same place and that we know the geometric dimensions of the current-carrying path. In our opinion, a proof of the macroscopic homogeneity of our structures is the agreement between the results of measurements of ρ_{xx} and ρ_{xy} obtained using different contacts for the same sample. Additional evidence of the macroscopic homogeneity of our silicon MOS structures is that all these structures exhibited a proportionality between σ_{xx} and $\partial\sigma_{xy}/\partial i$. In fact, the hypothesis of a macroscopic inhomogeneity would have given rise to a difference between the average electron densities between the contacts used to measure ρ_{xx} and ρ_{xy}. This would have unavoidably resulted in the absence of proportionality between σ_{xx} and $\partial\sigma_{xy}/\partial i$.

Using the Einstein relationship, we can write down the following expression for σ_{xx}:

$$\sigma_{xx}=\nu(\varepsilon_F)\beta(\varepsilon_F)e^2/h. \tag{5}$$

A comparison of Eqs. (3) and (5) shows that the proportionality of $\partial\sigma_{xy}/\partial i$ to σ_{xx} (Fig. 10) is equivalent to constancy of the ratio $\alpha(\varepsilon_F)/\beta\varepsilon_F$) if the density of states is constant in a range of energies corresponding to extended wave functions. Therefore, we can regard it as experimentally established that for electrons at the zeroth Landau level in the case of a short-period potential the contributions of an individual electron state to the conductivities σ_{xx} and σ_{xy} are proportional to one another.

One should mention particularly that this proportionality was observed for all four samples, including those for which the flow line pattern in the (σ_{xx}, σ_{xy}) plane was asymmetric. Consequently, the asymmetry of the flow lines cannot be related to the inhomogeneity of a sample. Moreover, the degree of asymmetry was different for different

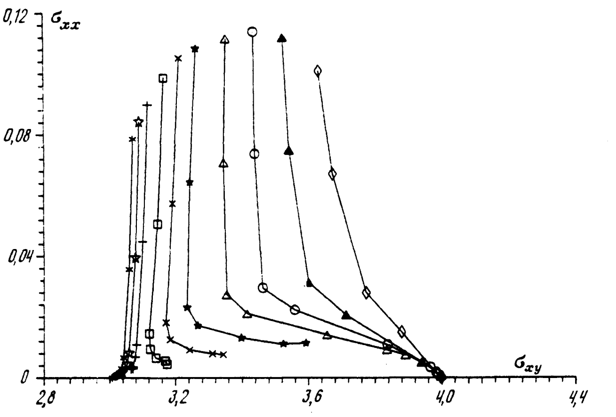

FIG. 12. Same as in Fig. 11, but for $i \approx 3.5$.

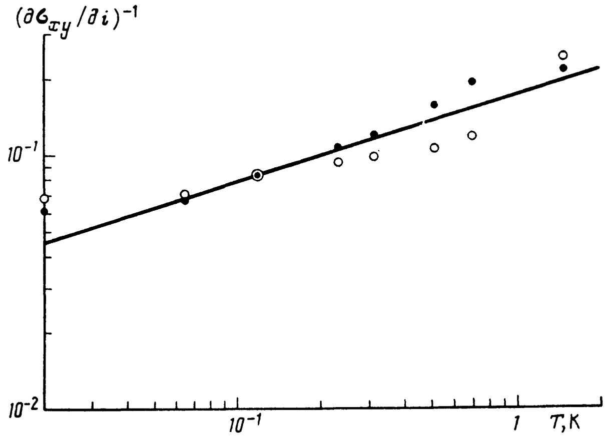

FIG. 13. Temperature dependence of the fraction of delocalized states α_0^{-1} at a quantum level: ○) $i \approx 1.5$, ●) $i \approx 3.5$; sample MP2.

quantum levels in the same sample: the pattern for each sample was more symmetric for the higher-valley levels (Figs. 5–7). Similar behavior was reported earlier in Refs. 5 and 8. In the case of our GaAs/AlGaAs samples the flow line pattern was also asymmetric.

The absence of the flow line symmetry together with a shift of the σ_{xx} peaks relative to the half-integral value of the occupancy factor (Figs. 3 and 4) indicated the absence of an electron–hole symmetry.

