# Stimulated light pressure on atoms in counterpropagating amplitude-modulated waves

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Theoretical and experimental investigations were made of the effect of a stimulated light pressure on a beam of Na atoms. Expressions were obtained for the force exerted by this pressure on a two-level atom in the field of counterpropagating short light pulses of arbitrary area and also for the coefficient of pulsed diffusion of atoms in the case of square-wave light pulses. It was found that the value of this force close to the ideal case of square-wave pulses can be achieved in the case of counterpropagating amplitude-modulated waves, i.e., two standing waves of different frequencies, which is more convenient from the point of view of experimental implementation. An experimental study was made of the effects of a light pressure force in the field of two standing waves exerted on Na atoms in a beam.

## 1. INTRODUCTION

There is considerable interest in the problem of the light pressure exerted on atoms because this provides an effective means for controlling their motion, particularly in connection with the feasibility of cooling to sub-Kelvin temperatures and formation of traps that can confine cold atoms for a long time.[1] The light pressure force exerted on atoms in the field of one traveling wave is limited to $\hbar k\gamma$, where $(2\gamma)^{-1}$ is the radiative lifetime and $\hbar k$ is the photon momentum.[2] This is because a directional change in the momentum of an atom occurs only when the absorption of a photon is followed by its spontaneous emission.

It would undoubtedly be of interest to develop such systems for the excitation of atoms in which the rate of transfer of the momentum from an electromagnetic field to an atom would not be related directly to $\gamma$. Obviously, this requires the presence of at least two waves propagating along different directions. We can then induce a directional change in the momentum of an atom mainly because of stimulated transitions (when photons are absorbed preferentially from one wave and emitted in a stimulated manner in the other wave) at a rate which is governed by the radiation intensity. The resultant force due to a stimulated light pressure (SLP) is not restricted by the rate of spontaneous emission and can exceed greatly the value $\hbar k\gamma$.

The possibility of appearance of an SLP as a result of ordering of the absorption and stimulated emission events is illustrated by the ideal case of the interaction of an atom with two trains of counterpropagating square-wave pulses.[3] Obviously, implementation of the proposal made in Ref. 3 would be a difficult experimental task. It was shown in Ref. 4 that SLP may be induced in a field of two standing waves which essentially represent counterpropagating amplitude-modulated waves and represent the simplest analog of pulse trains. This relatively simple experimental configuration was used in the first observations of an SLP.[5] The force which acts on an atom in the field of two standing waves was considered also independently in Refs. 6 and 7 on the basis of perturbation theory applied to the ratio of the Rabi frequency of a strong wave and its frequency offset from the center of an atomic transition line, but the problem of the maximum attainable force was not discussed.

In Sec. 2 we shall obtain an expression for the force acting on a two-level atom subjected to a train of short pulses of arbitrary area. Pulsed diffusion of atoms in the special case of square light pulses is analyzed in Sec. 3. The force acting on a two-level atom in the field of two standing waves of the same intensity is discussed in Sec. 4. The results of an experimental study of deflection of a beam of Na atoms in the field of two standing waves are reported in Sec. 5.

## 2. LIGHT PRESSURE ON ATOMS IN THE FIELD OF PERIODIC TRAINS OF SHORT LIGHT PULSES

The equations for the density matrix of a two-level atom with a dipole moment $d$ of a transition and an energy difference between the levels $\hbar\omega_0$ in the field of waves of the same frequency counterpropagating along the $z$ axis are

$$\begin{gathered}\frac{\partial\rho_{11}}{\partial t}=\frac{i}{\hbar}Ed(\rho_{21}-\rho_{12})+2\gamma\rho_{22},\\ \frac{\partial\rho_{12}}{\partial t}=i\omega_0\rho_{12}+\frac{i}{\hbar}Ed(\rho_{22}-\rho_{11})-\gamma\rho_{12},\\ \rho_{11}+\rho_{22}=1,\qquad \rho_{21}=(\rho_{12})^*,\\ E=E_1(t)\cos(\omega t-kz)+E_2(t)\cos(\omega t+kz)\end{gathered}\tag{1}$$

and they will be solved on the assumption that these counterpropagating waves are trains of short light pulses with a repetition period $T$ and durations $\tau_1$ and $\tau_2$ such that

$$\tau_{1,2}|\omega_0-\omega|\ll 1,\quad \gamma\tau_{1,2}\ll 1,\quad kv\tau_{1,2}\ll 1,$$

where $v$ is the component of the velocity of the atom in the $z$ direction. In this case we can ignore relaxation and evolution, during a pulse, of elements of the density matrix because of the Doppler frequency shift and offset of the field frequency $\omega$ from the frequency of the atomic transition $\omega_0$.

We assume that a wave propagating in the positive direction of the $z$ axis interacts with an atom at time intervals $mT$ and $mT+\tau_1$, whereas the opposite wave interacts at time intervals $mT+\tau$ and $mT+\tau+\tau_2$, where $m$ is an integer. Then, introducing $N=\rho_{22}-\rho_{11}$ and $\sigma=\rho_{12}\exp(-i\omega t)$, we find from Eq. (1) the relationship between the values of $N$, $\sigma$, and $\sigma^*$ after a pulse and the values before a pulse:

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$$N(mT+\tau_1)=N(mT)\cos\varphi_1+i\sin\varphi_1[\sigma(mT)e^{ikz}-\sigma^*(mT)e^{-ikz}],$$
$$2\sigma(mT+\tau_1)e^{ikz}=i\sin\varphi_1 N(mT)+\sigma(mT)e^{ikz}(1+\cos\varphi_1)+\sigma^*(mT)e^{-ikz}(1-\cos\varphi_1), \quad (2)$$

where the area under a pulse is

$$\varphi_1=\frac{d}{\hbar}\int_{mT}^{mT+\tau_1}E_1(t)dt, \quad (3)$$

and the values $\sigma$ and $N$ at the moment of arrival of the next pulse (propagating in the negative direction of the $z$ axis) are related to the values at the moment $mT+\tau_1$ at the end of the previous pulse:

$$N(mT+\tau)=-1+e^{-2\gamma\tau}+N(mT+\tau_1)e^{-2\gamma\tau},$$
$$\sigma(mT+\tau)=\sigma(mT+\tau_1)e^{-(\gamma+i(\omega-\omega_0))\tau}. \quad (4)$$

The equations relating $N$ and $\sigma$ at the times $mT+\tau+\tau_2$ and $mT+\tau$, and also the times $mT+T$ and $mT+\tau+\tau_2$ can be obtained from Eqs. (2) and (4) by the substitutions $\varphi_1\to\varphi_2$, $k\to-k$, and $\tau\to T-\tau$.

In the case under discussion here (when the field intensity is a function only of the coordinate $z$) the force acting on an atom is[8]

$$F(t)=d(\rho_{12}+\rho_{21})\frac{\partial E}{\partial z}. \quad (5)$$

Averaging this force over one pulse repetition period, we find that

$$F=\frac{1}{T}\int_{mT}^{mT+T}F(t)dt=\frac{\hbar k}{2T}[N(mT+\tau_1)-N(mT)-N(mT+\tau+\tau_2)+N(mT+\tau)]. \quad (6)$$

In general, the average force depends on $m$. We are interested in long durations of the interaction of an atom with the field, $t>1/\gamma$, when completion of the initial transient process is followed by the establishment of a quasisteady value of the population inversion during the time occurring in Eq. (6). These quasisteady values can be found from the requirement of the periodicity of $N$ and $\sigma$.

