Resonant excitation and stabilization of atomic Rydberg levels during multiphoton ionization in a strong laser field

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Resonant ionization of atoms in two fields—an exciting (weak) and an ionizing (strong) field—under resonant conditions in the region of the Rydberg levels, whose spectrum is restructured under the action of a strong ionizing field, is considered. The photoelectron energy spectrum and the dispersion dependence of the ionization rate on the frequency of the exciting field are found. It is shown that under certain conditions these functions have narrow minima in the vicinity of values of the electron energy and the exciting field frequency corresponding to resonances on the quasi-energy atomic levels. The nature of the effect is elucidated and it is shown that it is due to interference between the direct resonant and cascade ionization processes. On this basis a physical interpretation of the stabilization of multiphoton ionization of atoms in the ground state, located in a very strong field, is presented. The case of short laser pulses with a smooth envelope and the specific features of the excitation and interference stabilization of the Rydberg levels which thus arise are considered.

1. INTRODUCTION

The phenomenon of resonant multiphoton ionization of atoms has been studied in great detail both theoretically and experimentally[1-5] [see also the literature cited in Refs. 3 and 4 (review articles) and Ref. 5]. In the description of this phenomenon it is usually assumed that only one isolated discrete level of the atom or a very small number of close-lying levels take part in the resonance. The situation is qualitatively changed if some number k of the quanta of a strong laser field $k\hbar\omega$ (ω is the frequency) produce a resonant transition from the ground state of the atom (with energy E_0) to the region of Rydberg levels E_n (n is the principal quantum number, $n \gg 1$):

$$E_n \approx E_0 + k\hbar\omega.$$

If the interaction energy of the atom with the field is sufficiently large, then a large number of Rydberg levels can take part simultaneously in the resonance, which can bring about qualitative changes in the character of the process of resonant ionization compared to the case of an isolated level. This prediction is based on the recent theoretical discovery and description of a class of interference effects which arise in the photoionization of Rydberg atoms in a strong field.[6-10] In particular, the question of the single-photon ionization of an atom was considered, in which the atom at the initial instant of time is found at some highly excited level E_n, $n \gg 1$. It was shown that at a high enough intensity of the ionizing field, when the energy of its interaction with the atom becomes greater than the distance between neighboring Rydberg levels, the following unusual phenomena arise:

1) In a strong field the quasi-energy spectrum of the atom is redistributed. The quasi-energy levels $E^{(n)}$ are localized at the midpoints between the neighboring Rydberg levels of the free atom

$$E^{(n)} = {}^1/_2(E_n + E_{n+1}). \qquad (1)$$

2) The width Γ of the quasi-energy levels (1) in a strong field becomes a decreasing function of the field strength ε_0 and becomes smaller than the distance between neighboring levels. In other words, the quasi-energy levels $E^{(n)}$ (1) are narrow and are narrowed by the field instead of broadened.

3) The energy spectrum of the photoelectrons in a strong field acquires a multi-peaked appearance. These peaks are localized near $E^{(n)} + \hbar\omega$. If n_0 is the number of the solitary initially populated level, then the two main maxima, which are of equal height, are localized near $E^{(n_0)} + \hbar\omega$ and $E^{(n_0-1)} + \hbar\omega$. The heights of the remaining maxima fall off as $(n - n_0 + 1/2)^{-2}$. The widths of the maxima are equal to the width of the quasi-energy levels Γ and decay as the field ε_0 grows, at the same time that the heights of all the maxima grow.

4) Depending on the time t, the total photoionization probability is characterized by an ionization time $t_i = 1/\Gamma$, which in a strong field becomes an increasing function of ε_0, i.e., there takes place a strong-field-induced stabilization of the atom, or a slowing down of the ionization process, or trapping of the populations to discrete levels. Qualitatively similar phenomena also take place during the initial coherent population of many Rydberg levels. For example, as before, the photoelectron energy spectrum has a multi-peaked character with maxima localized near $E^{(n_0)} + \hbar\omega$. But the envelope of these maxima is different from that described above: it has a dip in the center, i.e., near $E^{(n_0)} + \hbar\omega$, where the level number n_0 corresponds to the center of the zone of the initially populated Rydberg levels.

It is of unquestioned interest to study the question of how these properties of the Rydberg levels are manifested in a strong ionizing field during multiphoton resonant ionization of an atom initially located in the ground state. Some aspects of this problem were considered in Ref. 10. However, many questions which are key for the theory of resonant ionization still remain unanswered. These include the energy spectrum of the photoelectrons formed during resonant multiphoton ionization, the dynamics of the processes of ionization and excitation of Rydberg levels, the dispersion

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dependence of these probabilities on the field frequency, etc. A determination and an analysis of these characteristics of the multiphoton resonant ionization process constitute the contents of the present work.

Note that in the literature there exist some indications that such an unusual effect as the slowing down of the ionization of an atom can appear in a very strong electromagnetic field. First of all, such a result follows from a consideration based on averaging the Hamiltonian in the asymptotic limit of very high field frequency.[11] Second, lowering of the ionization level in a very strong field was also detected in numerical modeling calculations.[12] In our opinion, under certain conditions the suppression of the ionization can be connected with the multiphoton excitation of the Rydberg levels and their interference stabilization[13,14] (see also Sec. 5 below).

We now formulate some basic approximations and assumptions which will be used below. For the matrix elements of the transitions between the Rydberg states and the states of the continuum under the action of the operator $\hat{V} = -(1/2)\mathbf{d}\cdot\boldsymbol{\varepsilon}_0$ ($\mathbf{d}$ is the dipole moment) we will use the quasiclassical expressions[15-17]

$$V_{nE}\sim\varepsilon_0 n^{-3/2}|E-E_n|^{-5/3}\approx\varepsilon_0 n^{-3/2}\omega^{-5/3}\sim Vn^{-3/2}, \qquad (2)$$

$$V\equiv H_{nE}n^{3/2}\sim\varepsilon_0\omega^{-5/3}=\text{const},$$

where $E>0$ is the energy of the electron in the continuum; the atomic system of units is used and in the second equality we have applied the energy conservation law $E-E_n\approx\omega$, which assumes $\omega>(1/2)n^2$ (single-photon transitions $E_n\to E$ are possible). Replacing E by $E_n+\omega$ in expression (2) for V_{nE} is justified because V_{nE} is a sufficiently smooth function of the energy E and the characteristic interval δE of continuum energy levels in the vicinity of $E_n+\omega$ which are effectively populated during the ionization process is much less than ω. The above strong-field criterion has the form $V>1$ or $\varepsilon_0>\omega^{5/2}$.

Within the framework of the approximation (2), for $V>1$ (i.e., in a strong field) the width Γ of the narrow quasienergy levels $E^{(n)}$ (1) is equal to[6,7]

$$\Gamma=\frac{2}{\pi^3 n^3 V^2}. \qquad (3)$$

We will formulate the problem of multiphoton resonant ionization as a problem of two-photon ionization in two fields—an exciting field and an ionizing field (Fig. 1). If the ionizing field has frequency ω and intensity ε_0, then for the exciting field the corresponding parameters are equal to ω' and ε_0'. We will assume that the interaction with both fields is turned on simultaneously and instantaneously at $t=0$. If the duration of the interaction with both fields is identical, then there is a complete correspondence of the problem as posed with the multiphoton ionization in one strong monochromatic field. To carry over to the solution of the latter it is sufficient to replace ω' by $k\omega$ and the matrix elements $V'_{n0}\equiv(-\mathbf{d}\cdot\boldsymbol{\varepsilon}_0'/2)_{n0}$ by the component matrix elements of order k, i.e., by $V^{(k)}_{n0}\equiv(-\mathbf{d}\cdot\boldsymbol{\varepsilon}_0/2)^{(k)}_{n0}$. However, we will also consider another formulation of the problem, in which the ionizing and exciting fields are different and can have not only different frequencies and intensities, but also act for different times. This formulation of the problem is completely realistic and, as will be shown below, is very interesting

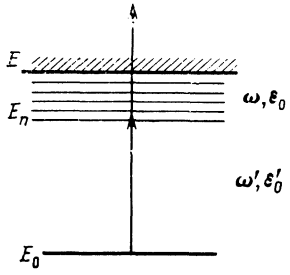

FIG. 1. Diagram of the process of two-photon resonant ionization of an atom in the presence of a resonance at the Rydberg levels.

from the point of view of an understanding of the physics of the results obtained.