The second conclusion which can be drawn from our experimental results is that the separatrix was not universal. In fact, the ordinate of the saddle point varied by an order of magnitude from sample to sample. In the case of the silicon transistors the different positions of the saddle point along the ordinate could be attributed to a change in the potential relief from sample to sample, whereas in the case of the GaAs/AlGaAs heterostructures the saddle point position was even different for different Landau levels in the same sample (MP2). Since the hypothesis of two-parameter scaling led to the conclusion of periodicity of the pattern along the σ_{xy} axis,[11,12] the experimental results obtained by us taken as a whole demonstrated that two parameters were insufficient to describe the flow lines in the (σ_{xx}, σ_{xy}) plane. This conclusion was supported also by observation of the fractional quantum Hall effect in silicon (see, for example, Refs. 25–27), since in these experiments the start points in the (σ_{xx}, σ_{xy}) plane were well above the separatrix observed by us (Figs. 5–7).

Single-parameter scaling observed in the vicinity of the separatrix allowed us, in accordance with Refs. 6 and 7, to determine one further parameter representing the two-dimensional electron gas. If the localization length ξ behaves as $\xi \propto |\varepsilon - \varepsilon_c|^{-s}$, the scaling parameter is the quantity $\varkappa$ depending on the ratio l_φ/ξ, or

$$\varkappa \propto T^{-p/2s} |\varepsilon - \varepsilon_c|. \tag{6}$$

Here, ε_c is the energy which corresponds to an infinitely extended wave function. Our experiments on the silicon MOS transistors showed that the difference $\varepsilon_F - \varepsilon_c$ was proportional to the difference $N_s - N_s^c$, whereas in the case of the GaAs/AlGaAs heterojunctions the corresponding difference obeyed $(\varepsilon_F - \varepsilon_c) \propto (H - H_c)$.

At sufficiently low temperatures the dephasing time of Si MOS transistors was limited by the electron–electron scattering processes, so that we could expect $p = 1$ (Ref. 28). The experiments on these silicon transistors showed that the scaling parameter $\varkappa$ was proportional to the product $(N_s - N_s^c) \cdot T^{-0.9 \pm 0.1}$. Comparing this quantity with Eq. (6), we concluded that the silicon MOS transistors were characterized by $s = 0.55 \pm 0.05$.

In the case of the MP0 sample, which also exhibited single-parameter scaling in the vicinity of the separatrix, we obtained results similar to those reported in Ref. 7 for a single InGaAs/InP heterojunction: $\varkappa \propto T^{-0.40 \pm 0.05}$ or $s = 1.2 \pm 0.1$ on condition that $p = 1$. Strictly speaking, the hypothesis that $p = 1$ was not experimentally justified because, for example, in the case of the silicon MOS transistors the weak localization experiments gave values of p ranging from 1.3 to 1.8, depending on the electron density.[29] It was very likely that the value of s was the same for different electron systems and the temperature dependence of the scaling parameter was different because of the different values of p.

All the power exponents of the temperature dependences discussed earlier were valid in a limited temperature interval. It is clear from Figs. 8 and 9 that in the case of the silicon transistors there were considerable deviations already at temperatures below 150 mK. Deviations exhibited by MP0 and MP2 samples were detectable at temperatures 20–50 mK.

As pointed out above, we are not in favor of attributing these deviations to a difference between the temperature of the electron system and the temperature of the heat-exchanging mixture. The differences could be due to inhomogeneity of our samples over distances of $\sim 1\ \mu$m. One should mention particularly that in all our experiments the height of the saddle point was several times less than the limiting value predicted by the theory of Refs. 11 and 13.

CONCLUSIONS

We shall now formulate the most important results of the present study. For the first time we demonstrated reliably by an experimental method the transfer of the flow lines to a separatrix and we discovered a saddle point. The saddle point was located well below that predicted theoretically and its position varied from sample to sample and even for different quantum levels within the same sample. It was found that the flow line pattern was asymmetric relative to the half-integral value $(h/e^2)\sigma_{xy}$ and nonperiodic along the σ_{xy} axis. At the lowest temperatures the values of σ_{xx} and $\partial\sigma_{xy}/\partial i$ were proportional to one another for the lowest Landau level in the investigated silicon MOS transistors.

The experimental results indicated that two parameters were insufficient to provide a complete description of the flow line pattern. However, single-parameter scaling was confirmed in the vicinity of the separatrix. The temperature dependence of the scaling parameter for the silicon MOS transistors differed from that obtained by us for our GaAs/AlGaAs heterostructures and that reported earlier for InGaAs/InP heterojunctions. The proportionality of σ_{xx} and $\partial\sigma_{xy}/\partial i$ demonstrated that the contributions of a single quantum state to the dissipative (σ_{xx}) and nondissipative (σ_{xy}) conductivities were also proportional.

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