The procedure for the determination of $N$ and $\sigma$ is as follows. Expressing first $N$, $\sigma$, and $\sigma^*$ at the moments $mT+T$ in terms of their values at the moments $mT+\tau+\tau_2$, and then the latter in terms of the values at $mT+\tau$ and so on, and applying the requirement of periodicity with a period $T$, we obtain a system of linear equations for $N(mT)$, $\sigma(mT)$, and $\sigma^*(mT)$. Solving this system and finding the values of the inversion at the times needed for the calculation of the force using Eq. (6) we obtain the average force acting on the atom.

We first give the solution for the case when $\varphi_1=\varphi$ and $\varphi_2=0$ (when there is only one wave in the positive direction of the $z$ axis) and there is no SLP force:

$$F=\frac{\hbar k}{2T}\frac{(1-\cos\varphi)(e^{2\gamma T}-1)}{e^{2\gamma T}+1-e^{\gamma T}(1+\cos\varphi)\cos(\omega_0-\omega)T}. \quad (7)$$

We note the periodic dependence of the force on $\omega$. Since in this case the frequency $\omega$ describes the field in a reference system linked to the investigated atom, the Doppler shift should also result in a periodic dependence of the force on the velocity. As expected, in the case of a single traveling wave the force is less than $\hbar k\gamma$. It is maximal at the frequency

$$\omega=\omega_0+\frac{2\pi}{T}j,$$

where $j$ is an integer, i.e., in the case when one of the frequencies in the spectrum is in resonance with an atomic transition. Such behavior was observed experimentally and reported in Ref. 9.

It is of interest to compare the force of Eq. (7) with the force that appears in the field of a traveling monochromatic wave. According to Ref. 2, when the frequency $\omega$ of the wave coincides with $\omega_0$, we have

$$F_t=\hbar k\gamma\frac{G}{1+G}, \quad (8)$$

where $G=\Omega_0^2/2\gamma^2$ and $\Omega_0$ is the Rabi frequency for a traveling wave. At low values of $\Omega_0$ and $\varphi$ it follows from Eqs. (7) and (8) that in the most interesting case when $\omega=\omega_0$ and $\gamma T\ll 1$, we have

$$F=\hbar k\gamma\frac{\varphi^2}{2(\gamma T)^2}, \quad F_t=\hbar k\gamma\frac{\Omega_0^2}{2\gamma^2}.$$

Since $\varphi=T\tilde{\Omega}_0$, where $\tilde{\Omega}_0$ is the Rabi frequency for the central component of the spectrum of a train of light pulses, the expression for $F=\hbar k\gamma\tilde{\Omega}_0^2/2\gamma^2$ is completely identical, as expected, with $F_t$ (at low values of $\varphi$ only the central component of the spectrum interacts effectively with an atom). For arbitrary values of $\tilde{\Omega}_0=\Omega_0$ the forces $F$ and $F_t$ are close, with a high degree of precision, for practically all the values of $\Omega_0$ with the exception of narrow intervals in the vicinity of the points $\tilde{\Omega}_0=2\pi n/T$ (i.e., when the Rabi frequency $\tilde{\Omega}_0$ is a multiple of the pulse repetition frequency $\Omega_{rep}$). According to Eq. (7), the value of $F$ falls to zero in these intervals and the width of a resonance is $\Delta\tilde{\Omega}_0\propto\gamma$. Rigorous periodicity in $\varphi$ and the fall of $F$ to zero at $\varphi=2\pi n$ are no longer observed if we allow for relaxation of the density matrix of an atom during the action of a pulse. In particular, for $\tilde{\Omega}_0=2\pi n/T$, then apart from terms linear in $\gamma\tau_1$, we have $F=7\hbar k\gamma\tau_1/4T$.

We now consider two counterpropagating waves consisting of trains of pulses of the same area $\varphi_{1,2}=\varphi$. We discuss only the most interesting special cases. In the case of two trains of counterpropagating square pulses, such that $\varphi=\pi$, the force $F$ is maximal and is governed by

$$F=\frac{2\hbar k}{T}\frac{\mathrm{sh}[\gamma(T-2\tau)]}{\mathrm{sh}(\gamma T)}. \quad (9)$$

The force (9) vanishes at $\tau=T/2$ and is maximal at $\tau=0$ and $\tau=T$. Its maximal value

$$F=\hbar k\gamma\frac{2}{\gamma T}$$

corresponding to $\gamma T\ll 1$ is much greater than the maximum force $F_t=\hbar k\gamma$ in the field of one traveling wave.

The force acting on an atom in the case of an arbitrary $\varphi$ will be now considered for the case of large and small values of $\gamma T$. At a low pulse repetition frequency ($\exp(\gamma T)\gg 1$) the SLP force may be observed if the delay $\tau$ between the pulses is not too long. Assuming that $\exp(\gamma\tau)\ll\exp(\gamma T)$, we find that

$$F=\frac{\hbar k}{2T}e^{-2\gamma\tau}[(1-\cos\varphi)^2-e^{\gamma\tau}\sin^2\varphi\cos(2kz+(\omega-\omega_0)\tau)]. \tag{10}$$

Since the force $F$ is independent of the coordinate $z$, the above expression is valid at low velocities $v$ along the $z$ direction. In fact, Eq. (10) is derived on the assumption of quasisteady conditions. This means that the value of $kz$ should change little in the time $1/\gamma$, i.e., we should have $kv<\gamma$, when the force "tracks" the coordinate and the average SLP force per atom can be obtained by averaging Eq. (10) over $z$. In order to remain within the quasisteady approximation we must assume that the momentum acquired by an atom in a time $1/\gamma$ does not change the velocity too much, which is generally true for atoms of mass $M>\hbar k^2/\gamma^2T$. It should be mentioned here that, since in the case of square pulses the system of equations (2) is independent of $z$, a quasisteady state can be reached also at much higher values of the velocity $v>\gamma/k$.

In the other limiting case when $\gamma T\ll 1$, which is of the greatest interest, the average force acting on an atom along the $z$ direction is

$$F=\frac{2\hbar k}{T}\left(1-\frac{2\tau}{T}\right)\Phi\left(\sin^2\frac{\varphi}{2}\right), \tag{11}$$

where

$$\Phi(x)=\frac{x[(1-x)^{1/2}+(1+x)^{1/2}]}{(1+x)^{1/2}(1+(1-x^2)^{1/2})}. \tag{12}$$

We can see that the condition $\varphi=\pi$ is not essential for the existence of a large SLP force. When $\varphi$ differs little from $\pi$, the SLP force decreases linearly on increase in $|\pi-\varphi|$:

$$F=\frac{2\hbar k}{T}\left(1-\frac{2\tau}{T}\right)\left(1-\frac{|\pi-\varphi|}{2^{3/2}}\right). \tag{13}$$

We must stress once again that the expression for the SLP force is valid when the duration of interaction with the field exceeds the spontaneous relaxation time $1/\gamma$. If the duration of the interaction with the field is shorter, so that $t<1/\gamma$, the force acting on an atom is governed by the initial conditions. The influence of relaxation on the force experienced by an atom can be followed most clearly in the case of counterpropagating square pulses. In this case the momentum of an atom changes by $\hbar k$ on absorption of light from one wave and by a further amount $\hbar k$ as a result of stimulated emission to the other wave. The force which acts then is $2\pi\hbar k/T$ and its direction is governed by the direction of propagation of the first square pulse interacting with an atom.