Section 5 gives a qualitative description of the smooth switching-on of a strong field which both excites and ionizes the atom.

We will restrict the discussion in the present article to the case in which the exciting field can be assumed to be weak and it is not necessary to take into account the depletion of the ground state. This assumption allows us to use perturbation theory in the first step of the process, the excitation of the Rydberg levels. At the same time, the interaction of the Rydberg levels with the continuum is described without assuming anything about the weakness of the ionizing field, i.e., the parameter V in Eq. (2) can be arbitrary: $V\gtrless 1$.

Note that for $V>1$ it is necessary to allow for transitions not only from the Rydberg levels to the continuum, but also between the states of the continuum. In Refs. 6 and 7 such transitions were taken into account by means of the model of significant states.[18] The main result of such a generalization of the theory is the renormalization of the constant V: $V\to(V/\pi)^{1/2}$ for $V\gg 1$. This substitution has no effect on the qualitative conclusions of the theory. We will therefore not dwell on this problem of taking account of transitions within the continuum, nor on the question of the applicability of the model of essential states (see in regard to this Refs. 19 and 20).

In the present work we will limit the discussion to the model of the one-dimensional spectrum of the Rydberg levels, neglecting the degeneracy in the orbital momentum in a three-dimensional Coulomb potential. According to Refs. 6 and 7, an account of this degeneracy, and also of the Stark level splitting, does not alter the qualitative conclusions obtained in the one-dimensional model, although it complicates the theory and the details of the behavior of the investigated quantities considerably.

Some other approximations which we will use will be described below within the body of the exposition of the main material.

Finally, to close out this section, we cite some numerical estimates of the main parameters which affect the formulation of the problem. If the atom is found initially in the ground state, and an exciting field with parameters ε_0' and ω' creates a resonance with the highly excited states with principal quantum number $n\approx 20$, then the frequency satisfies $\omega'\approx|E_0|\sim 10$ eV, where $|E_0|$ is the ionization potential of the atom. The binding energy of the states with $n\approx 20$ is $|E_n|\approx 0.034$ eV, so that the frequency of the ionizing field

should satisfy the condition $\omega > 0.034$ eV. In particular, the discussion may be about CO_2-laser radiation ($\omega \approx 0.1$ eV). The strong-ionizing-field criterion $V > 1$ in this case has the form $\varepsilon_0 > 10^6$ V/cm, whereas the inverse criterion $\varepsilon_0 \ll 10^6$ V/cm corresponds to the case of a weak field. Finally, the characteristic time scale for the resonant states is determined by the Kepler period $2\pi n^3$ and for $n \approx 20$ is equal to 1 ps.

2. RESONANT IONIZATION IN THE CASE OF IDENTICAL EXCITING AND IONIZING PULSE LENGTHS (GENERAL EQUATIONS)

Let us now consider the process of two-photon ionization, a diagram of which is shown in Fig. 1. We write the wave function of the atom in a field in the form of a superposition of wave functions of the free atom φ_0 (the ground state), φ_n (the Rydberg levels), and φ_E (the continuous spectrum)

$$\Psi = \exp(-iE_0 t)\left\{\varphi_0 + \exp(-i\omega' t)\sum_n A_n(t)\varphi_n + \exp[-i(\omega+\omega')t]\int_0^\infty dE\, A_E(t)\varphi_E\right\}, \quad (4)$$

where E_0 is the energy of the ground state of the atom, $A_n(t)$ and $A_E(t)$ are the unknown amplitudes of the probability of finding the atom in the states φ_n and φ_E. In line with the assumption that depletion is absent from the ground state, the coefficient of the term $\varphi_0 \exp(-iE_0 t)$ in the expansion (4) is set equal to unity. The equations for $A_n(t)$ and $A_E(t)$ follow directly from the Schrödinger equation:

$$\dot{A}_n - (E_n - E_0 - \omega')A_n = 2V_{n0}'\cos(\omega' t)\exp(i\omega' t) + 2\int dE\, A_E(t)\, V_{nE}\cos(\omega t)\exp(-i\omega t), \quad (5)$$

$$\dot{A}_E - (E - E_0 - \omega - \omega')A_E = 2\sum_n A_n(t)\, V_{En}\cos(\omega t)\exp(i\omega t).$$

In what follows we will use the resonant approximation or the so-called rotating wave approximation, in which the terms $2\cos(\omega' t)\exp(i\omega' t)$ and $2\cos(\omega t)\exp(\pm i\omega t)$ are replaced by unity. The meaning of this approximation is that the energy regions in the vicinity of $E_0 + \omega'$ and $E_0 + \omega + \omega'$, where the amplitudes $A_n(t)$ and $A_E(t)$ are different from zero, are assumed to be small in comparison with ω and ω'. Taking into account the initial conditions $A_n(0) = A_E(0) = 0$, we solve the resulting system of equations by the Laplace transform method. As a result, for example, we find the equations which are satisfied by the Laplace transforms $\tilde{A}_n(p)$ of the functions $A_n(t)$:

$$(ip - E_n + E_0 + \omega')\tilde{A}_n(p) - \sum_{n'}\int_0^\infty \frac{dE V_{nE}V_{En'}\tilde{A}_{n'}(p)}{ip - E + E_0 + \omega + \omega'} = \frac{V_{n0}'}{p}. \quad (6)$$

Here

$$\tilde{A}_E(p) = \sum_n V_{En}\tilde{A}_n(p)/(ip - E + E_0 + \omega + \omega'). \quad (7)$$

After taking the inverse Laplace transform, the functions $A_n(t)$ and $A_E(t)$ are represented in the form of integrals over the variable p to the right of the branch cut in the complex p plane extending from the point $i(E_0 + \omega + \omega')$ to $-i\infty$.[6,7,19] On this contour we have $p = -iz + 0$ with $-(E_0 + \omega + \omega') \leqslant z \leqslant \infty$, and the integral over E in the second term on the left side of Eqs. (6) is equal to

$$\int dE \frac{V_{nE}V_{En'}}{ip - E + E_0 + \omega + \omega'} = \int dE \frac{V_{nE}V_{En'}}{z - E + E_0 + \omega + \omega'} - i\pi V_{nE}V_{En'}|_{E = z + E_0 + \omega + \omega'}. \quad (8)$$

Further simplification of Eqs. (6) is based on the so-called pole approximation.[18] In this approximation, in relations of the above type the principal value of the integral is neglected in comparison with the "pole" term [the second term on the right side of Eq. (8)]. Questions concerning the possibility of justifying the pole approximation and the conditions of its applicability are discussed in detail in Refs. 6, 7, and 19. Without dwelling on this problem, we note that, on the one hand, at the present time it has not been analyzed exhaustively, nor is there an absolutely convincing proof of the validity of the pole approximation. On the other hand, the results of numerical calculations are available[21] in which the contributions of the pole term and the principal value of the integral in the two-photon matrix element of the bound-free transition

$$E_n \to E = E_n + 2\omega$$

are compared under the condition that the intermediate states $E' \approx E_n + \omega$ are located in the continuum. The results of these calculations show that for $n \approx 20$ and $\omega \sim 10^{-1}$ eV the contribution of the principal value of the integral is a few percent of that of the pole term. Therefore there are grounds for assuming that the condition for the applicability of the pole approximation is the smallness of the field frequency ω and the energy of the electrons in the E continuum.[19] Note also that although the parameter z stands in the denominator of expression (8) in linear combination with ω, formally varying within infinite limits, in fact in Eq. (6) only the values $z \sim \Delta E \ll \omega$ are important, where $\Delta E \ll \omega$ is the effectively populated energy region in the continuum.