It therefore follows that in the case of short durations of the interaction with the field an atomic beam should split into two and the fraction of the atoms in the two beams is naturally governed by the time interval $\tau$ between the arrival of the counterpropagating square pulses. At $\tau=+0$ or $\tau=T-0$ an atom interacting with short pulses of the kind considered here is practically all the time in the ground state and the relaxation can be ignored even after a time $t>1/\gamma$, so that the force acting on an atom is $+2\hbar k/T$ or $-2\hbar k/T$. At $\tau=T/2$ the numbers of atoms accelerated in the positive and negative directions of the $z$ axis are equal. "Activation" of relaxation for interaction times longer than $1/\gamma$ has the effect that the force $2\hbar k/T$ acting on an atom changes its direction randomly. Since the probability that the force acts in the positive and negative directions at $\tau=T/2$ is the same, the average force acting on an atom in a time interval which is long compared with $1/\gamma$ naturally vanishes. In the general case of arbitrary values of $\tau$ the dependence of the force on $\tau$ is described in Eqs. (11) and (13) by a factor $(1-2\tau/T)$, which is equal to the difference between the probabilities that an atom is accelerated at a given moment in the positive direction of the $z$ axis and the probability that it is accelerated in the negative direction of $z$.

## 3. PULSED DIFFUSION OF ATOMS IN THE FIELD OF SQUARE PULSE TRAINS

When a field of square pulses resonant with an atomic transition is applied, the off-diagonal elements of the density matrix vanish and the interaction of an atom with the optical field can be described in terms of populations.

We introduce the distribution of the population densities of atoms in the ground and excited states $n_1(p,t)$ and $n_2(p,t)$ in terms of the momentum $p$ normalized to that fraction of the atoms which are in these states:

$$N_1(t)=\int n_1(p,t)dp,\quad N_2(t)=\int n_2(p,t)dp,\quad N_1+N_2=1. \tag{14}$$

Bearing in mind that the interaction of an atom in the ground state with a square pulse transfers the atom to an excited state, whereas an atom in an excited state drops to the ground state, and allowing for the change in the momentum of an atom as a result of such a transition, we can write down

$$\begin{aligned} &n_1(p,jT+\tau_1)=n_2(p+\hbar k,jT),\\ &n_2(p,jT+\tau_2)=n_1(p-\hbar k,jT). \end{aligned} \tag{15}$$

Hence, we can derive equations describing the change in the zeroth-first- and second-order moments of the functions representing the distribution of atoms in the ground and excited states when these changes occur during the interaction of an atom with a square pulse propagating in the positive direction of the $z$ axis.

After the interaction with the field there is a relaxation period during which the changes $n_1(p,t)$ and $n_2(p,t)$ with time are described by

$$\begin{aligned} &\frac{\partial n_1(p,t)}{\partial t}=2\gamma\int n_2(p+\hbar q,t)\varphi(q)dq,\\ &\frac{\partial n_2(p,t)}{\partial t}=-2\gamma n_2(p,t), \end{aligned} \tag{16}$$

where the even function $\varphi(q)$ describes the probabilistic nature of spontaneous emission.

We first consider the interaction of an atom only with one traveling wave, which is a train of square pulses propagating in the positive direction of the $z$ axis. The force acting on an atom and the coefficient of pulsed diffusion of atoms can be found, provided we know the momentum of an atom

$$P(t)=\int p(n_1(p,t)+n_2(p,t))dp$$

and the average square of the momentum

$$R(t)=\int p^2(n_1(p,t)+n_2(p,t))dp$$

in the case when $\gamma t \gg 1$, i.e., after the transient processes cease. Obviously, in this case the initial value of the population inversion

$$\Delta N(0) = \int (n_2(p,0) - n_1(p,0))\,dp$$

is of no significance. We shall assume that an atomic beam is perpendicular to the direction of propagation of the pulses and select $\Delta N(0) = -\tanh(\gamma T)$. Then, for even values of $j$, we obtain

$$P(jT) = \hbar k j \,\mathrm{th}(\gamma T),$$

$$R(jT) = P^2(jT) + j\hbar^2 \,\mathrm{th}(\gamma T)\left[q_0^2 + \frac{k^2}{\mathrm{ch}^2(\gamma T)}\right] + \frac{\hbar^2 k^2}{2\,\mathrm{ch}^2(\gamma T)}(1 - e^{-2\gamma T j}), \tag{17}$$

where the mean-square value of the projection along the $z$ axis of the wave vector of a spontaneously emitted photon is

$$q_0^2 = \int q^2 \varphi(q)\,dq.$$

It therefore follows that the pulsed-diffusion coefficient is

$$D = \frac{1}{2}\lim_{t\to\infty}\frac{R(t) - P^2(t)}{t} = \frac{\hbar^2}{2T}\,\mathrm{th}(\gamma T)\left(q_0^2 + \frac{k^2}{\mathrm{ch}^2(\gamma T)}\right). \tag{18}$$

The first term in the parentheses is responsible for the diffusion in momentum space because of the random direction of a spontaneously emitted photon, while the other is related to the random number of spontaneous transitions which are, in the final analysis, responsible for the change in the momentum of an atom in the case when only one traveling wave is present. For $\exp(\gamma T) \gg 1$, the number of spontaneous emission events is fixed: it is equal to the number of the square pulses and the term containing $k$ tends to zero. The change in the momentum of an atom at right-angles to the $z$ axis involves only spontaneous emission and the average force acting on an atom is then zero, whereas the diffusion coefficient is given by Eq. (18) subject to $k = 0$.

We now consider the case of two counterpropagating waves in the form of trains of square pulses. The equations describing the change in the distribution of the population densities during the interaction with the field from $jT + \tau$ to $jT + \tau + \tau_2$ (i.e., during the interaction with a wave traveling in the negative direction of the $z$ axis) can be derived from Eq. (15) by the substitutions $k \to -k, jT \to jT + \tau$, and $\tau_1 \to \tau_2$, and the relaxation process is then described by the system (16).

As in the preceding case, we assume that an atomic beam is perpendicular to the direction of propagation of the pulses and we postulate that

$$\Delta N(0) = -(1 + e^{2\gamma T} - 2e^{2\gamma\tau})/(e^{2\gamma T} - 1).$$

We then obtain

$$R(jT) = P^2(jT) + 2\hbar^2 e^{-4\gamma\tau}(e^{2\gamma\tau} - 1)(e^{2\gamma T} - e^{2\gamma\tau})(e^{2\gamma T} - 1)^{-1} \times \left\{ j q_0^2 e^{2\gamma\tau} + \frac{2k^2}{(e^{2\gamma T} - 1)^2}\left[j\left((e^{2\gamma T} + 1)(e^{2\gamma T} + e^{4\gamma\tau}) + 4e^{2\gamma T + 2\gamma\tau}\right) - \frac{2e^{2\gamma T}}{e^{2\gamma T} - 1}(1 - e^{2\gamma T j})(e^{2\gamma\tau} + 1)(e^{2\gamma T} + e^{2\gamma\tau})\right]\right\}, \tag{19}$$

where

$$P(jT) = \frac{2\hbar k j}{(e^{2\gamma T} - 1)}(e^{2\gamma T - 2\gamma\tau} - e^{2\gamma\tau}).$$

Hence, we find that the expression for the pulsed-diffusion coefficient is

$$D = \frac{\hbar^2 (e^{2\gamma\tau} - 1)}{T(e^{2\gamma T} - 1)}(e^{2\gamma(T-\tau)} - 1) \times \left\{ q_0^2 + \frac{2k^2}{(e^{2\gamma T} - 1)^2}\left[(e^{2\gamma T} + 1)(e^{2\gamma(T-\tau)} + e^{2\gamma\tau}) + 4e^{2\gamma T}\right]\right\}. \tag{20}$$

As expected, the expression for $D$ is symmetric in the deviation of $\tau$ from $T/2$. The diffusion coefficient peaks at $\tau = T/2$ when the force vanishes, but is equal to zero at $\tau = 0$ and $\tau = T$, when the force is maximal. In the case of interest to us when $\gamma T \ll 1$ and the force is maximal, the diffusion coefficient is given by

$$D = 2\gamma\tau\frac{\hbar^2}{T}\left(1 - \frac{\tau}{T}\right)\left(q_0^2 + \left(\frac{2k}{\gamma T}\right)^2\right), \tag{21}$$

and we can ignore pulsed diffusion due to the change in the momentum of an atom in the course of spontaneous emission. Comparing in this case the average value of the directional change in the momentum $P = Ft$ and the scatter of the momentum of atoms around this value $\Delta P = (2Dt)^{1/2}$, we can see that the process of diffusion has practically no influence on the motion of atoms at times

$$t > \frac{4\tau}{\gamma T}\left(1 - \frac{\tau}{T}\right)\left(1 - \frac{2\tau}{T}\right)^{-2}.$$

## 4. LIGHT PRESSURE ON ATOMS IN THE FIELD OF TWO STANDING WAVES

The condition for the action of SLP on atoms is the presence of at least two amplitude-modulated waves differing in the direction of propagation, but the simplest to interpret is the case of two counterpropagating square pulses discussed here. It is shown in Ref. 4 that results close to the ideal case of square pulses can be obtained even in the simplest case of counterpropagating amplitude-modulated waves, in the form of a superposition of two standing waves of frequencies $\omega + \Omega/2$ and $\omega - \Omega/2$:

$$E = \mathscr{E}\cos(\omega + \Omega/2)t\cos(kz + \psi/2) + \mathscr{E}\cos(\omega - \Omega/2)t \times \cos(kz - \psi/2) = E^{(+)}e^{i\omega t} + E^{(-)}e^{-i\omega t}, \tag{22}$$

where

$$E^{(+)} = \frac{\mathscr{E}}{2}\left[e^{i\Omega t/2}\cos(kz + \psi/2) + e^{-i\Omega t/2}\cos(kz - \psi/2)\right], \quad E^{(-)} = E^{(+)*}. \tag{23}$$

Here, $\psi$ describes the spatial shift of these waves and $\psi/k$ is the distance between their nodes. We shall assume that $\Omega \ll \omega$, so that the difference between the wave vectors of the counterpropagating waves can be ignored. Then, in the approximation of a rotating wave, we find from the system (1) subject to the field of Eq. (22) that the following equations apply to $N$ and $\sigma$:

$$\frac{\partial N}{\partial t}=\frac{2id}{\hbar}[\sigma E^{(-)}-\sigma^{*}E^{(+)}]-2\gamma(1+N),$$
$$\frac{\partial \sigma}{\partial t}=\frac{id}{\hbar}E^{(+)}N-\gamma\sigma+i\delta\sigma, \tag{24}$$

where the offset of the atomic transition frequency from the field frequency is $\delta=\omega_0-\omega$. In the same approximation the expression for the force is

$$F=d\left[\sigma\frac{\partial E^{(-)}}{\partial z}+\sigma^{*}\frac{\partial E^{(+)}}{\partial z}\right]. \tag{25}$$

The expression (25) represents the force acting on an atom at a point whose coordinate is $z$. When the atom is traveling at a velocity $v$, this coordinate is $z=z_0+vt$. We are interested in the average force during the time which is long compared with $1/kv$ and $1/\Omega$, and which determines the momentum transferred to the atom as a result of its prolonged interaction with the field.

In the case of weak saturation [Eq. (24)] we can obtain the solution by expanding in the small parameter $\Omega_r/\gamma$, where $\Omega_r=d\mathscr{E}/\hbar$ is the Rabi frequency. In the general case $v\neq 0$ and $\delta\neq 0$, the force acting on an atom is a quadratic function of the Rabi frequency and appears even in the first order of expansion of the density matrix:

$$F=\frac{\hbar k\gamma}{8}\Omega_r^2\left[\frac{1}{\gamma^2+(\delta+kv+\Omega/2)^2}-\frac{1}{\gamma^2+(\delta-kv+\Omega/2)^2}+\frac{1}{\gamma^2+(\delta+kv-\Omega/2)^2}-\frac{1}{\gamma^2+(\delta-kv-\Omega/2)^2}\right]. \tag{26}$$

The first two terms describe the interaction of an atom with a standing wave of frequency $\omega-\Omega/2$, whereas the third and fourth terms describe the interaction with a wave of frequency $\omega+\Omega/2$. If $\delta=0$ or $v=0$, Eq. (26) vanishes and the force must be calculated including third-order terms, Obviously, in this case the force is due to the simultaneous interaction of an atom with both standing waves (in the case of one standing wave if $v=0$ or $\delta=0$ the force vanishes in any order of an expansion in $\Omega_r/\gamma$—see Ref. 10) and, in general, it should depend on the phase shift $\psi$ between the standing waves.

We now give the expressions for the force when $v=0$ and $\delta=0$. If $\delta\neq 0$ and $v=0$, we obtain

$$F=-\frac{1}{8}\hbar k\gamma^2\Omega_r^4\Omega\sin 2\psi\frac{\gamma^2+(\Omega/2)^2+\delta^2}{(\gamma^2+(\Omega/2+\delta)^2)^2(\gamma^2+(\Omega/2-\delta)^2)^2}. \tag{27}$$

At low values of $\Omega$ the force is a maximum for the "symmetric" ($\delta=0$) tuning. An increase in the difference between the frequencies of the standing waves $\Omega$ shifts the maximum to the region where $|\delta|\sim\Omega/2$. In this case the frequency of one of the standing waves is close to the frequency of an atomic transition and the magnitude of the maximum is proportional to $\Omega^{-1}$ if $\Omega$ is large. The optimum frequency difference ensuring that the force of Eq. (27) is maximal amounts to $\Omega=2\times5\times10^{-1/2}\gamma$.