In the pole approximation the solution of Eq. (6) is easily found. It has the form

$$\tilde{A}_n(-iz) = \frac{i}{(z+i0)(z - E_n + E_0 + \omega' + i0)}\left\{V_{n0}' - i\pi \times V_{n\Sigma}\sum_{n'}\left[\frac{V_{En'}V_{n'0}}{E - E_{n'} - \omega}\left(1 + i\pi\sum_{n''}\frac{|V_{En''}|^2}{E - E_{n''} - \omega}\right)^{-1}\right]_{E = z + E_0 + \omega + \omega'}\right\}. \quad (9)$$

In the calculation of $\tilde{A}_E(p)$ by means of Eq. (7) we make one more approximation: we neglect the difference between the matrix elements V_{nE} and $V_{n,z+E_0+\omega+\omega'}$, which is justified by the relatively slow dependence of V_{nE} (2) on E and the smallness of the region δE of energies E in the vicinity of $E_0 + \omega + \omega'$, where the terms $\tilde{A}_E(p)$ are different from zero, and $\delta E \ll \omega$, E holds. In this approximation we find

$$\tilde{A}_E(-iz) = \frac{i}{(z+i0)(z - E + E_0 + \omega + \omega' + i0)} \times \sum_n \frac{V_{En}V_{n0}'}{z - E_n + E_0 + \omega'}\left(1 + i\pi\sum_{n'}\frac{|V_{n'E}|^2}{z - E_{n'} + E_0 + \omega'}\right)^{-1}. \quad (10)$$

The time-dependent functions $A_n(t)$ and $A_E(t)$ are expressed in terms of $\tilde{A}_n(z)$ and $\tilde{A}_E(z)$ by means of the equations

$$A_{n,E}(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}dz\exp(-itz)\tilde{A}_{n,E}(-iz). \tag{11}$$

The extension of the integration region over z to the entire axis $(-\infty, \infty)$ [instead of the half-axis $(-(E_0+\omega+\omega'), \infty)$] is justified in essence by the previously used assumption of the smallness of the energy intervals δE in which $A_E(t)$ and $A_n(t)$ [or $\tilde{A}_E(p)$ and $\tilde{A}_n(t)$] are different from zero in comparison with $E\sim\omega$. Indeed, the main contribution to $A_{n,E}(t)$ in Eq. (11) comes from the poles of the integrand function, which are located in the region $z\approx 0$, or more precisely, the intervals $\sim\delta E$ in the region of small z. By virtue of the condition $\delta E\ll E\sim E_n+\omega\sim E_0+\omega+\omega'\sim\omega$, this region in z is located far from the lower boundary of the integration region $z_{\min}=-(E_0+\omega+\omega')$, which allows us to extend the lower limit of integration in (11) to $-\infty$.

In the calculation of the ionization probability density over the energy

$$w(E,t)=|A_E(t)|^2$$

let us consider the case $\Gamma t\gg 1$, where Γ is the ionization width, which in a weak field ($V\ll 1$) is equal to

$$\Gamma=2\pi|V_{nE}|^2=2\pi V^2/n^3,$$

and in a strong field ($V>1$) is given by Eq. (3). In this approximation only two poles of the function $\tilde{A}_E(-iz)$ given in Eq. (10) contribute to the integral (11): $z=-i0$ and $z=E+E_0+\omega+\omega'-i0$. The poles at the points $E_n-i\Gamma/2$ (for $V\ll 1$) or $E^{(n)}-i\Gamma/2$ (for $V>1$) give an exponentially small contribution which can be neglected. The result of the calculation has the form

$$w(E,t)=\frac{1}{(E-E_0-\omega-\omega')^2}\left|\sum_n V_{En}V_{n0}'\right.$$
$$\times\left\{\frac{\exp[-it(E-E_0-\omega-\omega')]}{E-E_n-\omega}\left(1+i\pi\sum_{n'}\frac{|V_{n'E}|^2}{E-E_{n'}-\omega}\right)^{-1}\right.$$
$$\left.\left.-\frac{1}{E_0+\omega'-E_n}\left(1+i\pi\sum_{n'}\frac{|V_{n'E}|^2}{E_0+\omega'-E_{n'}}\right)^{-1}\right\}\right|^2. \tag{12}$$

The total ionization probability after a time t is given by

$$w_i(t)=\int dE\,w(E,t). \tag{13}$$

In the approximation $\Gamma t\gg 1$ the probability $w_i(t)$ (13) can be calculated not only using Eq. (12), but also as a result of an initial integration over E in the general formulas obtained with the help of Eqs. (10) and (11), in which it is then necessary to make use of the condition $\Gamma t\gg 1$. The result has the form $w_i(t)\approx t\dot{w}_i$, where

$$\dot{w}_i=2\pi\left|\sum_n\frac{V_{En}V_{n0}'}{\omega'+E_0-E_n}\left(1+i\pi\sum_{n'}\frac{|V_{n'E}|^2}{\omega'+E_0-E_{n'}}\right)^{-1}\right|^2. \tag{14}$$

In the calculation of $A_n(t)$ using Eqs. (9) and (11) it must be borne in mind that the point $z=E_n-E_0-\omega'$ is not a pole of the function $\tilde{A}_n(-iz)$. Therefore the only pole contributing to $A_n(t)$ in the approximation $\Gamma t\gg 1$ is the pole at the point $z=-i0$, which gives

$$w_n(t)=\frac{1}{(\omega'+E_0-E_n)^2}\left|V_{n0}'-i\pi\sum_{n'}\frac{V_{nE}V_{En'}V_{n'0}'}{\omega'+E_0-E_{n'}}\right.$$
$$\left.\times\left(1+i\pi\sum_{n''}\frac{|V_{n''E}|^2}{\omega'+E_0-E_{n''}}\right)^{-1}\right|^2. \tag{15}$$

Note that in contrast to $w(E,t)$ and $w_i(t)$, the probabilities $w_n(t)$ (15) do not depend on the time. This result is easily understood: in the case $\Gamma t\gg 1$ a steady flow of particles through the Rydberg levels takes place. The number of particles arriving at the Rydberg levels from the ground state is equal to the number of particles departing from it for the continuum. Such a regime, obviously, does not contradict the presence at the Rydberg levels of the stationary population given by Eq. (15).