The other case ($\delta=0$, $v\neq 0$), when the force is proportional to the fourth power of the Rabi frequency, was considered by us earlier.[4] The expression obtained in Ref. 4 for the average force is

$$F=-\frac{1}{8}\hbar k\Omega_r^4\gamma^2\Omega\times\frac{[\gamma^2+(\Omega/2)^2-(kv)^2]\sin 2\psi}{[(\gamma^2+(\Omega/2)^2)^2+(kv)^4+2\gamma^2(kv)^2-{}^1/_2\Omega^2(kv)^2]^2}. \tag{28}$$

At low values of $\Omega$ the force reaches its maximum at $v=0$ and an increase in the frequency difference between the standing waves shifts this maximum to $|kv|\sim\Omega/2$.

The average force acting on an atom in the case of arbitrary values of $\Omega_r/\gamma$ can be considered at low velocities, assuming however that during the time that an atom crosses a light beam it travels more than one wavelength, so that in the calculation of the force we can assume $v=0$ and then average the results obtained over $z$. Strictly speaking, this approach is valid if $kv\lesssim\gamma$, but using the above analysis based on perturbation theory (according to which the characteristic velocity scale is $v\sim\Omega/k$), we can expect the results obtained to be valid (at least to the nearest order of magnitude) also when $kv>\gamma$. This is supported also by the results of numerical integration of the system (24) when $z=z_0+vt$ and $\delta=0$ (Ref. 4).

If we represent $N$, $\sigma$, and $\sigma^*$ in Eq. (24) by Fourier series

$$N=\sum_n N_n\exp\left(\frac{in\Omega t}{2}\right), \tag{29}$$

we obtain a recurrence relation between the Fourier components $N_n$:

$$A_{n+2}N_{n+2}+B_nN_n+A_{n-2}N_{n-2}=-\frac{4i\gamma}{\Omega_r^2}\delta_{n0}, \tag{30}$$

where

$$A_n=\left(\frac{1}{\gamma^{*}_{-n+1}}-\frac{1}{\gamma_{n-1}}\right)\cos\left(kz+\frac{\psi}{2}\right)\cos\left(kz-\frac{\psi}{2}\right),$$
$$B_n=-\frac{n\Omega+4i\gamma}{\Omega_r^2}+\left(\frac{1}{\gamma^{*}_{-n+1}}-\frac{1}{\gamma_{n+1}}\right)\cos^2\left(kz+\frac{\psi}{2}\right)+\left(\frac{1}{\gamma^{*}_{-n-1}}-\frac{1}{\gamma_{n-1}}\right)\cos^2\left(kz-\frac{\psi}{2}\right), \tag{31}$$
$$\gamma_n=\delta-\frac{n}{2}\Omega+i\gamma.$$

Assuming that $N_{n+2}=N_nq_n$, we obtain

$$q_n=-\frac{A_{n+2}}{A_{n+4}q_{n+2}+B_{n+2}} \tag{32}$$

for even values of $n$ and $q_n=0$ for odd values. If $n=0$, it follows from Eq. (30) that

$$N_0=-\frac{4i\gamma}{\Omega_r^2(B_0+A_0q_0^{*}-A_0^{*}q_0)}, \tag{33}$$

where $q_0$ is given by the continued fraction (32). The force acting on an atom is expressed in terms of $q_0$ and $N_0$ as follows:

$$F=\frac{\hbar k}{2}N_0\Omega_r^2\,\mathrm{Re}\left\{\frac{1}{\gamma_{-1}}\sin\left(kz-\frac{\psi}{2}\right)\left[\cos\left(kz-\frac{\psi}{2}\right)+q_0^*\cos\left(kz+\frac{\psi}{2}\right)\right]+\frac{1}{\gamma_1}\sin\left(kz+\frac{\psi}{2}\right)\left[\cos\left(kz+\frac{\psi}{2}\right)+q_0\cos\left(kz-\frac{\psi}{2}\right)\right]\right\}. \quad (34)$$

As in the weak saturation case, the force $F$ is an odd function of $\psi$ with a period $\pi$, but at high values of the ratio $\Omega_r/\gamma$ the dependence $F(\psi)$ is very different from sinusoidal, as demonstrated in Fig. 1 (this figure gives the values of $F/\hbar k\gamma$ averaged over $z$). The maximal force corresponds to $\psi\approx\pi/4$, $\Omega\approx\Omega_r$, and $\delta=0$. Figure 2 gives the dependence of the force, averaged over $z$, on the square of the Rabi frequency calculated for different values of $\Omega$ and $\delta$. In the range $\Omega_r<\gamma$, which corresponds to perturbation theory, the force is proportional to $\Omega_r^4$, i.e., it is proportional to the product of the intensities of the standing waves. An increase in $\Omega_r$ in the range $\gamma<\Omega_r<\Omega$ shows that for $\delta=\Omega/2$ (which corresponds to the conditions in our experiments) the force is proportional to $\Omega_r^2$, which is the intensity of laser radiation. A change in the nature of the dependence on $\Omega_r$ is due to saturation of an atomic transition by one of the standing waves, whereas because of a large offset the field of the other wave represents a small perturbation. In this case an expression for the force can be obtained from Eqs. (31)–(34) on the assumption that $q_2=0$ and this can be done by averaging over $z$:

$$F=-\hbar k\frac{\Omega_r^2}{4\Omega}\sin 2\psi. \quad (35)$$

In the symmetric tuning case when $\delta=0$ the dependence of the force on $\Omega_r^2$ is nonlinear, because an atomic transition is far from saturation (Fig. 3).

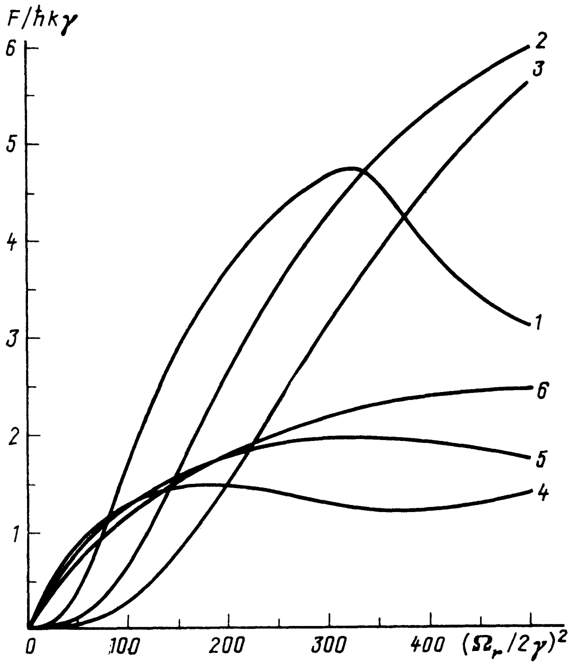


FIG. 2. Dependence of the SLP force on the saturation parameter $(\Omega_r/2\gamma)^2$: $\delta=0$ (curves 1–3), $\Omega/2$ (curves 4–6); $\Omega/2\gamma=15$ (curves 1 and 4), 20 (curves 2 and 5), and 25 (curves 3 and 6).

## 5. EXPERIMENTAL RESULTS

The first observation of SLP was reported by us in Ref. 5. We shall now give the results of a more detailed investigation of the SLP characteristics.

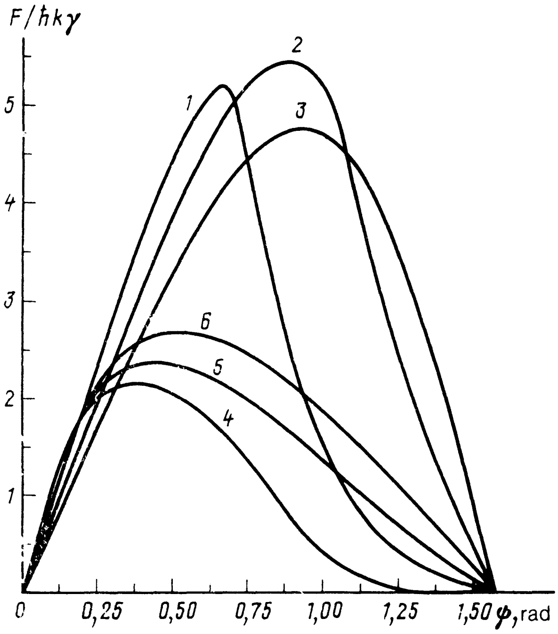


FIG. 1. Dependence of the SLP force on the phase difference $\psi$ between the standing waves: $\delta=0$ (curves 1–3), $\Omega/2$ (curves 4–6); $\Omega=0.75\Omega$ (curves 1 and 4), $1.25\Omega_r$ (curves 3 and 6), and $\Omega_r$ (curves 2 and 5).

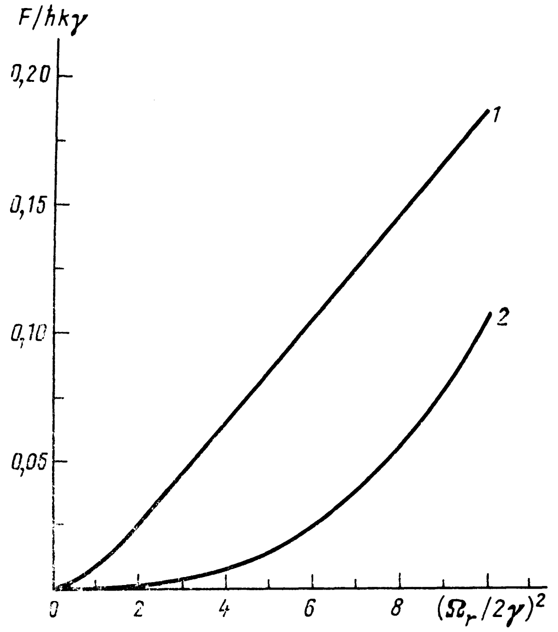


FIG. 3. Dependence of the SLP force on the saturation parameter $(\Omega_r/2\gamma)^2$ at low values of $\Omega_r$; $\Omega/2\gamma=20$; $\delta=0$ (curve 1), $\Omega/2$ (curve 2). In the case of curve 2 the values on the ordinate should be multiplied by 0.01.

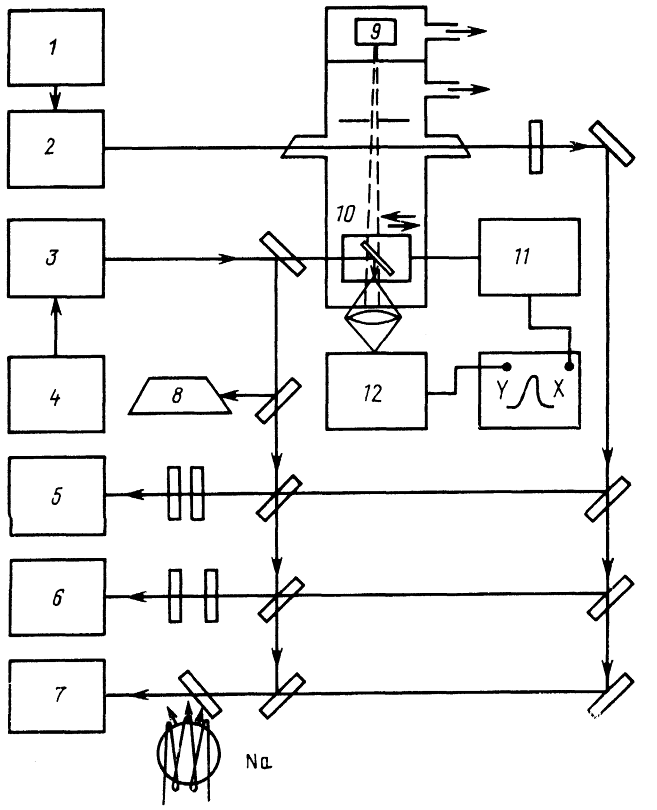


FIG. 4. Block diagram of the apparatus: 1), 4) pump lasers; 2) two-frequency deflecting laser; 3) probing laser; 5), 6) optical radiation detectors; 7) spectrograph; 8) cell containing Na vapor; 9) source of the beam of Na atoms; 10) system for scanning the beam of the probing laser; 11) unit for control of the scanning system; 12) photomultiplier.

Figure 4 is a block diagram of the apparatus used to study the SLP produced by the field of two standing waves interacting with a beam of sodium atoms traveling at right-angles to the wave vectors of the field.

The necessary high intensity and the required spectral and spatial characteristics were ensured by the interaction of the atomic beam with an intracavity field of a two-frequency cw dye laser. The resonator of this laser included a scheme for compensation of the astigmatism. The longitudinal modes were selected and the frequency was tuned to the absorption line of sodium by a three-component Lyot filter and a Fabry–Perot etalon which was 2 mm thick. The laser radiation wavelength was monitored using a spectrograph, scanning Fabry–Perot interferometers, and a comparison cell containing sodium vapor; precise tuning to the $D_2$ line of sodium was made using a fluorescence signal emitted by the atomic beam.

A spatially inhomogeneous depletion of the population inversion in the adopted selectors ensured stable two-frequency lasing throughout the investigated range of pump powers (the maximum pump power was $P_{max} = 2$ W and it represented all the lines of an ILA-120 argon ion laser). The frequency interval $\Delta\nu$ between the modes was $\Delta\nu = \nu_1 - \nu_2 = c/4l_0$, where $l_0$ is the distance from the dye jet to the nearest mirror. In our experiments this distance was $l_0 = 4.5$ cm, corresponding to $\Delta\nu = 1.67$ GHz. For a given value of $\Omega = 2\pi\Delta\nu$ a change in the distance between the standing-wave nodes by $\lambda/2$ (or, which was equivalent, a change in the difference between the phases $\psi$ of the amplitude-modulated counterpropagating waves by $\pi$) resulted from a displacement by $\Delta z = 9$ cm along the resonator axis.