3. DISPERSION DEPENDENCE WHEN THE EXCITING AND IONIZING RADIATION PULSE LENGTHS ARE EQUAL

Let us begin a description of the dispersion dependence with a description of the energy spectrum of the photoelectrons. It can be easily seen that, depending on E, the function $w(E,t)$ (12) has the form which is qualitatively depicted in Fig. 2. By virtue of the condition $\Gamma t\gg 1$, the curve in Fig. 2 has one high and narrow principal maximum at $E=E_0+\omega+\omega'$. The width of the maximum is of the order of $1/t$. For $E=E_0+\omega+\omega'$, the height of the main peak in the dependence of $w(E,t)$ on E according to Eq. (12) is equal to

$$w(E,t)|_{E=E_0+\omega+\omega'}=\frac{t^2\dot{w}_i}{2\pi}, \tag{16}$$

where $\dot{w}_i$ is given by Eq. (14). Hence the rate of ionization of the atom $\dot{w}_i$ to within a factor of $t^2/2\pi$ coincides with the maximum value of expression (16) for the photoelectron energy probability density distribution (12). Both of these expressions are functions of the frequency of the exciting field ω'. Before going on to a study of the dispersion dependence $\dot{w}_i(\omega')$, note that the photoelectron spectrum depicted in Fig. 2 differs significantly from that which arises in the photoionization of an atom initially found in the high Rydberg levels.[6,7]

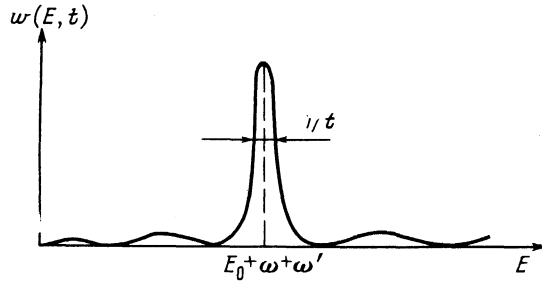

FIG. 2. Energy spectrum of the photoelectrons formed in the process of resonant ionization in two fields (diagram in Fig. 1) for long duration of the exciting and ionizing radiation pulses $\Gamma t\gg 1$.

Let us now find an explicit form of the dependence $\dot{w}_i(\omega')$ in the asymptotic limit of a strong ionizing field ($V>1$) in the vicinity of the values $\omega' = E^{(n)} - E_0$. We note that under these conditions[6,7]

$$1+i\pi\sum_{n'}\frac{|V_{n'E}|^2}{\omega'+E_0-E_{n'}}\approx 1-\frac{2i}{\Gamma}(\omega'+E_0-E^{(n)}) \qquad (17)$$

and, in addition,

$$(E_0+\omega'-E_{n'})^{-1}\approx(E^{(n)}-E_{n'})^{-1}-(E_0+\omega'-E^{(n)})(E^{(n)}-E_{n'})^{-2}. \qquad (18)$$

With the help of these approximate relations Eq. (14) can be put in the form

$$w_i(\omega')=\pi^2\Gamma V_{n_0}{}^2 n^6\frac{(x+q)^2}{x^2+1}, \qquad (19)$$

where

$$x=2(\omega'+E_0-E^{(n)})/\Gamma,$$

$$q=\frac{2}{\Gamma}\frac{\sum_{n'}d_{En'}d_{n'0}(E_{n'}-E^{(n)})^{-1}}{\sum_{n'}d_{En'}d_{n'0}(E_{n'}-E^{(n)})^{-2}}\approx\pi V^2\sum_{n'}\frac{d_{n'0}d_{En'}}{d_{n0}d_{En}}\frac{n^{-3}}{E_{n'}-E^{(n)}}. \qquad (20)$$

The symbols x and q in (20), used in the last form of writing the dependence $\dot{w}_i(\omega')$ (19), are the dimensionless resonance detuning on the quasi-energy level $E^{(n)}$ and the dimensionless Fano parameter, which characterizes the degree of asymmetry $\dot{w}_i(\omega')$ [or $\dot{w}_i(x)$] about the point $\omega' = E^{(n)} - E_0$ (or $x = 0$). In the calculation of q in expression (20) we have noted that the sum standing in the denominator converges rapidly and is approximately equal to $\pi^2 n^6 d_{n0} d_{En}$, and in place of Γ we have substituted the explicit expression (3).

The behavior of the curve $\dot{w}_i(\omega')$ given by (19) is completely determined by the magnitude of the Fano parameter q. From the definitions (20) it can be seen that $q = V^2\sigma$, where σ is a constant independent of the field strength ε_0, determined only by the spectrum of the atom. In essence, q and σ are determined by the principal value of the integral over $E_{n'}$ of the product of matrix elements $d_{En'}d_{n'0}$ divided by $E_{n'} - E^{(n)}$ (20). The magnitude of this integral is determined by the behavior of the matrix elements over a wide range of values $E_{n'}$, including also the continuous spectrum, where simple quasiclassical formulas of type (2) can already be applied.

Without addressing the problem of actually calculating σ, we note only that $\sigma = 0$ in the approximation of equidistant Rydberg levels and constant matrix elements $d_{n'0}$ and $d_{En'}$ independent of n'. This fact, which is due to the cancellation of many of the terms in the definition of q (and by implication, σ) [Eq. (20)], indicates that σ may be small, although, strictly speaking, $\sigma\neq 0$. In light of the indicated calculational difficulties, we will treat the two cases $\sigma\ll 1$ and $\sigma\sim 1$ on an equal footing. Since we have $q = V^2\sigma$, for fixed $\sigma\ll 1$ the Fano parameter q in a strong field $V>1$ can take on values both $\ll 1$ and $\gg 1$. For $\sigma\sim 1$, values of $q\gg 1$ are attained almost at once upon entering the strong field region. We will consider both cases: $q\ll 1$ and $q\gg 1$ (under the condition $V>1$), which completely exhaust both possibilities $\sigma\ll 1$ and $\sigma\sim 1$.

The dependence $\dot{w}_i(\omega')$ in (19) is qualitatively depicted in Fig. 3 for small (a) and large (b) values of the Fano parameter q given by (20). The asymmetric dependence $\dot{w}_i(\omega')$ from (19) is similar to the Fano curves that characterize the processes of excitation and decay of auto-ionized states.[4] According to Eq. (19), we have $\dot{w}_i = 0$ for $x = -q$ or $\omega' = E^{(n)} - E_0 - (1/2)q\Gamma$. The maximum of the function $\dot{w}_i(\omega')$ is attained at $x = 1/q$ or $\omega' = E^{(n)} - E_0 + \Gamma/q$. Far from these singular points ($|x|\gg 1, q, 1/q$)

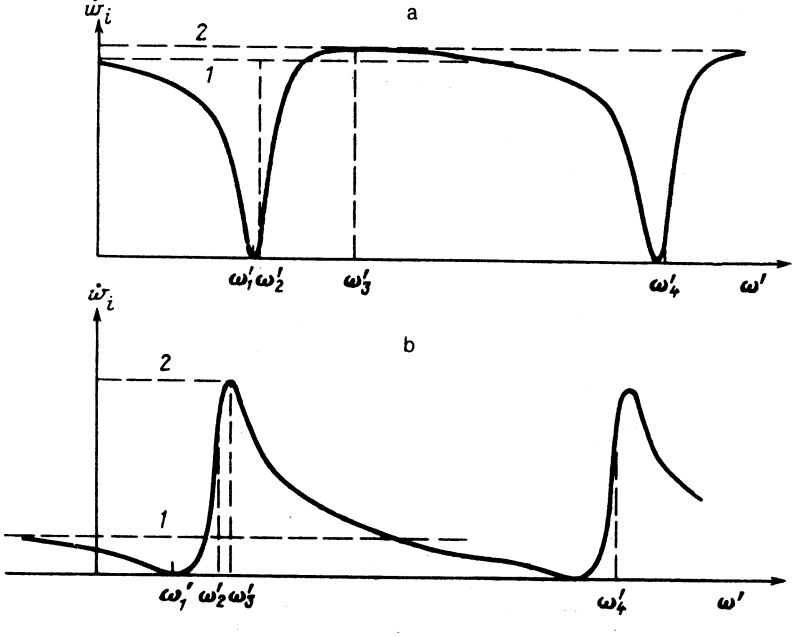

FIG. 3. Dispersional dependence of the rate of resonant ionization on the frequency of the ionizing field ω' for small (a) and large (b) values of the parameter q (20) for the case of a strong ($V>1$) ionizing field. The dashed line 1 corresponds to the value $\dot{w}_i(\infty)$, and 2—to the value $(1+q^2)\dot{w}_i(\infty)$; $\omega_1' = (E^{(n)} - E_0 - q\Gamma/2)$, $\omega_2' = E^{(n)} - E_0$, $\omega_3' = (E^{(n)} - E_0 - \Gamma/2q)$, $\omega_4' = E^{(n+1)} - E_0$.