The spatial distribution of the atoms in the beam was deduced from a fluorescence signal excited by a second (probing) dye laser. The radiation from this laser was directed at right-angles to the atomic beam and to the resonator axis of the "deflecting" laser, reaching a point separated by $L = 35$ cm from the region of interaction between the atomic beam and the deflecting laser beam. The necessary spatial resolution was ensured by focusing the laser radiation, so that in the region of intersection of the probing beam with the atomic beam the radius of a spot did not exceed 80 $\mu$m. The probing beam could be shifted parallel to itself in a plane perpendicular to the atomic beam axis. The phosphorescence signal was recorded with a photomultiplier and the lowest detectable density of the sodium atoms was about $3\times10^5$ cm$^{-3}$ when the signal/noise ratio was 2.

The sodium atomic beam was created in a two-section vacuum chamber (residual pressure $\lesssim 10^{-6}$ torr). The first section contained the atomic beam source whose temperature could be varied from 90 to 400 °C and could be kept constant at the selected value to within 0.5 °C. The second (transit) section had windows (oriented at the Brewster angle) for injection of the deflecting laser radiation, windows for injection of the probing laser radiation and for detection of fluorescence of the atomic beam excited by both the probing and deflecting lasers. The atomic beam divergence was less than $5\times10^{-3}$ rad, the density of atoms in the beam in the region of interaction with the field of the two standing waves was $2\times10^7$ cm$^{-3}$ at the working frequencies, and the beam diameter was 0.5 mm. Under these conditions the change in the phase difference $\psi$ across the beam diameter was less than $2\times10^{-2}$ rad.

In the course of our experiments the vacuum chamber was inside the deflecting laser resonator. The distance from the exit mirror of the resonator to the atomic beam could be varied within a fairly wide range and this made it possible to set in a controlled manner the phase difference $\psi$ to within $2\times10^{-2}$ rad. In the course of our experiments the orthogonal orientation of the wave vector of the deflecting beam to the atomic beam axis was maintained to within $2\times10^{-3}$ rad.

The SLP altered the radial distribution function $f(r)$ of the atoms in the beam. This could be described by a shift of the center of gravity of this beam [by the first moment of the experimentally determined distribution function $f_e(r)$] and by an rms deviation from the center of gravity [the square root of the difference between the second and square of the first moments of $f_e(r)$], representing the broadening of the atomic beam.

Note that a comparison of the experimental and theoretical results was difficult under the selected conditions for a number of reasons. Since the atomic beam was not monochromatic and the distribution of the velocities of the atoms obeyed a function[11]

$$f(v)=\frac{2}{v_0}\left(\frac{v}{v_0}\right)^3\exp\left(-\frac{v^2}{v_0^2}\right), \tag{36}$$

where $v_0$ is the thermal velocity, the action of a constant SLP force $F$ not only shifted the center of gravity of the atomic beam, but also broadened it even in the absence of diffusion. In the case of low-divergence beams the distribution function $f(r)$ in the plane of observation was

$$f(x,y)=\frac{x_0^2}{x^3}\exp\left(-\frac{x_0}{x}\right)f_0(y), \quad (37)$$

where $x_0 = bFL/mv_0^2$ is the displacement of an atom traveling at the thermal velocity, $m$ is the mass of this atom, $b$ is the length of the region of interaction of the atoms with the field, $L$ is the distance from the interaction region to the plane of observation obeying the inequality $L \gg b$, $f_0(y)$ is the distribution function of the unperturbed beam along the $y$ axis. Equation (37) was derived on the assumption $x_0 \gg L\theta$, where $\theta$ is the divergence of the atomic beam and the $x$ axis is parallel to the wave vector of the field. Using Eq. (37), we found that $x_0$ was equal to deflection of the center of gravity of the beam. One could readily see that under the assumption that the force $F$ was constant throughout the length of the interaction region $b$, this was true also of a beam with a divergence $\theta \gtrsim x_0/L$. Therefore, we could assume that the displacement of the center of gravity of the atomic beam was proportional to the force acting on the atoms and equal to the displacement of the atoms at the thermal velocity $v_0$.

However, as demonstrated by Eq. (37), the second moment

$$M_2=\int_0^\infty x^2 f(x,y)\,dx\,dy$$

should not exist ($M_2 = \infty$). Since the experimental distribution function $f_e(x)$ was recorded only for $x < L_0$ and $x_0 \ll L_0$, the experimental second moment was

$$M_{2e}\approx x_0^2 \ln\frac{L_0}{x_0}.$$

An analysis of the broadening of the beam characterized by $M_{2e} \neq 0$ should allow not only for the contribution associated with the nonmonochromaticity of the atomic velocity distribution, but also for the role of the diffusion of atoms that accompanied the SLP effects and for the initial divergence of the beam. Therefore, the manifestation of the pulsed diffusion effects when the SLP acted on atoms under our experimental conditions was masked by these secondary effects, so that the experimental value $\Delta x = (M_{2e} - x_0^2)^{1/2}$ could not be related directly to the pulsed diffusion.

On the other hand, it is known that because of the presence of the hyperfine structure the $D_2$ line of sodium cannot be described by the two-level approximation. Therefore, in comparing the experimental results with the theoretical conclusions we could expect only qualitative agreement.

A theoretical analysis indicated that an important feature of the SLP effect was the dependence of the displacement $x_0$ of the center of gravity of the atomic beam on the phase difference $\psi$. The experimental dependence $x_0(\psi)$ plotted in Fig. 5 was in qualitative agreement with the theoretical predictions [see Eq. (35)]: the value of $x_0$ exhibited reversal of the sign when the difference between the phases $\psi$ was altered by $\pi/2$; the dependence $x_0(\psi)$ was nonmonotonic and the maximum value of $x_0$ corresponded to $\psi = \pm\pi/4$. The existence of a characteristic dependence $x_0(\psi)$ led us to the conclusion that we did indeed encounter the SLP effects and the observation that $x_0 \neq 0$ was unrelated to a possible difference between the amplitudes of the counterpropagating waves in the resonator (due to, for example, transmission by the mirrors) and it was also unrelated to

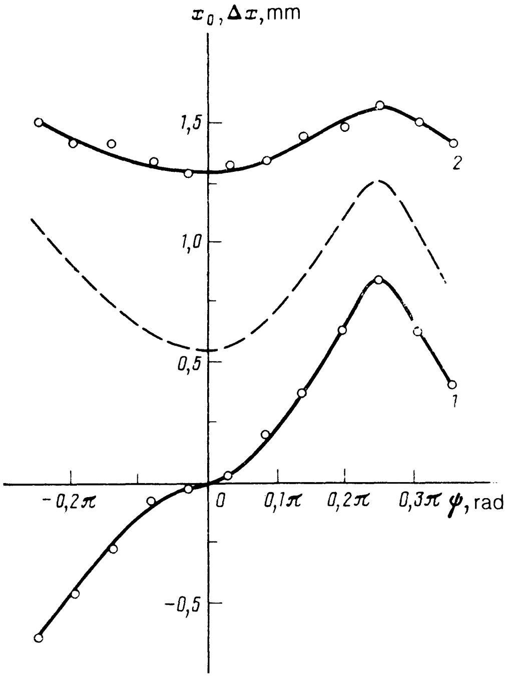


FIG. 5. Dependence of the displacement of the center of gravity $x_0$ (curve 1) and of the broadening $\Delta x$ (curve 2) of the atomic beam on the difference between the phases $\psi$ of amplitude modulation of the counterpropagating waves. The dashed curve shows the beam broadening without allowance for the pulsed diffusion.

nonorthogonality of the atomic beam relative to the resonator axis. These two factors could give rise to $x_0 \neq 0$, but the magnitude and the sign of $x_0$ should then be independent of $\psi$, i.e., they should be independent of the distance between the resonator mirror and the atomic beam.

The beam broadening $\Delta x$ also depended on $\psi$ and, as demonstrated in Fig. 5, the nature of this dependence $\Delta x(\psi)$ was in qualitative agreement with the dependence $x_0(\psi)$. Consequently, under our experimental conditions the main contribution to $\Delta x$ came from the beam divergence and its nonmonochromaticity. We plotted in Fig. 5 (dashed curve) a quantity $\Delta x_1 = (\Delta x_0^2 + x_0^2 \ln L_0/x_0)^{1/2}$, where $\Delta_x$ is the beam broadening in the absence of the deflecting laser field; $\Delta x_0 = 0.56$ mm; $L_0 = 5$ mm, which was calculated for the experimental values of $x_0$. The quantity $\Delta x_1$ was the expected beam broadening in the absence of pulsed diffusion. This diffusion clearly made a significant contribution to the beam broadening and the contribution was maximal at $\psi = 0$ when $F = 0$ and minimal at $\psi = \pm\pi/4$ and the SLP force was then maximal. Note also that for $\psi = \pm\pi/4$ the displacement $x_0$ of the center of gravity of the atomic beam was much greater than the contribution of the diffusion process to the atomic beam broadening. Therefore, the diffusion in the presence of SLP in the field of two standing waves was on the whole similar to the properties of diffusion in the field of two counterpropagating pulse trains discussed above if we allow for the correspondence between the phase $\psi$ and the delay $\tau$ between the counterpropagating pulses.

The dependences of $x_0$ and $\Delta x$ on the frequency of the deflecting laser radiation were also determined (Fig. 6). The value of $x_0$ attained a maximum when the frequencies of the laser modes coincided with the frequencies of the two main

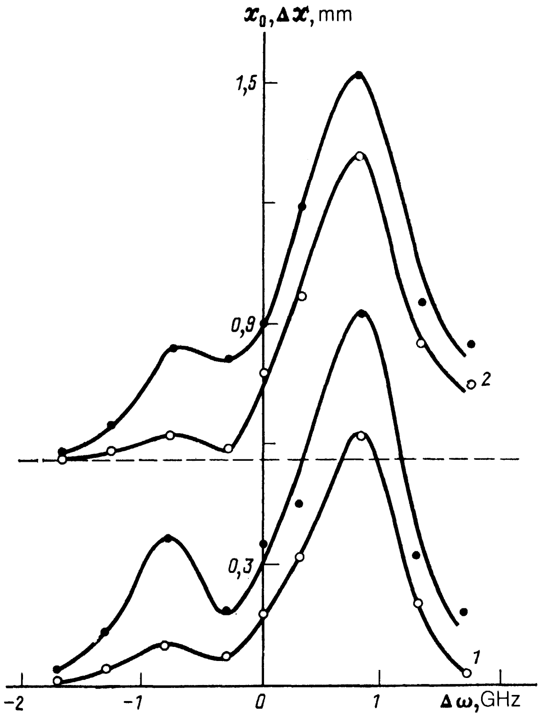


FIG. 6. Dependence of the displacement of the center of gravity $x_0$ (curve 1) and of the broadening $\Delta x$ (curve 2) of the atomic beam on the laser radiation frequency $\Delta\omega$: ●) intracavity laser power $P = 0.6$ W; ○) intracavity laser power $P = 0.4$ W.

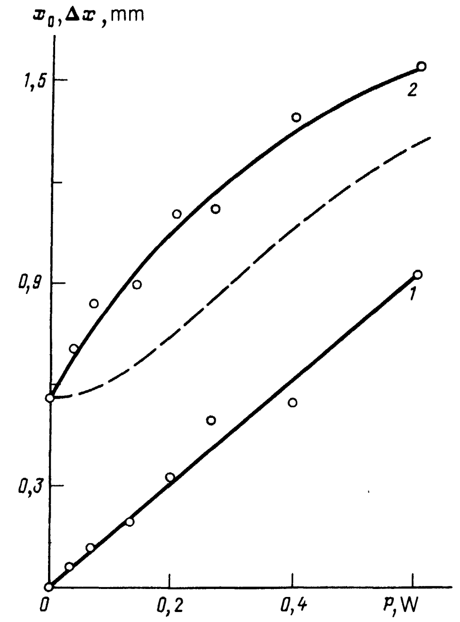


FIG. 7. Dependence of the displacement of the center of gravity $x_0$ (curve 1) and of the beam broadening $\Delta x$ (curve 2) on the intracavity laser power $P$. The dashed curve represents the beam broadening without allowance for pulsed diffusion.

components of the hyperfine structure of the $D_2$ line of sodium. Such a tuning was possible because of the proximity of the intermode spacing $\Delta\nu = 1.67$ GHz to the hyperfine splitting of the ground state $\Delta\nu_{hfs} = 1.77$ GHz. The second and weaker maximum corresponded to tuning of one of the laser modes to the stronger hyperfine-structure component, whereas the second mode was separated by $\Delta\nu$ from this component and by $\Delta\nu + \Delta\nu_{hfs}$ from the second component. The tuning of one of the modes to the weaker component was not investigated because the signal/noise ratio was then too low.

The relatively high value of $x_0$ in the case of exact tuning of the modes was due to an increase in the duration of interaction of atoms in the beam with the field since the undesirable effects of the optical pumping were avoided under these conditions. It should be stressed that the existence of the second maximum also supported the stimulated nature of the observed optical effects. When a similar tuning was studied in the case of traveling waves, the force due to the spontaneous light pressure was practically undetectable, because in the case of spontaneous reemission of even one or two photons, the atom ceased to interact with the field due to the optical pumping. This was a demonstration of the likely usefulness of the SLP in deflection of molecules, because under such tuning conditions an atom of sodium was essentially similar to a molecule in which spontaneous relaxation is accompanied by a transition to levels not coupled to the laser field.

The dependence of $x_0$ and $\Delta x$ on the intracavity power is plotted in Fig. 7. The tuning of the mode frequencies corresponded to the maximum force. The observed linear dependence of $x_0$ on the power was in agreement with the theory (Fig. 3), because in this range of powers we had $\gamma < \Omega_r < \Omega$. The dependence of the broadening $\Delta x_1$ in the absence of diffusion was also included in Fig. 7 (dashed curve). Clearly, the relative contribution of the diffusion process decreased on increase in the power and, consequently, on increase in the SLP force. The maximum SLP force was $F \approx 1.2\hbar k\gamma$, which was comparable with the maximum force acting on an atom in the field of just one traveling wave. An increase in the power of the pump laser and selection of the optimal value of $\Omega$ (in our case the value of $\Omega$ was governed by some technical parameters of the dye laser) should make it possible to reach the values of the force $F$ considerably in excess of $\hbar k\gamma$.

Our experimental investigation confirmed the stimulated nature of the observed optical pressure and demonstrated that it should be possible to generate an SLP force considerably greater than $\hbar k\gamma$ acting on both atoms and molecules.


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