$$\dot{w}_i(\omega') \equiv \dot{w}_i(\infty) = \pi^2 \Gamma V_{n0}' n^6. \quad (21)$$

The maximum value of $\dot{w}_i(\omega')$ is equal to

$$\dot{w}_{i\,max} = (1+q^2)\dot{w}_i(\infty). \quad (22)$$

The condition for the applicability of expansions (17) and (18) near $\omega' = E^{(n)} - E_0$ $(x=0)$ has the form $|\omega' + E_0 - E^{(n)}| \ll n^{-3}$ or $|x| \ll \pi^3 V^2$. By virtue of this constraint, the singular points $x = -q$ and $x = -1/q$, where we have $\dot{w}_i = 0$ and $\dot{w}_i = \dot{w}_{i,\max}$, respectively, are not found within the domain of applicability of the approximate formulas (17)–(19) for all q and V.

A comparison of the curves in Figs. 3a and b shows that the dispersion dependence $\dot{w}_i(\omega')$ has a substantially different form for $q \ll 1$ and $q \gg 1$. In the first of these two cases (small q, but, as before, a strong field, $V > 1$) the curve $\dot{w}_i(\omega')$ has narrow dips near the resonances on the narrow quasi-energy levels $E^{(n)}$ (1), i.e., $\dot{w}_i(\omega') = 0$ for $\omega' \approx E^{(n)} - E_0$. The width of each of the dips is equal to the width of the quasi-energy levels Γ (3). The maxima of the curve $\dot{w}_i(\omega')$ at $q \ll 1$ are weakly expressed and located a distance much greater than Γ from the resonance points $\omega' = E^{(n)} - E_0$.

On the contrary, at $q \gg 1$ the function $\dot{w}_i(\omega')$ has high and narrow maxima with width equal to Γ (3) near the resonance values of the frequency $\omega' \approx E^{(n)} - E_0$. The minima of the curve $\dot{w}_i(\omega')$, where $\dot{w}_i = 0$, are located far from the resonance values of ω'.

A physical interpretation of this behavior of the dispersion curve $\dot{w}_i(\omega')$ in the case $q \gg 1$ (Fig. 3b) does not cause any difficulties. The strong ionizing field redistributes the quasi-energy spectrum of the atom, leading to the formation of narrow quasi-energy levels $E^{(n)}$ of Eq. (1). The weak exciting field is a probe field, it "feels its way" through the quasi-energy spectrum of the atom arising in the strong ionizing field. The photoelectron yield attains its maximum when the exciting field is in resonance with the quasi-energy levels $E^{(n)}$ given by (1). As usual,[4,5] there are at the interresonance minima located between neighboring resonance maxima where the rate of ionization of the atom vanishes.

On the other hand, the behavior of the dispersion curve $\dot{w}_i(\omega')$ in the case of small q ($q \ll 1$, $V > 1$, see Fig. 3a) is quite unexpected and requires explanation. The appearance of dips in the vicinity of the resonances at the quasi-energy levels (instead of maxima) is associated with a specific kind of interference. In the case under consideration two resonant ionization channels are interfering: the direct and the cascade. The Fano parameter, in essence, is equal to the ratio of the component matrix elements of these transitions. For $q \gg 1$ the direct process of resonant ionization predominates over the cascade process, the interference is suppressed, and there appears a dispersion curve $\dot{w}_i(\omega')$ with maxima near the resonance values $\omega = E^{(n)} - E_0$ (Fig. 3b). For $q \ll 1$, on the other hand, the interference of the direct and cascade processes is very important and leads to the formation of dips in the dependence $\dot{w}_i(\omega')$ near the resonance values ω' (Fig. 3a). This interference effect is most clearly manifested in the different durations of the exciting and ionizing radiation pulses. Such a formulation of the problem will be considered below in Sec. 4.

Besides this dispersion dependence, which characterizes the probability of ionization of the atom, it is of interest to investigate the resonance structure of the probabilities of excitation of the discrete levels.

A general expression for the excitation probability w_n of the nth Rydberg level E_n of the free atom in the approximation $\Gamma t \gg 1$ is given by Eq. (15). Considering the case of a strong field $V > 1$ and using the approximate relations (17) and (18) near the resonance ($|\omega' - E^{(n)} + E_0| \ll n^{-3}$), we reduce Eq. (15) to the form

$$w_{n'} = \frac{\Gamma^2}{4} \frac{|V_{n'0}'|^2 (1+q^2)}{(E^{(n)} - E_{n'})^2 [(\omega' + E_0 - E^{(n)})^2 + {}^1/_4 \Gamma^2]}, \quad (23)$$

where q as before is defined by Eq. (20).

Here the index n' denotes the number of the exciting level $E_{n'}$ of the free atom, and the index n, the number of the quasi-energy level $E^{(n)}$ (1), at which the resonance takes place. The total probability of excitation of all the Rydberg levels is obtained from Eq. (23) by summing over n':

$$w_{ex} = \sum_{n'} w_{n'} = \frac{\pi^2 \Gamma^2 (1+q^2)}{4} \frac{n^6 |V_{n0}'|^2}{(\omega' + E_0 - E^{(n)})^2 + {}^1/_4 \Gamma^2}, \quad (24)$$

where it has been taken into account that the matrix elements V_{n0}' vary little when n varies by ± 1.

Equation (24) demonstrates that excitation of the discrete Rydberg levels proceeds more efficiently under conditions of exact resonance at the redistributed quasi-energy levels $E^{(n)}$ of Eq. (1). The dependence $\dot{w}_i(\omega')$ given by Eq. (24) is described by the usual Lorentz curve with width Γ [Eq. (3)]. The distribution over n' of the excitation probabilities of the levels of the free atom $w_{n'}$ [Eq. (23)] are determined by the weights with which the corresponding wave functions φ_n enter into the quasi-energy functions corresponding to the quasi-energy values $E^{(n)}$; the values $w_{n'}$ fall off increasing $|n' - n - 1/2|$ as $(n' - n - 1/2)^{-2}$. The fact that the efficiency of excitation of the discrete levels at $\omega' = E^{(n)} - E_0$ is at its maximum is found to be in agreement with the idea that narrow redistributed quasi-energy levels, $E^{(n)} - (1/2)\Gamma$, form in a strong field. The smallness of their width Γ indicates that the quasi-energy levels are stable with respect to photoionization and is found to be in agreement with the vanishing of the ionization rate of the atom $\dot{w}_i(\omega')$ [Eq. (19)] near $\omega' = E^{(n)} - E_0$.

4. DISPERSION CURVES FOR SHORT EXCITING PULSE LENGTH AND SOME OTHER AUXILIARY PROBLEMS

As noted above, the energy spectrum of the photoelectrons formed in the process of resonant ionization (Fig. 2) differs substantially from the spectrum that arises in the case of photoionization of an already excited atom, i.e., with an initial population of one or a few Rydberg levels.[6,7] In connection with this the question arises, do the results vary qualitatively when one goes over to a different formulation of the problem, specifically, to the case in which the exciting and ionizing radiation pulses are turned on simultaneously and instantaneously at $t = 0$, but under the condition that after a short time τ ($\tau < n^3$, $1/\Gamma$) the exciting pulse is turned off, while the ionizing field continues to act over a time $t \gg 1/\Gamma$?

At first glance, such a statement of the problem is very close to the statement of the problem with initial population of the Rydberg levels.[6,7] Therefore one might also expect qualitative variations, e.g., the appearance in the photoelectron spectra of narrow maxima at the points $E^{(n)} + \omega$ with a

smooth envelope determined by the short-lived excitation process. However, in reality the results obtained for small τ turn out to be much closer to the previous results of the solution of the problem with long-acting excitation than to the results of the solution of the problem with initial coherent population of the Rydberg levels.[6,7]

General formulas for $w(E,t)$ and $w_n(t)$ in the formulation of the problem with short exciting pulse length τ are obtained in analogy with the preceding. Therefore, not dwelling on the details of their derivation, we present the results:

$$w(E,t)\equiv w(E)=\frac{2}{\pi}\frac{\sin^2[{}^1/_2(E-E_0-\omega-\omega')\tau]}{(E-E_0-\omega-\omega')^2}\times\dot{w}_i(E-\omega-E_0), \tag{25}$$

where the function $\dot{w}_i$ is defined by the earlier equation (14). Here by virtue of the approximation $\Gamma t\gg 1$ and the fact that outside the limits of the interval $[0,\tau]$ no excitation of the Rydberg levels takes place, we have $w_n(t)=0$.

Equation (25) shows that as in the case of long-acting excitation the function $w(E)$ has its main maximum at $E=E_0+\omega=\omega'$. However, now the width of this maximum is $\sim 1/\tau$ greater than Γ, and can be greater than n^{-3} (Fig. 4).

Furthermore, from the properties of the function $\dot{w}_i(\omega')$ given by Eqs. (14) and (19), described in the previous section, it follows that in a strong field ($V>1$) for $q\ll 1$ the dependence $w(E)$ of Eq. (25) vanishes in the vicinity of the points $E^{(n)}+\omega$. The dips in the dependence $w(E)$ (Fig. 4) for $q\ll 1$, $V>1$ are narrow and have width Γ [Eq. (3)]. These conclusions differ radically from the results of the solution of the problem of initial population of the Rydberg levels, for which $w(E)$ has maxima at the points $E^{(n)}+\omega$.[6,7]

This difference shows that the simultaneous action of a weak exciting field and a strong ionizing field, even if their combined action lasts a very short time τ, is not equivalent to initial coherent population of the Rydberg levels. Since the ionizing field is strong, over the time τ not only the Rydberg levels, but also the levels of the continuum are populated. After the exciting field is turned on, an additional population of the continuum levels takes place due to the ionization of the Rydberg levels. Interference of the probability amplitudes of the population of the continuum states over the time interval $[0,\tau]$ and over the remaining time interval $[\tau,t]$ also determines the resultant photoelectron spectrum, which has been described above.

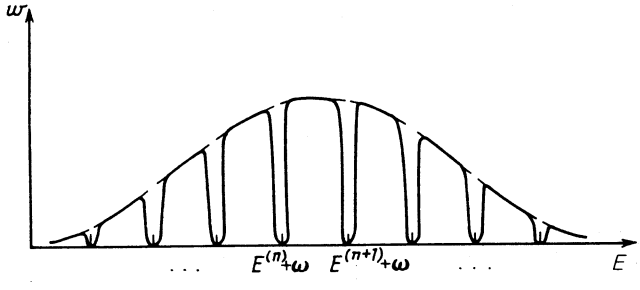

FIG. 4. Photoelectron spectrum for short duration τ of the exciting pulse, $\Gamma t\ll 1$, $V>1$, $q\ll 1$.

One can convince oneself of the validity of such an interpretation directly by considering first the auxiliary initial problem of the ionization of a Rydberg atom, assuming that only the ionizing field is turned on instantaneously at $t=0$ and that at the instant $t=0$ the atom has been prepared in the state

$$\Psi(0)=\sum_n C_n\varphi_n+\int dE C_E\varphi_E, \tag{26}$$

where C_n and C_E are the probability amplitudes of the initial coherent populations of the Rydberg levels and the levels of the continuum. This problem is a generalization of problems that have been considered earlier[6,7] in which it was assumed that $C_E=0$. The solution of this initial value problem is found in analogy with the previous problem. The energy probability density of ionization of the atom at times $t\gg 1/\Gamma$ can be represented in the form

$$w(E)=\left|C_E+\sum_n\frac{C_nV_{En}}{E-E_n-\omega}\left(1+i\pi\sum_{n'}\frac{|V_{n'E}|^2}{E-E_{n'}-\omega}\right)^{-1}+J(E)\right|^2, \tag{27}$$

where

$$J(E)=\sum_n V_{En}\int\frac{dE'}{E-E'}V_{nE'}C_{E'}\times\left[\frac{1}{E-E_n-\omega}\left(1+i\pi\sum_{n'}\frac{|V_{n'E}|^2}{E-E_{n'}-\omega}\right)^{-1}-\frac{\exp[i(E-E')t]}{E'-E_n-\omega}\left(1+i\pi\sum_{n'}\frac{|V_{n'E'}|^2}{E'-E_{n'}-\omega}\right)^{-1}\right]. \tag{28}$$

For $C_E=0$ Eq. (27) coincides with known results[6] which pertain to the case of initial coherent population of the Rydberg levels. In a strong field ($V>1$) for $C_E=0$ the probability density $w(E)$ given by Eq. (27) has a number of maxima in the vicinity of the values $E=E^{(n)}+\omega$. The sum $\Sigma_n C_n(E-E_n+\omega)$ determines the envelope of the maxima. For the curve $w(E)$ to have minima at the points $E=E^{(n)}+\omega$ instead of maxima, in the vicinity of these values of E the contributions from the initial amplitudes C_n and C_E of the probability of finding the atom in the levels E_n and in the continuum must cancel substantially. Below it will be clearly shown that for $q\ll 1$ and $V>1$ such a cancellation indeed takes place.

Noting that the integrand function in Eq. (28) has no singularity at $E'=E$, we replace $1/(E-E')$ by $1/(E+i\varepsilon-E')$, where $\varepsilon\to 0$, and integrate by parts. As a result, $J(E)$ [Eq. (28)] acquires the form

$$J(E)=\sum_n\frac{V_{En}}{E-E_n-\omega}\left(1+i\pi\sum_{n'}\frac{|V_{n'E}|^2}{E-\omega-E_{n'}}\right)^{-1}\times\int_{-\infty}^{+\infty}dE'\frac{V_{nE'}C_{E'}}{E+i\varepsilon-E'}+\sum_n\int_{-\infty}^{+\infty}dE'\,V_{En}\frac{d}{dE'}\left[\frac{\exp[i(E-E')t]}{E'-E_n-\omega}\times\left(1+i\pi\sum_{n'}\frac{|V_{n'E'}|^2}{E'-\omega-E_{n'}}\right)^{-1}\right]\int_{-\infty}^{E'}dE''\frac{V_{nE''}C_{E''}}{E+i\varepsilon-E''}. \tag{29}$$

By virtue of the assumption $\Gamma t \gg 1$, the second term on the right side of Eq. (29) is exponentially small and can be dropped. As for the first term, generally speaking, it does not vanish for arbitrary form of the function C_E. However, as will be shown below, if C_E (and C_n) result from the combined action of the exciting and ionizing pulses during a very short time τ, then in this case the first term on the right side of Eq. (29) will also be equal to zero, and, hence, so will their sum, $J(E)$.

Indeed, let C_E and C_n be the probability amplitudes of coherent population of the continuum and Rydberg levels under the action of two fields acting over the time τ, $\Gamma\tau \ll 1$. In their general form these amplitudes are given by Eqs. (9)–(11) (see Sec. 2). By virtue of the smallness of τ, the contributions from the poles of the integrand function at the points $z_k = E^{(k)} - E_0 - \omega' - (1/2)i\Gamma$, which are solutions of the equation

$$1 + \sum_n \frac{|V_{nE}|^2}{z - E_n + E_0 + \omega'} = 0, \tag{30}$$

can no longer be neglected, since $|\mathrm{Im}\ z_k|\tau \ll 1$.

Taking into account the obvious condition $C_E(\tau)$, $C_n(\tau) \to 0$ as $\tau \to 0$, we represent the resultant expressions for $C_E \equiv C_E(\tau)$ and $C_n \equiv C_n(\tau)$ in the form

$$C_E = -\frac{\exp[-i(E-E_0-\omega-\omega')\tau]-1}{E-E_0-\omega-\omega'} \sum_n \frac{V_{En}V_{n0}'}{E-E_n-\omega}$$
$$\times\left(1+i\pi\sum_{n'}\frac{|V_{n'E}|^2}{E-E_{n'}-\omega}\right)^{-1} + \sum_k \frac{\exp(-iz_k\tau)-1}{z_k(z_k-E+E_0+\omega+\omega')}$$
$$\times \sum_n \frac{V_{En}V_{n0}'}{z_k-E_n+E_0+\omega'}\left[-i\pi\sum_{n'}\frac{|V_{n'E}|^2}{(z_k-E_{n'}+E_0+\omega')^2}\right]^{-1}, \tag{31}$$

$$C_n = \sum_k \frac{\exp(-itz_k)-1}{z_k(z_k-E_n+E_0+\omega')} \sum_{n'} \frac{V_{nE}V_{En'}V_{n'0}}{z_k-E_{n'}+E_0+\omega'}$$
$$\times\left(\sum_{n''}\frac{|V_{n''E}|^2}{(z_k-E_{n''}+E_0+\omega')^2}\right)^{-1}. \tag{32}$$

Substituting C_E (31) in the first term on the right side of Eq. (29), in which we assume that $V_{nE} \approx \mathrm{const}$, and calculating the integral over E' by the method of residues, we find that for the given form of C_E

$$\int_{-\infty}^{+\infty} dE' \frac{V_{nE'}C_{E'}}{E+i\varepsilon-E'} = 0$$

and hence, in Eqs. (27)–(29) we have $J(E) \equiv 0$, as claimed.

We now show that the substitution of C_E from (31) and C_n from (32) in Eq. (27) [with $J(E) \equiv 0$] actually leads to a significant cancellation of the first and second terms inside the squared absolute value on the right side of Eq. (27), which in the end result leads to the appearance of dips in the dispersional dependence $w(E)$ for $q \ll 1$.

Assuming $V > 1$, $|E - E^{(n)} - \omega| \ll n^{-3}$, and $q \ll 1$, using the approximation of equidistant levels and constant matrix elements independent of n, we find from Eqs. (31) and (32)

$$C_E \approx -\frac{1}{2} i\pi^2 \Gamma V V_{n0}' n^{3/2} \frac{\exp[-i(E^{(n)}-E_0-\omega')t]-1}{E^{(n)}-E_0-\omega'}, \tag{33}$$

$$\sum_{n'} \frac{V_{En'}C_{n'}}{E-E_{n'}-\omega}$$
$$\approx -\frac{1}{4}\pi^2\Gamma^2 V V_{n0}' n^{3/2} \frac{\exp[-i(E^{(n)}-E_0-\omega')t]-1}{(E^{(n)}-E_0-\omega')(E-E^{(n)}-\omega+{}^1/_2 i\Gamma)}. \tag{34}$$

It can be easily seen that in the sum of expressions (33) and (34) the terms that do not vanish as $E \to E^{(n)} + \omega$ do indeed cancel. The substitution of expressions (33) and (34) in Eq. (27) brings $w(E)$ into a form which coincides with expressions (25) and (19) as $\sigma = q = 0$, i.e., in the approximation of equidistant levels and constant matrix elements.

Thus, we have shown that cancellation does take place of the probability amplitudes of the populations of the continuum levels, built up, on the one hand, during the short time τ ($\tau \ll 1/\Gamma$) of the combined action of the exciting and ionizing fields, and, on the other hand, during the long ionization time t ($t \gg 1/\Gamma$) after the exciting field has been turned off. This effect can be interpreted as the manifestation of interference between a direct resonance ionization process (taking place during the time τ) and a cascade ionization process (taking place during the time t). The obtained results show that in the strict sense of the word no separation into direct and cascade processes takes place despite the significant difference in the lengths of the exciting and ionizing pulses. To produce a purely cascade process, one would need a complete separation in time between the excitation and ionization processes: first only the exciting pulse should be turned on to populate the Rydberg levels, and only after it has been turned off should the ionizing pulse be turned on. Only in such a formulation of the problem do we arrive at the previously considered problem of ionization by a strong field of an already excited atom.[6,7] In such a formulation, in contrast with the situation considered in the present work, the photoelectron spectrum in a strong field acquires a multipeaked structure with maxima localized at the points $E \approx E^{(n)} + \omega$.

The difference between this result and the above regularities of the process of resonant ionization with resonance at the Rydberg levels is most clearly manifested in the photoelectron spectrum in the case of short duration of the exciting pulse (Fig. 4). In this case the dependence $w(E, t)$ has dips rather than maxima in the vicinity of $E = E^{(n)} + \omega$. When the pulse lengths of both fields (the exciting and the ionizing field) are long, this peculiarity of the resonant ionization process is manifested in dips in the dispersion curve $\dot{w}_i(\omega)$ at $\omega' \approx E^{(n)} - E_0$, $q \ll 1$ (Fig. 3a). We note, however, that since, as was shown above, these peculiarities of the behavior of $w(E, t)$ and $\dot{w}_i(\omega)$ are connected with the interference of the direct and cascade ionization processes, even small changes in the two-field ionization scheme can yield qualitatively different results. To illustrate this point, in contrast with the scheme depicted in Fig. 1 and the one considered so far, let us consider the case in which a weak exciting field of frequency ω' directly excites the levels of the contin-

uum in the energy region $E \sim E_0 + \omega'$, which are coupled with the Rydberg levels by a strong field of frequency ω (Fig. 5). The solution of this problem is similar to that of the problem of resonant ionization (Sec. 2). Therefore, not dwelling on details, we display the final expression for the energy probability density of ionization of the atom $w(E, t)$ with $\Gamma t \gg 1$, similar to expression (12):

$$w(E,t)=\frac{|V_{E_0}'|^2}{(E-E_0-\omega')^2}\left|\left(1+i\pi\sum_n\frac{|V_{nE}|^2}{E_0+\omega'-E_n-\omega}\right)^{-1}\right.$$
$$\left.-\exp[-it(E-E_0-\omega')]\left(1+i\pi\sum_n\frac{|V_{nE}|^2}{E-E_n-\omega}\right)^{-1}\right|^2. \tag{35}$$

again $w(E, t)$ as a function of E has one high and narrow principal maximum (Fig. 2) with width $\sim 1/t$ at the point $E = E_0 + \omega'$, and

$$w_{max}(E,t)=w(E,t)\,|_{E=E_0+\omega'}$$
$$=t^2|V_{E_0}'|^2\left[1+\pi^2\left(\sum_n\frac{|V_{nE}|^2}{E_0+\omega'-E_n-\omega}\right)^2\right]^{-1}$$
$$=\frac{\dot{w}_i(\omega')}{2\pi}t^2, \tag{36}$$

where $\dot{w}_i(\omega')$ is the total rate of ionization of the atom (analogous to Eq. (14)):

$$\dot{w}_i(\omega')=2\pi|V_{E_0}'|^2\left[1+\pi^2\left(\sum_n\frac{|V_{nE}|^2}{\omega'+E_0-E_n-\omega}\right)^2\right]^{-1}. \tag{37}$$

The total probability of ionization of the atom after the time t is equal to $w_i(t) = t\dot{w}_i$.

Despite the apparently minimal differences between Eqs. (12) and (35), (14) and (37) [the absence of the factor $1/(E - E_n - \omega)^2$ in Eq. (37)], it is precisely these minimal differences that dictate the completely different nature of the resonant dependence $\dot{w}_i(\omega')$: at arbitrary q [Eq. (20)] as a function of ω' the function $\dot{w}_i(\omega')$ [Eq. (37)] in a strong field ($V > 1$) has maxima in the vicinity of the values $\omega' \approx E_n + \omega - E_0$. The width of these maxima is equal to Γ, given by Eq. (3).

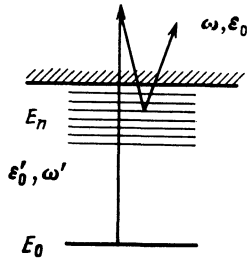

FIG. 5. Diagram of the process of direct excitation of the continuum levels by a weak field of frequency ω' in the presence of coupling of the continuous spectrum with the Rydberg levels, realized by a strong field of frequency ω.

5. CONTINUOUS SWITCHING ON OF THE INTERACTION

We now present a qualitative treatment of the process of multiphoton resonant ionization of an atom under the action of a short pulse of strong laser radiation of frequency ω, having a continuous amplitude envelope $f(t)$. We note two main peculiarities of the case of continuous switching on of the interaction of the atom with the field.[4,13,14] First of all, in a strong field with a variable amplitude $\varepsilon_0 f(t)$ there arises a time-dependent quadratic dynamic Stark shift of all the levels

$$\Delta E_n = -{}^1/_4\alpha_n(\omega)\,\varepsilon_0^2 f^2(t),$$

where $\alpha_n(\omega)$ is the dynamic polarizability of the level E_n. For the high levels ($n \gg 1$) for $\omega > |E_n| = 1/2n^2$ we have $\alpha_n \approx -1/\omega^2$ and the level shift

$$\Delta E(t) = \varepsilon_0^2 f^2(t)/4\omega^2 \tag{38}$$

is equal to the mean energy of oscillation of a free electron in the laser field. Equation (38) also describes the shift of the ionization threshold of the atom. The shift of the ground level E_0 for $\omega \ll 1$ is usually small in comparison with ΔE given by (38), and therefore Eq. (38) also determines the time-dependent variation of the binding energy of the unexcited electron in the atom.

Second, the presence of the time-dependent level shift (38) leads to the appearance at specified times of the so-called dynamic resonances.[4,13,14] In particular, if k is the minimum number of photons necessary for the ionization of the atom in a weak field ($\omega > E_0 + k\omega > 0$), then for

$$\Delta E_{max} = \varepsilon_0^2/4\omega^2 > E_0 + k\omega \sim \omega \tag{39}$$

there arise dynamic k-photon resonances at the Rydberg levels. The time t_n at which a resonance appears at the level E_n is found from the equation

$$E_0 + k\omega = E_n + \Delta E(t_n). \tag{40}$$

The appearance of dynamic resonances governs the possibility of fast and efficient population of the Rydberg levels. The interference effects described above and in Refs. 6 and 7 govern the stabilization of the population of the Rydberg levels, i.e., the stability of these levels with respect to photoionization. We think that the stabilization of the process of ionization of an atom in a strong field, which has been demonstrated, for example, in the numerical calculations carried out in Ref. 12, may be connected with this. We will indicate here the possibility of a somewhat different interpretation of this same result.

In a strong field (39) the difference $t_{n+1} - t_n$ between the times of appearance of the dynamic resonances at neighboring Rydberg levels E_{n+1} and E_n can become very small (less than $1/\Gamma$). Under these conditions the discrete structure of the Rydberg levels "smears out" and for a short pulse length $\tau < 2\pi n^3$ the band of Rydberg levels excited in the process of k-photon absorption can be considered as a quasi-continuous spectrum. From this point of view the difference between excitation of the high Rydberg levels and excitation of the continuum levels during the process of subthreshold ionization falls away. The meaning of condition (39) is that at least part of the lower band of levels excited in the process of "subthreshold" ionization turns out in fact to be below the

threshold, i.e., in the region of the discrete spectrum. By virtue of the stabilization of the Rydberg levels against photoionization in a strong field,[6,7] this fraction of the populations remains in the discrete spectrum even after the laser pulse is turned off and therefore does not contribute to the free electron yield.

6. CONCLUSION

Let us briefly sum up the results of this work. We have investigated the photoelectron energy spectrum and the dispersional dependence of the total probability of resonant ionization on the frequency of the exciting field ω' in the presence of resonance at the Rydberg levels (Fig. 1) in the instantaneous switching-on approximation for identical and different exciting and ionizing pulse lengths. It has been shown that the dependences have an unusual shape: they have narrow dips at the Rydberg quasi-energy levels $E^{(n)}$ given by Eq. (1) in a strong ionizing field (Figs. 3 and 4). We have elucidated the physical nature of this effect. We have shown that it is due to the interference of the direct and the cascade process of resonant ionization.

We have shown that in the alternative scheme (Fig. 5) with direct excitation of the continuum levels coupled by a strong field with the Rydberg levels, the following qualitatively distinct results arise: the ionization rate of the atom as a function of the frequency of the exciting field has maxima in the vicinity of the values of the frequencies ω' corresponding to Raman resonances at the quasi-energy levels $E^{(n)}$ of Eq. (1). We have shown that under conditions which minimize the rate of resonant ionization in the scheme depicted in Fig. 1, the probability of exciting the discrete Rydberg levels, on the other hand, has maxima. This result suggests that the Rydberg levels can be excited efficiently under resonance conditions at the quasi-energy levels $E^{(n)}$ of Eq. (1), and, thanks to the interference effects, the resulting population of the discrete levels is found to be stable against photoionization. This effect, it seems to us, may be the reason for the limitation of the electron yield in a very strong ionizing field, i.e., stabilization of the process of multiphoton ionization of the atom in a strong laser field. A similar result was obtained recently in numerical model calculations.[12]

In the last section we gave a qualitative analysis of the peculiarities and physics of stabilization of the process of multiphoton ionization of an atom which takes account of the smooth time-dependence of the envelope of the laser pulse and such effects as the time-dependent dynamic Stark shift of the levels and the ionization threshold and the appearance in connection with this of dynamic multiphoton resonances at the Rydberg levels. This analysis confirms the applicability of the model of stabilization of the atom based on the idea of efficient excitation of stable Rydberg quasi-energy levels of the atom in a strong field.

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